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PROCEEDINGS OF SCIENCE

Decay constants f_B and f_{B_s} from HISQ simulations

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We give a progress report on a project aimed at a high-precision calculation of the decay constants f_B and f_{B_s} from simulations with HISQ heavy and light valence and sea quarks. Calculations are carried out with several heavy valence-quark masses on ensembles with 2+1+1 flavors of HISQ sea quarks at five lattice spacings and several light sea-quark mass ratios m_{ud}/m_s, including approximately physical sea-quark masses. This range of parameters provides excellent control of the continuum limit and of heavy-quark discretization errors. We present a preliminary error budget with projected uncertainties of 2.2 MeV and 1.5 MeV for f_B and f_{B_s}, respectively.

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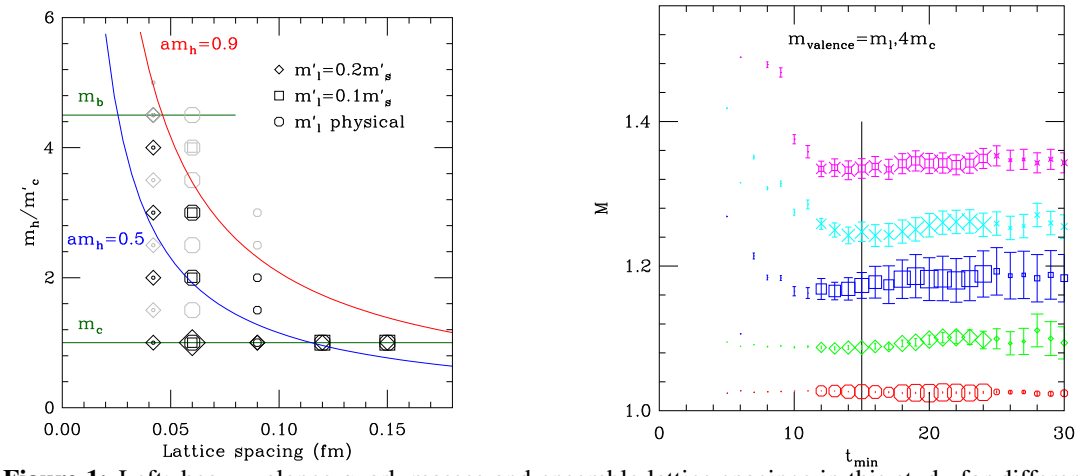

Figure 1: Left: heavy valence-quark masses and ensemble lattice spacings in this study for different light to strange sea-quark-mass ratios m'_l/m'_s. The symbol radius is proportional to the data sample size. The red line indicates the cut $am_h = 0.9$; the blue, $am_h = 0.5$. Faint symbols indicate omitted data. Right: Spectrum on the 0.042 fm physical-mass ensemble for a heavy-light meson with valence masses $4m'_c, m'_l$ *vs.* $t_{\rm min}$, the minimum fit-range distance, with $m'_l = m'_u = m'_d$. The symbol size is proportional to the p value. The inset magnifies the ground state. Priors have been introduced to constrain the excited-state contributions. The vertical line shows our choice for $t_{\rm min}$.

1. Introduction

We are searching for new physics by looking for discrepancies between precise calculations within the Standard Model and high-precision experimental measurements. Here we provide a progress report on an effort to improve Standard-Model calculations of the leptonic decay constants of the B and B_s mesons, f_B and f_{B_s}. These, in turn, provide accurate predictions for the charged-current leptonic decay $B \to \tau\nu$, for the FCNC decays $B_s \to \mu^+\mu^-$, and for neutral B mixing. Such calculations are also needed to probe the $V-A$ structure of the Wub vertex and to help address the present tension between inclusive and exclusive determinations of the CKM matrix element $|V_{ub}|$.

To achieve high precision, we employ highly improved staggered (HISQ) quarks [1, 2, 3] with masses heavier than the charm quark mass and adopt the HPQCD strategy of extrapolating to the b-quark mass [4]. The extrapolation is guided by heavy-quark effective theory, which we incorporate in our chiral/continuum analysis.

We reduce errors from previous calculations for three reasons. The staggered-fermion local pseudoscalar density does not require renormalization, one of the significant sources of systematic error with other heavy-quark formalisms [5, 6, 7, 8, 9]. Compared with HPQCD, our ensembles employ HISQ instead of asqtad sea quarks. The HISQ ensembles [10] have a large statistical sample size, and include lattices with nearly physical-mass light quarks, which reduces significantly chiral extrapolation errors.

