FERMILAB-CONF-15-414-T

Spontaneous Breaking of Scale Invariance in U(N) Chern-Simons Gauge Theories in Three Dimensions

William A. Bardeen

Fermilab, P.O. Box 500, Batavia, IL 60510, USA
email: bardeen@fnal.gov

Abstract

I explore the existence of a massive phase in a conformally invariant $U(N)$ Chern-Simons gauge theories in $D = 3$ with matter fields in the fundamental representation. These models have attracted recent attention as being dual, in the conformal phase, to theories of higher spin gravity on AdS_4. Using the ′t Hooft large N expansion, exact solutions are obtained for scalar current correlators in the massive phase where the conformal symmetry is spontaneously broken. A massless dilaton appears as a composite state, and its properties are discussed. Solutions exist for matters field that are either bosons or fermions.

Keywords:

1. Introduction

Scale invariant, or nearly scale invariant, interactions have played an important role in elementary particle physics. The Standard Model of particle physics is classically conformal as all gauge and Yukawa coupling constants are dimensionless. Masses for all of the elementary particles are generated by their interaction with an elementary Higgs field whose vacuum expectation value sets the electroweak scale, $v_{ew} = 175$ GeV. In the Standard Model, only the Higgs potential,

$$V_{\text{higgs}} = (1/2)\lambda(H^{+}H - v_{ew}^{2})^{2},$$

explicitly breaks the classical scale invariance and determines the scale of dynamical electroweak symmetry breaking.

However, there are two limits where the classical scale invariance is restored. In the limit, $v_{\text{ew}} \to 0$, there is no electroweak symmetry breaking , all particles are massless, and we recover the conformal phase. If we consider, instead, the limit, $\lambda \to 0$ with v_{ew} fixed, the potential becomes flat but the dynamical electroweak symmetry breaking remains with all Standard Model particles (top, W, Z, etc) having masses proportional to v_{ew}. Only the Higgs particle, itself, remains massless, the Goldstone mode of the dynamically broken scale symmetry, a dilaton. Quantum corrections will explicitly break the classical scale symmetry although the physical Higgs particle retains some features of its life as a dilaton.

$U(N)$ Chern-Simons gauge theories are also classically conformal in three dimensions as the gauge coupling constant is dimensionless. They are the analogs of quantum chromodynamics in four dimensions and can be studied with quarks as either scalar bosons or spinor fermions in the fundamental representation of the $U(N)$ gauge group. As in QCD, these theories can be studied using the ′t Hooft large N expansion where the planar diagrams dominate [1]. Unlike QCD, Chern-Simons theories retain their exact conformal invariance to leading order in the large N expansion and provide a laboratory for studying the possibility of the dynamical breaking of scale invariance.

These theories have been of considerable recent interest due to connections to meson effective theories and models of higher spin gravity. At leading order of the

Operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

1/N expansion, exact solutions have been found for both bosonic and fermionic matter fields in the massless conformal phase and at finite temperature [2]. These theories also have an infinite number of conserved higher spin currents and appear to have dual representations on AdS_4 corresponding to Vasiliev's theory of higher-spin gravity [3]. We will explore the possible existence of the massive phase for these theories where the dynamical breaking of the exact scale invariance results in a massless composite Goldstone boson, the dilaton.

2. Chern-Simons Gauge Theory

We will study the properties of $U(N)$ Chern-Simons gauge theories in three dimensions using the large N expansion. These theories are topological gauge theories no propagating gauge degrees of freedom. The Chern-Simons action on R^3 is given by

$$S_{CS} = \frac{i\kappa}{8\pi}\epsilon_{\mu\nu\rho}\int d^3x\{A^a_\mu\partial_\nu A^a_\rho + i\frac{2}{3}A^a_\mu A^b_\nu A^c_\rho f^{abc}\} \quad (1)$$

In three dimensions, the gauge coupling constant, $\lambda = N/\kappa$, is dimensionless.

