MUON BEAM TRACKING AND SPIN-ORBIT CORRELATIONS FOR PRECISION $g-2$ MEASUREMENTS *

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Abstract

The main goal of the Muon $g-2$ Experiment ($g-2$) at Fermilab is to measure the muon anomalous magnetic moment a_μ to unprecedented precision. This new measurement will allow to test the completeness of the Standard Model (SM) and to validate other theoretical models beyond the SM. The close interplay of the understanding of particle beam dynamics and the preparation of the beam properties with the experimental measurement is tantamount to the reduction of systematic errors in the determination of the muon anomalous magnetic moment. We describe progress in developing detailed calculations and modeling of the muon beam delivery system in order to obtain a better understanding of spin-orbit correlations, nonlinearities, and more realistic aspects that contribute to the systematic errors of the $g-2$ measurement. Our simulation is meant to provide statistical studies of error effects and quick analyses of running conditions for when $g-2$ is taking beam, among others. We are using COSY INFINITY, a differential algebra solver developed at Michigan State University that will also serve as an alternative to compare results obtained by other simulation teams of the $g-2$ Collaboration.

INTRODUCTION

The purpose of the Muon g-2 Experiment at Fermilab is to measure the muon anomalous magnetic moment, a_μ, to a precision of 0.14 ppm or less [1]. If such high precision level is achieved, the discrepancy between a_μ(Exp) measured at Fermilab and a_μ(SM) calculated from the Standard Model would have a significance equal or greater than 5σ (discovery threshold). A more precise measurement offers an opportunity to search for new physics beyond the standard model.

a_μ will be determined by measuring the muon spin precession relative to the momentum inside the magnetic storage-ring, ω_a, and the average magnetic field relative to the Larmor frequency of a free proton, ω_p. The motivation of the present work is to provide a computational model of the beamlines relevant to the muon $g-2$ experiment at Fermilab that would allow to understand and reduce systematic and statistical errors related to the measurement of ω_a.

Our simulations are performed using COSY INFINITY [2–4], a code that provides convenient computational tools to study spin dynamics and nonlinear effects. Results from COSY simulations regarding transmission fraction, dynamics, and distribution of the beam along the first part of the $g-2$ beamlines after the pion-production target (M2/M3 beamlines) are presented below. Also, preliminary studies of spin-orbit correlations are described.

M2 M3 BEAMLINES

Muons from which the anomalous magnetic moment a_μ will be determined are mainly produced from the weak decay of pions ($\pi^+ \rightarrow \mu^+ \nu_\mu$) that will be produced from the collision of 4×10^{12} batches of 8 GeV protons into a fixed target. The magnetic storage-ring is expected to receive clean bunches of highly polarized muons, since the original beam at the downstream face of the target station -composed of mostly protons, pions, positrons, and muons- will turn into a muon beam as it travels through the "muon campus beamlines" (see Fig. 1) [1]. The muon campus beam transport lines are designed to deliver $p_{magic} = 3.094$ GeV/c highly polarized muons with momentum spread $|\Delta p/p| < \pm0.5\%$ to the storage-ring.

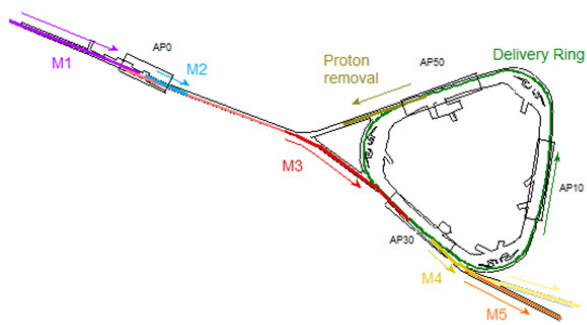

Figure 1: Muon campus beamlines at Fermilab.

M2 and M3 lines are specially designed to capture magic-momentum muons from pion decay. Secondaries from the target with a momentum 3.11 GeV/c ± ~ 10% will be selected with a 3° bending magnet and sent into the ~ 50 m long M2 beamline. A second 3° dipole with large aperture is placed at the intersection point between M2 and M3 beamlines. The M3 line has matching sections and a long transport FODO section, designed to maintain small beta functions to preserve the muon decay channel (see Fig. 2).

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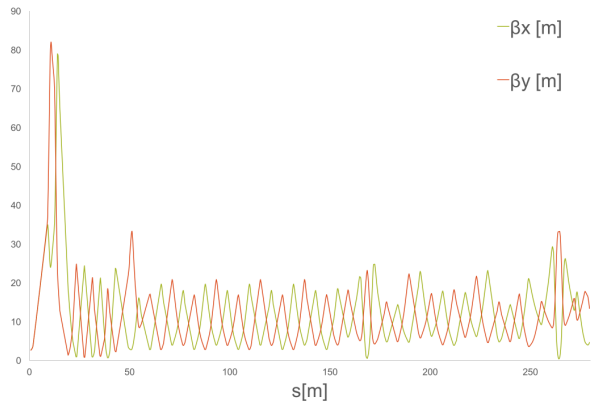

Figure 2: M2M3 beta functions.

