First-principles Study of Carbon and Vacancy Structures in Niobium

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Abstract

The interstitial chemical impurities hydrogen, oxygen, nitrogen, and carbon are important for niobium metal production, and particularly for the optimization of niobium SRF technology. These atoms are present in refined sheets and can be absorbed into niobium during processing treatments, resulting in changes to the residual resistance and the performance of SRF cavities. A first-principles approach is taken to study the properties of carbon in niobium, and the results are compared and contrasted with the properties of the other interstitial impurities. The results indicate that C will likely form precipitates or atmospheres around defects rather than strongly bound complexes with other impurities. Based on the analysis of carbon and hydrogen near niobium lattice vacancies and small vacancy chains and clusters, the formation of extended carbon chains and hydrocarbons is not likely to occur. Association of carbon with hydrogen atoms can, however, occur through the strain fields created by interstitial binding of the impurity.
atoms. Calculated electronic densities of states indicate that interstitial C may have a similar effect as interstitial O on the superconducting transition temperature of Nb.

1. INTRODUCTION

Pure niobium is a primary material for superconducting technology, including superconducting radio frequency (SRF) cavities, superconducting electronic devices, and superconducting magnet strands and cables. Manufacturing activities generally strive to reduce the content of interstitial impurities H, O, N, and C to low levels because they degrade the superconducting properties, increase electron scattering, and increase the hardening rate during metal working. Interstitial oxygen, for example, reduces the superconducting transition temperature ($T_c$) by ~0.9 K per at. %, and interstitial N and C are expected to have similar effects.\textsuperscript{1} Especially high purity, < 30 ppm by mass of O, N, and C and < 5 ppm by mass H, is specified for Nb grades used in SRF cavities\textsuperscript{2} to achieve high thermal conductivity at ~2 K.

Quite a bit of attention has been paid to the elements O and H in niobium, long ago due to applications of the metal in corrosive environments and in the presence of superheated steam\textsuperscript{3} and more recently because etching and polishing of Nb with aqueous solutions has been used to remove defects introduced during the fabrication of SRF cavities\textsuperscript{4}. Niobium forms a complex layering of oxides, including a diffusion gradient of interstitial oxygen, when the clean metal is exposed to air or water. The oxide is a good barrier against hydrogen at room temperature, but the structure and composition of the oxide layers changes upon heating to temperatures as low as 100 °C due to dissolution of oxygen into the metal.\textsuperscript{5,6} Niobium absorbs copious amounts of hydrogen during
electropolishing, in conjunction with the electrochemical stripping of the oxides and the injection of vacancies to form an abundance of vacancy-hydrogen clusters. \cite{8-10} If the hydrogen is not subsequently removed by annealing in high vacuum, niobium hydride precipitates can form at low temperature, leading to a sharp decrease of the cavity quality factor $Q$ at the onset of RF field. \cite{11} Even when hydrogen is de-gassed by annealing steps, small precipitates can still form, \cite{12} leading to losses at high RF fields. Previously we analyzed how, single H atoms may be trapped by single O or N atoms to prevent hydride phase formation. \cite{13}

Interstitial N has received vigorous attention very recently, due to the apparent suppression of residual resistance at low RF field and the appearance of a field-dependent BCS residual resistance component. \cite{14} The new behavior results in an increase of the cavity $Q$ up to moderate fields, and it implies a factor-of-2 reduction in cryogenic operation cost. Since the introduction of N at high temperature is followed by a brief electropolishing to remove traces of surface nitrides, it might be possible that the remaining solid solution of N in Nb completely immobilizes H impurities, N being more effective than O at trapping H, although this would not explain the unusual physics being reported. Two theoretical models have been presented \cite{15-16} to explain the unusual surface resistance, where \cite{16} pointed out that the addition of N may be crucial for changing Nb from clean-limit to dirty-limit regimes of BCS residual resistance in the presence of current flow. Besides nitrogen, titanium, which substitutes for Nb lattice sites and does not occupy interstitial lattice sites, has been shown to yield the new SRF cavity behavior. \cite{17}