Matter fields are quarks in the fundamental representation of $U(N)$ and are described by complex N vectors. The quarks can be either scalar bosons or spinor fermions. The action for scalar quarks is given by

$$\begin{aligned} S_{scalar} &= \int d^3x[(D_\mu\vec{\phi}^\dagger)(D_\mu\vec{\phi}) + m^2(\vec{\phi}^\dagger\vec{\phi}) \\ &+\frac{1}{2N}\lambda_4(\vec{\phi}^\dagger\vec{\phi})^2 + \frac{1}{6N^2}\lambda_6(\vec{\phi}^\dagger\vec{\phi})^3] \quad (2) \end{aligned}$$

while the action for fermions is

$$S_{fermion} = \int d^3x[\vec{\psi}^\dagger(\gamma_\mu D_\mu + m)\vec{\psi}] \quad (3)$$

and $D_\mu = \partial_\mu + iA^a_\mu T^a$ is the usual gauge covariant derivative. The fermions can be either two or four component spinors and the fermion mass operator in three dimensions is related to the axial-vector charge density in a dimensional reduction from four dimensions.

The recent studies of the massless phase of these theories [2] have made novel use of a Euclidean space light-cone gauge, $A^a_- = (A^a_1 - iA^a_2)/\sqrt{2} = 0$. In this gauge, the nonabelian interactions of the gauge field vanish and the gauge boson propagator becomes

$$G_{+3}(p) = -G_{3+}(p) = \frac{4\pi i}{\kappa}\frac{1}{p_-} = 4\pi i\frac{\lambda}{N}\frac{1}{p_-} \quad (4)$$

with $\lambda \equiv N/\kappa$ and all other components of $G(p)$ vanishing. In light-cone coordinates, we can make use of the following relations,

$$\begin{aligned} \frac{\partial}{\partial p_+}\frac{1}{(p-q)_-} &= 2\pi\delta^2(p-q), \quad (5) \\ \frac{\partial}{\partial p_+}(p_-) = 0, &\qquad \frac{\partial}{\partial p_+}p_s = \frac{p_-}{p_s}, \quad p_s^2 = 2p_+p_- \end{aligned}$$

3. Scalar quark theory at $\lambda = 0$

We first consider the scalar quark theory without gauge interactions [4]. This scale invariant theory is described by the action in Eq.(2) with the mass term and ϕ^4 coupling set to zero. In the symmetric phase, there are no mass scales, the scalar quark is massless and the vacuum expectation value of scalar current density, $< \vec{\phi}^+\vec{\phi} >$ vanishes. However, in the massive, phase, the vacuum expectation value of scalar current density is not zero and the ϕ^6 interaction generates both the mass term and the ϕ^4 coupling. The gap equation determining the scalar quark mass gives

$$\begin{aligned} M^2_{quark} &= M^2 = \frac{1}{2}\frac{\lambda_6}{N^2} < \vec{\phi}^\dagger\vec{\phi} >^2 = \frac{1}{2}\lambda_6(-\frac{M}{4\pi})^2 \\ &= \frac{\lambda_6}{32\pi^2}M^2 \quad (6) \end{aligned}$$

In general, there is no solution to Eq.(6) except for the massless quark case. However, for the critical coupling, $1 = \lambda_6/32\pi^2$, there can be a massive solution where the scalar quark mass is arbitrary. For the massive case, the induced four-point coupling is given by

$$L_{eff} = \frac{\lambda_4^{eff}}{N}(\vec{\phi}^\dagger\vec{\phi})^2, \quad \lambda_4^{eff} = \frac{\lambda_6}{N} < \vec{\phi}^\dagger\vec{\phi} > = -8\pi M$$

The scalar current two-point correlator is of particular interest as it is sensitive to vacuum structure of the theory. It is the place where we would expect to see a massless dilaton associated with the dynamical breaking of the scale symmetry. In the symmetric phase, this correlator is simply the bubble function, $B_S(\vec{k})$, associated with a simple scalar quark loop. However, in the massive phase, the induced four-point coupling generates a sum of bubble diagrams that is singular at low momenta reflecting the presence of the dilaton pole.

$$\begin{aligned} J_2^{scalar}(\vec{\phi})/N &= < \vec{\phi}^\dagger\vec{\phi}(k)\vec{\phi}^\dagger\vec{\phi}(-k) > /N \\ &= \frac{B_S(\vec{k})}{1 + \lambda_4^{eff}B_S(\vec{k})} \\ &\rightarrow \frac{3}{2\pi}\frac{M}{\vec{k}^2} = \frac{f_D^2}{N}\frac{1}{\vec{k}^2}, \vec{k}^2 \rightarrow 0 \quad (7) \end{aligned}$$

This result is exact at leading order of the large N expansion but all orders in the ϕ^6 coupling constant which remains unrenormalized. The dilaton also appears in correlators of the traceless energy-momentum tensor as expected for the dynamical breaking of an exact scale symmetry.

