

# THERMAL AND LORENTZ FORCE ANALYSIS OF BERYLLIUM WINDOWS FOR A RECTILINEAR MUON COOLING CHANNEL\*

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## Abstract

Reduction of the 6-dimensional phase-space of a muon beam by several orders of magnitude is a key requirement for a Muon Collider. Recently, a 12-stage rectilinear ionization cooling channel has been proposed to achieve that goal. The channel consists of a series of low frequency (325 MHz-650 MHz) normal conducting pillbox cavities, which are enclosed with thin beryllium windows (foils) to increase shunt impedance and give a higher field on-axis for a given amount of power. These windows are subject to ohmic heating from RF currents and Lorentz force from the EM field in the cavity, both of which will produce out of the plane displacements that can detune the cavity frequency. In this study, using the TEM3P code, we report on a detailed thermal and mechanical analysis for the actual Be windows used on a 325 MHz cavity in a vacuum ionization cooling rectilinear channel for a Muon Collider.

## INTRODUCTION

Recently, interest has increased in the possibility of using muon in high-energy physics as the colliding particles in  $\mu^+ - \mu^-$  colliders [1]. The liability of muons is that they are created in a diffuse phase-space. As a result, the volume of the 6-Dimensional (6D) phase-space must be rapidly reduced via ionization cooling [2] by several orders of magnitude in order to be able to further accelerate it.

To reduce the transverse emittance, the beam is strongly focused with high magnetic fields and subsequently sent through an absorber material to reduce the overall momentum. The beam regains longitudinal momentum in RF cavities, resulting in an overall loss in transverse emittance. Longitudinal emittance reduction is achieved by shaping the absorbers into wedges and providing a bending magnetic field, generating a dispersion such that particles with higher energy are sent through more material. Recently, a 12-stage tapered rectilinear scheme for cooling a muon beam sufficiently for use in a Muon Collider has been designed and simulated [3].

Thin Be windows may be utilized in the cooling channel to increase shunt impedance of the closed-cell rf cavities. In this study, using the TEM3P code, we report on a detailed thermal and mechanical analysis for the actual Be windows used on the first stage (stage A1), which is the most challenging due to its large beam aperture (30 cm).

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## RF CAVITY WITH BERYLLIUM WINDOW

The cavities in the rectilinear Muon Cooling channel are of pillbox type. The Be windows enclose the cavity on both sides to increase the cavity shunt impedance significantly.

The Be window needs to be thin enough to be almost transparent to the muon beam. However, the thinner the window, the poorer its thermal conduction. Besides, there is no extra cooling on the window with all the heat transferred out by thermal conduction. A “step” window design is proposed as a compromise between emittance dilution and the thermal heating. The window parameters of the first four stages are shown in Table 1.

Table 1: Window Parameters for the First 4 Stages of the Cooling Channel

Stg	f (MHz)	rWin (cm)	rStep (mm)	t0 (mm)	t1 (mm)
A1	325	30.0	16.0	0.300	1.40
A2	325	25.0	15.0	0.200	0.80
A3	650	19.0	10.0	0.200	0.60
A4	650	13.2	11.4	0.125	0.38

In this paper, we will focus on the cavity for A1 stage, which has the largest aperture, thus most challenging for the thermal heating. A schematic drawing of A1 stage lattice cell and its RF cavity model are shown in Figure 1.

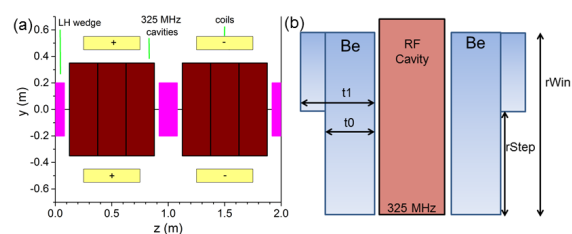


Figure 1: (a) Schematic drawing of Stage A1, and (b) RF cavity model.

Like MICE cavity, the Be windows are designed into a curved profile to control the direction of thermal expansion and reduce the thermal stress. Based on the parameters in Table 1 and the window curvature from MICE cavity, we build a 3D cavity model, as shown in Figure 2, for further study.

