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Is the Higgs boson composed of neutrinos?

Jens Krog[1,*] and Christopher T. Hill[2,†]
[1]*CP3-Origins, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*
[2]*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA*
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We show that conventional Higgs compositeness conditions can be achieved by the running of large Higgs-Yukawa couplings involving right-handed neutrinos that become active at $\sim 10^{13}$–10^{14} GeV. Together with a somewhat enhanced quartic coupling arising by a Higgs portal interaction to a dark matter sector, we can obtain a Higgs boson composed of neutrinos. This is a "next-to-minimal" dynamical electroweak symmetry breaking scheme.

I. INTRODUCTION

Many years ago, it was proposed that the top quark Higgs-Yukawa (HY) coupling y_t might be large and governed by a quasi-infrared-fixed-point behavior of the renormalization group [1,2]. This implied, using the minimal ingredients of the Standard Model, a top quark mass of order 220–240 GeV for the case of a Landau pole in y_t at a scale Λ of order the GUT-to-Planck scale. In light of the observed 173 GeV top quark mass, the fixed point prediction is seen to be within 25% of experiment. This suggests that small corrections from new physics might bring the prediction into a more precise concordance with experiment.

One of the main interpretations of the quasi-infrared fixed point was the compositeness of the Higgs boson. In its simplest form, the Higgs boson was considered to be a bound state containing a top and antitop quark [3–6]. This was amenable to a treatment in a large-N_c Nambu–Jona-Lasinio (NJL) model [7] defined by a four-fermion interaction at a scale Λ, with a large coupling constant, and a strong attractive 0^+ channel. The theory requires drastic fine-tuning of quadratic loop contributions, which is equivalent to a fine-tuning of the scale-invariant NJL coupling constant (and may require a novel insight into scale symmetry, such as [8]). By tuning the NJL coupling close to criticality, the Higgs boson mass becomes small, creating an infrared hierarchy between the compositeness scale Λ and the electroweak scale embodied in m_h. Tuning the coupling slightly supercritical yields a vacuum instability, and the Higgs boson acquires its vacuum expectation value (VEV).

Once the infrared hierarchy has been tuned, the remaining structure of the theory is controlled by renormalization group (RG) running of couplings [5]. The RG treatment indicates that a $\bar{t}t$ composite Higgs boson requires (i) a Landau pole at scale Λ in the running top HY coupling constant $y_t(\mu)$, (ii) the Higgs-quartic coupling λ_H must also have a Landau pole, and (iii) compositeness conditions must be met, such as $\lambda_H(\mu)/g_t^4(\mu) \to 0$ and $\lambda_H(\mu)/g_t^2(\mu) \to$ (constant) as $\mu \to \Lambda$, [5]. This predicts a Higgs boson mass of order $\sim$250 GeV with a heavy top quark of order $\sim$220 GeV, predictions that come within a factor of 2 of reality.

While the $\bar{t}t$ minimal composite Higgs model is ruled out, it remains of interest to ask, "can we rescue a NJL RG composite Higgs boson scenario with new physics?" and if so, "what are the minimal requirements of new physics needed to maintain a composite Higgs boson scenario?" In the present paper, we address this issue and revisit a composite Higgs boson model based upon an attractive idea of Martin [9] (this has also been considered in a supersymmetry context by Leontaris *et al.* [10]). Martin pointed out that the top quark HY is sensitive to right-handed neutrinos ν_R^i that become active in loops above the large Majorana mass scale, M. The right-handed neutrinos are assumed to have HY couplings $y_\nu^i \geq \mathcal{O}(1)$ and also have a Majorana mass of order $M \sim 10^{13}$, thus, leading to the neutrino seesaw model at low energies [11]. Turning on the neutrino loops will generally pull a large $y_t(m_t)$ to a Landau pole at a scale of order $\Lambda \sim 10^{15}$–10^{19} GeV, and the large top quark mass becomes intertwined with neutrino physics above M. The strong dynamics that forms the bound state Higgs boson for us is the dominant large coupling, y_ν^i.

Martin's model preserved some of the features of the $\bar{t}t$ composite Higgs model but extends the Higgs compositeness structure to become an entanglement of neutrinos and top quark. Martin's model considered one large neutrino HY interaction, while we will presently consider $N_f = 3$ right-handed neutrinos with degenerate HY couplings to the corresponding left-handed doublets, y_ν. Hence, our present model becomes a large-N_f fermion bubble NJL model as we approach the compositeness scale Λ. The y_ν^i become active above the scale M and are chosen to be large enough to have Landau poles at the scale Λ.

*krog@cp3-origins.net
†hill@fnal.gov

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The top quark HY coupling is pulled up by the large y_ν^i to a Landau pole, but we find that the ratio of $y_t(\mu)/y_\nu(\mu) \to$ (constant) as $\mu \to \Lambda$. This implies that the top quark couples to the dynamics that forms the Higgs boson at the scale Λ, which we treat as a Nambu–Jona-Lasinio model, but this is merely a comparatively weak extension of the dynamics to give mass to the top (and presumably, all other quarks and leptons). The Higgs doublet (Higgs scalar) in our scheme is primarily composed of $\sum_i \bar{L}_{iL}\nu_R^i$, $(\sum_i \bar{\nu}_i \nu^i)$, where $L_{iL} = (\nu_i, \ell_i)_L$ is the left-handed lepton doublet and summing over $N_f = 3$ generations. The top quark HY coupling is then only a spectator to this physics.

