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Leptonic Decays of Charged Pseudoscalar Mesons – 2015

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Abstract

We review the physics of purely leptonic decays of $\pi^{\pm}$, $K^{\pm}$, $D^{\pm}$, $D_s^{\pm}$, and $B^{\pm}$ pseudoscalar mesons. The measured decay rates are related to the product of the relevant weak-interaction-based CKM matrix element of the constituent quarks and a strong interaction parameter related to the overlap of the quark and antiquark wave-functions in the meson, called the decay constant f_P. The leptonic decay constants for $\pi^{\pm}$, $K^{\pm}$, $D^{\pm}$, $D_s^{\pm}$, and $B^{\pm}$ mesons can be obtained with controlled theoretical uncertainties and high precision from *ab initio* lattice-QCD simulations. The combination of experimental leptonic decay-rate measurements and theoretical decay-constant calculations enables the determination of several elements of the CKM matrix within the standard model. These determinations are competitive with those obtained from semileptonic decays, and also complementary because they are sensitive to different quark flavor-changing currents. They can also be used to test the unitarity of the first and second rows of the CKM matrix. Conversely, taking the CKM elements predicted by unitarity, one can infer "experimental" values for f_P that can be compared with theory. These provide tests of lattice-QCD methods, provided new-physics contributions to leptonic decays are negligible at the current level of precision. This review is the basis of the article in the Particle Data Group's 2016 edition, updating the versions in Refs. [120–122].

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I. INTRODUCTION

Charged mesons formed from a quark and antiquark can decay to a lepton-neutrino pair when these objects annihilate via a virtual W boson. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.

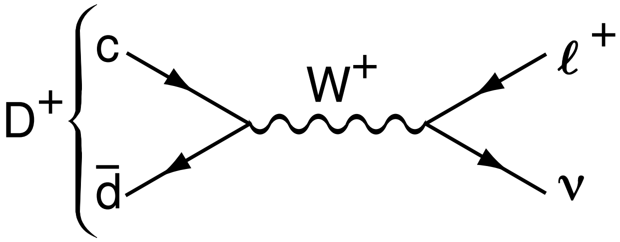

FIG. 1. The annihilation process for pure D^+ leptonic decays in the standard model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+, K^+, D_s^+, and B^+ mesons. (Charge-conjugate particles and decays are implied.) Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \; m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 \;\; . \tag{1}$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1\bar{q}_2$ in P, and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, proportional to the matrix element of the axial current between the one-P-meson state and the vacuum, and is related to the wave-function overlap of the quark and antiquark.

The decay $P^\pm$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width. Radiative corrections are needed when the final charged particle is an electron or muon.

Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1 q_2}| f_P$. If the decay constant f_P is known to sufficient precision from theory, one can obtain the corresponding CKM element within the standard model. If, on the other hand, one takes the value of $|V_{q_1 q_2}|$ assuming CKM unitarity, one can infer an "experimental measurement" of the decay constant that can then be compared with theory.

The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. Leptonic decays of charged pseudoscalar mesons occur at tree level within the standard model. Thus one does not expect large new-physics contributions to measurements of $\Gamma(P \to \ell\nu)$ for the lighter mesons $P = \pi^+, K^+$, and these processes in principle provide clean standard-model determinations of V_{ud} and V_{us}. The situation is different for leptonic decays of charm and bottom mesons. The presence of new heavy particles such as charged Higgs bosons or leptoquarks could lead to observable effects in $\Gamma(P \to \ell\nu)$ for $P = D_{(s)}^+, B^+$ [132–136]. Thus the determination of $|V_{ub}|$ from $B^+ \to \tau\nu$ decay, in particular, should be considered a probe of new physics. More generally, the ratio of leptonic decays to $\tau\nu$ over $\mu\nu$ final states probes lepton universality [132, 137].

The determinations of CKM elements from leptonic decays of charged pseudoscalar mesons provide complementary information to those from other decay processes. The decay $P \to \ell\nu$ proceeds in the standard model via the axial-vector current $\bar{q}_1\gamma_\mu\gamma_5 q_2$, whereas semileptonic pseudoscalar meson decays $P_1 \to P_2\ell\nu$ proceed via the vector current $\bar{q}_1\gamma_\mu q_2$. Thus the comparison of determinations of $|V_{q_1q_2}|$ from leptonic and semileptonic decays tests the $V - A$ structure of the standard-model electroweak charged-current interaction. More generally, a small right-handed admixture to the standard-model weak current would lead to discrepancies between $|V_{q_1q_2}|$ obtained from leptonic pseudoscalar-meson decays, exclusive semileptonic pseudoscalar-meson decays, exclusive semileptonic baryon decays, and inclusive semileptonic decays [138, 139].

Both measurements of the decay rates $\Gamma(P \to \ell\nu)$ and theoretical calculations of the decay constants f_P for $P = \pi^+, K^+, D^+_{(s)}$ from numerical lattice-QCD simulations are now quite precise. As a result, the elements of the first row of the CKM matrix $|V_{ud}|$ and $|V_{us}|$ can be obtained to sub-percent precision from $\pi^+ \to \ell\nu$ and $K^+ \to \ell\nu$, where the limiting error is from theory. The elements of the second row of the CKM matrix $|V_{cd(s)}|$ can be obtained from leptonic decays of charged pseudoscalar mesons to few-percent precision, where here the limiting error is from experiment. These enable stringent tests of the unitarity of the first and second rows of the CKM matrix.

This review is organized as follows. Because the experimental and theoretical issues associated with measurements of pions and kaons, charmed mesons, and bottom mesons differ, we discuss each one separately. We begin with the pion and kaon system in Sec. II. First, in Sec. II A we review current measurements of the experimental decay rates. We provide tables of branching-ratio measurements and determinations of the product $|V_{ud(s)}|f_{\pi^+(K^+)}$, as well as average values for these quantities including correlations and other effects needed to combine results. Then, in Sec. II B we summarize the status of theoretical calculations of the decay constants. We provide tables of recent lattice-QCD results for f_{π^+}, f_{K^+}, and their ratio from simulations including dynamical u, d, s, and (in some cases c) quarks, and present averages for each of these quantities including correlations and strong SU(2)-isospin corrections as needed. We note that, for the leptonic decay constants in Secs. II B, III B, and IV B, when available we use preliminary averages from the Flavor Lattice Averaging Group [140, 141] that update the determinations in Ref. [142] to include results that have appeared since their most recent review, which dates from 2013. We next discuss the charmed meson system in Sec. III, again reviewing current experimental rate measurements in Sec. III A and theoretical decay-constant calculations in Sec. III B. Last, we discuss the bottom meson system in Sec. IV, following the same organization as the two previous sections.

After having established the status of both experimental measurements and theoretical calculations of leptonic charged pseudoscalar-meson decays, we discuss some implications for phenomenology in Sec. V. We combine the average $\mathcal{B}(P \to \ell\nu)$ with the average f_P to obtain the relevant CKM elements from leptonic decays, and then compare them with determinations from other processes. We also use the CKM elements obtained from leptonic decays to test the unitarity of the first and second rows of the CKM matrix. Further, as in previous reviews, we combine the experimental $\mathcal{B}(P \to \ell\nu)$s with the associated CKM elements obtained from CKM unitarity to infer "experimental" values for the decay constants; the comparison with theory provides a test of lattice and other QCD approaches assuming that new-physics contributions to these processes are not significant.

II. PIONS AND KAONS

A. Experimental rate measurements

The expression for $\Gamma(P \to \ell\nu)$ in Eq. (1) is for the decay rate in the absence of radiative corrections, and is modified by both electroweak and hadronic contributions (cf. Refs. [143, 144], and references therein). These corrections can be combined into an overall factor that multiplies the rate in the presence of only the strong interaction ($\Gamma^{(0)}$) as follows:

$$\Gamma(P \to \ell\nu) = \Gamma^{(0)} \left[1 + \frac{\alpha}{\pi} C_P\right] \quad , \tag{2}$$

where C_P differs for $P = \pi, K$. The inclusion of these corrections is numerically important given the level of precision achieved on the experimental measurements of the $\pi^\pm \to \mu^\pm\nu$ and $K^\pm \to \mu^\pm\nu$ decay widths. The explicit expression for the term in brackets above including all known electroweak and hadronic contributions is given in Eq. (114) of Ref. [145]. It includes the universal short-distance electroweak correction obtained by Sirlin [146], the universal long-distance correction for a point-like meson from Kinoshita [147], and effects from hadronic structure [148]. We evaluate $\delta^P_{\rm EM} \equiv (\alpha/\pi)C_P$ using the latest experimentally-measured meson and lepton masses and coupling constants from the Particle Data group [122], and taking the low-energy constants (LECs) that parameterize the hadronic contributions from Refs. [145, 149, 150]. The finite non-logarithmic parts of the LECs were estimated within the large-N_C approximation assuming that contributions from the lowest-lying resonances dominate. We therefore conservatively assign a 100% uncertainty to the LECs, which leads to a ±0.9 error in $C_{\pi,K}$.[1] We obtain the following correction factors to the individual charged pion and kaon decay widths:

$$\delta^\pi_{\rm EM} = 0.0176(21) \quad \text{and} \quad \delta^{\rm K}_{\rm EM} = 0.0107(21)\,. \tag{3}$$

The error on the ratio of kaon-to-pion leptonic decay widths is under better theoretical control because the hadronic contributions from low-energy constants estimated within the large-N_c framework cancel at lowest order in the chiral expansion. For the ratio, we use the correction factor

$$\delta^{K/\pi}_{\rm EM} = -0.0069(17)\,, \tag{4}$$

where we take the estimated error due to higher-order corrections in the chiral expansion from Ref. [152].

The sum of branching fractions for $\pi^- \to \mu^-\bar\nu$ and $\pi^- \to \mu^-\bar\nu\gamma$ is 99.98770(4)% [122]. The two modes are difficult to separate experimentally, so we use this sum. Together with the lifetime 26.033(5) ns [122] this implies $\Gamma(\pi^- \to \mu^-\bar\nu[\gamma]) = 3.8408(7) \times 10^7\ {\rm s}^{-1}$. The right-hand side of Eq. (1) is modified by the factor 1.0176 ± 0.0021 mentioned above to include photon emission and radiative corrections [151, 153]. The decay rate together with the masses from the 2014 PDG review [122] gives

$$f_{\pi^-}|V_{ud}| = (127.13 \pm 0.02 \pm 0.13)\ {\rm MeV}\,, \tag{5}$$

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[1] This uncertainty on $C_{\pi,K}$ is smaller than the error estimated by Marciano and Sirlin in Ref. [151], which predates the calculations of the hadronic-structure contributions in Refs. [145, 148–150]. The hadronic LECs incorporate the large short-distance electroweak logarithm discussed in Ref. [151], and their dependence on the chiral renormalization scale cancels the scale-dependence induced by chiral loops, thereby removing the dominant scale uncertainty of the Marciano–Sirlin analysis [151].

where the errors are from the experimental rate measurement and the correction factor $\delta^{\pi}_{\rm EM}$ in Eq. (3), respectively. The uncertainty is dominated by that from theoretical estimate of the electroweak and hadronic corrections, which include next-to-leading order contributions of $\mathcal{O}(e^2 p^2_{\pi,K})$ in chiral perturbation theory [145].

The data on $K_{\mu 2}$ decays have been updated recently through a global fit to branching ratios and lifetime measurements [154]: $\mathcal{B}(K^- \to \mu^- \bar\nu[\gamma]) = 63.58(11)\%$ and $\tau_{K^\pm} = 12.384(15)$ ns. The improvement in the branching ratio is primarily due to a new measurement of $\mathcal{B}(K^\pm \to \pi^\pm\pi^+\pi^-)$ from KLOE-2 [155], which is correlated with $\mathcal{B}(K^\pm_{\mu 2})$ through the constraint that the sum of individual branching ratios must equal unity. The sum of branching fractions for $K^- \to \mu^-\bar\nu$ and $K^- \to \mu^-\bar\nu\gamma$ and the lifetime imply $\Gamma(K^- \to \mu^-\bar\nu[\gamma]) = 5.134(11) \times 10^7~\text{s}^{-1}$. Again taking the 2014 PDG masses [122], this decay rate implies

$$f_{K^+}|V_{us}| = (35.09 \pm 0.04 \pm 0.04)~\text{MeV}\,, \tag{6}$$

where the errors are from the experimental rate measurement and the correction factor $\delta^{K}_{\rm EM}$, respectively.