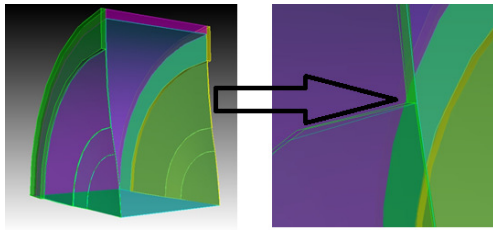


Figure 2: 3D model of 325 MHz cavity with stepped Be window. Left: a quarter of the whole cavity; right: the zoom-in view at window step.

## TEM3P SIMULATION OF THE BERYLLIUM WINDOWS

The thermal and Lorentz force simulation of Be window is carried out by TEM3P [4]. In the first step, the EM field of the accelerating mode is solved by Omega3P [4]. Then EM field results are loaded into TEM3P for temperature and mechanical stress analysis. The E and B field in the cavity are shown in Figure 3.

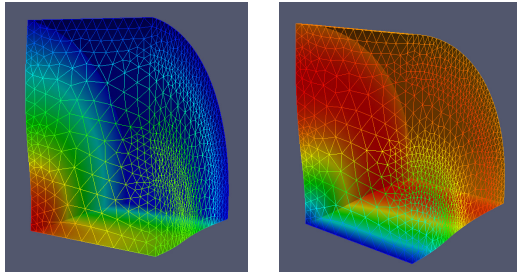


Figure 3: The RF field in the cavity solved by Omega3P. Left: E field; right: B field.

### Cavity Operating Condition

The nominal operation of the Stage A1 cavity is of 21.6 MV/m peak gradient and 0.093% duty factor. For a better electric and thermal conductivity, the cavity will be cooled by liquid nitrogen instead of water. Thus the copper torus is kept at 80 K in the simulation.

### Beryllium Material Properties

To calculate the thermal and mechanic behavior of Be window, one needs accurate material properties of Be, some of which strongly depend on the Be purity, composition, structure (single crystal or amorphous), temperature, etc. The Be properties we apply in this simulation are as follows:

- Temperature dependent electric resistivity:

$$\rho(T) = \rho_0 \sqrt{R_{12}^{-2} + R_3^{-2}},$$

where  $R_{12} = (R_1^{-5} + R_2^{-5})^{0.2}$ ,  $R_1 = 0.035 \times (T/100)^{-3.8}$ ,  $R_2 = 1.005 \times (T/273)^{-2.5}$ ,  $\rho_0 = 5.8 \Omega \cdot m$  and  $R_3$  is the Residual Resistance Ratio (RRR) merit, which we choose to be 6 as a conservative estimation;

- Temperature dependent thermal conductivity:

$$k(T) = k_0 \frac{T}{273} \frac{\rho_0}{\rho(T)},$$

where  $k_0 = 200 \text{ W/m/K}$ ;

- Mechanical properties are approximated as temperature independent constants: coefficient of thermal expansion  $8.0 \times 10^{-6}/\text{K}$ , Young's module  $3.1 \times 10^{11} \text{ Pa}$  and Poisson's ratio 0.06.

The surface resistance and thermal conductivity of Be and Cu applied in the simulation are plotted in Figure 4:

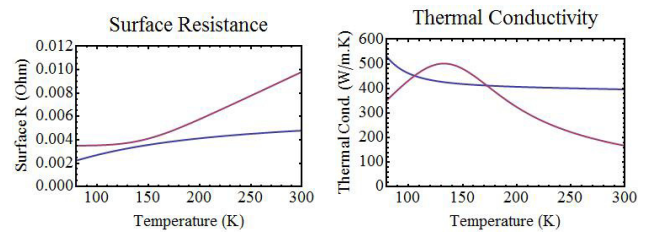


Figure 4: Surface resistance and thermal conductivity of beryllium (magenta) and copper (blue) from 80 K to 300 K.

### Thermal Heating on Be Window

With a peak field of 21.6 MV/m and duty factor 0.093%, we have calculated the temperature rise (Fig 5), the thermal deformation (Fig 6) and Von-Mises stress (Fig 7) on the Be windows. The simulation shows that:

1. The highest temperature rise is about 70 K, located at the center of the curved-in window.
2. Under the RF heating, both windows expand in the same direction. However, the curved-in window heats up more and expand further than the curved-out window, thus the cavity vacuum volume changes and the resonant frequency shifts down  $\delta f = 263 \text{ kHz}$ . With quality factor estimated by OMEGA3P to be about 62000, the cavity bandwidth (BW) under critical coupling is

$$BW = \frac{f_0}{Q_L} = \frac{2f_0}{Q_0} \approx 10 \text{ kHz},$$

which is much smaller than the frequency drift caused by thermal deformation.