A second and important demand of a NJL-composite Higgs model is the behavior of the running of the Higgs-quartic coupling, λ_H. It is shown in Ref. [5] that λ_H will have a Landau pole at the scale Λ, but $\lambda_H(\mu)/y_t^4(\mu) \to 0$ and $\lambda_H(\mu)/y_t^2(\mu) \to$ (constant) as $\mu \to \Lambda$. Engineering this is a more challenging issue than that of the Landau poles in the Yukawa couplings, as we are confronted by the small value of λ_H in the Standard Model and the apparent RG behavior $\lambda_H \to 0$ for scale of order $\sim 10^{12}$ GeV. For general gauge Yukawa theories containing a scalar bilinear, the divergent behavior is readily obtainable [12], but for the single doublet of the Standard Model, this is not easily constructed.

There is, however, a simple remedy available to us here: the Higgs portal interaction. The main point is that if there exists new physics coupled to the Higgs via a portal interaction, e.g., a sterile dark matter boson, then the Higgs-Yukawa coupling that we observe λ is actually only effective and is replaced typically by a larger value near the TeV scale due to the mixing via the portal interaction [13]. We presently exploit this mechanism.

We note that many authors have considered various neutrino-composite Higgs boson schemes, many in the context of a fourth generation and some with overlap to our present case [14]. We turn presently to a Nambu–Jona-Lasinio schematic model of our mechanism.

II. NJL MODEL

The effective UV model we have in mind is a variation on the Nambu–Jona-Lasinio model [7] and top condensation models [3–5]. We adapt this to a neutrino condensate with the four-fermion interaction Lagrangian:

$$\mathcal{L}' = \frac{g^2}{\Lambda^2}(\bar{L}_{Li}\nu_R^i)(\bar{\nu}_{Rj}L_L^j) + \frac{h^2}{\Lambda^2}(\bar{L}_{Li}\nu_R^i)(\bar{t}_{Ra}T_L^a) + \text{H.c.}, \tag{1}$$

where $L_L^i = (\nu^i, \ell^i)_L$ (ν_{Ri}) are left-handed lepton doublets (right-handed neutrino singlets), and T_L (t_R) is the top quark doublet (singlet), $(i, j, \ldots)$ are generation indices running to $N_f = 3$, and $(a, b, \ldots)$ are color indices running to $N_c = 3$. The dominant large coupling constant in our scheme is g and $h < g/N_f$. We will have additional smaller couplings involving the other quarks associated with light fermion mass generation and flavor physics, as well as charge conjugated terms like $(\bar{L}_{Li}\nu_R^i)g^{jk}(\bar{\nu}_{Rj}L_{Lk})^C$. These generate the charged lepton and quark masses and mixing angles, which we presently ignore.

We follow [5] and factorize the NJL interactions to write

$$\mathcal{L}'_\Lambda = g\bar{L}_{iL}H\nu_R^i + g'\bar{T}_{aL}Ht_R^a - \Lambda^2 H^\dagger H. \tag{2}$$

Here we define $g' = h^2/g$, and we have introduced an auxiliary field H that regenerates Eq. (1) by H equation of motion. This is the Lagrangian at the scale Λ, where the auxiliary field H will become the dynamical Higgs boson bound state at lower energies. We have ignored terms of order g'^2 which are generated when H is integrated out to recover Eq. (1).

We now use the RG to run the Lagrangian down to the Majorana mass scale M of the right-handed neutrinos using only fermion loops. The result is formally

$$\begin{aligned}\mathcal{L}'_M = {} & Z_H|DH|^2 - \tilde{M}^2 H^\dagger H + \frac{\tilde{\lambda}_H}{2}(H^\dagger H)^2 \\ & + [g\bar{L}_{iL}H\nu_R^i + g'\bar{T}_{aL}Ht_R^a + \bar{\nu}_{Ri}^C M_{ij}\nu_R^j + \text{H.c.}],\end{aligned} \tag{3}$$

where the Majorana mass matrix M_{ij} is now incorporated by hand. The Higgs boson has acquired a logarithmic kinetic term and a quartic interaction due to the fermion loops, and the Higgs mass has run quadratically:

$$\begin{aligned} Z_H &= (4\pi)^{-2}(g^2 N_f + g'^2 N_c)\ln(\Lambda^2/M^2), \\ \tilde{M}^2 &= \Lambda^2 - (4\pi)^{-2}(2g^2 N_f + 2g'^2 N_c)(\Lambda^2 - M^2), \\ \tilde{\lambda}_H &= (4\pi)^{-2}(2g^4 N_f + 2g'^4 N_c)\ln(\Lambda^2/M^2). \end{aligned} \tag{4}$$

The quantities appearing in Eq. (3) are, of course, unrenormalized. The renormalized couplings at the present level of approximation are

$$y_\nu = \frac{g}{\sqrt{Z_H}}, \qquad y_t = \frac{g'}{\sqrt{Z_H}}, \qquad \lambda_H = \frac{\tilde{\lambda}_H}{Z_H^2}, \tag{5}$$

and we see that in the large-(N_f, N_c) limit, the ratio y_ν^2/y_t^2 is a constant.

For simplicity, we take the Majorana mass matrix to be diagonal, $M = \text{diag}(M_1, M_2, M_3)$. In the large-$M/v_{\text{weak}}$ limit, where $v_{\text{weak}} \sim 175$ GeV, the masses of the three light neutrino states are given by the seesaw mechanism:

$$m_\nu^i = \frac{y_\nu^2 v_{\text{weak}}^2}{M_i}. \tag{6}$$

Assuming that y_ν is $\sim\mathcal{O}(1)$ and $\sim$eV masses for the light neutrinos, we expect $M_i \sim 10^{13}$ GeV. Thus, in the RG

evolution of the system, loops containing right-handed neutrinos occur only above the scales M_i. As an approximation, take the threshold of the ν_R^i in loops to be at a common Majorana mass scale M.