COSY SIMULATIONS FOR M2M3 BEAMLINES

Fringe Field Effects on Beta Functions

COSY INFINITY [2–4] allows to calculate fringe field effects to substantially high precision. It describes fringe field maps based on a six parameter Enge function that contains the information of the arc length dependence of multipole strengths or other appropriate models.

Fringe fields distort beta functions along the M2/M3 beamlines up to ∼ 4% (see Fig. 3). As the fringe field falloff close to the edges of the bending dipoles and magnetic quadrupoles in M2/M3 is connected to the aperture and geometry of each element, our simulation takes this information into account. The distortion exhibits periodicity along the FODO section in M3 line. It is worth mentioning that beta functions are affected by fringe fields in no special way at the end of the M3 beamline, where several bends are rotated ±90° around the optic axis in order to bend the beam vertically for injection into the Delivery Ring (see Fig. 1).

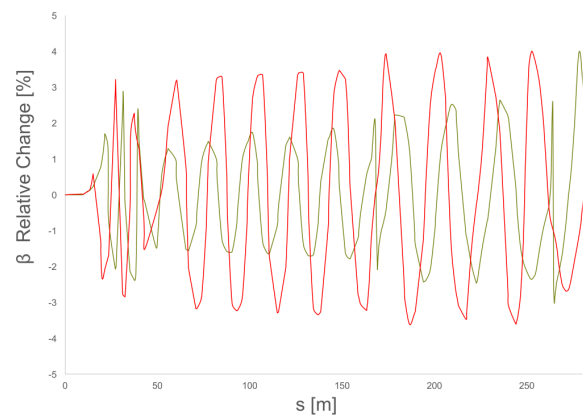

Figure 3: Fringe field effects on beta functions along M2/M3 beamlines. The graphs correspond to the relative difference between calculated x (green) and y (red) beta functions with and without fringe fields.

Pion Transmission Fraction

The muon campus beamlines have relatively large apertures in order to avoid unacceptable muon loss, allowing a beam with large size and intensity to properly accommodate as it reaches the storage-ring. Several magnets were redesigned at Fermilab to attain the $g-2$ acceptance goal of 40π mm-mr.

Star-shaped magnetic quadrupoles, rectangular, elliptical, circular, and other special aperture shapes in the M2/M3 beamlines were incorporated into our COSY codes, which allowed us to calculate the pion transmission fraction (without pion decay) along the M2/M3 beamlines (see Fig. 4).

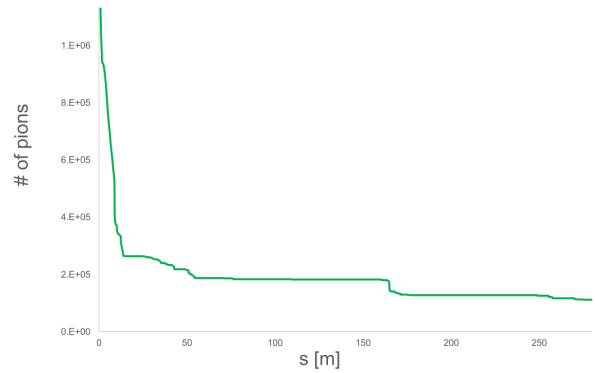

Figure 4: Pions transmission in M2/M3 beamlines. Our results are in good agreement with G4BL and BMAD simulations performed by D. Stratakis, M. Korostelev *et al.* (the Muon $g-2$ Collaboration).

The initial distribution at the downstream face of the pion-production target was obtained by V. Tishckenko from MARS simulations [5], which resulted from 10^9 protons on target. Initially, ∼ 20% of the number of pions produced on target are lost after hitting the apertures of the PMAG bending magnet. Approximately half of the transmitted pions after PMAG are lost in the 7 m-long drift downstream. A fraction of 1.8×10^{-4} per proton on target is maintained along the FODO section in M3. Overall, dipoles are the elements in that cause the highest losses of pions in M2/M3. At the end of these beamlines, a total fraction of 1.1×10^{-4} pions without decay per proton on target is estimated to be transmitted.

The effect of fringe fields on the pion transmission fraction was performed using COSY. The magnetic quadrupoles and other particle optical elements located in the M2 beamline downstream of the pulsed dipole contribute to maintain slightly more pions per proton on target within the muon campus beamlines, which otherwise would have been lost due to interaction with element apertures.