This work extends our earlier calculation of f_D, f_{D_s}, which also used the HISQ formalism for charm quarks [11]. Here we include valence fermions heavier than charm.

The range of valence heavy-quark masses and lattice spacings for the HISQ ensembles in this study is indicated in the left panel of Fig. 1. HISQ quarks show large lattice artifacts in the meson dispersion relations when $am_h > 0.9$. Discretization effects in the heavy-light meson mass and

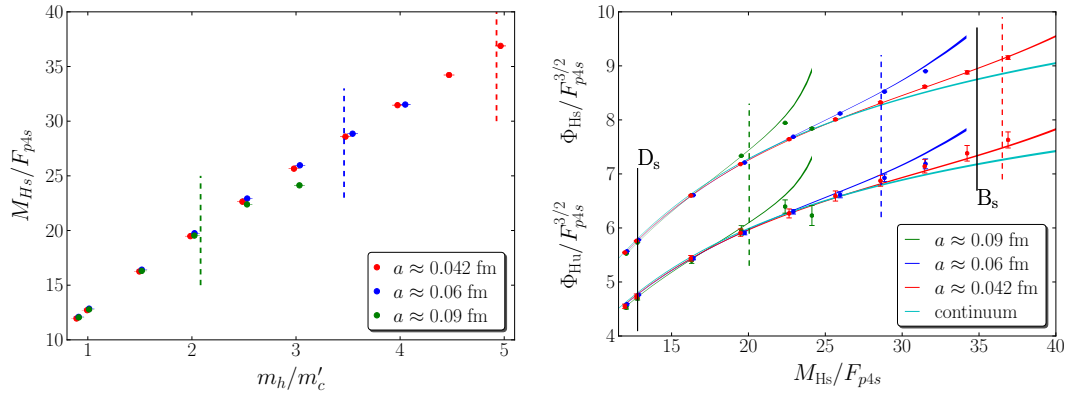

Figure 2: Illustration of heavy-quark discretization effects. Left: heavy-strange meson mass in units of $F_{p4s} \approx 154$ MeV *vs.* the ratio of the heavy-quark mass to the simulation charm quark mass m_h/m'_c for three lattice spacings at physical sea-quark masses. The small differences in the heavy-strange mass for different lattice spacings gives an indication of the heavy-quark discretization error. Right: decay constants Φ_{H_q} (defined below) plotted in units of F_{p4s} *vs.* the heavy-strange meson mass for three lattice spacings and the continuum extrapolation. In both cases the dashed vertical lines indicate the cut $am_h = 0.9$ for each lattice spacing. Such a pattern of deviations was first reported by HPQCD [4].

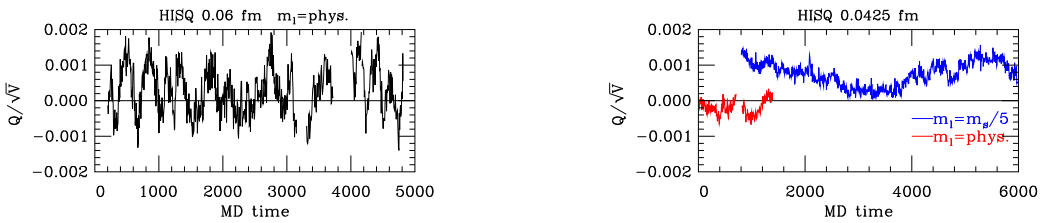

Figure 3: Topological charge *vs.* molecular dynamics time unit. Only fluctuations in Q^2 are relevant for the (parity-conserving) matrix elements here.

decay constant, shown in Fig. 2, also increase dramatically for larger am_h. So we drop data with $am_h > 0.9$ and parameterize the heavy-quark mass dependence in our fits at smaller values with the help of heavy-quark effective theory. Because of strong correlations, we also discard some m_h/m'_c points. (Primes on the masses indicate the simulation mass values.)

An important question is whether at the smallest lattice spacings our ensembles are ergodic with respect to the topological charge. In Fig. 3 we show the evolution of topological charge for our 0.06 fm and 0.042 fm ensembles. It is clear that tunneling has slowed at the smaller lattice spacing. Tunneling is less frequent when the light sea quark is unphysically heavy. We are currently investigating the implications for various observables.