Note that the renormalized $\lambda_H = \tilde{\lambda}_H/Z_H^2$ has the limits $\lambda_H/y_\nu^4 \to 0$ and $\lambda_H/y_\nu^2 \to$ (constant) as $\mu \to \Lambda$. The extent to which the top quark participates in the binding of the Higgs boson relative to the neutrinos is determined by $g'^2 N_c/g^2 N_f$ which we assume is of order $1/N_f$, and, thus, the dominant coupling at the UV scale is g^2. While we could keep the order g'^2 terms in the factorization of Eq. (3), this would make a weakly bound state doublet H' composed mainly of $\bar{t}t$, but since g'^2 is subcritical, this state would remain a heavy dormant doublet with $m^2 \sim \Lambda^2$.

Below the Majorana mass scale M, the neutrinos decouple, and the only significant running in the fermion loop approximation is the top quark. The electroweak scale is tuned by the choice of critical couplings. The quadratic running to a zero mass Higgs boson $\tilde{M}^2 = 0$ defines the critical coupling:

$$g^2 N_f + g'^2 N_c = 8\pi^2\left(1 + \frac{M^2}{\Lambda^2}\right). \tag{7}$$

The criticality, we assume, is due principally to the large value of g^2 and is only slightly modified by the top quark. We then choose g^2 slightly supercritical to produce the phenomenological tachyonic Higgs potential, $\tilde{M}^2 = -M_H^2 Z_H(\Lambda/M_H)$.

The NJL model is schematic and must itself be an approximation to some new dynamics in the UV. This structure suggests a new gauge interaction which leads to Eq. (1) upon Fierz rearrangement, in analogy to top color models [15] as

$$\frac{g^2}{\Lambda^2}\bar{L}_{Li}\nu_R^i\bar{\nu}_{Rj}L_L^j = -\frac{g^2}{\Lambda^2}\bar{L}_L\frac{\lambda^A}{2}\gamma^\mu L_L\bar{\nu}_R\frac{\lambda^A}{2}\gamma_\mu\nu_R + \cdots, \tag{8}$$

where the Gell-Mann matrices λ^A now act on the flavor indices. The g' term then requires some extension of the theory. A model such as this assigns an $SU(3)$ gauge group to lepton family number, and, therefore, gauge charges to the ν_{Ri}, i.e., the ν_{Ri} are no longer sterile. This would imply that the Majorana mass matrix must be generated by a VEV associated, e.g., with additional $SU(3)$ scalar fields. With ν_{Ri} in the triplet representation, this requires $\{\bar{3}\}$ and/or $\{6\}$ scalar condensates and would dictate the neutrino mass and mixing angle structure. Construction of this kind of model will be done elsewhere.

III. YUKAWA SECTOR

The above discussion is the Wilsonian renormalization group approach. To improve the calculation, we turn to the full RG equations which are used below the scale Λ, together with the matching conditions dictated by the fermion bubble approximation [5]. The full RG equations (for $N_f = 3$, these are a slight modification of Ref. [9]) take the form

$$\begin{aligned}
(4\pi)^2\frac{\beta_{y_t}}{y_t} &= \frac{9}{2}y_t^2 - 8g_3^2 + 3\theta_M(\mu - M)y_\nu^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2,\\
(4\pi)^2\frac{\beta_{y_\nu}}{y_\nu} &= \theta_M(\mu - M)\left(\frac{9}{2}y_\nu^2 + 3y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2\right),\\
(4\pi)^2\frac{\beta_{g_1}}{g_1} &= \frac{41}{6}g_1^2, \qquad (4\pi)^2\frac{\beta_{g_2}}{g_2} = -\frac{19}{6}g_2^2,\\
(4\pi)^2\frac{\beta_{g_3}}{g_3} &= -7g_3^2,
\end{aligned} \tag{9}$$

where g_1, g_2, and g_3 are the gauge couplings of the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ symmetries, respectively, y_t is the top HY coupling, and y_ν the HY coupling of the lepton doublets to right-handed neutrinos. We have introduced a step function $\theta_M = \theta(\mu - M)$, where $\theta(x) = 1; x \geq 0$ and $\theta(x) = 0; x < 0$. The step function models the threshold of the turning on of the right-handed neutrinos at the scale of the Majorana mass matrix.

In Fig. 1, we demonstrate the running of the HY coupling for the top quark and the neutrino as described. We use the initial conditions for the gauge couplings $g_1(m_Z) = 0.36$, $g_2(m_Z) = 0.65$, $g_3(m_Z) = 1.16$, for the HY couplings $y_t(m_t) = 0.99$ and $y_\nu(M) = 1$, and for the masses $m_Z = 91.2$ GeV, $m_t = 173.2$ GeV, and $M = 10^{13}$ GeV.

The evolution in Fig. 1 clearly indicates the existence of a Landau pole for the HY couplings at a scale $\Lambda \sim 10^{20}$ GeV, in accord with what one would expect if the Higgs is a fermion pair condensate.