Short-distance radiative corrections cancel in the ratio of pion-to-kaon decay rates [156]:

$$\frac{\Gamma_{K_{\ell 2[\gamma]}}}{\Gamma_{\pi_{\ell 2[\gamma]}}} = \frac{|V^2_{us}|f^2_{K^-}}{|V_{ud}|^2 f^2_{\pi^-}} \frac{m_K(1-m_\ell^2/m_K^2)^2}{m_\pi(1-m_\ell^2/m_\pi^2)^2}(1+\delta^{K/\pi}_{EM})\,, \tag{7}$$

where $\delta^{K/\pi}_{EM}$ is given in Eq. (4). The left-hand side of Eq. (7) is 1.3367(28), yielding

$$\frac{|V_{us}|f_{K^-}}{|V_{ud}|f_{\pi^-}} = 0.27599 \pm 0.00029 \pm 0.00024\,, \tag{8}$$

where the first uncertainty is due to the branching fractions and the second is due to $\delta^{K/\pi}_{EM}$. Here the error due to electroweak and hadronic corrections is commensurate with the experimental error.

In summary, the main experimental results pertaining to charged pion and kaon leptonic decays are

$$|V_{ud}|f_{\pi^-} = (127.13 \pm 0.02 \pm 0.13)~\text{MeV}\,, \tag{9}$$
$$|V_{us}|f_{K^+} = (35.09 \pm 0.04 \pm 0.04)~\text{MeV}\,, \tag{10}$$
$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^-}} = 0.27599 \pm 0.00029 \pm 0.00024\,, \tag{11}$$

where the errors are from the experimental uncertainties in the branching fractions and the theoretical uncertainties in the radiative correction factors δ^{P}_{EM}, respectively.

B. Theoretical decay-constant calculations

Table I presents recent lattice-QCD calculations of the charged pion and kaon decay constants and their ratio from simulations with three ($N_f = 2{+}1$) or four flavors ($N_f = 2{+}1{+}1$) of dynamical quarks. The results have been obtained using several independent sets of gauge-field configurations, and a variety of of lattice fermion actions that are sensitive to different

TABLE I. Recent lattice-QCD results for f_{π^+}, f_{K^+}, and their ratio. The upper and lower panels show $(2+1+1)$-flavor and $(2+1)$-flavor determinations, respectively. When two errors are shown, they are statistical and systematic, respectively. Results obtained in the isospin-symmetric limit $m_u = m_d$ are quoted directly from the papers before applying the strong-isospin correction via Eq. (12), and are noted with an "*". Unpublished results noted with a "†" or "‡" are not included in the averages.

Reference	N_f	f_{π^+}(MeV)	f_{K^+}(MeV)	f_{K^+}/f_{π^+}
ETM 14 [159][§]	2+1+1	–	154.4(1.5)(1.3)	1.184(12)(11)
FNAL/MILC 14 [160][§]	2+1+1	–	$155.92(13)(^{+42}_{-34})$	$1.1956(10)(^{+26}_{-18})$
HPQCD 13 [161][§]	2+1+1	–	155.37(20)(28)	1.1916(15)(16)
FLAG 15 average [140, 141][¶]	2+1+1	–	155.6(0.4)	1.193(3)
RBC/UKQCD 14 [162][*,†]	2+1	130.19(89)	155.51(83)	1.1945(45)
RBC/UKQCD 12 [163][*]	2+1	127(3)(3)	152(3)(2)	1.199(12)(14)
Laiho & Van de Water 11 [164][‡]	2+1	130.53(87)(210)	156.8(1.0)(1.7)	1.202(11)(9)(2)(5)
MILC 10 [165]	2+1	129.2(0.4)(1.4)	$156.1(4)(^{+6}_{-9})$	$1.197(2)(^{+3}_{-7})$
BMW 10 [166][*]	2+1	–	–	1.192(7)(6)
HPQCD/UKQCD 07 [167][*]	2+1	132(2)	157(2)	1.189(2)(7)
FLAG 15 average [140, 141][¶]	2+1	130.2(1.4)	155.9(0.9)	1.192(5)
Our average	Both	130.2(1.4)	155.6(0.4)	1.1927(26)

[§] PDG 2014 value of $f_{\pi^+} = 130.41(21)$ MeV used to set absolute lattice scale.

[¶] Preliminary numbers shown here may change if further new lattice-QCD calculations are published before the deadline for inclusion in the final 2015 FLAG review.

[†] Preprint submitted to Phys. Rev. D. Published RBC/UKQCD 12 results included in $N_f = 2+1$ average.

[‡] Lattice 2011 conference proceedings.

systematic uncertainties.[2] The lattice uncertainties on both the individual decay constants and their ratio have now reached sub-percent precision. The SU(3)-breaking ratio f_{K^+}/f_{π^+} can be obtained with especially small errors because statistical errors associated with the Monte Carlo simulations are correlated between the numerator and denominator, as are some systematics. The good agreement between these largely independent determinations indicates that the lattice-QCD uncertainties are controlled and that the associated error estimates are reliable.[3]

All of the results in Table I were obtained using isospin-symmetric gauge-field configurations, *i.e.*, the dynamical up and down quarks have the same mass. Most calculations of pion and kaon decay constants now include, however, the dominant strong isospin-breaking contribution, by evaluating the masses of the constituent light (valence) quarks in the pion at the physical up- and down-quark masses, respectively, and evaluating the mass of the va-

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[2] See the PDG review on "Lattice Quantum Chromodynamics" [157] for a general review of numerical lattice-QCD simulations. Details on the different methods used in modern lattice calculations are provided in Appendix A of the FLAG "Review of lattice results concerning low energy particle physics" [142].

[3] The recent review [158] summarizes the large body of evidence validating the methods employed in modern lattice-QCD simulations.

lence light quark in the kaon at the physical m_u. In computing the preferred decay-constant values to be compared with experiment and used for phenomenology, these results are corrected for the leading isospin-breaking effects using chiral perturbation theory (χPT). The next-to-leading order χPT corrections are given by [152, 168]

$$f_\pi = f_{\pi^+} \tag{12}$$

$$f_K = f_{K^+}\left(1 - \delta_{\mathrm{SU}(2)}/2\right) \tag{13}$$

$$\frac{f_K}{f_\pi} = \frac{1}{\sqrt{\delta_{\mathrm{SU}(2)} + 1}}\frac{f_{K^+}}{f_{\pi^+}} \tag{14}$$

where the expression for $\delta_{\mathrm{SU}(2)}$ in terms of the quark masses, meson masses, and decay constants, is given in Eq. (37) of Ref. [142]. Numerically, values of $\delta_{\mathrm{SU}(2)} \approx -0.004$ were employed to obtain the (2+1)-flavor average in Table I, but some direct lattice-QCD calculations of $\delta_{\mathrm{SU}(2)}$ give larger values [159, 161, 169] and further studies are needed.

The Flavour Lattice Averaging Group (FLAG) compiles and averages lattice-QCD results for numerous hadronic parameters including pseudoscalar-meson decay constants [142]. The FLAG averages include only results that are published in refereed journals, or are straightforward conference updates of published analyses, with complete systematic error budgets. Because the precise correlations between the different lattice calculations are not known, if there is reason to believe that a source of uncertainty is correlated between two results, FLAG conservatively takes the correlation to be 100% when calculating the average. Here we quote preliminary updates of the 2013 FLAG averages [142] that include all available results that satisfy the FLAG criteria. Table I shows the three- and four-flavor preliminary 2015 FLAG averages for the pion and kaon decay constants and their ratio [140, 141] in the lines labeled "FLAG 15 average." The errors in the (2+1+1)-flavor averages include statistical correlations between the results of HPQCD and FNAL/MILC (which were obtained using the same gauge-field configurations) and have been increased by the $\sqrt{\chi^2/\mathrm{dof}}$ to reflect a slight tension between the two results. All of the available four-flavor lattice-QCD calculations in Table I use the quantity f_{π^+} to fix the absolute lattice scale (needed to convert from lattice-spacing units to GeV), and therefore only provide determinations of f_{K^+} and f_{K^+}/f_{π^+} [159–161].

Within the current level of precision, there is no observable difference between the three- and four-flavor results. Therefore, for this review, we take an average of the two for our final preferred theoretical values:

$$\text{Our averages}: \quad f_{\pi^+} = 130.2(1.4)\ \mathrm{MeV}\,, \quad f_{K^+} = 155.6(0.4)\ \mathrm{MeV}\,, \quad \frac{f_{K^+}}{f_{\pi^+}} = 1.1927(26)\,. \tag{15}$$

We take a simple weighted average of the (2+1)- and (2+1+1)-flavor FLAG values from Table I because they are each obtained from a sufficient number of independent calculations that we do not expect there to be significant correlations.

III. CHARMED MESONS

A. Experimental rate measurements

Measurements have been made for $D^+ \to \mu^+\nu$, $D_s^+ \to \mu^+\nu$, and $D_s^+ \to \tau^+\nu$. Only an upper limit has been determined for $D^+ \to \tau^+\nu$. Both CLEO-c and BES have made measurements of D^+ decay using e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D's, say the D^-. Counting the number of these events provides the normalization for the branching fraction measurement. They then find a candidate μ^+, and then form the missing-mass squared, $MM^2 = (E_{\rm CM} - E_{D^-})^2 - \left(\overrightarrow{p}_{\rm CM} - \overrightarrow{p}_{D^-} - \overrightarrow{p}_{\mu^+}\right)^2$, taking into account their knowledge of the center-of-mass energy, $E_{\rm CM}$, and momentum, $p_{\rm CM}$, that equals zero in e^+e^- collisions. A peak at zero MM^2 inplies the existence of a missing neutrino and hence the $\mu^+\nu$ decay of the D^+. CLEO-c does not explicitly identify the muon, so their data consists of a combination of $\mu^+\nu$ and $\tau^+\nu$, $\tau^+ \to \pi^+\nu$ events. This permits them to do two fits: in one they fit for the individual components, and in the other they fix the ratio of $\tau^+\nu/\mu^+\nu$ events to be that given by the standard-model expectation. Thus, the latter measurement should be used for standard-model comparisons and the other for new-physics searches. Our average uses the fixed ratio value. The measurements are shown in Table II.

To extract the value of $|V_{cd}|f_{D^+}$ we use the well-measured D^+ lifetime of 1.040(7) ps. The $\mu^+\nu$ results include a 1% correction (lowering) of the rate due to the presence of the radiative $\mu^+\nu\gamma$ final state based on the estimate by Dobrescu and Kronfeld [136].

We now discuss the D_s^+. Measurements of the leptonic decay rate have been made by several groups and are listed in Table III [173–179]. We exclude older values obtained by normalizing to D_s^+ decay modes that are not well defined. Many measurements, for example, used the $\phi\pi^+$ mode. This decay is a subset of the $D_s^+ \to K^+K^-\pi^+$ channel which has interferences from other modes populating the K^+K^- mass region near the ϕ, the most prominent of which is the $f_0(980)$. Thus the extraction of the effective $\phi\pi^+$ rate is sensitive to the mass resolution of the experiment and the cuts used to define the ϕ mass region [180].[4]

To find decays in the $\mu^+\nu$ signal channels, CLEO, BaBar and Belle rely on fully reconstructing all the final state particles except for neutrinos and using a missing-mass technique to infer the existence of the neutrino. CLEO uses $e^+e^- \to D_sD_s^*$ collisions at 4170 MeV, while Babar and Belle use $e^+e^- \to DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$. CLEO does a similar analysis as was done for the D^+ above. Babar and Belle do a similar MM^2 calculation by using the reconstructed hadrons, the photon from the D_s^{*+} decay and a detected μ^+. To get the normalization they do a MM^2 fit without the μ^+ and use the signal

TABLE II. Experimental results for $\mathcal{B}(D^+ \to \mu^+\nu)$, $\mathcal{B}(D^+ \to \tau^+\nu)$, and $|V_{cd}|f_{D^+}$. Numbers for $|V_{cd}|f_{D^+}$ have been extracted using updated values for masses (see text). Radiative corrections are included. Systematic uncertainties arising from the D^+ lifetime and mass are included. For the average $\mu^+\nu$ number we use the CLEO-c result for $\mu^+\nu^+ + \tau^+\nu$.

