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Reply to "Comment on 'Axion induced oscillating electric dipole moments' " [1]

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A recent paper of Flambaum, Roberts and Stadnik, [1], claims there is no induced oscillating electric dipole moment (OEDM), *e.g.*, for the electron, arising from the oscillating cosmic axion background via the anomaly. This claim is based upon the assumption that electric dipoles always be defined by their coupling to *static* (constant in time) electric fields. The relevant Feynman diagram, as computed by [1], then becomes a total divergence, and vanishes in momentum space. However, an OEDM does arise from the anomaly, coupled to time dependent electric fields. It shares the decoupling properties with the anomaly. The full action, in an arbitrary gauge, was computed in [2, 3]. It is nonvanishing with a time dependent outgoing photon, and yields physics, *e.g.*, electric dipole radiation of an electron immersed in a cosmic axion field.

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In recent papers [2, 3] we have computed the effect of a coherent oscillating axion dark matter field, via the electromagnetic anomaly, upon the magnetic moment of a static electron. Figure (1) has been computed in several ways: (1) in standard radiation gauge, using Bjorken and Drell conventions; (2) in a general gauge using Pauli-Schroedinger nonrelativistic formalism; (3) in a general gauge using Georgi's effective heavy fermion formalism applied to the electron [4]. The result is nontrivial, has potentially several physical implications, and leads to the consistent interpretation that the electron develops an effective oscillating electric dipole moment (OEDM) in the background oscillating cosmic axion field.

For example, in the Georgi formalism with the axion field $a(t)/f_a = \theta(t) = \theta_0 \cos(m_a t)$, we obtain the effective interaction [3]:

$$\propto \frac{1}{2} g\mu_{Bohr}\theta(t)\overline{\psi}_v\gamma_5\sigma_{\mu\nu}\psi_v F^{\mu\nu} + ... \qquad (1)$$

This reduces in the rest-frame to the Pauli-Schroedinger result:

$$g\mu_{Bohr}\theta(t)\psi^\dagger\sigma_i\psi E^i + ... \qquad (2)$$

The ellipsis represents a nonlocal term, similar to the transverse current in QED, [5], and will be described below. Given the form of this result, which arises from Fig.(1) as a contact term, we interpret this as an oscillating EDM, or "OEDM." This interaction produces electric dipole radiation from a static electron immersed in, and absorbing energy from, the oscillating cosmic axion field. The radiation is formally that of an oscillating (Hertzian) electric dipole. Such radiation is physically interesting, and may be detectable in experiment [3].

In ref.[1], however, it is claimed that the results of the analysis [2, 3] are wrong. The authors claim that the Feynman diagram of Fig.(1) "when properly computed" vanishes.

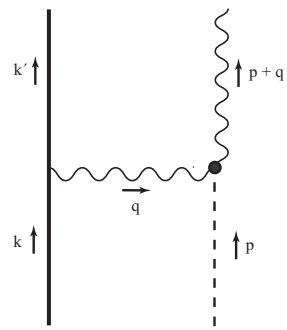

FIG. 1: The dotted vertex is the axion-anomaly, $\theta F\widetilde{F}$, and the solid vertical line is the electron. The electron–photon vertex is the magnetic moment of the electron. The incoming axion with 4-momentum $(m_a, \vec{0})$ absorbs a spacelike photon of 4-momentum $(0, \vec{k})$ with $|\vec{k}| = m_a$ to produce an outgoing photon of momentum $\sim (m_a, \vec{k})$. The electron barely recoils, since $m_e >> m_a$.

We firmly disagree with the conclusions of Flambaum, *et al.* In fact, we had already given the full action for a stationary electron in an arbitrary gauge, [2, 3] and from it one can readily see that Flambaum *et al.* have computed a total divergence, which vanishes in momentum space. For a general time-dependent electric field our action is that of an OEDM of the electron, formally indistinguishable from *e.g.*, the OEDM of a neutron.