Although interstitial carbon in niobium has some similar properties to interstitial
oxygen and nitrogen,\textsuperscript{13} it has not been studied as intensively, perhaps because it has a much lower solubility.\textsuperscript{18-19} The lack of attention paid to carbon in niobium may be an important oversight, however. Recent secondary ion mass spectroscopy measurements of SRF niobium samples indicate local carbon concentrations of $10^{-1}$-$10^{-2}$ at.% depending on the specimen history and heat treatment,\textsuperscript{17} and recent Raman spectroscopy studies indicate that large amounts of carbon are present in various forms as local clusters on niobium cavity and sample surfaces.\textsuperscript{20} How the carbon collects is not known; the reports above emphasize clean handling and rule out casual environmental contributions. Dislocations seem to play a role, since material working coupled with chemical polishing and annealing is a reproducible pathway toward the results above. Some carbide precipitates exhibit enhanced superconducting gaps, and Raman signatures suggested extremely strong electron-phonon coupling.\textsuperscript{21} How different carbon configurations affect electronic and phonon properties is of interest to understand superconducting properties of technological Nb materials.

The superconducting properties of bulk niobium carbides and carbo-nitrides have been studied extensively in the past, as they present an interesting theoretical case study of electron-phonon superconductivity that is strongly dependent on composition and structure. The $T_c$ of NbC has been shown to decrease with increasing concentration of carbon vacancies\textsuperscript{22} and increase with the substitution of carbon for nitrogen.\textsuperscript{23-24} NbN and NbC are very difficult to make at the stoichiometric ratio, which might reflect on structural instabilities at the root of superconductive electron pairing. Additionally, NbC$_{1-x}$ up to $x \approx 0.25$ with disordered carbon vacancies has been shown to have a lower $T_c$ than the ordered compound at a comparable concentration.\textsuperscript{25}
In addition to carbon or carbide clusters in Nb and the bulk niobium carbide phases, the interaction of C with Nb is itself an interesting physical and metallurgical question. NbC precipitates are tremendously important to high-strength corrosion-resistant steels, because the compound remains stable in austenitic stainless steels at high temperatures and the strain fields associated with NbC nanoprecipitates act as effective dislocation pinners.26-27

In the Nb-C system, C atoms might not necessarily precipitate as intermetallic phases or remain dissolved as a solid solution, but instead they can form atmospheres around lattice defects such as metal lattice vacancies, dislocations, or grain boundaries. C may be unique among the interstitial impurities by having a tendency to form dimers or chains, as C-C dimers in metal vacancies have been predicted for bcc Fe28-30. The formation of carbon clusters around dislocations has been observed as early as the 1960’s,31-32 and the migration of carbon atoms to grain boundaries upon heat treatment has recently been observed in SRF niobium.33 Study of the interactions of carbon impurities with other defects can have a major effect on understanding the kinetic mechanisms in Nb processing.

In this study, we examine potential structures of carbon impurity and metal vacancy complexes in bcc niobium. Density functional theory (DFT) is used to compute the total energies and electronic structures of carbon atoms in bcc niobium interstitial sites, carbon atoms near niobium lattice vacancies, di-vacancies, and tri-vacancies, and several niobium carbides. The C-H interaction in niobium is also investigated. The results are discussed in terms of thermodynamic stability and electronic properties.
2. METHODS

Most of the impurity structures were modeled in a 4x4x4 unit cell of bcc niobium (128 atoms) with Nb atoms removed to create lattice vacancies, and C atoms added in interstitial or vacancy binding sites. Some of the larger defect configurations required the use of 5x5x5 unit cells. The unit cells were chosen to be computationally efficient while minimizing interactions between periodic images. They are not intended to represent a specific concentration of defects. Several tests were performed for larger unit cells and the conclusions drawn in this paper were upheld. Nb$_2$C was modeled in the $\beta$-LT structure, which has hexagonal symmetry and contains four formula units per unit cell, and NbC was modeled in the rock-salt structure with four formula units per unit cell.