Experiment	Mode	$\mathcal{B}$	$\lvert V_{cd}\rvert f_{D^+}$ (MeV)
CLEO-c [170, 171]	$\mu^+\nu$	$(3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$	$47.07 \pm 2.10 \pm 0.57$
CLEO-c [170, 171]	$\mu^+\nu$+$\tau^+\nu$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$	$46.41 \pm 1.94 \pm 0.57$
BES [172]	$\mu^+\nu$	$(3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$	$45.73 \pm 1.17 \pm 0.38$
Our average	Lines 2+3	$(3.74 \pm 0.17) \times 10^{-4}$	45.91 ± 1.05
CLEO-c [173, 174]	$\tau^+\nu$	$< 1.2 \times 10^{-3}$	

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[4] We have not included the BaBar result for $\mathcal{B}(D_s^+ \to \mu^+\nu)$ reported in Ref. [181] because this measurement determined the ratio of the leptonic decay rate to the hadronic decay rate $\Gamma(D_s^+ \to \ell^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$.

TABLE III. Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu)$, $\mathcal{B}(D_s^+ \to \tau^+\nu)$, and $|V_{cs}|f_{D_s^+}$. Numbers for $|V_{cs}|f_{D_s^+}$ have been extracted using updated values for masses (see text). The systematic uncertainty for correlated error on the D_s^+ lifetime is included. The mass uncertainties are also common, but negligible. Common systematic errors in each experiment have been taken into account in the averages.

Experiment	Mode	$\mathcal{B}(\%)$	$\lvert V_{cs}\rvert f_{D_s^+}$ (MeV)
CLEO-c [173, 174]	$\mu^+\nu$	$(0.565 \pm 0.045 \pm 0.017)$	$250.8 \pm 10.0 \pm 4.2$
BaBar[a] [179]	$\mu^+\nu$	$(0.602 \pm 0.038 \pm 0.034)$	$258.9 \pm 8.2 \pm 7.5$
Belle [175]	$\mu^+\nu$	$(0.531 \pm 0.028 \pm 0.020)$	$243.1 \pm 6.4 \pm 4.9$
Our average	$\mu^+\nu$	(0.556 ± 0.024)	248.8 ± 5.8
CLEO-c [173, 174]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$(6.42 \pm 0.81 \pm 0.18)$	$270.8 \pm 17.1 \pm 4.2$
CLEO-c [176]	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$(5.52 \pm 0.57 \pm 0.21)$	$251.1 \pm 13.0 \pm 5.1$
CLEO-c [177, 178]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$(5.30 \pm 0.47 \pm 0.22)$	$246.1 \pm 10.9 \pm 5.4$
BaBar [179]	$\tau^+\nu$ $(e^+(\mu^+)\nu\overline{\nu})$	$(5.00 \pm 0.35 \pm 0.49)$	$239.0 \pm 8.4 \pm 11.9$
Belle [175]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$(6.04 \pm 0.43^{+0.46}_{-0.40})$	$262.7 \pm 9.3^{+10.2}_{-8.9}$
Belle [175]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$(5.37 \pm 0.33^{+0.35}_{-0.31})$	$247.7 \pm 7.6^{+8.3}_{-7.4}$
Belle [175]	$\tau^+\nu$ $(\mu^+\nu\overline{\nu})$	$(5.86 \pm 0.37^{+0.34}_{-0.59})$	$258.7 \pm 8.2^{+7.7}_{-13.2}$
Our average	$\tau^+\nu$	(5.56 ± 0.22)	252.1 ± 5.2
Our average	$\mu^+\nu + \tau^+\nu$		250.9 ± 4.0

[a] We do not use a previous unpublished BaBar result from a subsample of data that uses a different technique for obtaining the branching fraction normalization [182].

at the D_s^+ mass squared to determine the total D_s^+ yield.

When selecting the $\tau^+ \to \pi^+\bar{\nu}$ and $\tau^+ \to \rho^+\bar{\nu}$ decay modes, CLEO uses both the calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s^- and the τ^+ decay products. The $\tau^+ \to e^+\nu\bar{\nu}$ mode, however, uses only extra energy. Babar and Belle also use the extra energy to discriminate signal from background in their $\tau^+\nu$ measurements.

We extract the decay constant times the CKM factor from the measured branching ratios using the D_s^+ mass of 1.96830(11) GeV, the τ^+ mass of 1.77682(16) GeV, and a lifetime of 0.500(7) ps [122]. CLEO has included the radiative correction of 1% in the $\mu^+\nu$ rate listed in the Table [136] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that the $\gamma\mu^+\nu$ rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [123–131, 183]. As this is a small effect we do not attempt to correct the other measurements. The values for $f_{D_s^+}|V_{cs}|$ are in good agreement for the two decay modes. Our average value for both the $\mu^+\nu$ and $\tau^+\nu$ final states is 250.9 ± 4.0 MeV.

B. Theoretical decay-constant calculations

Table IV presents recent theoretical calculations of the charged D^+- and D_s-meson decay constants and their ratio. The upper two panels show results from simulations with three ($N_f = 2+1$) or four flavors ($N_f = 2+1+1$) of dynamical quarks. Although there are fewer

TABLE IV. Recent theoretical determinations of f_{D^+}, f_{D_s}, and their ratio. The upper panels show results from lattice-QCD simulations with $(2+1+1)$ and $(2+1)$ dynamical quark flavors, respectively. Statistical and systematic errors are quoted separately. Lattice results obtained in the isospin-symmetric limit $m_u = m_d$ are noted with an "*". The bottom panel shows estimates from QCD sum rules (QCD SR) and the light-front quark model (LFQM). These are not used to obtain our preferred decay-constant values.

Reference	Method	N_f	f_{D^+}(MeV)	f_{D_s}(MeV)	f_{D_s}/f_{D^+}
ETM 14 [159]*	LQCD	2+1+1	207.4(3.7)(0.9)	247.2(3.9)(1.4)	1.192(19)(11)
FNAL/MILC 14 [160]	LQCD	2+1+1	$212.6(0.4)(^{+1.0}_{-1.2})$	$249.0(0.3)(^{+1.1}_{-1.5})$	$1.1712(10)(^{+29}_{-32})$
Average	LQCD	2+1+1	212.2(1.1)	248.8(1.3)	1.172(3)
χQCD 14 [184]*	LQCD	2+1	–	254(2)(4)	–
HPQCD 12 [185]*	LQCD	2+1	208.3(1.0)(3.3)	246.0(0.7)(3.5)	1.187(4)(12)
FNAL/MILC 11 [186]	LQCD	2+1	218.9(9.2)(6.6)	260.1(8.9)(6.1)	1.188(14)(21)
HPQCD 10 [187]*	LQCD	2+1	–	248.0(1.4)(2.1)	–
Average	LQCD	2+1	209.2(3.3)	249.8(2.3)	1.187(12)
Our average	LQCD	Both	211.9(1.0)	249.1(1.1)	1.173(3)
Wang 15 [188]§	QCD SR		208(10)	240(10)	1.15(6)
Gelhausen 13 [189]	QCD SR		$201\left(^{+12}_{-13}\right)$	$238\left(^{+13}_{-23}\right)$	$1.15\left(^{+0.04}_{-0.05}\right)$
Narison 12 [190]	QCD SR		204(6)	246(6)	1.21(4)
Lucha 11 [191]	QCD SR		206.2(8.9)	245.3(16.3)	1.193(26)
Hwang 09 [192]	LFQM		–	264.5(17.5)¶	1.29(7)

§ Obtained using $m_c^{\overline{\mathrm{MS}}}$; results using m_c^{pole} are also given in the paper.

¶ Obtained by combining PDG value $f_D = 205.8(8.9)$ MeV [193] with f_{D_s}/f_D from this work.

available results than for the pion and kaon sector, both f_{D^+} and f_{D_s} have been obtained using multiple sets of gauge-field configurations with different lattice fermion actions, providing independent confirmation. We average these lattice results to obtain our preferred decay-constant values in Eq. (18). For comparison, the bottom panel of Table IV shows non-lattice determinations from QCD sum rules and the light-front quark model; only results which include uncertainty estimates are shown. The lattice and non-lattice results agree, but the uncertainties on $D^+_{(s)}$-meson decay constants from lattice QCD have now reached significantly greater precision than those from other approaches.

The results in Table IV were all obtained using isospin-symmetric gauge-field configurations. The two calculations by the Fermilab Lattice and MILC Collaborations [160, 186], however, include the dominant strong isospin-breaking contribution by evaluating the mass of the valence light quark in the D^+-meson decay constant at the physical down-quark mass. Reference [160] provides a determination of the size of this correction,

$$f_{D^+} - f_D = 0.47(1)(^{+25}_{-6})\ \mathrm{MeV}\,, \tag{16}$$

where f_D is the value of the D-meson decay constant evaluated at the average up-down quark mass. Equation (16) implies that the correction to the SU(3)$_f$-breaking ratio is

$$\frac{f_{D_s}}{f_{D^+}} - \frac{f_{D_s}}{f_D} = -0.0026\,, \tag{17}$$

taking the central values for f_{D^+} and f_{D_s} from the same work. Inspection of Table IV shows that the errors on the calculations that neglect this effect are still about 5–8 × larger than the sizes of the shifts in Eqs. (16)–(17). Thus we do not correct any results *a posteriori* for this effect in the current review. Nevertheless, we strongly encourage future lattice-QCD publications to present results for both f_{D^+} and f_{D^0}. New lattice calculations that reach the level of precision in Eqs. (16)–(17), will, of course, need to include the effect of isospin breaking.

Preliminary 2015 averages for $D^+_{(s)}$-meson decay constants from the Flavour Lattice Averaging Group are not yet available, so we construct our own for this review, but follow the FLAG criteria for inclusion and treatment of uncertainties [142].[5] Some lattice-QCD decay-constant calculations analyze the same gauge-field configurations or use some identical input parameters. Thus, when computing the averages for this review, we consider possible correlations between the different results. Following the approach established by Laiho *et al.* in Ref. [194] and adopted by FLAG for the 2013 review [142], whenever we have reason to believe that a source of uncertainty is correlated between two results, we conservatively take the correlation to be 100% when calculating the average. We construct the correlation matrix for the set of lattice results using the prescription of Schmelling [195].

We first consider the (2+1)-flavor results. Inspection of Table IV shows that there have been no new (2+1)-flavor lattice-QCD calculations of f_{D^+} or $f_{D_s^+}/f_{D^+}$ since 2013. We therefore take the (2+1)-flavor average from Ref. [142], which treats the statistical errors as 100% correlated between the results of FNAL/MILC and HPQCD in Refs. [185, 186]. For $f_{D_s^+}$, however, we compute our own (2+1)-flavor average which includes the recent result from the χQCD collaboration [184]. Although the HPQCD Collaboration has published two calculations of f_{D_s}, we include only the earlier result from 2010 [187] in our average because it is based on an analysis of finer lattice spacings and is more precise. We treat the statistical errors as correlated between the results of the HPQCD and Fermilab Lattice and MILC collaborations in Refs. [186, 187] because they analyze some of the same ensembles of gauge-field configurations, and take the χQCD result [184] to be independent. Next we consider the (2+1+1)-flavor results. The two calculations by the ETM [159] and FNAL/MILC Collaborations [160] use different light-quark and gluon actions and different treatments of the chiral-continuum extrapolation. Thus we expect them to be independent, and take a simple weighted average. Given these considerations, we obtain the three- and four-flavor averages for the charged $D^+_{(s)}$-meson decay constants and their ratio shown in Table IV, where the error on the (2+1)-flavor f_{D_s} average has been rescaled by the factor $\sqrt{(\chi^2/\text{dof})} = 1.1$.

It is useful to have a single determination of the $D^+_{(s)}$-meson decay constants for phenomenology applications. The three- and four-flavor results are compatible within the current level of precision. Therefore, for this review, we quote the weighted average of the entries in the two lines labeled "Average" in Table IV for our final preferred theoretical values:

$$\text{Our averages :}\quad f_{D^+} = 211.9(1.0)\ \text{MeV}\,,\quad f_{D_s} = 249.1(1.1)\ \text{MeV}\,,\quad \frac{f_{D_s}}{f_{D^+}} = 1.173(3)\,. \tag{18}$$

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[5] After this article was submitted for review, preliminary (2+1)- and (2+1+1)-flavor FLAG averages for f_D, f_{D_s}, and f_{D_s}/f_D were presented in Ref. [**?**] that are identical to our separate averages in Table IV.

In practice, the errors on the (2+1+1)-flavor averages are so much smaller than on the (2+1)-flavor averages that the combination in Eq. (18) is almost identical to the (2+1+1)-flavor average in Table IV.