4. Chern-Simons gauge theory for scalar quarks

The Chern-Simons gauge interactions modify the calculations of the previous section [5]. In the special light-cone gauge discussed in Section 2, there is considerable simplification as there are no self-couplings of the gauge fields and the matter fields interact only through a coupling between the scalar quark currents, $J_3(x)$ and $J_-(x)$ and the seagull term, $J_{33}(x)$.

The gap equation now includes additional terms induced by the gauge interactions. While the individual terms appear to be nonlocal, the effect of the new gauge interactions is captured by a renormalization of the local ϕ^6 interaction.

$$\begin{aligned} M^2_{quark} &= M^2 = \frac{1}{2}\frac{1}{N^2}(8\pi^2\lambda^2 + \lambda_6) < \vec{\phi}^\dagger\vec{\phi} >^2 \\ &= (\frac{1}{4}\lambda^2 + \frac{\lambda_6}{32\pi^2})M^2 \end{aligned} \tag{8}$$

In general, only the massless solution exists. However, as in Section 3, a massive solution can exist at a certain critical coupling for λ_6 as a function of the gauge coupling constant λ,

$$1 = (1/4)\lambda^2 + \lambda_6/32\pi^2 = \lambda_6^{eff}/32\pi^2.$$

The gauge interactions also modify the computation of the two-point correlator of the scalar currents. The scalar current vertex function has new contributions from ladder diagrams involving the exchange gluons between $J_3(p)$ and $J_-(p)$. These terms can be exactly summed and result in a nontrivial function of the longitudinal momentum, k_3, and the transverse momentum, p_-. There are also contributions involving the seagull interactions which can be resummed as a renormalization of the local ϕ^6 interaction.

Repeating the calculation of Section 3, the two scalar current correlator is given by a bubble sum involving the renormalized ϕ^6 coupling and a renormalized bubble function, $B_{CS}(\vec{k}, \lambda)$,

$$B_{CS}(\vec{k}, \lambda) = \tan(\lambda \arctan(\sqrt{\vec{k}^2}/2M))/(4\pi\lambda\sqrt{\vec{k}^2}\,) \tag{9}$$

As before, scalar current correlator is singular at low momenta reflecting the presence of the dilaton pole in the massive phase of the theory,

$$\begin{aligned} J_2^{scalar}(\vec{\phi})/N &= \frac{B_{CS}(\vec{k}, \lambda)}{1 + \lambda_4^{eff} B_{CS}(\vec{k}, \lambda)} \\ &\rightarrow \frac{3}{2\pi}\frac{M}{1-\lambda^2}\frac{1}{\vec{k}^2} = \frac{f_D^2}{N}\frac{1}{\vec{k}^2}, \vec{k}^2 \rightarrow 0 \end{aligned} \tag{10}$$

The decay constant for the dilaton is renormalized by the gauge interactions,

$$f_D = \sqrt{(3NM/2\pi(1-\lambda^2)}$$

and becomes singular as $\lambda \rightarrow 1$ reflecting the expected boundary for the range of physical gauge couplings for the scalar quark theory.