3. The maximum thermal stress is along the edge of the window, which is about 84 MPa. It is only a quarter of the beryllium yield strength of 345 MPa. There is a local maximum stress at the thickness step, which can be alleviated by tapering.

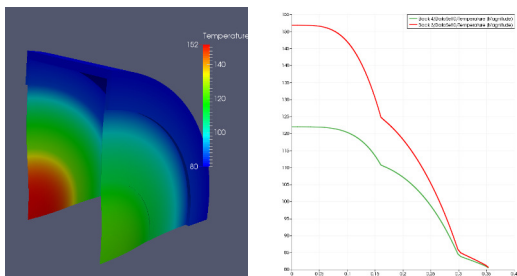


Figure 5: Cavity temperature result. Left: temperature on the cavity surface; Right: temperature along the radius on curved-in (red) and curved-out (green) window.

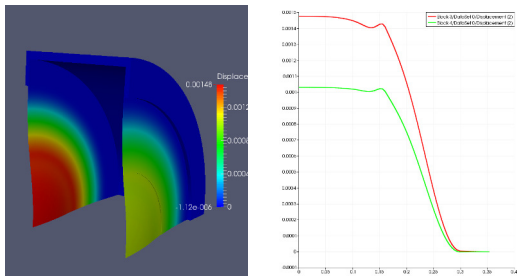


Figure 6: Cavity deformation along beam direction, while deformation along radian direction is negligible. Left: Von-Mises stress on the cavity surface; Right: Von-Mises stress along the radius on curved-in (red) and curved-out (green) window.

*Lorentz Force on Beryllium Window*

The Lorentz force from the EM field in the cavity exerts an equivalent press on the cavity inner surface:

$$P = \frac{1}{4}(\mu_0 H^2 - \epsilon_0 E^2),$$

where position sign means the outwards direction. While it is usually negligible for the normal conducting cavity, due to the thin window and the narrow bandwidth, it might become a concern for this cavity. Following the similar procedure of previous thermal analysis, the frequency shift due to Lorentz force deformation is calculated to be 10 KHz, comparable to the cavity BW.

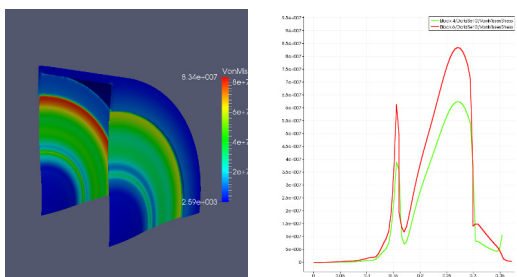


Figure 7: Cavity Von-Mises stress results. Left: Von-Mises stress on the cavity surface; Right: Von-Mises stress along the radius on curved-in (red) and curved-out (green) window.

The Lorentz force has the same time pattern of the RF pulse, with a duration of about 1 ms and a repetition rate at 1Hz. The lowest mechanic mode of each Be window calculate by TEM3P, as shown in Figure 8, are of 22.502 KHz (curved-out) and 23.087 KHz (curved-in). Thus the Lorentz force at 1 Hz is unlikely to cause the resonant vibration of Be windows.

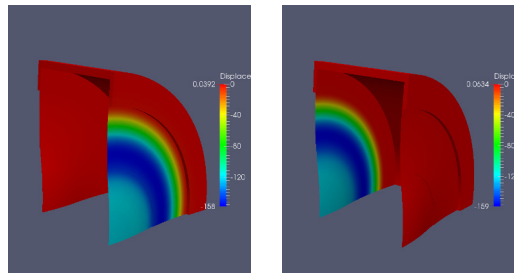


Figure 8: The lowest mechanic mode of Be windows. Left: curved out window at 22.502 KHz; right: curved in window at 23.087 KHz.

**CONCLUSION**

In this paper, we have presented the thermal and mechanical analysis for the Be windows of VCC A1 stage RF cavity. Based on an conservative estimation of Be properties, under the nominal operation, the thermal expansion of the Be window will cause a frequency shift tens of times of the cavity bandwidth. The thermal stress is within the manageable level. The Lorentz might cause a frequency shift comparable to the cavity bandwidth. It won't cause the resonant vibration of the Be window.

The properties of the Be window of this cavity are very similar to the MICE cavity. With the RF commissioning at Fermilab MTA going on, we will learn more about their properties.

**ACKNOWLEDGMENT**

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