IV. BOTTOM MESONS

A. Experimental rate measurements

The Belle and BaBar collaborations have found evidence for $B^- \to \tau^- \bar{\nu}$ decay in $e^+e^- \to B^- B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and photon candidates, and examining the extra energy in the event which should be close to zero for a real τ^- decay to $e^-\nu\bar{\nu}$ or $\mu^-\nu\bar{\nu}$ opposite a B^+ tag. While the BaBar results have remained unchanged, Belle reanalyzed both samples of their data. The branching fraction using hadronic tags changed from $1.79\,^{+0.56}_{-0.49}\,^{+0.46}_{-0.51} \times 10^{-4}$ [196] to $0.72^{+0.27}_{-0.25} \pm 0.11 \times 10^{-4}$ [197], while the corresponding change using semileponic tags was from $1.54^{+0.38}_{-0.37}{}^{+0.29}_{-0.31}$ to $1.25 \pm 0.28 \pm 0.27$. These changes demonstrate the difficulty of the analysis. The results are listed in Table V.

TABLE V. Experimental results for $\mathcal{B}(B^- \to \tau^- \overline{\nu})$ and $|V_{ub}| f_{B^+}$.

Experiment	Tag	$\mathcal{B}$ (units of 10^{-4})	$\|V_{ub}\| f_{B^+}$ (MeV)
Belle [197]	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$	
Belle [198]	Semileptonic	$1.25 \pm 0.28 \pm 0.27$	
Belle [198]	Average	0.91 ± 0.22	0.72 ± 0.09
BaBar [199]	Hadronic	$1.83\,^{+0.53}_{-0.49} \pm 0.24$	
BaBar [200]	Semileptonic	$1.7 \pm 0.8 \pm 0.2$	
BaBar [199]	Average	1.79 ± 0.48	1.01 ± 0.14
Our average		1.06 ± 0.20	0.77 ± 0.07

There are large backgrounds under the signals in all cases. The systematic errors are also quite large. Thus, the significances are not that large. Belle quotes 4.6σ for their combined hadronic and semileptonic tags, while BaBar quotes 3.3σ and $2.3\ \sigma$, for hadronic and semileptonic tags. More accuracy would be useful, especially to investigate the effects of new physics.

To extract the value of $|V_{ub}| f_{B^+}$ we use the PDG 2014 value of the B^+ lifetime of 1.638 ± 0.004 ps, and the τ^+ and B^+ masses of 1.77684 and 5.27926 GeV, respectively.

B. Theoretical decay-constant calculations

Table VI presents theoretical calculations of the B^+- and B_s-meson decay constants and their ratio. The upper two panels show results from simulations with three ($N_f = 2 + 1$) or four flavors ($N_f = 2 + 1 + 1$) of dynamical quarks. For both f_{B^+} and f_{B_s}, calculations using different gauge-field configurations, light-quark actions, and b-quark actions provide

TABLE VI. Recent theoretical determinations of f_{B^+}, f_{B_s}, and their ratio. The upper panels show results from lattice-QCD simulations with $(2+1+1)$ and $(2+1)$ dynamical quark flavors, respectively. For some of the lattice results, statistical and systematic errors are quoted separately. Lattice results for f_B and f_{B_s}/f_B in the isospin-symmetric limit $m_u = m_d$ are noted with an “*”; they are corrected by the factors in Eq. (19) and (20), respectively, before computing the averages. Preliminary conference results noted with a “†” are not included in the lattice averages. The bottom panel shows estimates from QCD sum rules and the light-front quark model, which are not used to obtain our preferred decay-constant values.

Reference	Method	N_f	f_{B^+}(MeV)	f_{B_s}(MeV)	f_{B_s}/f_{B^+}
ETM 13 [201]*,†	LQCD	2+1+1	196(9)	235(9)	1.201(25)
HPQCD 13 [202]	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Average	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Aoki 14 [203]*,‡	LQCD	2+1	218.8(6.5)(30.8)	263.5(4.8)(36.7)	1.193(20)(44)
RBC/UKQCD 14 [204]	LQCD	2+1	195.6(6.4)(13.3)	235.4(5.2)(11.1)	1.223(14)(70)
HPQCD 12 [205]*	LQCD	2+1	191(1)(8)	228(3)(10)	1.188(12)(13)
HPQCD 12 [205]*	LQCD	2+1	189(3)(3)*	–	–
HPQCD 11 [206]	LQCD	2+1	–	225(3)(3)	–
FNAL/MILC 11 [186]	LQCD	2+1	196.9(5.5)(7.0)	242.0(5.1)(8.0)	1.229(13)(23)
Average	LQCD	2+1	189.8(4.2)	226.3(2.8)	1.210(15)
Our average	LQCD	Both	187.0(2.9)	226.0(2.2)	1.215(7)
Wang 15 [188]§	QCD SR		194(15)	231(16)	1.19(10)
Baker 13 [207]	QCD SR		186(14)	222 (12)	1.19(4)
Lucha 13 [208]	QCD SR		192.0(14.6)	228.0(19.8)	1.184(24)
Gelhausen 13 [189]	QCD SR		$207\left(^{+17}_{-9}\right)$	$242\left(^{+17}_{-12}\right)$	$1.17\left(^{+3}_{-4}\right)$
Narison 12 [190]	QCD SR		206(7)	234(5)	1.14(3)
Hwang 09 [192]	LFQM		–	270.0(42.8)¶	1.32(8)

[†] Lattice 2013 conference proceedings.

[‡] Obtained with static b quarks (*i.e.* $m_b \to \infty$).

[⋆] Obtained by combining f_{B_s} from HPQCD 11 with f_{B_s}/f_B from this work. Approximate statistical (systematic) error obtained from quadrature sum of individual statistical (systematic) errors.

[§] Obtained using $m_b^{\overline{\rm MS}}$; results using $m_b^{\rm pole}$ are also given in the paper.

[¶] Obtained by combining PDG value $f_B = 204(31)$ MeV [193] with f_{B_s}/f_B from this work.

independent confirmation. These lattice results are averaged to obtain our preferred decay-constant values in Eq. (21). For comparison, the bottom panel of Table VI shows non-lattice determinations of the $B_{(s)}$-meson decay constants which include error estimates. These are consistent with the lattice values, but with much larger uncertainties.

The results in Table VI were all obtained using isospin-symmetric gauge-field configurations. The two most recent calculations of f_{B^+} by the HPQCD and FNAL/MILC Collaborations [160, 186], however, include the dominant strong isospin-breaking contribution by evaluating the decay constant with the valence light-quark mass fixed to the physical up-quark mass. Reference [202] also presents results for f_B and f_{B_s}/f_B (*i.e.*, with the B-meson decay constant evaluated at the average up-down quark mass), from which one can infer the sizes of the leading corrections:[6]

$$f_{B^+} - f_B = -1.9(5)\ \mathrm{MeV}\,, \tag{19}$$

$$\frac{f_{B_s}}{f_{B^+}} - \frac{f_{B_s}}{f_B} = 0.012(4) \tag{20}$$

Inspection of Table VI shows that, for the ratio f_{B_s}/f_B, the error on the ETM result [201] is only a factor of two bigger than the correction in Eq. (20), and the error on the HPQCD result is comparable [205]. Therefore, to enable comparison with experimental measurements, in this review we correct those lattice results for B-meson decay constants obtained in the isospin limit *a posteriori* by the empirically-obtained factors in Eqs. (19) and (20) before computing our averages.

Preliminary 2015 averages for $B_{(s)}$-meson decay constants from the Flavour Lattice Averaging Group are not yet available. There is, however, only a single published (2+1+1)-flavor lattice-QCD calculation of these quantities. We therefore take the results of Ref. [202] from the HPQCD collaboration as our (2+1+1)-flavor "average." There are several published (2+1)-flavor lattice-QCD calculations of $f_{B_{(s)}}$, however, with various sources of statistical and systematic correlations. We treat the statistical errors as correlated between the results of Aoki *et al.* and RBC/UKQCD because they employ the same gauge-field configurations [203, 204]. We also treat the statistical errors as correlated between the results of the HPQCD and Fermilab Lattice and MILC collaborations because they analyze an overlapping set of gauge-field configurations [186, 205, 206]. We note that there may be mild correlations between some sub-dominant systematic errors of Aoki *et al.* and RBC/UKQCD, who use the same determinations of the absolute lattice scale and the physical light- and strange-quark masses from Ref. [209], and who use the same power-counting estimates for the light-quark and gluon discretization errors. The effects of any correlations between these systematics, however, would be too small to impact the numerical values of the averages.

The HPQCD collaboration presents (2+1)-flavor results for $B_{(s)}$-meson decay constants in two publications. The 2011 [206] and 2012 [205] calculations of HPQCD employ different b-quark actions, but use the same light- and strange-quark action, and analyze an overlapping set of gauge-field configurations. Further, HPQCD presents two determinations of f_B in Ref. [205]: the more precise value is obtained by combining the ratio f_{B_s}/f_B from this work with f_{B_s} from Ref. [206]. Because the (2+1)-flavor results from HPQCD are not independent, but the correlations between them are complex and difficult to estimate, we include only the most precise HPQCD determination of each of the three quantities f_B, f_{B_s}, and f_{B_s}/f_B in our averages.

Given these considerations, we obtain the three- and four-flavor averages for the B^+- and B_s-meson decay constants and their ratio shown in Table VI, where the error on the (2+1)-flavor f_{B_s} average has been rescaled by the factor $\sqrt{(\chi^2/\mathrm{dof})} = 1.2$ to account for the tension among results. We also present a single determination of the B^+- and B_s-meson decay constants for phenomenology applications. Because the four-flavor "average" is obtained from only a single result, we do not simply combine the two lines labeled "Average" in Table VI, which would weight the four-flavor result too heavily. Instead, noting that the three- and four-flavor results are compatible within the current level of precision, we form a single average including the published (2+1)-flavor results and the (2+1+1)-flavor result

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[6] The correlated errors on Eqs. (19) and (20) were provided by HPQCD via private communication.

from HPQCD 13. We do not expect any significant correlations between HPQCD's four-flavor calculation and their three-flavor ones because they differ in most important aspects including the gauge-field configurations, b-quark actions, and the input quantity used to fix the lattice scale. Our final preferred theoretical values for the B^+- and B_s-meson decay constants and their ratio are

$$\text{Our averages}: \quad f_{B^+} = 187.0(2.9)\text{ MeV}\,, \quad f_{B_s} = 226.0(2.2)\text{ MeV}\,, \quad \frac{f_{B_s}}{f_{B^+}} = 1.215(7)\,, \tag{21}$$

where the error on f_{B_s} has been rescaled by the factor $\sqrt{(\chi^2/\text{dof})} = 1.1$.

V. PHENOMENOLOGICAL IMPLICATIONS

A. $|V_{ud}|$, $|V_{us}|$, and status of first-row unitarity

Using the average values for $f_{\pi^+}|V_{ud}|$, $f_{K^+}|V_{us}|$, and their ratio from Eqs. (9)–(11) and for f_{π^+}, f_{K^+}, and their ratio from Eq. (15), we obtain the following determinations of the CKM matrix elements $|V_{ud}|$, $|V_{us}|$, and their ratio from leptonic decays within the standard model:

$$|V_{ud}| = 0.9764(2)(105)(10)\,, \quad |V_{us}| = 0.2255(3)(6)(3), \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2314(2)(5)(2)\,, \tag{22}$$

where the errors are from the experimental branching fraction(s), the pseudoscalar decay constant(s), and radiative corrections, respectively. These results enable a precise test of the unitarity of the first row of the CKM matrix from leptonic decays alone (the contribution from $|V_{ub}|$ is negligible). Using the values of $|V_{ud}|$ and $|V_{us}|$ from Eq. (22), we find

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.004(21)\,, \tag{23}$$

which is consistent with three-generation unitarity at the sub-percent level.

The determinations of $|V_{ud}|$ and $|V_{us}|$ from leptonic decays in Eq. (22) can be compared to those obtained from other processes. The result above for $|V_{ud}|$ agrees with the determination from superallowed β-decay, $|V_{ud}| = 0.97417(21)$ [210], but with an approximately fifty times larger error that is primarily due to the uncertainty in the theoretical determination of f_{π^+}. The CKM element $|V_{us}|$ can be determined from semileptonic $K^+ \to \pi^0\ell^+\nu$ decay. Here experimental measurements provide a value for the product $f_+^{K\pi}(0)|V_{us}|$, where $f_+^{K\pi}(0)$ is the form-factor at zero four-momentum transfer between the initial state kaon and the final state pion. Taking the most recent experimental determination of $|V_{us}|f_+^{K\pi}(0) = 0.2165(4)$ from Moulson [154][7] and the preliminary 2015 (2+1+1)-flavor FLAG average for $f_+(0)^{K\pi} = 0.9704(24)(22)$ [140, 141][8] gives $|V_{us}| = 0.22310(74)_{\text{thy}}(41)_{exp}$ from $K_{\ell 3}$ decay. The determinations of $|V_{us}|$ from leptonic and semileptonic kaon decays are both quite precise (with the error from leptonic decay being about 20% smaller), but the central values differ by 2.2σ. Finally, the combination of the ratio $|V_{us}|/|V_{ud}|$ from leptonic decays [Eq. (22)] with $|V_{ud}|$

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[7] This is an update of the 2010 Flavianet review [156] that includes new measurements of the K_s lifetime [211, 212], $\text{Re}(\epsilon'/\epsilon)$ [212], and $\text{BR}(K^\pm \to \pi^\pm\pi^+\pi^-)$ [155]. The latter measurement is the primary source of the reduced error on $\text{BR}(K_{\ell 3})$, via the constraint that the sum of all BRs must equal unity.