5. Chern-Simons gauge theory for spinor quarks

We now turn to the consideration of quarks as fermions in the fundamental representation of $U(N)$ [6]. The matter action is given in Eq.(3) where we have included a bare mass term that may be required to tune to the critical point. We will consider the quarks to be two component spinors with the gamma matrices identified with the Pauli spin matrices. In light-cone gauge, gap equation for the fermion self-energy contains both covariant and noncovariant terms,

$$S^{-1}(p) = (i\gamma_\mu p_\mu + \Sigma(p)),\ \ \Sigma(p) = \gamma_-\Sigma_+(p) + \Sigma_0(p) \tag{11}$$

where Σ_+ and Σ_0 depend only on the transverse momenta, (p_+, p_-). The self-energy functions satisfy the integral equation,

$$\Sigma(p) = m - 2\pi\lambda i \int \frac{d^3q}{(2\pi)^3}\frac{1}{(p-q)_-}\{\gamma_3 S(q)\gamma_- - \gamma_- S(q)\gamma_3\} \tag{12}$$

From the structure of this equation, we find the dynamical quark mass is independent of the transverse momentum and given by

$$M^2_{quark} = M^2 = -2ip_-\Sigma_+(p) + \Sigma_0^2(p) \tag{13}$$

Using these relations, we can solve for the covariant and noncovariant self-energy functions,

$$\begin{aligned} \Sigma_0(p) &= m - 4\pi\lambda i \int \frac{d^3q}{(2\pi)^3}\frac{1}{(p-q)_-}\frac{iq_-}{q^2+M^2} \\ &= m - \frac{\lambda}{2}\int_{p_s^2}^{\Lambda^2} dq_s^2 \frac{1}{\sqrt{q_s^2+M^2}} \\ &\rightarrow m + \lambda\sqrt{p_s^2+M^2},\ \ p_s^2 = 2p_-p_+ \end{aligned} \tag{14}$$

and

$$2ip_-\Sigma_+(p) = \Sigma_0^2(p) - M^2 \tag{15}$$
$$\rightarrow \lambda^2 p_s^2 + 2\lambda m\sqrt{p_s^2 + M^2} + m^2 - (1-\lambda^2)M^2$$

Using these solutions we can compute the exact solutions for the scalar current vertex functions and the two point scalar current correlator. As with the fermion self-energies, there are both covariant and noncovariant contributions to the vertex functions that are gauge dependent. The scalar current correlator should be a gauge invariant and Lorentz invariant function of the square of the total three momentum. Using a conformal cutoff, the correlator takes the form

$$< J_0(\vec{k})J_0(-\vec{k}) > /N = -\frac{1}{2\pi}\frac{1}{\lambda}m - \frac{1}{2\pi}(\frac{1}{\lambda} - \lambda)(\sqrt{\vec{k}^2}/2)T, \tag{16}$$

$$T = \frac{1 + (\sqrt{\vec{k}^2}/2(m+\lambda M))\tan(\lambda \arctan(\sqrt{\vec{k}^2}/2M))}{\tan(\lambda \arctan(\sqrt{\vec{k}^2}/2M)) - (\sqrt{\vec{k}^2}/2(m+\lambda M))}$$

In the conformal limit, this becomes

$$< J_0(\vec{k})J_0(-\vec{k}) > /N = \frac{1}{4\pi}(\frac{1}{\lambda} - \lambda)\sqrt{\vec{k}^2}\tan(\lambda\frac{\pi}{2})$$
$$\text{for} \quad \lambda < 2, \ \vec{k} \rightarrow \infty \tag{17}$$

This result is in some tension with computations done strictly in the conformal phase which have a different prefactor, $(1/\lambda)$ vs $(1/\lambda - \lambda)$ as above, and a different critical coupling, $\lambda_c = 1$ vs $\lambda_c = 2$ [2, 7].

If the quark mass is dynamically generated from a spontaneously broken scale symmetry, then the two point current correlation function in Eq.(16) should reveal an infrared singularity associated with the presence of a dilaton pole. At zero momentum, we find

$$< J_0(\vec{k})J_0(-\vec{k}) > /N \rightarrow \frac{1}{2\pi}(1 - \frac{1}{\lambda^2})\frac{M(m+\lambda M)}{m - (\frac{1}{\lambda} - \lambda)M} \tag{18}$$

which is singular if we tune the bare mass parameter to satisfy $m = (1/\lambda - \lambda)M$. At this critical point, we can expand the denominator of Eq.(16) to second order in the momentum, $\vec{k}$, to expose the dilaton pole.

$$< J_0(\vec{k})J_0(-\vec{k}) > /N \rightarrow \frac{3}{2\pi}\frac{1}{\lambda^2}\frac{4M^3}{\vec{k}^3} + finite, \quad \vec{k} \rightarrow 0 \tag{19}$$

From the near critical behavior we can determine the dilaton mass and decay constant

$$< J_0(\vec{k})J_0(-\vec{k}) > \rightarrow \frac{f_D^2}{\vec{k}^2 + \mu^2}, \quad f_D = \sqrt{\frac{6NM^3}{\pi\lambda^2}}, \tag{20}$$

and

$$\mu^2 = 12M(M(\frac{1}{\lambda} - \lambda) - m)\frac{\lambda}{1-\lambda^2}$$

.