The Landau pole of the neutrino HY coupling is seen to pull the top HY towards a Landau pole at Λ. The neutrino

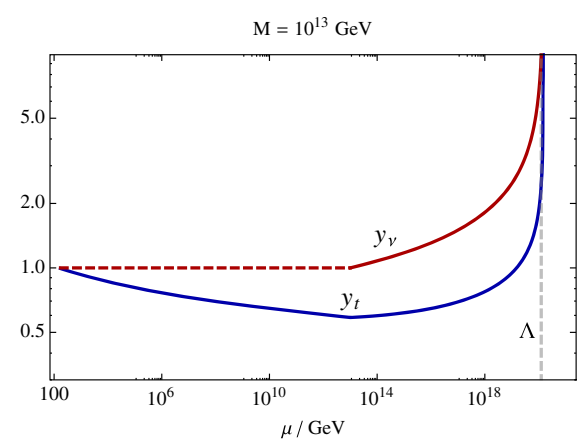

FIG. 1 (color online). The RG evolutions of the top (solid) and neutrino (dashed) HY couplings with contributions from right-handed neutrinos for renormalization scales above the neutrino Majorana mass; $\mu > M = 10^{13}$ GeV.

HY coupling is always significantly larger than the top coupling for the scales where the perturbative result is valid. For the displayed example, we find the ratio $y_\nu/y_t \geq 3$ for the region very close to the Landau pole.

To verify the consistency of this behavior of the top quark, consider the region below, but near, Λ. Here the RG equations for top and neutrino HY couplings can be approximated in the large-(N_f, N_c) limit by

$$(4\pi)^2 \frac{d\ln y_t}{d\ln\mu} \approx (4\pi)^2 \frac{d\ln y_\nu}{d\ln\mu} \approx N_f y_\nu^2 + N_c y_t^2, \tag{10}$$

hence,

$$(4\pi)^2 \frac{d\ln(y_t/y_\nu)}{d\ln\mu} = 0. \tag{11}$$

This implies that $y_\nu(\mu)/y_t(\mu) \to$ (constant) as we approach the scale Λ. The ratio $y_t/y_\nu \sim g'/g$, so the role of the top quark is only that of a spectator.

In this simplified setup, inserting an experimental neutrino mass in (6) yields $y_\nu(M)$ as a function of M. For a chosen M, this value may be used as an initial condition in the RG equation for y_ν, and the scale Λ may be read off from the solution to the RG equations. A simple analytic estimate is given by setting to zero all couplings except y_ν, in which case one finds for the one loop solution

$$\Lambda = M \exp\left[\frac{(4\pi v_{\text{weak}})^2}{9 m_\nu^{\text{exp}} M}\right]. \tag{12}$$

Here v is again the Higgs VEV, and m_ν^{exp} is the experimentally measured neutrino mass. The estimate (12) is in good agreement with the full numerical solution due to the fact that the neutrino coupling itself is what drives the divergence at Λ. The relation (12) also gives a lower bound on the possible compositeness scale Λ_{min} for any neutrino mass given by

$$\Lambda_{\text{min}} \simeq 1.5 \times \left(\frac{m_\nu^{\text{exp}}}{\text{eV}}\right)^{-1} \times 10^{15}\ \text{GeV}. \tag{13}$$

We perform the numerical analysis as before using the RG equations above and obtain the scale associated with the Landau pole for different values of M given a specific mass of the light neutrino states in the eV range. In Fig. 2, we show numerical results concerning the relation between the Majorana mass and the Λ scale for different values of the neutrino mass. The perturbative nature of our analysis does not allow us to extrapolate to infinite coupling values, so we instead take the naive estimate of the Λ scale to be defined by $y_\nu(\Lambda) = 30$. We stress that this analysis is meant to provide a demonstration of principles rather than high precision results.

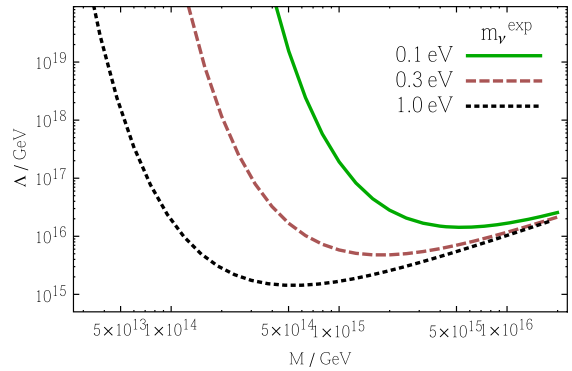

FIG. 2 (color online). Numerical results displaying the relation between the Majorana mass and the scale associated with the Landau pole for the neutrino HY coupling for different values of the neutrino mass.

Two distinct behaviors are exhibited in Fig. 2: For smaller values of the Majorana mass, the scale Λ is very sensitive to the choice of neutrino and Majorana mass. This is due to the fact that $y_\nu(m_t)$ is quite small for these values, and more RG time is needed to run to the Landau pole. For larger values of the masses, $y_\nu(m_t)$ also grows large in accordance with (6), and the Landau pole is shifted closer to the scale where the neutrino coupling becomes active in the RG equations.

IV. SCALAR SECTOR

In the minimal version of a single composite Higgs boson, the physical Higgs mass prediction is larger than the observed ~125 GeV. The Higgs mass is controlled by the electroweak VEV v_{weak} and the quartic coupling. The Higgs compositeness conditions predict a Landau pole for the quartic scalar coupling at the compositeness scale Λ [5]. However, the quartic coupling constant in the Standard Model is to be too low to match these conditions, and indeed, appears to decrease with scale potentially, becoming negative at $\sim 10^{12}$ GeV [16].

To achieve compositeness of the Higgs boson, we employ a simple modification by which the observed Higgs-quartic coupling λ becomes only a low energy effective coupling, while the true quartic coupling λ_H is larger and can have the requisite Landau pole. The actual quartic coupling needs only be about 2× the observed λ to achieve this but requires additional physics at the ~1 TeV scale.

