

COSMIC REIONIZATION ON COMPUTERS II. REIONIZATION HISTORY

NICKOLAY Y. GNEDIN^{1,2,3} AND ALEXANDER A. KAUROV²
Draft version March 4, 2014

ABSTRACT

We compare the results from several sets of cosmological simulations of cosmic reionization, produced under Cosmic Reionization On Computers (CROC) project, with existing observational data on the high-redshift Ly α forest and the abundance of Ly α emitters. We find good consistency with the observational measurements and the previous simulation work. By virtue of having several independent realizations for each set of numerical parameters, we are able to explore the effect of cosmic variance on observable quantities. One unexpected conclusion we are forced into is that cosmic variance is unusually large at $z > 6$, with both our simulations and, most likely, observational measurements are still not fully converged for even such basic quantities as the average Gunn-Peterson optical depth or the volume-weighted neutral fraction.

Subject headings: cosmology: theory – cosmology: large-scale structure of universe – galaxies: formation – galaxies: intergalactic medium – methods: numerical

1. INTRODUCTION

If cosmic reionization can be called the current frontier of extragalactic astronomy, then, in historic terms, we live in the middle of XIX century. I.e., the frontier is being settled...

Ultra Deep Field campaigns with the Hubble Space Telescope pushed the search for the most likely reionization sources - young star-forming galaxies - to double digit values of cosmic redshift (Bouwens et al. 2007, 2011; Oesch et al. 2012; Bradley et al. 2012; Schenker et al. 2013; Willott et al. 2013; Oesch et al. 2013a; Bowler et al. 2013; Oesch et al. 2013b). Observations of Ly α emitters at $z \sim 7$ (Hu et al. 2010; Ouchi et al. 2010; Pentericci et al. 2011; Kashikawa et al. 2011; Schenker et al. 2012; Ono et al. 2012; Caruana et al. 2012, 2013) indicate rapid change in their abundance as one pushed deeper into the frontier territory. Recent mind-blowing progress of the first generation experiments for detecting the redshifted 21cm signal from the epoch of reionization (Parsons et al. 2013; Dillon et al. 2013) promises a major observational breakthrough well before the end of this decade. Even along the well-trodden “Oregon Trail” of Ly α absorption spectroscopy of high redshift quasars new advances are expected in the nearest future, as discoveries of several $z \sim 6.5$ quasars are expected to be announced soon (R. Simcoe, private communication).

Theoretical studies did not stay behind the observational strides, rejuvenating a somewhat slowed-down progress of the second half of the last decade. The major push on the theory side was galvanized by the pioneering idea of Furlanetto et al. (2004), who realized that the standard lore of large-scale structure theory, Excursion Set formalism, can be applied to studying the reionization process. That idea generated a large following of semi-analytical and semi-numerical approaches for modeling reionization (Furlanetto & Oh 2005; Furlanetto et al. 2006; Mesinger & Furlanetto 2007; Alvarez & Abel 2007; Zahn et al. 2011; Mesinger et al. 2011; Alvarez & Abel 2012; Zhou et al. 2013; Battaglia et al. 2013;

Kaurov & Gnedin 2013; Sobacchi & Mesinger 2014), and more traditional models were pursued as well (Choudhury & Ferrara 2005, 2006; Shull & Venkatesan 2008; Mitra et al. 2011; Venkatesan & Benson 2011; Mitra et al. 2012; Kuhlen & Faucher-Giguère 2012; Robertson et al. 2013a). Unfortunately, on the numerical simulation front the progress was less dramatic, although important advances in the simulation technology did take place (e.g. Iliev et al. 2006; Zahn et al. 2007; McQuinn et al. 2007; Trac et al. 2008; Shin et al. 2008; Croft & Altay 2008; Lee et al. 2008; Iliev et al. 2009; Aubert & Teyssier 2010; Friedrich et al. 2011; Ahn et al. 2012; Shapira et al. 2012, for a complete review see Trac & Gnedin (2009)). However, the primary brake on the simulation progress - insufficient computing power - is finally being released, thanks to Moore’s Law.

Modern High Performance Computing platforms have crossed an important threshold of “sustained peta-scale” performance. This level of performance, currently available on about a dozen or so (non-classified) supercomputers across the globe, offers a unique opportunity for reionization theorists to make a substantial breakthrough in our ability to model cosmic reionization with high physical fidelity, and some of the most recent simulation work already took advantage of that opportunity (Iliev et al. 2014; So et al. 2013; Norman et al. 2013; Hutter et al. 2014).

Cosmic Reionization On Computers (CROC) project is another effort in producing peta-scale simulations of reionization in sufficiently large volumes (above 100 Mpc in comoving units), with spatial resolution reaching down to 100 pc, and including most (if not all) of the relevant physical processes, from star formation and feedback to radiative transfer.

In the first paper in the series (Gnedin 2014, hereafter Paper I) we described in complete detail the design and the calibration of numerical parameters. In this paper we explore the overall process of cosmic reionization as captured by CROC simulations, and compare our theoretical predictions to several observational constraints.

We deliberately limit the scope of this paper to relatively easily computable quantities, which give only a global, broad-brush view of reionization, due to the limited human effort available for the analysis of the rich, but complex simulation data. We intend to continue this paper series as more detailed,

¹ Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA; gnedin@fnal.gov

² Department of Astronomy & Astrophysics, The University of Chicago, Chicago, IL 60637 USA; kaurov@uchicago.edu

³ Kavli Institute for Cosmological Physics, The University of Chicago, Chicago, IL 60637 USA

labor-intensive analysis gets completed.