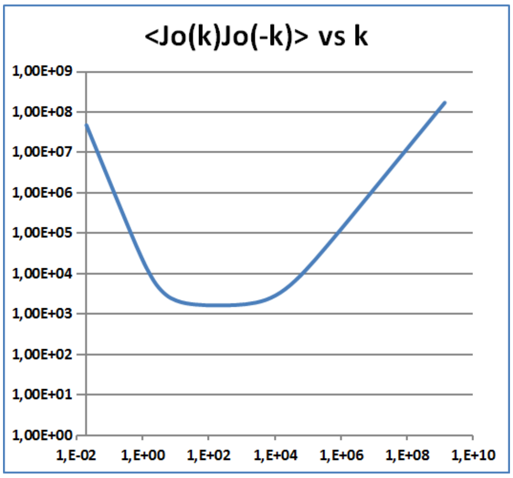

Figure 1: The scalar correlation function:[$\lambda = 0.01,\ \mu = 0$].

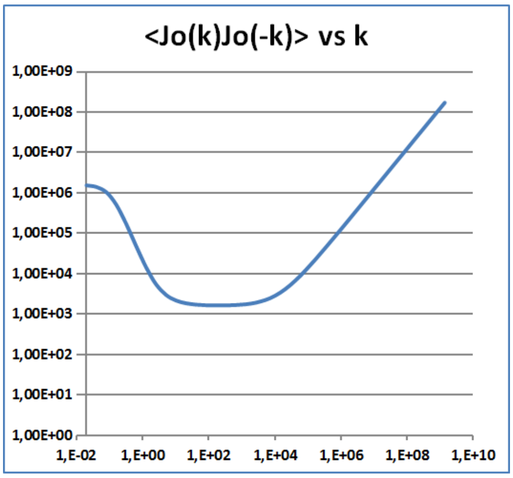

Figure 2: The scalar correlation function: [$\lambda = 0.01,\ \mu \neq 0$].

The scalar current correlation function of Eq.(16) is plotted versus momentum in Figures (1,2,3). Figures 1 and 2 show the behavior at weak coupling, $\lambda = 0.01$, for critical and near critical values of μ. Figure 3 shows the critical case for a moderate coupling, $\lambda = 0.5$. In all cases, the presence of the dilaton pole is seen in the infrared behavior of the correlator while the normal scaling behavior emerges in the ultraviolet region.

This critical behavior depends crucially on the infrared behavior of the noncovariant self-energy function derived in Eq.(15) using our expression for the dynam-

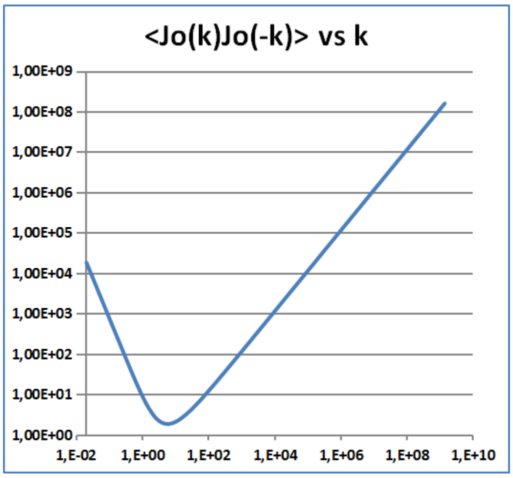

Figure 3: The scalar correlation function: $[\lambda = 0.5,\ \mu = 0]$.