We extend the scalar sector to include a complex singlet [13] S, and the new Higgs potential becomes

$$V = \frac{\lambda_H}{2}(H^\dagger H - v^2)^2 + \frac{\lambda_S}{2}(S^\dagger S - u^2)^2 + \lambda_{HS}(H^\dagger H - v^2)(S^\dagger S - u^2), \tag{14}$$

where we have assigned the vacuum expectation values

$$\langle H^\dagger H\rangle = v^2, \qquad \langle S^\dagger S\rangle = u^2. \tag{15}$$

The VEVs (15) are the global minima of the potential when $\lambda_H, \lambda_S > 0$, and $\lambda_H\lambda_S > \lambda_{HS}^2$.

Expanding about the minimum of Eq. (15), one finds the mass matrix for the massive scalars to be

$$\frac{\partial^2 V}{\partial\phi_i\phi_j} = 2\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} vu \\ \lambda_{HS} vu & \lambda_S u^2 \end{pmatrix},$$

where ϕ_i refers to the direction of the VEV in H and S.

The eigenvalues are

$$m_\pm^2 = \lambda_H v^2 + \lambda_S u^2 \pm \kappa,$$

where $\kappa = \sqrt{(\lambda_H v^2 - \lambda_S u^2)^2 + 4\lambda_{HS} v^2 u^2}$. In the limit where $\lambda_H v^2 \ll \lambda_S u^2$, the lightest state mostly resides within H, and the mass can be approximated by

$$m_H^2 = m_-^2 = 2\left(\lambda_H - \frac{\lambda_{HS}^2}{\lambda_S}\right)v^2 + \mathcal{O}\left(\frac{\lambda_H v^2}{\lambda_S u^2}\right). \tag{16}$$

The effective quartic coupling measured from the Higgs mass is now

$$\lambda = \lambda_H - \frac{\lambda_{HS}^2}{\lambda_S}, \tag{17}$$

which is intrinsically smaller than the coupling λ_H. Thus, the composite picture with a suitable Landau pole in λ_H is now possible.

A. Singlet scalar extension

We now analyze the RG evolution of the full theory with an eye to the Landau pole in Λ_H. Assuming S is an electroweak $SU(2)$ singlet, and $U(1)_Y$ sterile, the RG equations for the scalar sector are given by

$$\beta_{\lambda_H} = (12y_t^2 + 12\theta_M y_\nu - 3g_1^2 - 9g_2^2)\lambda_H - 12(y_t^4 + \theta_M y_\nu^4) + \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2 g_2^2 + \frac{9}{4}g_2^2 + 12\lambda_H^2 + 2\theta_u\lambda_{HS}^2, \tag{18}$$

$$\beta_{\lambda_{HS}} = \left(6y_t^2 + 12\theta_M y_\nu - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_H\right)\lambda_{HS} + 4\theta_u(\lambda_S + \lambda_{HS})\lambda_{HS}, \tag{19}$$

$$\beta_{\lambda_S} = 4\lambda_{HS}^2 + 10\theta_u\lambda_S^2, \tag{20}$$

where we have included the Heaviside function $\theta_u = \theta(\mu - u)$ to adjust for the fact that loops involving the S states are not taken into account scales below the VEV $\langle S\rangle = u$, which generates the mass for the S state.

To accommodate the composite scenario as first described in [5], both the quartic coupling and the HY coupling for the condensating fermions must diverge at a scale Λ. Furthermore, the nature with which the scalar becomes propagating at lower energy scales sets the requirement

$$\lim_{\mu\to\Lambda}\lambda_H/y_\nu^4 = 0, \tag{21}$$

and we expect the common divergence to yield

$$\lim_{\mu\to\Lambda}\lambda_H/y_\nu^2 = \mathcal{O}(1). \tag{22}$$

In Fig. 3, we demonstrate the evolution of the quartic coupling for a specific choice of initial conditions. We choose a mass for the active neutrino $m_\nu^{\rm exp} = 1$ eV, which yields a divergence of y_ν around $\Lambda = 10^{18}$ GeV, under the assumption that $M = 5\times 10^{13}$ GeV. At the scale where $y_\nu(\mu) = 10$, we then define the initial conditions for the quartic couplings $\lambda_H(\mu) = 98$, in accordance with (22), and the somewhat arbitrary choices $\lambda_{HS}(\mu) = 23$, $\lambda_S(\mu) = 1.7$. The assumed value for $u = 1$ TeV. The IR phenomenology features a large value for the Higgs-quartic coupling $\lambda_H \sim 0.7$, while the effective coupling is considerably smaller $\lambda \sim 0.28$ corresponding to a Higgs mass $m_H \sim 130$ GeV.

The RG system involves some degree of tuning to ensure the proper behavior of the two new quartic couplings. Specifically, we must tune λ_S to be small to ensure a large correction in λ as seen in (16) while λ_{HS} is also tuned, such that $\lambda_H > \lambda_{HS} > \lambda_S$ is satisfied for all RG scales in order to ensure a valid value of λ at small scales.