All calculations were performed with the Vienna Ab initio Simulation Package (VASP), using DFT, periodic boundary conditions, and a plane wave basis set with a 400 eV kinetic energy cutoff. The generalized gradient approximation (GGA) was used with the Perdew, Burke, Ernzerhof (PBE) exchange–correlation functional, and the core electrons were described by the projector-augmented-wave (PAW) pseudopotentials. All of the structure optimizations were calculated with all of the atom and cell degrees of freedom relaxed. Forces were converged to 0.02 eV/A. K-points were selected by the Monkhorst-Pack scheme with 2x2x2 gamma-centered grids for the geometry optimizations of the defect structures in niobium, 3x10x6 gamma-centered for Nb$_2$C, and 9x9x9 for NbC. Denser grids were used for the electronic densities of states (DOS) calculations: 6x6x6 gamma-centered grids for Nb defects, 5x19x12 gamma-centered for Nb$_2$C, and 13x13x13 for NbC. The partial occupancies for the one-electron levels were
determined by the first order Methfessel–Paxton method with a smearing width of 0.2 eV for the geometry optimizations, while the tetrahedron method with Blöchl corrections was used for the DOS calculations. Spin-polarization was not used for the calculations, after check for selected configurations resulted in zero magnetic moment. The optimized lattice parameters for Nb, Nb$_2$C and NbC are within 1 % of their experimentally determined values.40-42

Several energetic properties are discussed in the results section for comparison between structures. Binding energies were calculated from various reference states as were appropriate for the discussion, and these are specified in the results section. Negative (−) binding energies indicate that the bound state is lower in energy than the unbound state or reference state. Strain energies were calculated by subtracting the energy of the niobium lattice that was deformed from defects or impurities from the energy of the ideal bcc niobium lattice. In the cases of impurity absorption, the geometry was optimized with the impurity, then the impurity was removed and a single-point energy calculation was done to determine the energy of the deformed lattice.

3. RESULTS AND DISCUSSION

Carbon impurity atoms occupy octahedral interstitial sites in bcc niobium. The binding configuration for a single carbon atom in Nb is shown in Figure 1a. As calculated previously in ref$^{13}$, absorption of carbon into the octahedral site imparts a large strain on the niobium lattice substantiated by a strain energy of 0.96 eV, a 25% expansion of the two “short” C-Nb bonds compared to what they would be in an undeformed octahedral site, and a 5% contraction of the four “long” C-Nb bonds. This indicates that
C should preferentially bind to sites near structural lattice defects such as vacancies, dislocations, grain boundaries, interfaces, and surfaces; where the lattice strain imparted by the C atom can be reduced.

Figure 1. Configurations of one carbon atom in (a) an octahedral lattice interstice in bcc niobium, (b) an interior vacancy binding site, and (c) an exterior vacancy binding site. The insets show the configurations in a larger section of the Nb lattice. Niobium atoms are represented as blue spheres, carbon atoms as small black spheres, and niobium lattice vacancies as grey cubes. Black arrows indicate the direction of relaxation of Nb atoms upon absorption of a C atom into the binding site.

3.1 Interstitial Impurity Atoms Near Single Lattice Vacancies

The calculated vacancy formation energy for niobium is 2.71 eV, which is in good agreement with the experimentally determined range 2.6–3.1 eV given in a review by Schultz and Ehrhart\(^\text{43}\) and other first-principles calculations\(^\text{44-45}\). We explored several binding sites for a C atom near a single niobium lattice vacancy and show two configurations in Figure 1: the site preferred by H and O impurity atoms (ref \(^\text{13}\) and references therein) is depicted in Figure 1b and the site that we found to be the most energetically favorable for C is depicted in Figure 1c. The binding energies of C to the
vacancy for these two configurations relative to C in an octahedral site far from a
vacancy are 0.00 eV and -0.39 eV, respectively (Table 1). The niobium lattice strain
energies for these two vacancy-binding configurations are 0.30 eV and 0.68 eV,
respectively, which compares to the strain energy of 0.96 eV for C binding in an
octahedral site far away from the vacancy. Although the reduction in strain energy is
larger for C to migrate to a site where one of the short Nb-C bonds is eliminated (Figure
1b), C exhibits a strong preference for octahedral binding and maximizing the electronic
interaction with Nb. This may be understood considering the elemental data provided in
Table 1. The groundstate electron configuration of the C atom has four empty 2p states,
resulting in the ability to form up to four covalent bonds in molecules. Carbon, however,
does not form covalent bonds with Nb atoms in the bcc lattice, as can be seen in the
charge density difference plots in Figure 2. Ionic nature of bonding is observed, as
indicated by the large region of electron density around the C atom, small regions of
decreased electron density between the C and Nb atoms and increased electron density
near the Nb atoms in the opposite direction of the C atom. Carbon has a higher
electronegativity than Nb, so it draws some of the Nb electrons onto itself, resulting in a
partially anionic state. The charge on the carbon atom in three binding configurations
shown in Figure 1 was calculated with the Bader partitioning scheme and is given in
Table 1. The most favorable configuration for carbon near a Nb lattice vacancy allows
for a greater charge on the carbon atom than does the less favorable configuration, even
though the less favorable configuration provides more physical space for the C atom to
occupy.
Table 1. Elemental data for Nb, C, N, O, and H; interstitial site preference, charge on the interstitial atom in the Nb lattice, and binding energy (B.E.) for a C, N, O, and H atom in a Nb lattice vacancy referred to the binding energy for that atom in its preferred interstitial binding site. Vacancy site b. and vacancy site c. are depicted in Figure 1b and 1c, respectively. The lowest energy is in bold, and n.a. stands for not applicable.