2. Correlator Analysis

We compute the decay constant from the asymptotic behavior of the heavy-light pseudoscalar density-density correlator in the usual way:

$$C_{\mathrm{pt-pt}}(t) \to A_{\mathrm{pt-pt}} \exp(-M_{H_q} t) \tag{2.1}$$

where M_{H_q} is the heavy-light meson mass and q labels the light valence quark. The decay constant

is then obtained without the need for matching factors from

$$f_{H_q} = (m_h + m_q)\sqrt{\frac{3VA_{\mathrm{pt-pt}}}{2M_{H_q}^3}} \tag{2.2}$$

The correlators are calculated with random and Coulomb-gauge wall sources and point sinks. We compared fits to 3+2 states (3 nonoscillating and 2 oscillating) and to 2+1 states and conclude that 3+2 states incorporate adequately the effects of excited states. We can get a sense of the stability of the 3+2 fit from a plot of the resulting spectrum of ground and excited states as a function of t_{min}, shown in the right panel of Fig. 1 for the 0.042 fm, physical-mass ensemble.

3. Chiral-continuum Extrapolation

We use heavy-quark effective theory (HQET) to model the heavy-quark mass dependence of f_{H_q}. We start by relating the QCD current to the HQET current:

$$J^{\mathrm{QCD}} \equiv C_i(m_Q/\mu, \alpha_s(\mu)) J_i^{\mathrm{HQET}}(\mu) + \mathscr{O}(1/m_Q)\,, \tag{3.1}$$

where the Wilson coefficient has the perturbative expression

$$C_i(m_Q/\mu, \alpha_s(\mu)) = \left[\frac{\alpha_S(m_Q)}{\alpha_S(\mu)}\right]^{-6/25} [1 + \mathscr{O}(\alpha_S(m_Q), \alpha_S(\mu))]\,. \tag{3.2}$$

So, with M_{H_s} at physical sea-quark masses and physical valence s-quark mass, we remove the scaling factors from the decay constant and write it in terms of Φ_{H_q}:

$$\begin{aligned} f_{H_q}\sqrt{M_{H_q}} &= \Phi_{H_q} = [\alpha_S(M_{H_s})]^{-6/25}\,[1 + \mathscr{O}(\alpha_S(M_{H_s}))]\;\tilde{\Phi}_{H_q}\,, \\ \tilde{\Phi}_{H_q} &= \tilde{\Phi}_0\left(1 + k_1\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} + k_2\frac{\Lambda_{\mathrm{HQET}}^2}{M_{H_s}^2} + k_1'\frac{\Lambda_{\mathrm{HQET}}}{m_c'}\right)(1 + \text{log/analytic terms})\,. \end{aligned} \tag{3.3}$$

We then introduce the heavy-quark discretization correction for the low energy constant Φ_0 following HPQCD [4]

$$\tilde{\Phi}_0 \to \tilde{\Phi}_0[1 + c_1\alpha_S(a\Lambda)^2 + c_2(a\Lambda)^4 + c_3\alpha_S(am_h)^2 + c_4(am_h)^4 + \ldots]\,. \tag{3.4}$$

Finally, for the chiral logarithms and analytic terms in Eq. (3.3), we use heavy-meson rooted all-staggered χPT from Bernard and Komijani [12], which parameterizes the light-quark mass dependence and incorporates taste-breaking and generic discretization errors from the light-quark and gluon actions. Because of space limitations we give only a schematic representation:

$$\begin{aligned} (1 + \text{log/analytic terms}) &= 1 + \text{NLO staggered chiral logarithms} \\ &\quad + \text{NLO + NNLO + NNNLO analytic terms} \\ \text{NLO analytic terms} &= \left(L_s + L_{s,M}\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}}\right)(2m_l + m_s) + \left(L_q + L_{q,M}\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}}\right)(m_q) + c_{a,\Xi}a^2\,. \end{aligned} \tag{3.5}$$

Note that the coefficients of the NLO analytic terms include a heavy-quark mass correction. A heavy-quark mass dependence also appears implicitly through the $M_{H_q^*} - M_{H_q}$ hyperfine splitting and heavy-light flavor splittings.