[8] This result comes from the calculation of FNAL/MILC in Ref. [213]. For comparison, the 2015 preliminary (2+1)-flavor FLAG average based on the calculations of FNAL/MILC [214] and RBC/UKQCD [215] is $f_+(0)^{K\pi} = 0.9677(37)$.

from β decay implies an alternative determination of $|V_{us}| = 0.2254(6)$ which agrees with the value from leptonic kaon decay, but disagrees with $K_{\ell 3}$-decay result at the 2.2σ level. Collectively, these results indicate that that there is some tension between theoretical calculations and/or measurements of leptonic pion and kaon decays, semileptonic kaon decays, and superallowed β-decay. Although this may be due to the presence of new physics, it is also important to revisit the quoted uncertainties on both the theoretical and experimental inputs.

Finally, we combine the experimental measurements of $f_{\pi^+}|V_{ud}|$, $f_{K^+}|V_{us}|$ from leptonic psuedoscalar-meson decays in Eqs. (9) and (10) with determinations of the CKM elements from other decays or unitarity to infer "experimental" values for the decay constants. Assuming that there are no significant new-physics contributions to any of the input processes, the comparison of these results with theoretical calculations of the decay constants, enables a test of lattice-QCD methods. Taking $|V_{ud}|$ from superallowed β-decay [216] leads to

$$f_{\pi^-}^{\rm exp} = 130.50(1)(3)(13) \text{ MeV}\,, \tag{24}$$

where the uncertainties are from the errors on Γ, $|V_{ud}|$, and higher-order corrections, respectively. This agrees with the theoretical value $f_{\pi^+} = 130.2(1.4)$ MeV in Eq. (15) obtained from an average of recent (2+1)-flavor lattice-QCD results [163, 165, 167]. We take the value $|V_{us}| = 0.22534(65)$ from the most recent global unitarity-triangle fit of the UTfit Collaboration [217] because there is tension between the values of $|V_{us}|$ obtained from leptonic and semileptonic kaon decays. This implies

$$f_{K^-} = 155.72(17)(45)(16) \text{ MeV} \tag{25}$$

where the uncertainties are from the errors on Γ, $|V_{us}|$, and higher-order corrections, respectively. This agrees with the theoretical value $f_{K^+} = 155.6(0.4)$ MeV in Eq. (15) obtained from an average of recent three and four-flavor lattice-QCD results [159–161, 163, 165, 167].

B. $|V_{cd}|$, $|V_{cs}|$, and status of second-row unitarity

Using the average values for $|V_{cd}|f_{D^+}$ and $|V_{cs}|f_{D_s^+}$ from Tables II and III, and for f_{D^+} and $f_{D_s^+}$ from Eq. (18), we obtain the following determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$, and from leptonic decays within the standard model:

$$|V_{cd}| = 0.217(5)(1) \quad \text{and} \quad |V_{cs}| = 1.007(16)(4)\,, \tag{26}$$

where the errors are from experiment and theory, respectively, and are presently limited by the measured uncertainties on the decay rates. The central value of $|V_{cs}|$ is greater than one, but is compatible with unity within the error. The above results for $|V_{cd}|$ and $|V_{cs}|$ do not include higher-order electroweak and hadronic corrections to the rate, in analogy to Eq. (2). These corrections have not been computed for $D^+_{(s)}$-meson leptonic decays, but are estimated to be about to be about 1–2% for charged pion and kaon decays (see Sec. II A). Now that the uncertainties on $|V_{cd}|$ and $|V_{cs}|$ from leptonic decays are at this level, we hope that the needed theoretical calculations will be undertaken.

The CKM elements $|V_{cd}|$ and $|V_{cs}|$ can also be obtained from semileptonic $D^+ \to \pi^0 \ell^+ \nu$ and $D_s^+ \to K^0 \ell^+ \nu$ decays, respectively. Here experimental measurements determine the product of the form factor times the CKM element, and theory provides the value for the form factor at zero four-momentum transfer between the initial $D_{(s)}$ meson and the final pion or kaon. We combine the latest experimental averages for $f_+^{D\pi}(0)|V_{cd}| = 0.1425(19)$ and $f_+^{D_sK}(0)|V_{cs}| = 0.728(5)$ from the Heavy Flavor Averaging Group (HFAG) [218] with the zero-recoil form factors $f_+^{D\pi}(0) = 0.666(29)$ and $f_+^{D_sK}(0) = 0.747(19)$ calculated in (2+1)-flavor lattice QCD by the HPQCD Collaboration [219, 220] to obtain $|V_{cd}| = 0.2140(97)$ and $|V_{cs}| = 0.9746(257)$ from semileptonic $D_{(s)}$-meson decays. The values of $|V_{cd}|$ from leptonic and semileptonic decays agree, while those for $|V_{cs}|$ are marginally compatible at the 1.1σ level. The determinations of $|V_{cd}|$ and $|V_{cs}|$ from leptonic decays in Eq. (26), however, are 2.0× and 1.6× more precise than those from semileptonic decays, respectively.

The results for $|V_{cd}|$ and $|V_{cs}|$ from Eq. (26) enable a test of the unitarity of the second row of the CKM matrix. We obtain

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.063(34)\,, \tag{27}$$

which is in slight tension with three-generation unitarity at the 2σ level. Because the contribution to Eq. (27) from $|V_{cb}|$ is so small, we obtain the same result taking $|V_{cb}|^{\text{incl.}} \times 10^3 = 42.21(78)$ from inclusive $B \to X_c\ell\nu$ decay [221] or $|V_{cb}|^{\text{excl.}} \times 10^3 = 39.04(75)$ from exclusive $B \to D^*\ell\nu$ decay at zero recoil [222].

We can also combine the experimental measurements of $f_{D^+}|V_{cd}| = 45.91(1.05)$ MeV and $f_{D_s^+}|V_{cs}| = 250.9(4.0)$ MeV from leptonic pseudoscalar-meson decays from Tables II and III with determinations of $|V_{cd}|$ and $|V_{cs}|$ from CKM unitarity to infer "experimental" values for the decay constants within the standard model. For this purpose, we obtain the values of $|V_{cd}|$ and $|V_{cs}|$ by relating them to other CKM elements using the Wolfenstein parameterization [223]. We take $|V_{cd}|$ to equal the value of $|V_{us}|$ minus the leading correction [224]:

$$|V_{cd}| = |V_{us}| \left| -1 + \frac{|V_{cb}|^2}{2}(1 - 2(\rho + i\eta)) \right| \tag{28}$$

$$= |V_{us}| \left(\left[-1 + (1 - 2\rho)\frac{|V_{cb}|^2}{2} \right]^2 + \eta^2 |V_{cb}|^4 \right)^{1/2} \,. \tag{29}$$

Using $|V_{us}| = 0.2255(2)(6)(2)$ from leptonic kaon decay [Eq. (22)], inclusive $|V_{cb}|$ as above, and $(\rho, \eta) = (0.136(24), 0.361(14))$ from CKM unitarity [217] $|V_{cd}| = 0.2254(7)$. We take $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2$ [224], using $|V_{ud}| = 0.97417(21)$ from β decay [210], giving $|V_{cs}| = 0.9733(2)$. Given these choices, we find

$$f_{D^+}^{\text{exp.}} = 203.7(4.7)(0.6)\ \text{MeV} \quad \text{and} \quad f_{D_s^+}^{\text{exp.}} = 257.8(4.1)(0.1)\ \text{MeV}\,, \tag{30}$$

where the uncertainties are from the errors on Γ and $|V_{us}|$ (or $|V_{ud}|$), respectively. These disagree with the theoretical values $f_{D^+} = 211.9(1.0)$ MeV and $f_{D_s^+} = 249.1(1.1)$ MeV in Eq. (18) obtained from averaging recently published three and four-flavor lattice-QCD results at the 1.7σ and 2.0σ levels, respectively. The significances of the tensions are sensitive, however, to the choices made for $|V_{us}|$ and $|V_{ud}|$. Thus resolving the inconsistencies between determinations of elements of the first row of the CKM matrix discussed previously in Sec. V A may also reduce the mild tensions observed here.

C. $|V_{ub}|$ and other applications

Using the average value for $|V_{ub}|f_{B^+}$ from Table V, and for f_{B^+} from Eq. (21), we obtain the following determination of the CKM matrix element $|V_{ub}|$ from leptonic decays within the standard model:

$$|V_{ub}| = 4.12(37)(6) \times 10^{-3}\,, \tag{31}$$

where the errors are from experiment and theory, respectively. We note, however, that decays involving the third generation of quarks and leptons may be particularly sensitive to new physics associated with electroweak symmetry breaking due to their larger masses [132, 134], so Eq. (31) is more likely to be contaminated by new physics than the determinations of the elements of the first and second rows of the CKM matrix in the previous sections.

The CKM element $|V_{ub}|$ can also be obtained from semileptonic B-meson decays. Over the past several years there has remained a persistent 2-3σ tension between the determinations of $|V_{ub}|$ from exclusive $B \to \pi\ell\nu$ decay and from inclusive $B \to X_u\ell\nu$ decay, where X_u denotes all hadrons which contain a constituent up quark [122, 218, 225–227]. The currently most precise determination of $|V_{ub}|^{\text{excl.}} = 3.72(16)\times 10^{-3}$ is obtained from a joint z-fit of the vector and scalar form factors $f_+^{B\pi}(q^2)$ and $f_0^{B\pi}(q^2)$ calculated in (2+1)-flavor lattice QCD by the FNAL/MILC Collaboration [228] and experimental measurements of the differential decay rate from BaBar [229, 230] and Belle [231, 232]. On the other hand, the most recent HFAG determination of $|V_{ub}|^{\text{incl.}} = 4.45(15)\left({}^{+20}_{-21}\right) \times 10^{-3}$ [218] using the theoretical framework of Bosch, Lange, Neubert and Paz (BLNP) [233]. The result for $|V_{ub}|$ from leptonic $B \to \tau\nu$ decay in Eq. (31) is compatible with determinations from both exclusive and inclusive semileptonic B-meson decays.

The CKM element $|V_{ub}|$ can now also be obtained from semileptonic Λ_b decays. Specifically, the recent LHCb measurement of the ratio of decay rates for $\Lambda_b \to p\ell\nu$ over $\Lambda_b \to \Lambda_c\ell\nu$ [234], when combined with the ratio of form factors from (2+1)-flavor lattice QCD [235], enables the first determination of the ratio of CKM elements $|V_{ub}|/|V_{cb}| = 0.083(4)(4)$ from baryonic decay. Taking $|V_{cb}|^{\text{incl.}} = 42.21(78) \times 10^{-3}$ [221] for the denominator,[9] we obtain $|V_{ub}| = 3.50(17)(17)(6) \times 10^{-3}$ from exclusive Λ_b semileptonic decays, where the errors are from experiment, the form factors, and $|V_{cb}|$, respectively. The result for $|V_{ub}|$ from leptonic $B \to \tau\nu$ decay in Eq. (31) is 1.4σ higher than the determination from b-baryon decays.

Given these results, the "V_{ub}" puzzle still stands, and the determination from leptonic B^+-meson decay is not yet sufficiently precise to weigh in on the discrepancy. New and improved experimental measurements and theoretical calculations of other $b \to u$ flavor-changing processes, however, are providing additional information and sharpening the picture of the various tensions. Further, the error on $|V_{ub}|$ from $B \to \tau\nu$ decay will shrink once improved rate measurements from the Belle II experiment are available.