The discrepancy with our results appears to stem from a definition that the authors of [1] give, which they claim is valid for any EDM. They state:

"(1) The EDM of an elementary particle is defined by the linear energy shift that it produces through its interaction with an applied *static* electric field: $\delta\epsilon = \vec{d}\cdot\vec{E}$. As we show explicitly, the interaction of an electron with an applied *static* electric field, in the presence of the axion electromagnetic anomaly, in the lowest order does not produce an energy shift in the limit $v/c \to 0$. This implies that no electron EDM is generated by this mechanism in the same limit."

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While this definition may be applicable to a static EDM, in an introductory course in electromagnetism, it is generally inapplicable to a time dependent one, such as an oscillating EDM in quantum mechanics. With an OEDM we are dealing with a dynamical situation and must resort to a more general definition, typically phrased in the context of an action.

We can define the EDM of any object as a covariant action of the form:

$$S = g \int d^4x \; S_{\mu\nu}(x) F^{\mu\nu}(x) \tag{3}$$

where $S_{\mu\nu}$ is an antisymmetric odd parity dipole density (*e.g.*, $S_{\mu\nu} \sim \overline{\psi}\sigma_{\mu\nu}\gamma^5\psi$ for a relativisitic particle). g is an overall coupling that will remain unspecified.

For concreteness, let us consider the case of the axion induced neutron OEDM. The neutron OEDM is believed to arise in QCD from instantons and the θ-term, which contains the axion field. It is being sought in a proposed experiment (see ref.[6] and references therein). We thus consider the neutron to allow a side-by-side comparison with our result for the anomaly induced electron OEDM.

In the common rest frame of the neutron and axion, the OEDM action of eq.(3), reduces to:

$$S = g \int d^4x \; \theta(t) \overrightarrow{P} \cdot \overrightarrow{E}(t) \tag{4}$$

where $\vec{P}(x) = (e/m_N)\psi^\dagger \vec{\sigma}\psi(x)$ is the dipole spin density, written in terms of two-component spinors. Here we assume $\vec{P}(x)$ is localized in space and static (time independent), and the oscillating aspect of the EDM comes from the axion $\theta(t)$. [Ref.[1] may be confused on this point: we are interested in a static $\vec{P}$ and not a static $\vec{E}$. Note that a non-recoiling neutron, or electron, is the kinematically favored limit, *e.g.*, as in Fig.(1). We have the incoming axion with 4-momentum $(m_a, \vec{0})$. The neutron (or electron) is very heavy compared to the axion, (like a truck being hit by a ping-pong ball) and can only acquire an insignificant kinetic energy, $m_a^2/2m_N << m_a$. Therefore, the radiated photon must carry off the full energy of the incident axion, with a 4-momentum of $(m_a, \vec{k})$, and $|\vec{k}| = m_a$ (and the exchange photon 4-momentum is essentially spacelike, $(0, \vec{k})$).]

For a static electric field the action of eq.(4) averages to zero; moreover it is a total divergence, since $\theta(t) = -(\theta_0/m_a)\partial_t \sin(m_a t)$, and therefore does not contribute to any dynamics. The radiated photon, kinematically, is necessarily time dependent with frequency m_a and it is on-shell with $m_a^2 - \vec{k}^2 = 0$. This will happen for any oscillating EDM.

Now consider the electron. We have computed the full action for the electron OEDM in an arbitrary background electric field from the anomaly, as in Fig(1) in refs.[2],[3]. We obtain in the static $\vec{P}(x)$ limit :

$$S = g \int d^4x \; \theta(t) \left(\overrightarrow{P} \cdot \overrightarrow{E} + \overrightarrow{\nabla} \cdot \overrightarrow{P} \left(\frac{1}{\overrightarrow{\nabla}^2} \right) \overrightarrow{\nabla} \cdot \overrightarrow{E} \right) \tag{5}$$

We emphasize that this is fully gauge invariant. This differs from eq.(4) by the appearance of the nonlocal term. This has been written in a convenient shorthand notation to avoid writing double integrals, where $\frac{1}{\overrightarrow{\nabla}^2}$ is a static Green's function, *i.e.*,

$$A(x) \frac{1}{\overrightarrow{\nabla}^2} B(x) = \int d^4y \; A(x) \frac{\delta(x_0 - y_0)}{4\pi|\vec{x} - \vec{y}|} B(y) \tag{6}$$