Altogether our HQET-chiral continuum-chiral function, Eq. (3.3) with Eqs. (3.4) and (3.5) including NLO, NNLO, and NNNLO terms has 26 parameters. We use this parameterization for our central fit to our values of Φ_{H_q}. An example of the resulting fit is shown in the left panel of Fig. 4. From the continuum extrapolation, we obtain values of the decay constants as a function of M_{H_s} and the light valence-quark mass.

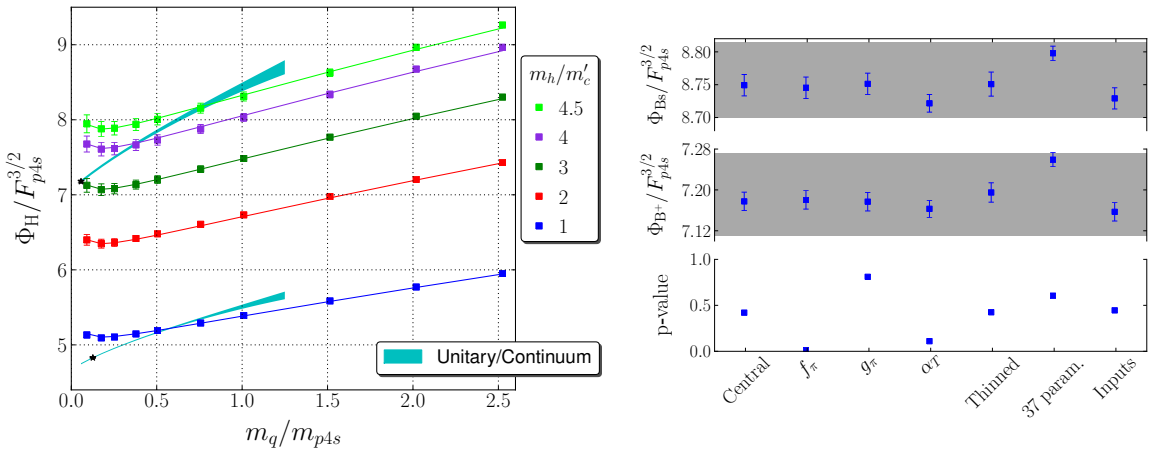

Figure 4: Left: sample chiral-continuum fit result for the $a \approx 0.042$ fm, $m_l'/m_s' = 0.2$ data. The unitary/continuum result at physical s-, c-, and b-quark masses is shown in cyan. The stars indicate the fully physical results for the B^+ and D^+ mesons. Right: stability plot showing the sensitivity to model choices as described in the text. The error band shows the error taken for systematics associated with the chiral-continuum fit.

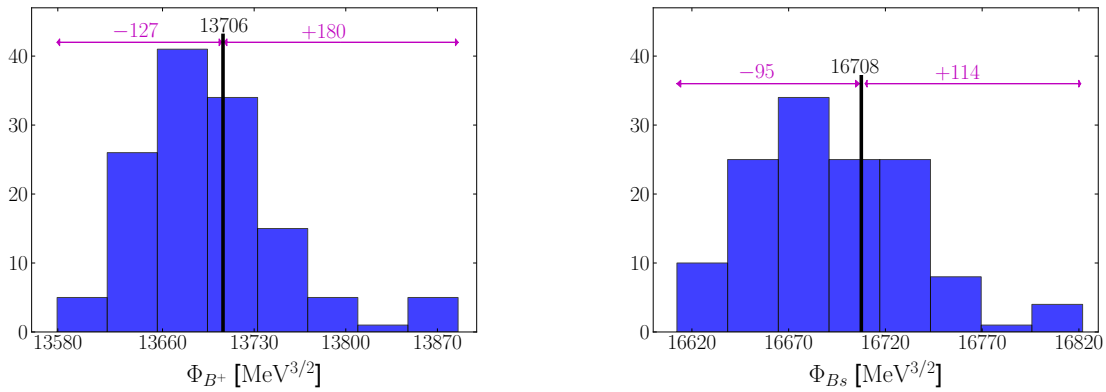

Figure 5: Histogram of central values resulting from varying choices of the fit function and input data. We estimate the associated systematic error from the extrema.