ical quark mass in Eq.(13). However, the noncovariant self-energy function should also satisfy an integral equation following from Eq.(12) which gives

$$\begin{aligned}\Sigma_+(p) &= -4\pi\lambda i \int \frac{d^3q}{(2\pi)^3}\frac{1}{(p-q)_-}\frac{\Sigma_0(q)}{q_3^2+q_s^2+M^2} \\ &= -\frac{i}{p_-}\frac{\lambda}{2}\int_0^{p_s^2} d^2q\frac{\Sigma_0(q)}{\sqrt{q_s^2+M^2}} \qquad (21) \\ &\to -\frac{i}{p_-}(\frac{\lambda^2}{2}p_s^2+\lambda m(\sqrt{p_s^2+M^2}-M))\end{aligned}$$

which differs from the result found in Eq.(15) by the presence of a zero mode contribution to the gap equation.

$$i\Sigma_+(p) = \frac{R_+}{p_-} + (\lambda^2 p_+ + 2\lambda m p_+/(\sqrt{p_s^2+M^2}+M) \quad (22)$$

where $R_+ = (m + \lambda M)^2 - M^2$. The zero mode appears to be essential to the presence of the dilaton pole and a consistent solution for the massive phase but may be a particular artifact of the light-cone gauge. The residue, R_+, does not vanish at the critical point found Eqs.(18,19,20) unless $\lambda = 1$. Without the zero mode contribution, at $\lambda = 1$ and $R_+ = 0$, the two point scalar current correlator of Eq.(16) is nonsingular and there is no dilaton in the massive phase. This could imply a hidden explicit breaking of the scale symmetry in the massive phase or that the massive phase is not the true groundstate of the fermion theory [8].

6. Conclusions

$U(N)$ Chern-Simons gauge theories are an interesting laboratory for studying the mechanism of the dynamical breaking of scale or conformal symmetry. As in quantum chromodynamics in four dimensions, the gauge coupling constant is dimensionless and the theories are classically conformal. Unlike QCD, Chern-Simons gauge theories remain conformal to leading order in the large N expansion for quarks in the fundamental representation of $U(N)$.

We obtain exact solutions for the massive phase in theories with quarks as scalar bosons or spinor fermions. The scalar current density represents the order parameter of the spontaneous breaking of conformal symmetry. The explicit computation of the scalar current correlator reveals the presence of the dilaton pole in the massive quark phase.

A zero mode pole seems essential to the existence of the massive fermion phase and remains a puzzling artifact of the light-cone gauge calculations.

It would be interesting to explore theories containing both scalar bosons and fermions with additional portal interactions connecting the two sectors. In three dimensions, the portal interactions preserve the scale invariance to leading order in the large N expansion. For particular values of the coupling constants, these theories should display $N = 1$ or $N = 2$ superconformal symmetry [9].

Acknowledgements

I wish to thank M. Moshe, O. Aharony, C. Hill and E. Eichten for many useful discussions and critical comments.

References

[1] S. Jain, et al., JHEP10(2012)194, arXiv:1207.4750 [hep-th]; S. Giombi et al, Eur.Phys.J. C72(2012)2112, arxiv:1110.4386; G. Gur-Ari and R. Yacoby, JHEP1302(2013)150, arXiv:1211.1866; O. Aharony et al, JHEP1203(2012)037, arXiv:1110.4382.

[2] E. Fradkin and M. Vasiliev, Phys.Lett. B189(1987)89; M. Vasiliev, The Many Faces of the Superworld, M. Shifman, ed., World Scientific (1999), pg. 533.

[3] G. ‘t Hooft, Nucl.Phys. B**72**(3), 461(1974); M. Moshe and J. Zinn-Justin, Phys.Rept. 385:69-228,2003, hep-th/0306133.

[4] W. Bardeen, M. Moshe and M. Bander, Phys.Rev.Lett. 52(1984)1188.

[5] W. Bardeen and M. Moshe, JHEP1406 (2014)113, arXiv:1402.4196.

[6] W. Bardeen, arXiv:1404.7477, JHEP, to be published.

[7] M. Moshe and J. Zinn-Justin, private communication.

[8] Chen Zhang, private communication.

[9] J.H. Schwarz, JHEP0411(2004)078; arXiv:hep-th/0411077.