Finally, we can combine the experimental measurement of $|V_{ub}|f_{B^+}$ from leptonic B^+-meson decays in Table V with a determination of the CKM element $|V_{ub}|$ from elsewhere to infer an "experimental" values for f_{B^+} within the standard model. This, of course, assumes that there are no significant new-physics contributions to $B^+ \to \tau\nu$, which may turn out not to be the case. Further, one does not know *a priori* what value to take for $|V_{ub}|$ given the inconsistencies between the various determinations discussed above. We therefore take a weighted average of the determinations from inclusive [218] and exclusive [228] semileptonic B-meson decays and rescale the error by the $\sqrt{\chi^2/\text{dof}} = 2.4$ to account for the disagreement, giving $|V_{ub}|^{\text{excl.+incl.}} = 3.93(33) \times 10^{-3}$. Using this result we obtain

$$f_{B^+}^{\text{exp.}} = 196(18)(16)~\text{MeV}\,, \tag{32}$$

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[9] This differs from the choice for $|V_{cb}|$ made by LHCb [234], who use the determination from exclusive $B \to D^{(*)}\ell\nu$ decays at zero recoil [236]. The Belle Experiment recently announced a new preliminary measurement of the $B \to D\ell\nu$ differential decay rate [237] and determination of $|V_{cb}|$ [238]. They find that the inclusion of experimental and theoretical nonzero-recoil information increases the value for $|V_{cb}|$ compared to when only zero-recoil information is used, and leads to agreement with the inclusive result.

where the uncertainties are from the errors on Γ and $|V_{ub}|$, respectively. This agrees within large uncertainties with the theoretical value $f_{B^+} = 186.8(2.9)$ MeV in Eq. (21) obtained from an average of recent three and four-flavor lattice-QCD results [186, 202, 204, 205].

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[1] K. Nakamura *et al.* (Particle Data Group), J. Phys. **G37**, 075021 (2010)
[2] J. Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012)
[3] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014)
[4] G. Burdman, J. T. Goldman, and D. Wyler, Phys. Rev. **D51**, 111 (1995), arXiv:hep-ph/9405425 [hep-ph]
[5] D. Atwood, G. Eilam, and A. Soni, Mod. Phys. Lett. **A11**, 1061 (1996), arXiv:hep-ph/9411367 [hep-ph]
[6] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. **B372**, 331 (1996), arXiv:hep-ph/9506332 [hep-ph]
[7] A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. **B358**, 129 (1995), arXiv:hep-ph/9506242 [hep-ph]
[8] G. Eilam, I. E. Halperin, and R. R. Mendel, Phys. Lett. **B361**, 137 (1995), arXiv:hep-ph/9506264 [hep-ph]
[9] C. Q. Geng, C. C. Lih, and W.-M. Zhang, Phys. Rev. **D57**, 5697 (1998), arXiv:hep-ph/9710323 [hep-ph]
[10] C. Q. Geng, C. C. Lih, and W.-M. Zhang, Mod. Phys. Lett. **A15**, 2087 (2000), arXiv:hep-ph/0012066 [hep-ph]
[11] G. P. Korchemsky, D. Pirjol, and T.-M. Yan, Phys. Rev. **D61**, 114510 (2000), arXiv:hep-ph/9911427 [hep-ph]
[12] C.-W. Hwang, Eur. Phys. J. **C46**, 379 (2006), arXiv:hep-ph/0512006 [hep-ph]
[13] W.-S. Hou, Phys. Rev. **D48**, 2342 (1993)
[14] A. G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003), arXiv:hep-ph/0210376 [hep-ph]
[15] A. G. Akeroyd and S. Recksiegel, J. Phys. **G29**, 2311 (2003), arXiv:hep-ph/0306037 [hep-ph]
[16] A. G. Akeroyd, Prog. Theor. Phys. **111**, 295 (2004), arXiv:hep-ph/0308260 [hep-ph]
[17] B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008), arXiv:0803.0512 [hep-ph]

[18] J. L. Hewett, in *Heavy flavor physics. Proceedings, LISHEP 95, LAFEX International School on High-Energy Physics, session C, cbt Workshop, Rio de Janeiro, Brazil, February 21-23, 1995* (1995) arXiv:hep-ph/9505246 [hep-ph], `http://www-public.slac.stanford.edu/sciDoc/docMeta.aspx?slacPubNumber=SLAC-PUB-6821`

[19] A. Crivellin, Phys. Rev. **D81**, 031301 (2010), arXiv:0907.2461 [hep-ph]

[20] F. U. Bernlochner, Z. Ligeti, and S. Turczyk, Phys. Rev. **D90**, 094003 (2014), arXiv:1408.2516 [hep-ph]

[21] A. Jüttner, "Light Flavour Physics," `https://indico2.riken.jp/indico/contributionDisplay.py?sessionId=0&contribId=347&confId=1805` (2015), plenary talk presented at Lattice 2015

[22] P. A. Boyle, T. Kaneko, and S. Simula (Flavour Lattice Averaging Group), private communication (2015)

[23] S. Aoki *et al.* (Flavour Lattice Averaging Group), Eur. Phys. J. **C74**, 2890 (2014), arXiv:1310.8555 [hep-lat]

[24] W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004), arXiv:hep-ph/0402299 [hep-ph]

[25] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, and J. Portoles, Rev. Mod. Phys. **84**, 399 (2012), arXiv:1107.6001 [hep-ph]

[26] V. Cirigliano and I. Rosell, JHEP **10**, 005 (2007), arXiv:0707.4464 [hep-ph]

[27] A. Sirlin, Nucl. Phys. **B196**, 83 (1982)

[28] T. Kinoshita, Phys. Rev. Lett. **2**, 477 (1959)

[29] M. Knecht, H. Neufeld, H. Rupertsberger, and P. Talavera, Eur. Phys. J. **C12**, 469 (2000), arXiv:hep-ph/9909284 [hep-ph]

[30] B. Ananthanarayan and B. Moussallam, JHEP **06**, 047 (2004), arXiv:hep-ph/0405206 [hep-ph]

[31] S. Descotes-Genon and B. Moussallam, Eur. Phys. J. **C42**, 403 (2005), arXiv:hep-ph/0505077 [hep-ph]

[32] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993)

[33] V. Cirigliano and H. Neufeld, Phys. Lett. **B700**, 7 (2011), arXiv:1102.0563 [hep-ph]

[34] V. Cirigliano and I. Rosell, Phys. Rev. Lett. **99**, 231801 (2007), arXiv:0707.3439 [hep-ph]

[35] M. Moulson, in *8th International Workshop on the CKM Unitarity Triangle (CKM2014) Vienna, Austria, September 8-12, 2014* (2014) arXiv:1411.5252 [hep-ex], `http://inspirehep.net/record/1328784/files/arXiv:1411.5252.pdf`

[36] D. Babusci *et al.* (KLOE KLOE-2), Phys. Lett. **B738**, 128 (2014), arXiv:1407.2028 [hep-ex]

[37] M. Antonelli *et al.* (FlaviaNet Working Group on Kaon Decays), Eur. Phys. J. **C69**, 399 (2010), arXiv:1005.2323 [hep-ph]

[38] S. Hashimoto, J. Laiho, and S. R. Sharpe, "Lattice Quantum Chromodynamics," `http://pdg.lbl.gov/2014/reviews/rpp2014-rev-lattice-qcd.pdf` (2014), review prepared for PDG 2014 edition

[39] A. S. Kronfeld, Ann. Rev. Nucl. Part. Sci. **62**, 265 (2012), arXiv:1203.1204 [hep-lat]

[40] N. Carrasco *et al.* (ETM), Phys. Rev. **D91**, 054507 (2015), arXiv:1411.7908 [hep-lat]

[41] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. **D90**, 074509 (2014), arXiv:1407.3772 [hep-lat]

[42] R. Dowdall, C. Davies, G. Lepage, and C. McNeile (HPQCD), Phys.Rev. **D88**, 074504 (2013), arXiv:1303.1670 [hep-lat]

[43] T. Blum *et al.* (RBC/UKQCD)(2014), arXiv:1411.7017 [hep-lat]

[44] R. Arthur *et al.* (RBC/UKQCD), Phys. Rev. **D87**, 094514 (2013), arXiv:1208.4412 [hep-lat]

[45] J. Laiho and R. S. Van de Water, *Proceedings, 29th International Symposium on Lattice field theory (Lattice 2011)*, PoS **LATTICE2011**, 293 (2011), arXiv:1112.4861 [hep-lat]
[46] A. Bazavov *et al.* (MILC), *Proceedings, 28th International Symposium on Lattice field theory (Lattice 2010)*, PoS **LATTICE2010**, 074 (2010), arXiv:1012.0868 [hep-lat]
[47] S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, A. Ramos, and K. K. Szabo (BMW), Phys. Rev. **D81**, 054507 (2010), arXiv:1001.4692 [hep-lat]
[48] E. Follana, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu (HPQCD, UKQCD), Phys. Rev. Lett. **100**, 062002 (2008), arXiv:0706.1726 [hep-lat]
[49] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985)
[50] G. M. de Divitiis *et al.*, JHEP **04**, 124 (2012), arXiv:1110.6294 [hep-lat]
[51] M. Artuso *et al.* (CLEO), Phys. Rev. Lett. **95**, 251801 (2005), arXiv:hep-ex/0508057 [hep-ex]
[52] B. I. Eisenstein *et al.* (CLEO), Phys. Rev. **D78**, 052003 (2008), arXiv:0806.2112 [hep-ex]
[53] M. Ablikim *et al.* (BESIII), Phys. Rev. **D89**, 051104 (2014), arXiv:1312.0374 [hep-ex]
[54] M. Artuso *et al.* (CLEO), Phys. Rev. Lett. **99**, 071802 (2007), arXiv:0704.0629 [hep-ex]
[55] J. P. Alexander *et al.* (CLEO), Phys. Rev. **D79**, 052001 (2009), arXiv:0901.1216 [hep-ex]
[56] A. Zupanc *et al.* (Belle), JHEP **1309**, 139 (2013), arXiv:1307.6240 [hep-ex]
[57] P. Naik *et al.* (CLEO), Phys. Rev. **D80**, 112004 (2009), arXiv:0910.3602 [hep-ex]
[58] K. M. Ecklund *et al.* (CLEO), Phys. Rev. Lett. **100**, 161801 (2008), arXiv:0712.1175 [hep-ex]
[59] P. U. E. Onyisi *et al.* (CLEO), Phys. Rev. **D79**, 052002 (2009), arXiv:0901.1147 [hep-ex]
[60] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. **D82**, 091103 (2010), [Erratum: Phys. Rev.D91,no.1,019901(2015)], arXiv:1008.4080 [hep-ex]
[61] J. P. Alexander *et al.* (CLEO), Phys. Rev. Lett. **100**, 161804 (2008), arXiv:0801.0680 [hep-ex]
[62] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **98**, 141801 (2007), arXiv:hep-ex/0607094 [hep-ex]
[63] J. P. Lees *et al.* (BaBar)(2010), arXiv:1003.3063 [hep-ex]
[64] C.-D. Lu and G.-L. Song, Phys. Lett. **B562**, 75 (2003), arXiv:hep-ph/0212363 [hep-ph]
[65] Y.-B. Yang *et al.* (χQCD), Phys. Rev. **D92**, 034517 (2015), arXiv:1410.3343 [hep-lat]
[66] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D86**, 054510 (2012), arXiv:1206.4936 [hep-lat]
[67] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. **D85**, 114506 (2012), arXiv:1112.3051 [hep-lat]
[68] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu (HPQCD), Phys. Rev. **D82**, 114504 (2010), arXiv:1008.4018 [hep-lat]
[69] Z.-G. Wang(2015), arXiv:1506.01993 [hep-ph]
[70] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys.Rev. **D88**, 014015 (2013), arXiv:1305.5432 [hep-ph]
[71] S. Narison, Phys. Lett. **B718**, 1321 (2013), arXiv:1209.2023 [hep-ph]
[72] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. **B701**, 82 (2011), arXiv:1101.5986 [hep-ph]
[73] C.-W. Hwang, Phys. Rev. **D81**, 054022 (2010), arXiv:0910.0145 [hep-ph]
[74] C. Amsler *et al.* (Particle Data Group), Phys. Lett. **B667**, 1 (2008), and 2009 partial update for the 2010 edition
[75] J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. **D81**, 034503 (2010), arXiv:0910.2928 [hep-ph]
[76] M. Schmelling, Phys. Scripta **51**, 676 (1995)
[77] K. Ikado *et al.* (Belle), Phys. Rev. Lett. **97**, 251802 (2006), arXiv:hep-ex/0604018 [hep-ex]

[78] I. Adachi *et al.* (Belle), Phys. Rev. Lett. **110**, 131801 (2013), arXiv:1208.4678 [hep-ex]