4. Error Budget

To estimate the systematic error in our methodology arising from the variety of possible analysis choices and to test the stability of our results, we tried the following alternatives. (1) Replaced f_K with f_π in the chiral terms; (2) Let the $B^* - B - \pi$ coupling g_π that enters the chiral logarithms float with a prior 0.53(8). (3) Replaced α_S with a value determined empirically from the measured taste splittings. (4) Dropped half the light quark masses and refit; (5) Increased the number of

Table 1: Preliminary error budget. Note that the first error, estimated from the histogram in Fig. 5, includes the chiral-continuum fit and g_π errors, the light- and heavy-quark discretization errors, and the error arising from excited state contamination.

	f_{B^+} (MeV)	f_{B_s} (MeV)
χPT$\oplus g_\pi \oplus$ HQ-LQ disc. $\oplus$ 2-pt fit	2.1	1.4
Statistics	0.6	0.5
Finite volume	0.3	0.2
Electromagnetic effects	0.1	0.1
Exp. f_π	0.3	0.4
Total	2.2	1.5

parameters to 37 by adding 11 higher-order heavy-quark terms and do not make any SVD cuts in the fit. (6) Used an alternative determination of the F_{p4s} scale and quark-mass ratios.

The effect of these variations on the decay constant is plotted in the right panel of Fig. 4. Selecting 132 combinations of such variations that yielded $p > 0.05$ gives us the histograms shown in Fig. 5 for Φ_B and Φ_{B_s}. We take the *extrema* of the histograms as the systematic error, noting that the majority of fit variations lie well within this conservative choice. The result is shown in the preliminary error budget in Table 1. The remaining sub-dominant systematic uncertainties from finite-volume effects, electromagnetic effects, and the experimental error in f_π used to set the absolute lattice scale are estimated following Ref. [11]. Adding these errors in quadrature gives the estimated total uncertainty shown there.

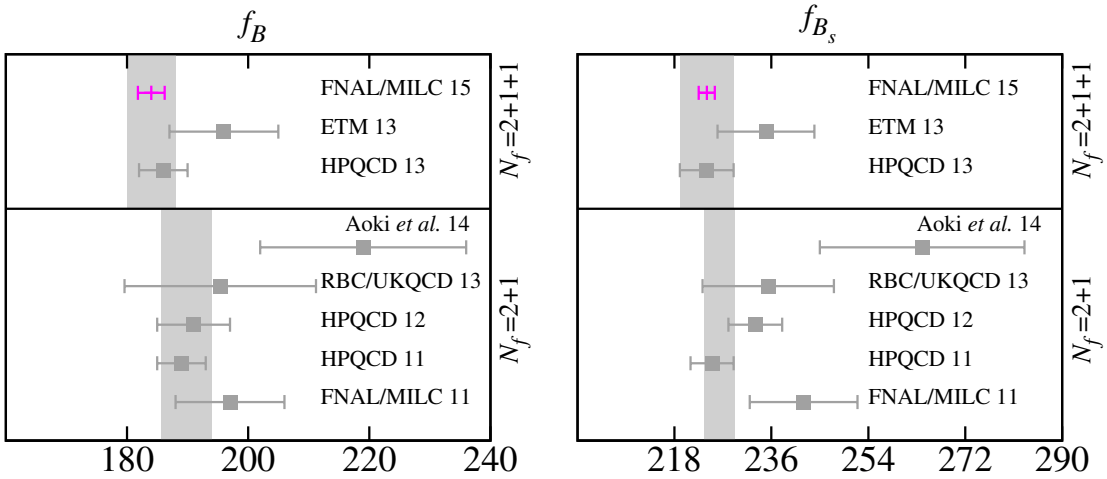

Figure 6: Comparison with other recent results [4, 5, 6, 7, 8, 9, 13]. Vertical bands show the recent PDG lattice averages [14]. Here we illustrate *only* our projected error, and take the (2+1+1)-flavor average as our central value.

5. Results and Outlook

We have presented the status of our analysis of f_B and f_{B_s} from a calculation with all HISQ

quarks using the HPQCD scheme of extrapolating from lighter heavy-quark masses to the b-quark mass. In Fig. 6 we compare our projected total error with the recent compilation of results from the Particle Data Group [14]. We anticipate that our calculations when completed will be the most precise to date. We are presenty generating and analyzing a 0.03 fm, ($m_l'/m_s' = 0.2$) ensemble for which $am_b = 0.6$. Here no extrapolation from lighter heavy-quark masses is needed.

Acknowledgments

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