In an arbitrary gauge, $\overrightarrow{E} = \overrightarrow{\nabla}\varphi - \partial_t \overrightarrow{A}$ we see, after some integrations by parts, the action of eq.(5) takes the form:

$$S = g \int d^4x \; \theta(t) \overrightarrow{\nabla} \cdot (\overrightarrow{P}\varphi)$$
$$+ g \int d^4x \; \partial_t \theta(t) \left(\overrightarrow{P} \cdot \overrightarrow{A} + \overrightarrow{\nabla} \cdot \overrightarrow{P} \left(\frac{1}{\overrightarrow{\nabla}^2} \right) \overrightarrow{\nabla} \cdot \overrightarrow{A} \right) \tag{7}$$

Flambaum *et al.* have computed this in the case $\vec{A} = 0$ and $A_0 = \varphi$. In this case we see that only the first term will be nonzero in eq.(7), but that term is a spatial total divergence, and hence it contributes nothing to the physics (a total divergence is zero in momentum space).

In the case of a physical radiation gauge photon, we have $A_0 = 0$ and a non-zero $\vec{A}$ with $\vec{\nabla} \cdot \vec{A} = 0$. In this case our action for the electron OEDM reduces to:

$$S = g \int d^4x \; \theta(t) \overrightarrow{P} \cdot \overrightarrow{E} \tag{8}$$

This is indistinguishable from the OEDM of the neutron. It requires a time dependent $\vec{E}$, as does the neutron or any other OEDM, and it is $\propto \partial_t \theta(t)$ upon integration by parts in time.

Let us now address some frequently asked questions concerning this result.

(1) What about allowing A_0 to be time dependent but $\vec{A} = 0$?

A_0 is a non-propagating field and cannot represent a physical out-going on-shell photon. The equation of motion for A_0 is $\vec{\nabla}^2 A_0 = -\rho(x)$, where $\rho(x)$ is a charge density. If we want to allow time dependent A_0, then $\nabla^2 \partial_0 A_0 = -\partial_0 \rho(x,t)$, but from current conservation we have $\partial_0 \rho(x) = \nabla \cdot \vec{j}$ where $\vec{j}$ is the 3-current. Hence, we have $\partial_0 A_0 = -(1/\vec{\nabla}^2)\vec{\nabla} \cdot \vec{j}$. This means that if A_0 is to be time dependent, then there must necessarily be a 3-current, hence a vector potential, $\vec{A}$.

Let us impose the condition $\vec{\nabla} \cdot \vec{A} = 0$. $\vec{A}$ satisfies $(\partial_0^2 - \nabla^2)\vec{A} - \vec{\nabla}\partial_0 A_0 = \vec{j}$ and the equation of motion of A_0 is unmodified. This is often written as $(\partial_0^2 - \nabla^2)\vec{A} = \vec{j}_T$. where $\vec{j}_T$ is the "transverse current" [5] which takes the form $\vec{j}_T = \vec{j} - \vec{\nabla}(1/\nabla^2)\vec{\nabla} \cdot \vec{j}$. Thus, introducing A_0 time dependence requires a vector potantial. Note that the nonlocal term we obtained in eq.(7) is the analogue of the transverse current [3].

Note that the calculation in ref.[1] is restricted to a 4-vector potential of the pure Coulomb form, $A_\mu = (A_0, \vec{0})$

i.e., $\vec{E} = \vec{\nabla} A_0$. Flambaum *et al.* stated that this is a "gauge choice." However, this is *not a gauge choice*, since a general 4-vector potential, $A_\mu(x,t)$, cannot be brought to the pure timelike form by a gauge transformation.

(2) What about the axion decoupling limit whereby the anomaly induced OEDM must vanish as $m_a \to 0$?

Our results have engendraned false criticism in some corridors because they do not *manifestly* vanish in the limit $m_a \to 0$. In fact, we readily see that eq.(7) becomes a total divergence in the limit $\partial_t \theta(t) \to 0$, hence they do vanish as a total divergence as $m_a \to 0$. In particular, the first term is a total divergence in space for a spatially constant $\theta(t)$, while the second term is manifestly zero in the limit. This is why the nonlocal term appears in eqs.(5,7). This is parallel to how the anomaly itself behaves in this limit. The nonlocal term can be completely determined by demanding a shift symmetry of the axion (a full discussion is given in section III of [3]).