This model should merely serve as a proof of concept displaying the possibility that the UV behavior of the Higgs-quartic coupling can include a Landau pole. In this setup, we have looked at the simplest possible scalar

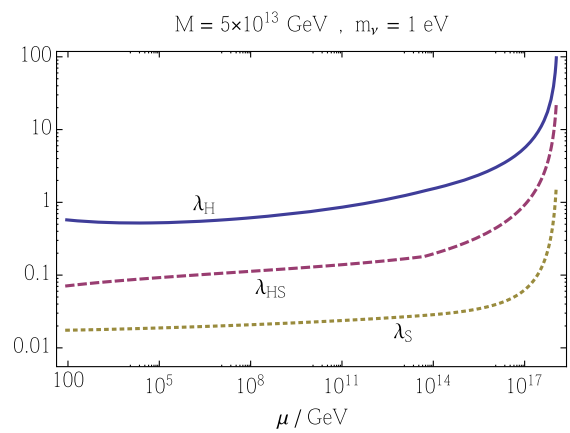

FIG. 3 (color online). The RG evolution for the quartic couplings for a specific choice of initial conditions at the UV scale. The IR phenomenology features a large quartic for the Higgs, while the effective coupling leads to a light Higgs mass $m_H \sim 130$ GeV.

extension of the Standard Model with the standard Higgs mechanism in play for both scalars. The issues of tuned scalar couplings may then be alleviated if a different mechanism for symmetry breaking or a more complex scalar sector is considered. For a large class of more general gauge Yukawa theories, a composite limit due to four-fermion interactions at high energies is easily obtainable, as shown in [12], while we will focus on the simplest alternative solutions below.

V. ALTERNATIVE SCALAR EXTENSIONS

As we have introduced a tuning between the dimensionless coupling constants of the scalar sector in addition to the usual tuning for the Higgs mass parameter, it would be beneficial to find a mechanism to stabilize the IR phenomenology towards changes in the initial UV conditions. We expect that this might be found by connecting the symmetry breaking mechanisms for the scalar sector.

In the previous example, the role of the "portal" coupling λ_{HS} was to supply a correction to the quartic Higgs coupling in the effective coupling by connecting the two scalar sectors, while the symmetry breaking mechanism is that of the standard Higgs boson for both the H and S scalars.

A. Negative portal coupling

We can expand the role of the portal coupling by letting the portal interaction communicate symmetry breaking in the dark S sector to the Standard Model. Setting $\lambda_{HS} < 0$ and assuming $\langle S \rangle \neq 0$ can trigger spontaneous symmetry breaking in the Standard Model, even if the mass term for the Higgs $m_H^2 \geq 0$, since the portal interaction will add a negative squared mass contribution for H. If the portal coupling is very small, then there can be a large hierarchy between the VEVs of S and H, and the validity of Eq. (16) is guaranteed.

The change from a positive portal coupling to a negative one can, thus, change the nature of the symmetry breaking for the Higgs particle. It allows for other values of the Higgs mass parameter, and, specifically, one can choose $m_H^2 = 0$ and still obtain a second order phase transition due to the portal interaction. The actual analysis of this alternative model is, however, almost identical to the original, since the stability constraint and mass prediction only involve λ_{HS}^2. The measured Higgs mass is still obtainable together with a Landau pole for λ_H, albeit tuning between the scalar couplings is needed.

B. Communicated Coleman-Weinberg (CW) symmetry breaking

Common to the scalar sectors discussed so far has been the feature that a mass scale has been inserted by hand into the potential either for both scalars or for one of them. This enables the generation of a vast interval of possible scalar masses but intrinsically means that these are very sensitive to the input parameters. An alternative way to generate mass scales is the dynamical one, where the mass scales arise directly from the RG evolution. We will show in the following that Landau poles in the quartic couplings, in accordance with a composite picture, may also accommodate spontaneous symmetry breaking due to the CW mechanism as demonstrated for elementary scalars in [17].

It is central to the success of this model that we now consider a dark[1] scalar doublet S gauged under a new $SU(2)_X$ group.[2] Since we want all mass scales to be generated dynamically, the potential is given as

$$V = \frac{\lambda_H}{2}(H^\dagger H)^2 + \lambda_{HS}(H^\dagger H)(S^\dagger S) + \frac{\lambda_S}{2}(S^\dagger S)^2, \quad (23)$$

where we will investigate the cases where $\lambda_{HS} < 0$. Just as before, the requirement for stability of the potential is

$$\lambda_H > 0, \qquad \lambda_S > 0, \qquad \lambda_H \lambda_S > \lambda_{HS}^2. \quad (24)$$

Spontaneous symmetry breaking then occurs dynamically in this setup via the Coleman-Weinberg mechanism when the RG evolution brings the system of coupling constants into violation of the stability conditions (24).

The driving force behind the symmetry breaking in this setup is the new gauge coupling g_x related to the $SU(2)_X$ gauge symmetry. As this coupling becomes large at some scale due to asymptotic freedom, the quartic coupling λ_S will be driven negative in the IR, due to the form of its beta function which is positive for any nonzero value of the couplings:

$$\beta_{\lambda_S} = 4\lambda_{HS}^2 + 12\lambda_S^2 + \frac{9}{4}g_x^4 - 9g_x^2\lambda_S. \quad (25)$$

Denoting by s^* the scale at which $\lambda_S = 0$ and performing the approximation close to this scale that $\lambda_S \simeq \beta_{\lambda_S} \ln(\frac{s}{s^*})$, the estimated value for the VEV of S coming from the associated Coleman-Weinberg symmetry breaking mechanism is given by

$$\langle S \rangle = u = s^* e^{-1/4}. \quad (26)$$

In return, the negative portal coupling λ_{HS} induces a VEV for H:

$$\langle H \rangle = v = u\sqrt{\frac{-\lambda_{HS}}{\lambda_H}}. \quad (27)$$

At this minimum, the mass matrix takes the form

[1]Similar models with a portal coupling to another scalar sector are often used to probe dark matter phenomenology.