<table>
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<tr>
<th></th>
<th>atomic radius (Å)</th>
<th>Pauling electronegativity</th>
<th>ground state electronic configuration</th>
<th>interstitial site preference</th>
<th>charge on interstitial atom (e⁻)</th>
<th>charge on interstitial atom (e⁻)</th>
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<td>Nb</td>
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<td>1.60</td>
<td>[Kr] 4d⁵ 5s¹</td>
<td>n.a.</td>
<td>n.a.</td>
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<td>C</td>
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<td>[He] 2s² 2p²</td>
<td>octahedral</td>
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<td></td>
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<tr>
<td>N</td>
<td>0.65</td>
<td>3.04</td>
<td>[He] 2s² 2p³</td>
<td>octahedral</td>
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<td>3.44</td>
<td>[He] 2s² 2p⁴</td>
<td>octahedral</td>
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<tr>
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<td>2.20</td>
<td>1s¹</td>
<td>tetrahedral</td>
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<th>charge on interstitial atom (e⁻)</th>
<th>B.E. (eV) for vacancy site c.</th>
<th>charge on interstitial atom (e⁻)</th>
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<td>-0.56</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
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</table>

*a Reference 47
*b Reference 48-49
*c Reference 13

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Figure 2. Charge density difference plots referred to the deformed niobium lattice with carbon atom(s) removed corresponding to the configurations in Figure 1a (a), Figure 1c (b), Figure 3a (c), and Figure 3b (d). The isosurface levels are 0.027 e/ Å³. The charge density difference referred to the overlapping atomic density is also shown for the configurations in (c) and (d), with isosurface levels at 0.101 e/ Å³. Niobium atoms are represented as light blue spheres and carbon atoms as small black spheres. The blue regions of charge density represent charge depletion and the red regions represent charge enhancement.

The basic elemental data, interstitial site preferences, vacancy binding energies, and charges in the bcc niobium lattice for a single H, O, and N atom are also reported in

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Table 1. Both interstitial and vacancy site preferences are correlated with the ground state electronic configuration of the impurity atom and not with its size. H is s-shell and prefers the sites in the Nb lattice with lower electron density (e.g. tetrahedral as opposed to octahedral interstitial sites). The other impurities are p-shell and prefer the octahedral interstitial site. Their preference for binding near Nb vacancies – Figure 1b or 1c – depends on the number of 2p electrons in their ground state electron configuration: more electrons in the p-shell indicates a stronger preference for the configuration depicted in Figure 1b. H, O, and N are also more electronegative than Nb, therefore become partially anionic in the bcc niobium lattice, as discussed above for C. The ability of each impurity atom to maintain its charge when binding near a vacancy correlates with its vacancy site preference. For example, O is able to maintain approximately the same charge in either of the vacancy binding sites considered as it had in an octahedral site far from the vacancy, therefore will bind in an interior vacancy site (Figure 1b) to minimize the lattice strain. C, on the other hand, loses 0.16 e⁻ when binding in an interior binding site compared to an octahedral site far from the vacancy, whereas it maintains the same charge when binding in an exterior binding site close to the vacancy. N represents an intermediate case. The magnitude of charge on the impurity atoms is ordered as C>N>O>H, which follows the order of the atom sizes and the number of empty states in the atom’s ground state electron configuration rather than the electronegativity.