This issue is actually somewhat subtle in axion physics in general. Even the induced electric field in an RF cavity does not *manifestly* vanish as $m_a \to 0$. It is a solution to the Maxwell equations, and requires a boundary condition in time. This implies that the solution should be written as, $\vec{E} \propto \theta(t) - \theta(t_0)$, where t_0 is an earlier time at which $\vec{E}$ is zero. The difference $\theta(t) - \theta(t_0) = \int_{t_0}^{t} \partial_t \theta \to 0$ as $\partial_t \theta \propto m_a \to 0$ [3].

(3) So much for formalism, what is the physics of the OEDM of the electron?

In fact, there is a lot of physics in the calculation of Fig.(1), when done correctly. In ref.[2] we obtained the amplitude of Fig.(1) in radiation gauge for an emitted photon of polarization $\vec{\epsilon}$ and three momentum $\vec{k}$: $2 g_{A\gamma\gamma} \theta_0 \ \mu_{Bohr} \cdot m_a \vec{\epsilon} \cdot \vec{S}$, where $\mu_{Bohr} = e/2m_e$. and $S^i = \chi^\dagger (\sigma_i/2) \chi$ is the spin vector. The emitted power by a free electron immersed in the axion field is obtained by squaring and doing the appropriate phase space integral. Assume the spin vector $\vec{S} = \hat{z}$, hence only the polarization in the $\vec{S}$-$\vec{k}$ plane contributes, with $\vec{\epsilon} \cdot \vec{S} = \frac{1}{2}\cos(\theta)$ (here θ is the polar angle). Hence the radiated power is:

$$
\begin{aligned}
P &= \frac{1}{2}(g_{A\gamma\gamma}\theta_0 \ \mu_{Bohr} m_a)^2 \times \\
&\qquad \int \frac{k^2 dk \sin(\theta) d\theta d\phi}{(2\pi)^3 2|k|} |k| \cos^2(\theta) 2\pi \delta(m_a - k) \\
&= \frac{1}{12\pi}(g_{A\gamma\gamma}\theta_0 \ \mu_{Bohr} m_a^2)^2 \qquad (9)
\end{aligned}
$$

(the collision rate is just P/m_a). This result is equivalent to that obtained from the classical Maxwell equations, (note: this is a preliminary result).

In ref. [3] we have examined in detail the classical solutions for this radiation obtained with retarded Green's functions. It can be seen that this is radiation from an oscillating (Hertzian) electric dipole $2 g_{A\gamma\gamma} \theta_0 \cos(m_a t) \ \mu_{Bohr} \vec{S}$. It corresponds to an emitted power of $\sim 10^{-74}$ watts for $f_a = 10^{10}$ GeV. This may be accessible to experiment in coherent arrays of magnets [3]. It is likely there are other effects of the induced OEDM of the electron that might be addressed experimentally.

In conclusion, Flambaum, *et al.* have argued that Fig.(1) is zero in a static (time independent) electric field, which is trivially correct because they are computing a total divergence. However, they then argue that there can be no OEDM for the electron, which is non-sequitur and utterly false.

The diagram of Fig.(1) actually represents real physics, the effective action of the electron OEDM when immersed in an oscillating axion field. It leads to an effective interaction that is indistinguishable from that of the neutron OEDM in a time dependent background electric field. It also produces electric dipole radiation emanating from any magnet placed in the oscillating cosmic axion field. We claim that this has the valid interpretation of an axion-induced effective oscillating electric dipole moment of the electron.

It is unfortunate that the authors of ref.[1] have apparently not even tried to reproduce our result, and substituted a different calculation of a total divergence that trivially yields zero, to claim our overall result is zero. If ref.[1] had carried out a more thoughtful analysis, and still maintained their position, then we might have a more substantive discussion.

Our claim stands, and has certainly not been falsified by the authors of ref.[1].

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