[79] B. Kronenbitter *et al.* (Belle)(2015), arXiv:1503.05613 [hep-ex]

[80] J. P. Lees *et al.* (BaBar), Phys. Rev. **D88**, 031102 (2013), arXiv:1207.0698 [hep-ex]

[81] B. Aubert *et al.* (BaBar), Phys. Rev. **D81**, 051101 (2010), arXiv:0912.2453 [hep-ex]

[82] N. Carrasco et al. (ETM), *Proceedings, 31st International Symposium on Lattice Field Theory (Lattice 2013)*, PoS **LATTICE2013**, 313 (2014), arXiv:1311.2837 [hep-lat]

[83] R. Dowdall, C. Davies, R. Horgan, C. Monahan, and J. Shigemitsu (HPQCD), Phys.Rev.Lett. **110**, 222003 (2013), arXiv:1302.2644 [hep-lat]

[84] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner, and A. Soni, Phys. Rev. **D91**, 114505 (2015), arXiv:1406.6192 [hep-lat]

[85] N. H. Christ, J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water, and O. Witzel (RBC/UKQCD), Phys. Rev. **D91**, 054502 (2015), arXiv:1404.4670 [hep-lat]

[86] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D86**, 034506 (2012), arXiv:1202.4914 [hep-lat]

[87] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage (HPQCD), Phys. Rev. **D85**, 031503 (2012), arXiv:1110.4510 [hep-lat]

[88] M. J. Baker, J. Bordes, C. A. Dominguez, J. Penarrocha, and K. Schilcher, JHEP **07**, 032 (2014), arXiv:1310.0941 [hep-ph]

[89] W. Lucha, D. Melikhov, and S. Simula, Phys.Rev. **D88**, 056011 (2013), arXiv:1305.7099 [hep-ph]

[90] Y. Aoki *et al.* (RBC/UKQCD), Phys. Rev. **D83**, 074508 (2011), arXiv:1011.0892 [hep-lat]

[91] J. C. Hardy and I. S. Towner, Phys. Rev. **C91**, 025501 (2015), arXiv:1411.5987 [nucl-ex]

[92] F. Ambrosino *et al.* (KLOE), Eur. Phys. J. **C71**, 1604 (2011), arXiv:1011.2668 [hep-ex]

[93] E. Abouzaid *et al.* (KTeV), Phys. Rev. **D83**, 092001 (2011), arXiv:1011.0127 [hep-ex]

[94] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. Lett. **112**, 112001 (2014), arXiv:1312.1228 [hep-ph]

[95] A. Bazavov, C. Bernard, C. Bouchard, C. DeTar, D. Du, *et al.* (Fermilab Lattice and MILC), Phys.Rev. **D87**, 073012 (2013), arXiv:1212.4993 [hep-lat]

[96] P. A. Boyle *et al.* (RBC/UKQCD), JHEP **06**, 164 (2015), arXiv:1504.01692 [hep-lat]

[97] I. S. Towner and J. C. Hardy, Phys. Rev. **C91**, 015501 (2015), arXiv:1412.0727 [nucl-th]

[98] M. Bona *et al.* (UTfit), JHEP **10**, 081 (2006), updated results and plots available at: `http://www.utfit.org`, arXiv:hep-ph/0606167 [hep-ph]

[99] Y. Amhis *et al.* (Heavy Flavor Averaging Group)(2014), arXiv:1412.7515 [hep-ex]

[100] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D82**, 114506 (2010), arXiv:1008.4562 [hep-lat]

[101] H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D84**, 114505 (2011), arXiv:1109.1501 [hep-lat]

[102] A. Alberti, P. Gambino, K. J. Healey, and S. Nandi, Phys. Rev. Lett. **114**, 061802 (2015), arXiv:1411.6560 [hep-ph]

[103] J. A. Bailey *et al.* (Fermilab Lattice, MILC), Phys. Rev. **D89**, 114504 (2014), arXiv:1403.0635 [hep-lat]

[104] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983)

[105] J. Charles, A. Hocker, H. Lacker, S. Laplace, F. R. Le Diberder, J. Malcles, J. Ocariz, M. Pivk, and L. Roos (CKMfitter Group), Eur. Phys. J. **C41**, 1 (2005), updated results and plots available at: `http://ckmfitter.in2p3.fr`, arXiv:hep-ph/0406184 [hep-ph]

[106] M. Antonelli *et al.*, *5th International Workshop on the CKM Unitarity Triangle (CKM 2008) Rome, Italy, September 9-13, 2008*, Phys. Rept. **494**, 197 (2010), arXiv:0907.5386 [hep-ph]

[107] J. N. Butler *et al.* (Quark Flavor Physics Working Group), in *Community Summer Study 2013: Snowmass on the Mississippi (CSS2013) Minneapolis, MN, USA, July 29-August 6, 2013* (2013) arXiv:1311.1076 [hep-ex], `http://inspirehep.net/record/1263372/files/arXiv:1311.1076.pdf`

[108] A. J. Bevan *et al.* (Belle, BaBar), Eur. Phys. J. **C74**, 3026 (2014), arXiv:1406.6311 [hep-ex]

[109] J. A. Bailey *et al.* (Fermilab Lattice, MILC), Phys. Rev. **D92**, 014024 (2015), arXiv:1503.07839 [hep-lat]

[110] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. **D83**, 032007 (2011), arXiv:1005.3288 [hep-ex]

[111] J. P. Lees *et al.* (BaBar), Phys. Rev. **D86**, 092004 (2012), arXiv:1208.1253 [hep-ex]

[112] H. Ha *et al.* (Belle), Phys. Rev. **D83**, 071101 (2011), arXiv:1012.0090 [hep-ex]

[113] A. Sibidanov *et al.* (Belle), Phys.Rev. **D88**, 032005 (2013), arXiv:1306.2781 [hep-ex]

[114] S. W. Bosch, B. O. Lange, M. Neubert, and G. Paz, Phys. Rev. Lett. **93**, 221801 (2004), arXiv:hep-ph/0403223 [hep-ph]

[115] R. Aaij *et al.* (LHCb), Nature Phys. **11** (2015), doi:"bibinfo doi 10.1038/nphys3415, arXiv:1504.01568 [hep-ex]

[116] W. Detmold, C. Lehner, and S. Meinel, Phys. Rev. **D92**, 034503 (2015), arXiv:1503.01421 [hep-lat]

[117] R. Kowalewski and T. Mannel, "Semileptonic B-meson decays and the determination of V_{cb} and V_{ub}," `http://pdg.lbl.gov/2014/reviews/rpp2014-rev-vcb-vub.pdf` (2014), review prepared for PDG 2014 edition

[118] R. Glattauer (Belle Collaboration), "Semileptonic B and B_s decays at Belle," `https://indico.cern.ch/event/356420/session/3/contribution/306` (2015), talk presented at EPS-HEP 2015

[119] P. Gambino, "V_{cb} and V_{ub}: where do we stand?." `https://indico.cern.ch/event/356420/session/3/contribution/824` (2015), talk presented at EPS-HEP 2015

[120] K. Nakamura *et al.* (Particle Data Group), J. Phys. **G37**, 075021 (2010)

[121] J. Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012)

[122] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014)

[123] G. Burdman, J. T. Goldman, and D. Wyler, Phys. Rev. **D51**, 111 (1995), arXiv:hep-ph/9405425 [hep-ph]

[124] D. Atwood, G. Eilam, and A. Soni, Mod. Phys. Lett. **A11**, 1061 (1996), arXiv:hep-ph/9411367 [hep-ph]

[125] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. **B372**, 331 (1996), arXiv:hep-ph/9506332 [hep-ph]

[126] A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. **B358**, 129 (1995), arXiv:hep-ph/9506242 [hep-ph]

[127] G. Eilam, I. E. Halperin, and R. R. Mendel, Phys. Lett. **B361**, 137 (1995), arXiv:hep-ph/9506264 [hep-ph]

[128] C. Q. Geng, C. C. Lih, and W.-M. Zhang, Phys. Rev. **D57**, 5697 (1998), arXiv:hep-ph/9710323 [hep-ph]

[129] C. Q. Geng, C. C. Lih, and W.-M. Zhang, Mod. Phys. Lett. **A15**, 2087 (2000), arXiv:hep-ph/0012066 [hep-ph]

[130] G. P. Korchemsky, D. Pirjol, and T.-M. Yan, Phys. Rev. **D61**, 114510 (2000), arXiv:hep-ph/9911427 [hep-ph]

[131] C.-W. Hwang, Eur. Phys. J. **C46**, 379 (2006), arXiv:hep-ph/0512006 [hep-ph]

[132] W.-S. Hou, Phys. Rev. **D48**, 2342 (1993)

[133] A. G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003), arXiv:hep-ph/0210376 [hep-ph]

[134] A. G. Akeroyd and S. Recksiegel, J. Phys. **G29**, 2311 (2003), arXiv:hep-ph/0306037 [hep-ph]

[135] A. G. Akeroyd, Prog. Theor. Phys. **111**, 295 (2004), arXiv:hep-ph/0308260 [hep-ph]

[136] B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008), arXiv:0803.0512 [hep-ph]

[137] J. L. Hewett, in *Heavy flavor physics. Proceedings, LISHEP 95, LAFEX International School on High-Energy Physics, session C, cbt Workshop, Rio de Janeiro, Brazil, February 21-23, 1995* (1995) arXiv:hep-ph/9505246 [hep-ph], `http://www-public.slac.stanford.edu/sciDoc/docMeta.aspx?slacPubNumber=SLAC-PUB-6821`

[138] A. Crivellin, Phys. Rev. **D81**, 031301 (2010), arXiv:0907.2461 [hep-ph]

[139] F. U. Bernlochner, Z. Ligeti, and S. Turczyk, Phys. Rev. **D90**, 094003 (2014), arXiv:1408.2516 [hep-ph]

[140] A. Jüttner, "Light Flavour Physics," `https://indico2.riken.jp/indico/contributionDisplay.py?sessionId=0&contribId=347&confId=1805` (2015), plenary talk presented at Lattice 2015

[141] P. A. Boyle, T. Kaneko, and S. Simula (Flavour Lattice Averaging Group), private communication (2015)

[142] S. Aoki *et al.* (Flavour Lattice Averaging Group), Eur. Phys. J. **C74**, 2890 (2014), arXiv:1310.8555 [hep-lat]

[143] W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004), arXiv:hep-ph/0402299 [hep-ph]

[144] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, and J. Portoles, Rev. Mod. Phys. **84**, 399 (2012), arXiv:1107.6001 [hep-ph]

[145] V. Cirigliano and I. Rosell, JHEP **10**, 005 (2007), arXiv:0707.4464 [hep-ph]

[146] A. Sirlin, Nucl. Phys. **B196**, 83 (1982)

[147] T. Kinoshita, Phys. Rev. Lett. **2**, 477 (1959)

[148] M. Knecht, H. Neufeld, H. Rupertsberger, and P. Talavera, Eur. Phys. J. **C12**, 469 (2000), arXiv:hep-ph/9909284 [hep-ph]

[149] B. Ananthanarayan and B. Moussallam, JHEP **06**, 047 (2004), arXiv:hep-ph/0405206 [hep-ph]

[150] S. Descotes-Genon and B. Moussallam, Eur. Phys. J. **C42**, 403 (2005), arXiv:hep-ph/0505077 [hep-ph]

[151] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993)

[152] V. Cirigliano and H. Neufeld, Phys. Lett. **B700**, 7 (2011), arXiv:1102.0563 [hep-ph]

[153] V. Cirigliano and I. Rosell, Phys. Rev. Lett. **99**, 231801 (2007), arXiv:0707.3439 [hep-ph]

[154] M. Moulson, in *8th International Workshop on the CKM Unitarity Triangle (CKM2014) Vienna, Austria, September 8-12, 2014* (2014) arXiv:1411.5252 [hep-ex], `http://inspirehep.net/record/1328784/files/arXiv:1411.5252.pdf`

[155] D. Babusci *et al.* (KLOE KLOE-2), Phys. Lett. **B738**, 128 (2014), arXiv:1407.2028 [hep-ex]

[156] M. Antonelli *et al.* (FlaviaNet Working Group on Kaon Decays), Eur. Phys. J. **C69**, 399 (2010), arXiv:1005.2323 [hep-ph]

[157] S. Hashimoto, J. Laiho, and S. R. Sharpe, "Lattice Quantum Chromodynamics," `http://pdg.lbl.gov/2014/reviews/rpp2014-rev-lattice-qcd.pdf` (2014), review prepared for PDG 2014 edition