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Figure 3 shows three possible configurations of two carbon atoms near a niobium lattice vacancy. We calculate an binding energy of -0.14 eV for two C atoms in interior octahedral binding sites that are directly across from each other (Figure 3a) relative to C atoms in octahedral interstitial sites far away from each other and any lattice vacancies.
A C-C dimer with a bond length of 1.45 Å, which is in between the C-C single and double bond lengths, and a binding energy of -0.46 eV forms when two C atoms were placed in neighboring vacancy binding sites (Figure 3b). The electron density distributions for these two configurations are shown in Figure 2c and 2d, respectively. Small regions of reduced electron density between the Nb and C atoms grew when the isolevels were decreased below what is shown in the figure, indicating that no covalent bonding occurs between the Nb and C atoms. Although it appears that the C atoms as shown in the first panel in Figure 2c may contain some overlapping electron density, no indication of covalent bonding was found between any of the atoms in this configuration when the overlapping atomic densities is taken as the reference (second panel). When the overlapping atomic densities is taken as the reference for the configuration appearing to contain a C-C dimer, an increased electron density between the two C atoms is apparent (second panel of Figure 2d), depicting the covalent nature of their bonding. The regions of reduced electron density surrounding the Nb atoms in both configurations, when the overlapping atomic densities is taken as the reference, indicates the metallic nature of the Nb bonding. The energetically most favorable configuration for two C atoms near a vacancy is neither of these configurations, however; it is instead where each C atom maintains an octahedral configuration within the Nb lattice and one of the short Nb-C bonds is with a Nb atom at the corner of the cube surrounding the vacancy (Figure 3c). This configuration allow the C atoms to maintain the maximum electronic interaction with the Nb atoms, while reducing lattice strain, as discussed above for the configurations of single carbon atoms near single lattice vacancies. The binding energy for this configuration is -0.89 eV. Note, this configuration is preferred to configurations
where the C atoms are in octahedral binding sites near the vacancy and the short Nb-C bonds push the Nb atoms at the corners of the vacancy cube towards each other, which would cause repulsion between the Nb atoms; or both C atoms are bonded to the same vacancy-corner Nb, which would not provide sufficient electrons to the C atoms. The ability of niobium to accommodate C in vacancy binding sites as well as in the strain field around the vacancy suggests the possibility of Cottrell atmospheres\textsuperscript{50} around these defect sites.

**Figure 3.** Configurations of two carbon atoms near a niobium lattice vacancy where they are in interior vacancy binding sites directly across from each other (a), offset from neighboring interior vacancy binding sites such that they form a dimer (b) and in the lowest energy exterior binding site configuration (c). Niobium atoms are represented as blue spheres, carbon atoms as small black spheres, and niobium lattice vacancies as grey cubes.

Carbon is unique among the interstitial impurity atoms – H, O, N, and C – in that it can form dimers in Nb lattice defects. We attempted to place pairs of interstitial atoms – H-H, O-O, N-N, and C-C – with gas phase dimer bond lengths in configurations similar to Figure 3b in a Nb lattice vacancy, as well as with one interstitial atom in an interior
vacancy binding site and the other pointed towards the center of the vacancy. All pairs except for C-C dissociated. This may be rationalized by considering the basic elemental data shown in Table 1. Carbon can form up to four covalent bonds to fill its 2p orbitals, so if it is in a position in the Nb lattice where it is facing a lower electron density, such as next to a Nb vacancy, it will form a covalent bond with an available neighboring carbon atom. The other p-block interstitial atoms considered here have more electrons in their 2p orbitals, therefore requiring less bonding electrons from their neighbors. C-C single bonds are stronger than N-N and O-O single bonds. Additionally, N and O have higher electronegativities than carbon, enabling them to more easily extract electrons from each neighboring Nb atom. H also has a higher electronegativity than Nb, therefore preferring to bond with Nb over another H atom.