[2]The critical property of the gauge group is asymptotic freedom, so any other gauge group with this property could have been used.

$$v^2 \begin{pmatrix} 2\lambda_H & -2\sqrt{-\lambda_H \lambda_{HS}} \\ -2\sqrt{-\lambda_H \lambda_{HS}} & \lambda_{HS} - \beta_{\lambda_S} \frac{\lambda_H}{\lambda_{HS}} \end{pmatrix}. \tag{28}$$

Assuming that $v^2 \ll u^2$, which is to say $\frac{-\lambda_{HS}}{\lambda_H} \ll 1$, we may expand the eigenvalues to the leading order in $\frac{\lambda_{HS}}{\lambda_H}$ and obtain

$$m_1^2 = 2\lambda^2, \qquad m_2^2 = -\frac{\beta_{\lambda_S}\lambda_H}{\lambda_{HS}} v^2, \tag{29}$$

where the indices 1 and 2 relate to the state composed of mostly H and S, respectively, and $\lambda = \lambda_H - \frac{\lambda_{HS}^2}{\beta_{\lambda_S}}$. This naturally resembles (16), and we see once again how the effective quartic coupling is smaller than the true coupling for the Higgs.

So far, the setup seems to resemble the simple one given in the previous chapter. The key difference is that while a high degree of tuning was needed for the initial conditions in the simple setup to guarantee the correct hierarchy at smaller scales, this is no longer the case, since the dynamics at these scales is controlled mainly by the evolution of the new gauge coupling.

Our probes of the parameter space for this theory will follow the lines of logic from the previous section: Assuming a certain neutrino mass m_ν and Majorana mass M, the scale of compositeness scale Λ is determined uniquely. We will then impose the constraint (22), which fixes the quartic couplings at this scale.[3] The last remaining free parameter is the new gauge coupling g_X, which will be fixed at the mass of the Z boson. The only free parameters in our analysis are, thus, the two masses associated to the neutrino sector and the value $g_x(m_Z)$.

The RG equations for the remaining quartic coupling and the new gauge coupling are given to one loop by

$$\beta_{\lambda_H} = (12y_t^2 + 12\theta_M y_\nu - 3g_1^2 - 9g_2^2)\lambda_H - 12(y_t^4 + \theta_M y_\nu^4) + \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2 g_2^2 + \frac{9}{4}g_2^2 + 12\lambda_H^2 + 2\theta_u \lambda_{HS}^2, \tag{30}$$

$$\beta_{\lambda_{HS}} = \left(6y_t^2 + 6\theta_M y_\nu - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 6\lambda_H\right)\lambda_{HS} + \left(6\lambda_S - \frac{9}{2}g_X^2\right)\lambda_{HS} + 4\lambda_{HS}^2, \tag{31}$$

$$\beta_{g_X} = -\frac{43}{6}g_X^3. \tag{32}$$

[3]We will assume that all quartic couplings are large at this scale, which would be true in a theory where all scalars are composite in the sense we have described here. This is not a necessary assumption, and it may be relaxed if one wishes to consider elementary scalar dark matter extensions.

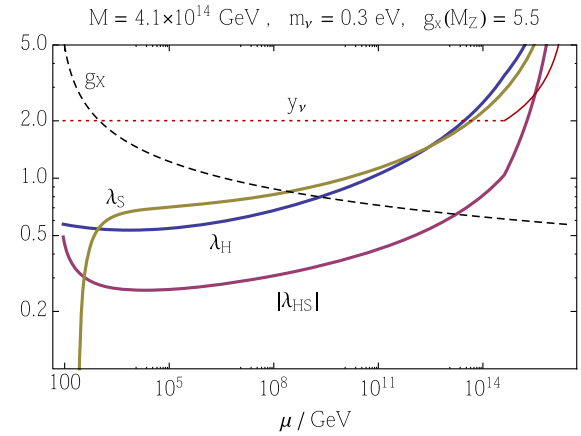

FIG. 4 (color online). The RG evolution for the quartic couplings in the communicated CW setup. Choosing the active neutrino mass and Majorana mass determines the compositeness scale, where the quartics are given values such that $\lambda_H = |\lambda_{HS}| = \lambda_S \approx y_\nu^2$ at this scale. The final assumption is that $g_X(m_Z) = 5.5$, which determines the IR behavior and symmetry breaking pattern. The evolution shown above yields $v \simeq 174$ GeV and $m_H \simeq 126$ GeV.

A numerical evaluation of the running of the couplings as described above will yield the VEVs of H and S as well as the masses of the respective eigenstates, through (26), (27), and (29), when the couplings are evaluated at the scale of symmetry breaking s^*.

A sample RG evolution yielding $v \simeq 175$ GeV and $m_H \simeq 125$ GeV is shown in Fig. 4, where the increase of the gauge coupling g_X in the IR is displayed alongside the decrease of the dark quartic λ_S, which is the source of the symmetry breaking. We warn the reader that the value for $\langle S \rangle = u \simeq 227$ GeV, such that $v^2/u^2 \sim 0.6$ such that the approximation used in (29) may be invalid, and a more complete analysis should be performed. Once again, we postpone this for other work, while aiming for a qualitative description for now.

For the RG evolution shown above, all quartic values are fixed to be equal at the compositeness scale, and the tuning between is no longer needed. Instead, having settled on a specific neutrino mass, only the Majorana mass M and $g_X(m_Z)$ require balancing in order to get the correct phenomenology in the Higgs sector. Keeping $g_X(m_Z)$ fixed while varying M with respect to the sample calculation above yields the Higgs VEV and mass depicted in Fig. 5. Interestingly, the Higgs mass seems to be stabilized around ~130 GeV for a range of different Majorana masses, while the VEV has a stronger dependence on M.