Pion Distribution

Studies of the beam optical coordinates' distributions considering apertures, fringe fields, and nonlinearities effects were recreated at the end of M2/M3 beamlines. Figure 5

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displays two-dim distributions of the horizontal, x, and vertical, y, coordinates with respect to the reference trajectory, as well as horizontal and vertical momentum offsets (a and b respectively). Apertures are the main source of deformation along the b. It was found that fringe fields do not have a significant impact on beam distributions, but it is worth performing similar analysis at the end of the muon campus beamlines.

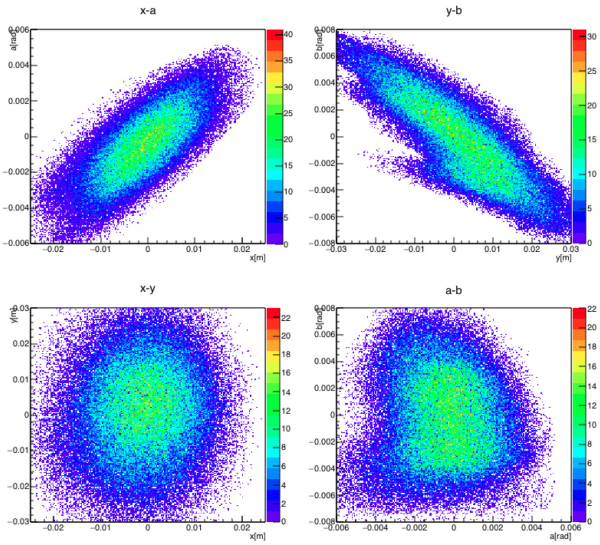

Figure 5: Beam distributions at the end of M2/M3 beamlines.

SPIN DYNAMICS SIMULATIONS USING COSY

Spin Tracking in a Homogeneous Ring

Using COSY INFINITY, fifth order 3×3 spin-orbit matrices that transport muon spin-vectors $\boldsymbol{s} = (s_x, s_y, s_z)$ through an homogeneous ring were calculated. The ring is comprised of 66 sector magnets, which have the same bending radius and arc length as the repeated bending magnet inside the muon campus's Delivery Ring. There is a specific spin-matrix for each muon, whose initial spatial and momentum-deviation coordinates with respect to the reference orbit determine, together with the initial spin polarization, the spin-matrix elements. The initial spin polarization of all muons at the entrance of the ring was set to be parallel to the forward motion direction z (i.e. $s_{x,0} = 0$, $s_{y,0} = 0$).

In Fig. 6, the distributions of muons in the precession angle ϕ_a versus momentum offset δ plane after 1 (n=1) and 4 (n=4) turns along the ring are shown.

The red lines correspond to linear functions where the intercept with $\delta = 0$ is estimated from the spin precession frequency equation. The slope, which is directly related to the spin-orbit correlation in the $\delta \ll 1$ regime, is based on an analytical equation that also depends on the spin precession

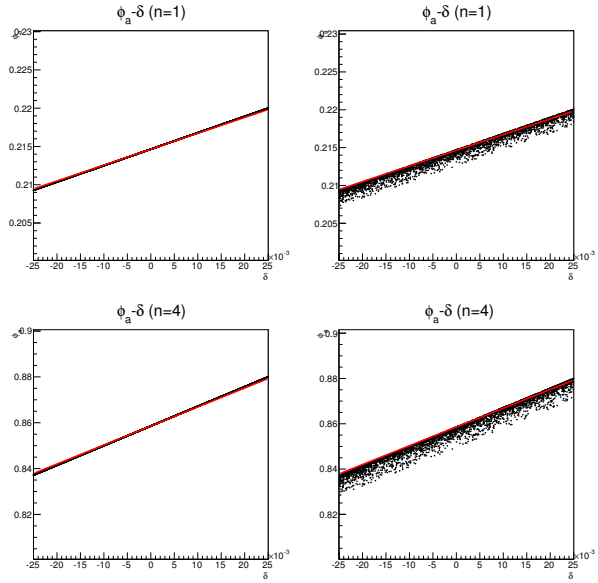

Figure 6: Spin precession angle versus momentum offset for muons travelling inside a homogeneous ring.

frequency and relevant transport-matrix orbit elements of the ring. On the left side of Fig. 6, all the muons were initially centered in the reference orbit except for a small offset δ. For the plots on the right, the initial conditions of the muon beam was set according to the expected beam distributions at the entrance of the Delivery Ring [1]. Figure 6 indicates that the main contribution to the spin-orbit correlation comes from momentum offsets, whereas the other beam coordinates affect the spreading of the precession angles relative to the $\phi_a - \delta$ correlation. Also, comparing between the results computed with COSY and the analytical estimations allows to verify the validity of the spin tracking map methods from COSY.

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