3.2 Niobium Vacancy Dimer and Trimer Configurations and Their Interaction with Carbon Impurity Atoms

Optimized geometries for calculated configurations of niobium dimer and trimer vacancy clusters are shown in Figure 4. We find the second-nearest neighbor di-vacancy to be the most stable di-vacancy configuration, in agreement with previous experimental\textsuperscript{51} and computational\textsuperscript{52} studies, with a binding energy of -0.38 eV. The binding energy for vacancy trimers is approximately equal to the sum of the binding energies of vacancy pairs, and a triangle with two sides formed from pairs of nearest-neighbor di-vacancies and the third side formed from a pair of second-nearest neighbor di-vacancies is the most stable tri-vacancy configuration. Vacancies tend to migrate to sinks, such as surfaces, when they are mobile; however if they find another vacancy or
vacancy cluster first, they may become trapped. Reported vacancy migration energies range from 0.6-1.0 eV,\textsuperscript{43,53} which is less than the 1.36-1.44 eV activation energy reported for C diffusion between octahedral interstitial sites.\textsuperscript{54-55} Therefore, it is possible to form small vacancy clusters prior to vacancy trapping by C atoms. This conclusion would still apply in the cases where interstitial O, with an activation energy for diffusion of 1.17 eV,\textsuperscript{55} or interstitial N, with an activation energy of 1.52 eV,\textsuperscript{55} are present.

\textbf{Figure 4.} Configurations of niobium lattice defects and their interaction energies (in eV) referred to single isolated vacancies. The formation energy of a single vacancy is 2.71 eV, and the lowest interaction energies for di- and tri- vacancies are in bold. Niobium atoms are represented as blue spheres and niobium lattice vacancies as grey cubes.

Additionally, vacancy dimers and linear trimers are reminiscent of edge dislocation cores, with the nearest-neighbor pair (chain) being in the [111] direction, the second-nearest neighbor pair (chain) being in the [100] direction, the third-nearest neighbor pair (chain) being in the [110] direction, and the fourth-nearest neighbor pair (chain) being in the [311] direction. Since the niobium atoms surrounding a vacancy relax towards the center of the vacancy, their Nb-Nb distances are shorter than in the
bulk, so we speculate that the strain-field around a vacancy dimer (linear trimer) may be more representative of the compressive side of an edge dislocation than the expansive side. The non-linear trimer configurations are similarly related to the lattice planes, and therefore planar defects.

We calculated several binding configurations of carbon near first- and second-nearest neighbor di-vacancies and again found that configurations in which the carbon atom maintains full octahedral coordination with niobium atoms are more stable than configurations in which one of the niobium atoms is replaced by a vacancy. In some cases the binding energy for a C atom near a di-vacancy was more favorable than for a C atom near a single vacancy because there is more space in the di-vacancy configuration for the Nb atom forming one of the short Nb-C bonds to be displaced. Examples of such a configuration for a first- and a second-nearest neighbor di-vacancy are shown in Figure 5. Both of these configurations have a binding energy of -0.49 eV, referred to a C atom in an octahedral site far away from the di-vacancy. Two configurations of C near a Nb tri-vacancy cluster are also shown in Figure 5. The configuration of C near the vacancy cluster in the (110) plane has a lower binding energy (-0.72 eV) than C near a single vacancy, whereas C near a (100) vacancy cluster has a comparable binding energy (-0.39 eV). The enthalpy of segregation to grain boundaries for C in Nb has been determined from the radio-tracer serial sectioning technique to be 49 kJ/mol, which is in-line with our calculation of multiple potential configurations of C near planar defects with binding energies of that magnitude. Bokstein and Razumovskii also measured C diffusion in niobium at 800–1173 K and report a lower activation energy and higher prefactor for grain boundary diffusion compared with bulk diffusion, which corroborates with our
finding of weaker binding energies for the configurations with less C-Nb bonds in extended lattice defects than for the configurations with octahedrally coordinated C in the strain field around the lattice defects. Carbon diffusion is accelerated by ~2 orders of magnitude in grain boundaries.56-57

Figure 5. Configurations of a C atom near Nb di- and tri-vacancy clusters that have a lower binding energy than a C atom near a single Nb vacancy (a) and a configuration of C near a Nb tri-vacancy that has a higher binding energy than a C near a single Nb vacancy (b). Niobium atoms are represented as blue spheres, carbon atoms as black spheres, and niobium lattice vacancies as grey cubes.