[158] A. S. Kronfeld, Ann. Rev. Nucl. Part. Sci. **62**, 265 (2012), arXiv:1203.1204 [hep-lat]

[159] N. Carrasco *et al.* (ETM), Phys. Rev. **D91**, 054507 (2015), arXiv:1411.7908 [hep-lat]

[160] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. **D90**, 074509 (2014), arXiv:1407.3772 [hep-lat]

[161] R. Dowdall, C. Davies, G. Lepage, and C. McNeile (HPQCD), Phys.Rev. **D88**, 074504 (2013), arXiv:1303.1670 [hep-lat]

[162] T. Blum *et al.* (RBC/UKQCD)(2014), arXiv:1411.7017 [hep-lat]

[163] R. Arthur *et al.* (RBC/UKQCD), Phys. Rev. **D87**, 094514 (2013), arXiv:1208.4412 [hep-lat]

[164] J. Laiho and R. S. Van de Water, *Proceedings, 29th International Symposium on Lattice field theory (Lattice 2011)*, PoS **LATTICE2011**, 293 (2011), arXiv:1112.4861 [hep-lat]

[165] A. Bazavov *et al.* (MILC), *Proceedings, 28th International Symposium on Lattice field theory (Lattice 2010)*, PoS **LATTICE2010**, 074 (2010), arXiv:1012.0868 [hep-lat]

[166] S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, A. Ramos, and K. K. Szabo (BMW), Phys. Rev. **D81**, 054507 (2010), arXiv:1001.4692 [hep-lat]

[167] E. Follana, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu (HPQCD, UKQCD), Phys. Rev. Lett. **100**, 062002 (2008), arXiv:0706.1726 [hep-lat]

[168] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985)

[169] G. M. de Divitiis *et al.*, JHEP **04**, 124 (2012), arXiv:1110.6294 [hep-lat]

[170] M. Artuso *et al.* (CLEO), Phys. Rev. Lett. **95**, 251801 (2005), arXiv:hep-ex/0508057 [hep-ex]

[171] B. I. Eisenstein *et al.* (CLEO), Phys. Rev. **D78**, 052003 (2008), arXiv:0806.2112 [hep-ex]

[172] M. Ablikim *et al.* (BESIII), Phys. Rev. **D89**, 051104 (2014), arXiv:1312.0374 [hep-ex]

[173] M. Artuso *et al.* (CLEO), Phys. Rev. Lett. **99**, 071802 (2007), arXiv:0704.0629 [hep-ex]

[174] J. P. Alexander *et al.* (CLEO), Phys. Rev. **D79**, 052001 (2009), arXiv:0901.1216 [hep-ex]

[175] A. Zupanc *et al.* (Belle), JHEP **1309**, 139 (2013), arXiv:1307.6240 [hep-ex]

[176] P. Naik *et al.* (CLEO), Phys. Rev. **D80**, 112004 (2009), arXiv:0910.3602 [hep-ex]

[177] K. M. Ecklund *et al.* (CLEO), Phys. Rev. Lett. **100**, 161801 (2008), arXiv:0712.1175 [hep-ex]

[178] P. U. E. Onyisi *et al.* (CLEO), Phys. Rev. **D79**, 052002 (2009), arXiv:0901.1147 [hep-ex]

[179] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. **D82**, 091103 (2010), [Erratum: Phys. Rev.D91,no.1,019901(2015)], arXiv:1008.4080 [hep-ex]

[180] J. P. Alexander *et al.* (CLEO), Phys. Rev. Lett. **100**, 161804 (2008), arXiv:0801.0680 [hep-ex]

[181] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **98**, 141801 (2007), arXiv:hep-ex/0607094 [hep-ex]

[182] J. P. Lees *et al.* (BaBar)(2010), arXiv:1003.3063 [hep-ex]

[183] C.-D. Lu and G.-L. Song, Phys. Lett. **B562**, 75 (2003), arXiv:hep-ph/0212363 [hep-ph]

[184] Y.-B. Yang *et al.* (χQCD), Phys. Rev. **D92**, 034517 (2015), arXiv:1410.3343 [hep-lat]

[185] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D86**, 054510 (2012), arXiv:1206.4936 [hep-lat]

[186] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. **D85**, 114506 (2012), arXiv:1112.3051 [hep-lat]

[187] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu (HPQCD), Phys. Rev. **D82**, 114504 (2010), arXiv:1008.4018 [hep-lat]

[188] Z.-G. Wang(2015), arXiv:1506.01993 [hep-ph]

[189] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys.Rev. **D88**, 014015 (2013), arXiv:1305.5432 [hep-ph]

[190] S. Narison, Phys. Lett. **B718**, 1321 (2013), arXiv:1209.2023 [hep-ph]

[191] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. **B701**, 82 (2011), arXiv:1101.5986 [hep-ph]

[192] C.-W. Hwang, Phys. Rev. **D81**, 054022 (2010), arXiv:0910.0145 [hep-ph]

[193] C. Amsler *et al.* (Particle Data Group), Phys. Lett. **B667**, 1 (2008), and 2009 partial update for the 2010 edition

[194] J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. **D81**, 034503 (2010), arXiv:0910.2928 [hep-ph]

[195] M. Schmelling, Phys. Scripta **51**, 676 (1995)

[196] K. Ikado *et al.* (Belle), Phys. Rev. Lett. **97**, 251802 (2006), arXiv:hep-ex/0604018 [hep-ex]

[197] I. Adachi *et al.* (Belle), Phys. Rev. Lett. **110**, 131801 (2013), arXiv:1208.4678 [hep-ex]

[198] B. Kronenbitter *et al.* (Belle)(2015), arXiv:1503.05613 [hep-ex]

[199] J. P. Lees *et al.* (BaBar), Phys. Rev. **D88**, 031102 (2013), arXiv:1207.0698 [hep-ex]

[200] B. Aubert *et al.* (BaBar), Phys. Rev. **D81**, 051101 (2010), arXiv:0912.2453 [hep-ex]

[201] N. Carrasco *et al.* (ETM), *Proceedings, 31st International Symposium on Lattice Field Theory (Lattice 2013)*, PoS **LATTICE2013**, 313 (2014), arXiv:1311.2837 [hep-lat]

[202] R. Dowdall, C. Davies, R. Horgan, C. Monahan, and J. Shigemitsu (HPQCD), Phys.Rev.Lett. **110**, 222003 (2013), arXiv:1302.2644 [hep-lat]

[203] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner, and A. Soni, Phys. Rev. **D91**, 114505 (2015), arXiv:1406.6192 [hep-lat]

[204] N. H. Christ, J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water, and O. Witzel (RBC/UKQCD), Phys. Rev. **D91**, 054502 (2015), arXiv:1404.4670 [hep-lat]

[205] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D86**, 034506 (2012), arXiv:1202.4914 [hep-lat]

[206] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage (HPQCD), Phys. Rev. **D85**, 031503 (2012), arXiv:1110.4510 [hep-lat]

[207] M. J. Baker, J. Bordes, C. A. Dominguez, J. Penarrocha, and K. Schilcher, JHEP **07**, 032 (2014), arXiv:1310.0941 [hep-ph]

[208] W. Lucha, D. Melikhov, and S. Simula, Phys.Rev. **D88**, 056011 (2013), arXiv:1305.7099 [hep-ph]

[209] Y. Aoki *et al.* (RBC/UKQCD), Phys. Rev. **D83**, 074508 (2011), arXiv:1011.0892 [hep-lat]

[210] J. C. Hardy and I. S. Towner, Phys. Rev. **C91**, 025501 (2015), arXiv:1411.5987 [nucl-ex]

[211] F. Ambrosino *et al.* (KLOE), Eur. Phys. J. **C71**, 1604 (2011), arXiv:1011.2668 [hep-ex]

[212] E. Abouzaid *et al.* (KTeV), Phys. Rev. **D83**, 092001 (2011), arXiv:1011.0127 [hep-ex]

[213] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. Lett. **112**, 112001 (2014), arXiv:1312.1228 [hep-ph]

[214] A. Bazavov, C. Bernard, C. Bouchard, C. DeTar, D. Du, *et al.* (Fermilab Lattice and MILC), Phys.Rev. **D87**, 073012 (2013), arXiv:1212.4993 [hep-lat]

[215] P. A. Boyle *et al.* (RBC/UKQCD), JHEP **06**, 164 (2015), arXiv:1504.01692 [hep-lat]

[216] I. S. Towner and J. C. Hardy, Phys. Rev. **C91**, 015501 (2015), arXiv:1412.0727 [nucl-th]

[217] M. Bona *et al.* (UTfit), JHEP **10**, 081 (2006), updated results and plots available at: `http://www.utfit.org`, arXiv:hep-ph/0606167 [hep-ph]

[218] Y. Amhis *et al.* (Heavy Flavor Averaging Group)(2014), arXiv:1412.7515 [hep-ex]

[219] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D82**, 114506 (2010), arXiv:1008.4562 [hep-lat]

[220] H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage, and J. Shigemitsu (HPQCD), Phys. Rev. **D84**, 114505 (2011), arXiv:1109.1501 [hep-lat]

[221] A. Alberti, P. Gambino, K. J. Healey, and S. Nandi, Phys. Rev. Lett. **114**, 061802 (2015), arXiv:1411.6560 [hep-ph]

[222] J. A. Bailey *et al.* (Fermilab Lattice, MILC), Phys. Rev. **D89**, 114504 (2014), arXiv:1403.0635 [hep-lat]

[223] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983)

[224] J. Charles, A. Hocker, H. Lacker, S. Laplace, F. R. Le Diberder, J. Malcles, J. Ocariz, M. Pivk, and L. Roos (CKMfitter Group), Eur. Phys. J. **C41**, 1 (2005), updated results and plots available at: `http://ckmfitter.in2p3.fr`, arXiv:hep-ph/0406184 [hep-ph]

[225] M. Antonelli *et al.*, *5th International Workshop on the CKM Unitarity Triangle (CKM 2008) Rome, Italy, September 9-13, 2008*, Phys. Rept. **494**, 197 (2010), arXiv:0907.5386 [hep-ph]

[226] J. N. Butler *et al.* (Quark Flavor Physics Working Group), in *Community Summer Study 2013: Snowmass on the Mississippi (CSS2013) Minneapolis, MN, USA, July 29-August 6, 2013* (2013) arXiv:1311.1076 [hep-ex], `http://inspirehep.net/record/1263372/files/arXiv:1311.1076.pdf`

[227] A. J. Bevan *et al.* (Belle, BaBar), Eur. Phys. J. **C74**, 3026 (2014), arXiv:1406.6311 [hep-ex]

[228] J. A. Bailey *et al.* (Fermilab Lattice, MILC), Phys. Rev. **D92**, 014024 (2015), arXiv:1503.07839 [hep-lat]

[229] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. **D83**, 032007 (2011), arXiv:1005.3288 [hep-ex]

[230] J. P. Lees *et al.* (BaBar), Phys. Rev. **D86**, 092004 (2012), arXiv:1208.1253 [hep-ex]

[231] H. Ha *et al.* (Belle), Phys. Rev. **D83**, 071101 (2011), arXiv:1012.0090 [hep-ex]

[232] A. Sibidanov *et al.* (Belle), Phys.Rev. **D88**, 032005 (2013), arXiv:1306.2781 [hep-ex]

[233] S. W. Bosch, B. O. Lange, M. Neubert, and G. Paz, Phys. Rev. Lett. **93**, 221801 (2004), arXiv:hep-ph/0403223 [hep-ph]

[234] R. Aaij *et al.* (LHCb), Nature Phys. **11** (2015), doi:“bibinfo doi 10.1038/nphys3415, arXiv:1504.01568 [hep-ex]

[235] W. Detmold, C. Lehner, and S. Meinel, Phys. Rev. **D92**, 034503 (2015), arXiv:1503.01421 [hep-lat]

[236] R. Kowalewski and T. Mannel, “Semileptonic B-meson decays and the determination of V_{cb} and V_{ub},” `http://pdg.lbl.gov/2014/reviews/rpp2014-rev-vcb-vub.pdf` (2014), review prepared for PDG 2014 edition

[237] R. Glattauer (Belle Collaboration), “Semileptonic B and B_s decays at Belle,” `https://indico.cern.ch/event/356420/session/3/contribution/306` (2015), talk presented at EPS-HEP 2015

[238] P. Gambino, “V_{cb} and V_{ub}: where do we stand?.” `https://indico.cern.ch/event/356420/session/3/contribution/824` (2015), talk presented at EPS-HEP 2015