Varying $g_X(m_Z)$, one sees that in order to get values of v and m_H close to the correct values, one has to remain within the interval $g_X(m_Z) \in [5;6]$ with $M \sim 4 \times 10^{14}$ GeV for the chosen value of $m_\nu = 0.3$ eV. Thus, the tuning problems within the parameters of the theory have been greatly reduced, and the interesting region of parameter space has

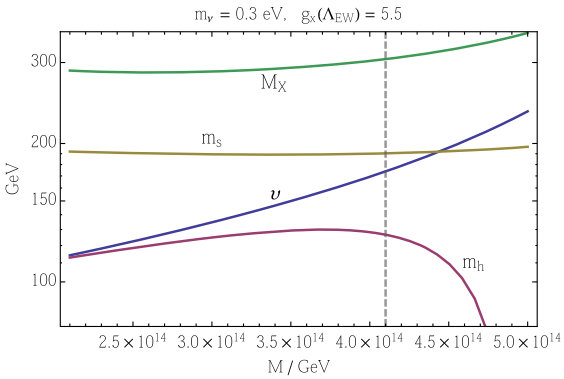

FIG. 5 (color online). Values of the Higgs VEV (v) and mass (m_h) along with the mass of the extra scalar (m_s) and the dark gauge bosons (M_X) as the Majorana mass M is varied, while the active neutrino mass $m_\nu = 0.3$ eV and $g_X(m_Z) = 5.5$. The grey dashes indicate the point where correct Higgs phenomenology is realized.

been discovered. For the higher neutrino mass $m_\nu = 1$ eV, the relevant values of M are centered at $M \sim 1.2 \times 10^{14}$ GeV, while for the lower mass $m_\nu = 0.1$ eV, realistic Higgs phenomenology requires $M \sim 1.2 \times 10^{15}$ GeV, while the value of $g_X(m_Z)$ is kept constant.

Along with the values for the Higgs mass and VEV, we obtain values for the mass of the other scalar state $m_s = m_2$ from (29) along with the mass for the dark matter candidate $M_X = g_X * u/2$, which are also shown in Fig. 5. For the choice of parameters corresponding to the values for the Higgs observables marked with a grey line, we obtain $m_S \sim 190$ GeV and $M_X \sim 300$ GeV. The predictions for these dark matter observables are fairly independent of the choice of neutrino and Majorana mass in the setup.

The phenomenology of the model presented here is by construction virtually identical to the one of its elementary counterpart, as reviewed in [17]. The main effect of imposing the composite picture is that the absolute value for the portal coupling λ_{HS} is larger in our setup.

VI. SUMMARY AND CONCLUSIONS

Our main goal in the present paper is to see how difficult it is to maintain the idea of a composite Higgs boson in the sense of Refs. [3–5], in light of modern Standard Model constraints. While the composite models fine-tune the scale v_{weak}, they are in rough concordance with the values of the Higgs boson and top quark masses as seen in nature, and offer potential predictivity.

Nature appears at face value to resist the idea of a strong, dynamical fermion condensate as the origin of the Higgs mechanism, given the apparent highly perturbative and critical behavior of the quartic coupling λ. It is, nonetheless, readily possible to construct a model that can yield the compositeness conditions at large scale $\Lambda \sim 10^{15}$–10^{19} GeV. Our main ingredient is the portal interaction that demotes λ to an effective low energy coupling, while the high energy theory is controlled by λ_H. We find a typical result that $\lambda_H \sim 2 \times \lambda$. This is sufficient to completely redefine the UV behavior of the theory. λ_H can easily have a Landau pole and satisfy the Higgs boson compositeness conditions [5]. Here we use a portal interaction near the TeV scale, which is popular in a large number of scale-invariant Higgs theories [13].

The constituents of the composite Higgs boson must couple with large Yukawa interactions to the Higgs doublet, and these couplings must also have a Landau pole at the scale Λ. The top quark in the large-N_c fermion loop approximation in the Standard Model has too weak a Higgs-Yukawa coupling to produce the Landau pole. This is easier to solve than the λ problem, and one can imagine a number of alternative theoretical fixes for it.

Presently, however, we essentially abandon the top quark as the constituent of the Higgs and have followed Martin [9] to adopt the neutrinos as the Higgs boson constituents. In the neutrino seesaw model [11], the Higgs will necessarily have Yukawa couplings to the conventional left-handed lepton doublets and right-handed neutrino singlets, $\sim y_\nu \bar{\psi}_L \nu_R \cdot H +$ H.c. These Yukawa couplings are not seen as $d = 4$ operators in the low energy theory below the scale M of the right-handed neutrino masses; rather, we observe only the $d = 5$ "Weinberg operators," $\sim y_\nu^2 (\bar{\psi}_L H)^c H \psi_L / M +$ H.c.

Above the scale of the Majorana mass M, the $d = 4$ operators materialize, and the Yukawa couplings y_ν begin to run. We assume that neutrino mixing is driven by M and assume degeneracy of three large Higgs-Yukawas, y_ν^i. Thus, our Higgs boson is engineered in a "large-$N_{\text{flavor}} = 3$-fermion bubble approximation." The y_ν have a Landau pole and can match the compositeness conditions for the Higgs. The top quark Yukawa is also pulled up to the Landau pole but remains a spectator to the new dynamics that forms the Higgs bound state.

The model has some nice features, lending a physical role to the right-handed neutrinos and demanding some new strong dynamics at Λ [e.g., a gauged $SU(3)$ neutrino flavor?]. New dynamics near the weak scale is relevant for this. As a proof of principle, there remains much to do to survey viable schemes and explore their phenomenological consequences.

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