C-C dimers are also more stable inside of a Nb nearest-neighbor di-vacancy than inside of a single vacancy. The configuration shown in Figure 6a, for example, has a binding energy of -0.60 eV, referred to isolated interstitial C atoms far from the di-vacancy. The binding energy reduces to -0.23 eV, however, when a trimer is formed (Figure 6b). Although we do not expect C chains to form in Nb defects, because the C-C dimer bond length is already longer than a double bond and the binding energy of a trimer in a di-vacancy is already reduced compared to a the binding energy of a dimer, recent Raman spectroscopy measurements20 indicate that we should consider the possibility more carefully. Therefore, we also selected two vacancy trimer configurations.

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– the nearest-neighbor linear trimer and the lowest energy trimer cluster – to examine the possibility of forming chains of C atoms in extended Nb lattice defects. We found that a C₃ chain inside of a vacancy chain is less stable than C atoms in nearby octahedral sites around the vacancy chain (similar to Figure 1c). Inside of the vacancy cluster, configurations of a C-C dimer and a separate C were also more stable than the C₃ chain.

![Figure 6. Configurations of a C-C dimer (a) and C₃ chain (b) near a nearest neighbor Nb di-vacancy. Niobium atoms are represented as blue spheres, carbon atoms as small black spheres, and niobium lattice vacancies as grey cubes.](image)

3.3 Comparison Among Niobium Carbide Phases

The binding energies of selected configurations of carbon in niobium as well as the ordered niobium carbide phases are shown in Figure 7. Although the configurations of carbon near niobium lattice defects are favorable compared to interstitial carbon far away from lattice defects, Nb₂C has the lowest energy so will preferentially form under favorable kinetic conditions. Nb₂C has been formed in sputter-deposited thin films with compositions as low as 1.9% C. Carbon has been shown to migrate to dislocations and replace oxygen in that region during strain aging experiments at 155 °C, which may convert to carbide phases once a sufficient local concentration has been reached.
Additional ordered phases that are not included in this study are present in experimental phase diagram\textsuperscript{18} and others have been predicted by first principles calculations\textsuperscript{58} as well.

\textbf{Figure 7.} Binding energies of selected configurations of C in Nb relative to isolated C atoms in octahedral interstitial sites and niobium lattice vacancies or nearest-neighbor (NN) di-vacancies.

3.4 Density of States and Trends in Superconductivity for NbC\textsubscript{x} Systems

The C and Nb partial electronic DOS for pure Nb, interstitial C in Nb, Nb\textsubscript{2}C, and NbC are shown in Figure 8 and corresponding DOS at the Fermi level ($N_F$) for these and selected other configurations of carbon in niobium are given in Table 2. The metallic band of pure bcc Nb in Figure 8 is ~9 eV wide with a peak at the Fermi level. As C is added to the octahedral interstitial sites in Nb, C 2p states appear around -5 eV and bind with Nb 5s electrons; this lowers $N_F$. Although $N_F$ is not clearly correlated with $T_c$. 

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because it has both a direct proportionality and an indirect effect on the log-mean phonon frequency.\textsuperscript{59} Koch, et al.\textsuperscript{60} has studied the Nb - interstitial O system with a variety of experimental techniques and showed that the superconducting transition temperature, $N_F$, and electron-phonon coupling constant decrease with increasing oxygen concentration. Additionally, Dynes and Varma analyzed data for a number of transition metal alloys, including Nb-O, and concluded that the electron-phonon coupling parameter varies linearly with $N_F$.\textsuperscript{61} We calculate a similar drop-off of $N_F$ for interstitial C in Nb as Koch, et al. report for interstitial O in Nb, and show in ref\textsuperscript{13} that interstitial C deforms the Nb lattice is a similar way as O, but to a larger extent. Therefore, we predict a similar effect of interstitial C on niobium superconductivity as interstitial O imparts. Collection of carbon atoms around niobium vacancy-type defects; however, mitigates this effect (Table 2), as the portion of the effect due to lattice deformation is reduced.

As shown in Figure 8, the electronic structure of the niobium carbide alloy changes considerably as the carbon content increases and the crystal structure of the Nb lattice changes from bcc. The rock-salt structure of NbC has a lower $N_F$ (Table 2) than Nb, but a higher $T_c$. Density functional theory calculations of the Fermi surface of NbC show that it contains nesting features that enhance the electron-phonon coupling.\textsuperscript{62} The DOS for Nb$_2$C is intermediate in appearance between Nb with interstitial C and NbC, with strong overlap between the Nb and C states below -3 eV, and mostly Nb states above. $N_F$ is lower for Nb$_2$C than for Nb and NbC. Therefore, we suggest that $T_c$ for Nb$_2$C is likely less than for Nb or NbC.
Figure 8. Partial electronic DOS of selected configurations of carbon in niobium, plotted with the Fermi energy at 0 eV.
Table 2. DOS at the Fermi level ($N_f$) for selected configurations of C in Nb.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$N_f$ (# / eV-atom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb</td>
<td>1.53</td>
</tr>
<tr>
<td>Nb with 0.78% vacancies</td>
<td>1.51</td>
</tr>
<tr>
<td>Nb with 1.56% vacancies as nearest-neighbor di-vacancies</td>
<td>1.64</td>
</tr>
<tr>
<td>Nb with 0.78% C in octahedral sites</td>
<td>1.37</td>
</tr>
<tr>
<td>Nb with single C atoms near single vacancies (0.78%)</td>
<td>1.47</td>
</tr>
<tr>
<td>Nb with 1.56% C as 2 atoms near single vacancies</td>
<td>1.43</td>
</tr>
<tr>
<td>Nb with 1.56% C as dimers near vacancies</td>
<td>1.40</td>
</tr>
<tr>
<td>Nb$_2$C</td>
<td>0.21</td>
</tr>
<tr>
<td>NbC</td>
<td>0.38</td>
</tr>
</tbody>
</table>

3.5 Interaction Between Carbon and Hydrogen in the Niobium Lattice

Next we calculate formation energies and structures of possible C-H complexes from C and H impurities in bcc Nb. We considered a C-H dimer inside of a single niobium vacancy and found that while it is stable, is less favorable by 0.20 eV compared to isolated interstitial C and H atoms far away from the vacancy. We also considered simple hydrocarbons C$_3$H$_8$, C$_3$H$_5$, and C$_3$H$_2$ in a chain of second-nearest neighbor Nb lattice vacancies, and in all cases the H atoms spontaneously dissociated from the C atoms during geometry optimization.

Interstitial carbon atoms may, however, serve as trapping centers for interstitial hydrogen atoms. We calculated the binding energy for an H atom in a tetrahedral interstitial site 2.8-5 Å away from a C atom in an octahedral site and show the results in Figure 9 compared to the profiles for H trapping by O and N. All three profiles are similar, and the trapping energy is slightly greater for C and N than for O. The configuration for the lowest energy C (N, O) – H pair is also shown in Figure 9. The C
atom shares one bonding Nb atom with the H atom, and this Nb atom is displaced in the same direction resulting from its interaction with the C atom and the H atom.

![Figure 9](image)

**Figure 9.** Plot of binding energy, referred to an H atom in a Nb tetrahedral interstitial site and a C, N, or O atom in an infinitely far away octahedral interstitial site, versus the distance between a C, N, or O atom in an octahedral site and an H atom in a nearby tetrahedral site. The O data series was previously published in ref13. The highest-energy configuration has a distance between the atoms of less than 2.5 Å and is not shown on the plot. An excerpt of lowest energy configuration is also shown; Nb atoms are represented as blue spheres, the C, N, or O is a small black sphere and the H is a small dark blue sphere.

4. Conclusions

First-principles calculations of carbon in Nb and its interactions with Nb vacancy-type defects, other C atoms, and H atoms show that C is likely to form precipitates or atmospheres around the vacancy-type defects rather than strongly bound complexes with
other impurities. While Nb₂C is calculated to be the most stable phase for C in Nb, carbon-vacancy complexes can form stable states, which are competitive with NbC and may be favored kinetically in Nb processing. These are likely to be carbon atoms in octahedral interstitial positions near vacancies or extended lattice defects.

Carbon is unique among the interstitial impurities – H, O, N, and C – in that it can form dimers inside of Nb lattice vacancies. Extended chains of carbon atoms and hydrocarbon inclusions, however, don’t seem to be favorable. Association of H with C atoms does occur in niobium through the strain field created by binding in interstitial sites, but a C-H bond does not form.

The electronic DOS at the Fermi level decreases for interstitial C, similar to interstitial O, indicating that a similar decrease in T_c may follow. Binding of C near lattice defects somewhat mitigates this effect.

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