2. CROC SIMULATIONS

All CROC simulations are performed with the Adaptive Refinement Tree (ART) code (Kravtsov 1999; Kravtsov et al. 2002; Rudd et al. 2008). The ART code is capable of modeling a diverse set of physical processes, from the dynamics of dark matter and gas to star formation, stellar feedback, and radiative transfer. A detailed description of all physical processes followed in the CROC simulations is presented in Paper I.

CROC simulations performed in volumes with $20h^{-1}$ Mpc and $40h^{-1}$ Mpc on a side, and $80h^{-1}$ Mpc boxes will be added to the full data set as the project progresses. All simulations have the same mass resolution of $5 \times 10^6 M_{\odot}$ ($20h^{-1}$ Mpc boxes use 512^3 dark matter particles, $40h^{-1}$ Mpc use 1024^3 particles, etc). Using the full Adaptive Mesh Refinement (AMR) functionality of ART, we reach formal spatial resolution (smallest simulation cell size) of 125 pc at $z = 6$, and even higher resolution at earlier times, as our resolution remains constant in comoving units. The real resolution of the simulations is a factor of 2-3 worse.

Paper I describes numerical parameters of CROC simulations, and how we calibrate their values. The only parameter that we vary in this paper is the “escape fraction of ionizing radiation up to the simulation resolution” ϵ_{UV} . In a numerical simulation with finite spatial resolution not all absorptions of ionizing photons can be accounted for, because some of the photons will be lost in structures that are not resolved in the simulation (like a parent molecular cloud). Hence, to account for those absorptions, we assign each stellar particle ionizing luminosity

$$L_{ion} = \epsilon_{UV} L_{ion}^{orig},$$

where L_{ion}^{orig} is unattenuated luminosity of a single-age stellar population and the parameter ϵ_{UV} accounts for unresolved photon losses.

Since the ionizing output of our model galaxies is proportional to ϵ_{UV} , that parameter critically controls the whole process of reionization in the intergalactic medium (IGM).

For each value of simulation parameters (box size, ϵ_{UV} , etc) we perform a set of simulations that start from independent realizations of initial conditions and properly account for the

TABLE 1
SIMULATION SETS

Set Id	ϵ_{UV}	Stopping redshift	Number of realizations
<i>20h⁻¹ Mpc boxes</i>			
B20.uv1	0.1	5	6 [A-F]
B20.uv2	0.2	5	6 [A-F]
B20.uv4	0.4	5	3 [D-F]
<i>40h⁻¹ Mpc boxes</i>			
B40.uv1	0.1	5	3 [A-C]
B40.uv2	0.2	5.5	3 [A-C]

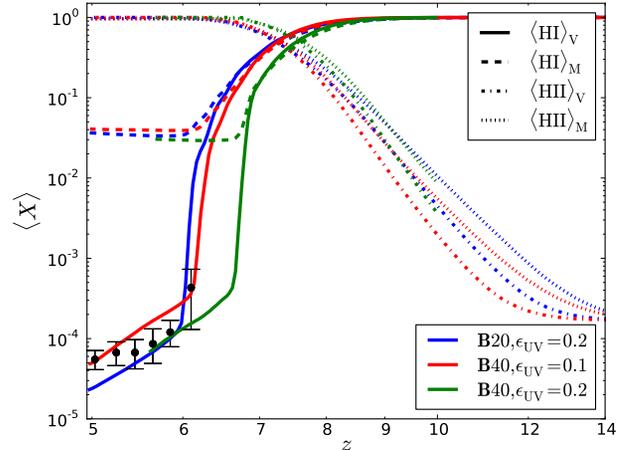


FIG. 1.— Evolution of mass- and volume-weighted hydrogen fractions with redshift in the best-fit $20h^{-1}$ Mpc simulation set B20.uv2 and both $40h^{-1}$ Mpc sets. Data points are from Fan et al. (2006).

fluctuations outside the box using the so-called “DC mode” (Gnedin et al. 2011). Hence, we can use a given simulation set to quantify the effect of the cosmic variance on our results.

Table 1 lists simulation sets that we use in this paper. Star formation and stellar feedback parameters in these simulations are calibrated so that the observed galaxy UV luminosity functions are matched to the observations at all redshifts from $z = 6$ to $z = 10$. Hence, for all simulation sets used in this paper stellar sources of cosmic reionization are followed accurately (at least for $z \leq 10$).

3. RESULTS

3.1. Reionization History

In Paper I it was showed that the value of ϵ_{UV} between 0.1 and 0.2 provides the best match to the observed evolution of the Ly α forest at $z < 6$ (with 0.2 giving a better fit). This is again illustrated in Figure 1, where we show the evolution of the average mass- and volume-weighted hydrogen fractions for the best fit $20h^{-1}$ Mpc set (B20.uv2) and both $40h^{-1}$ Mpc sets (in both cases we average over all independent realizations).

Shapes of various curves in the Figure are highly expected, and are consistent between almost all previous simulations of reionization. Ionized fractions grow steadily with time, as ionized bubbles expand in the neutral IGM. Neutral fractions decrease in response until the moment of overlap, when the volume-weighted neutral hydrogen fraction decreases rapidly (Gnedin 2000). In the post-overlap stage the IGM is highly ionized, and the evolution of neutral fractions is governed by the mean free path of ionizing photons and the level of cosmic ionizing background (see also Trac & Gnedin 2009, for a general overview of reionization process).

Lack of numerical convergence that we discussed in Paper I is also visible in Figure 1 - in the set B40.uv1 the overlap of ionized bubbles (indicated by the rapid drop in the average volume-weighted HI fraction just before $z = 6$) occurs at about the same time as in the smaller box set B20.uv2, but the post-reionization Ly α forest is better matched by the set B40.uv2, which has a significantly earlier overlap.

As we discussed in Paper I, that lack of convergence is caused by cosmic variance⁴ - having multiple independent re-

⁴ Under the term “cosmic variance” we understand the difference between

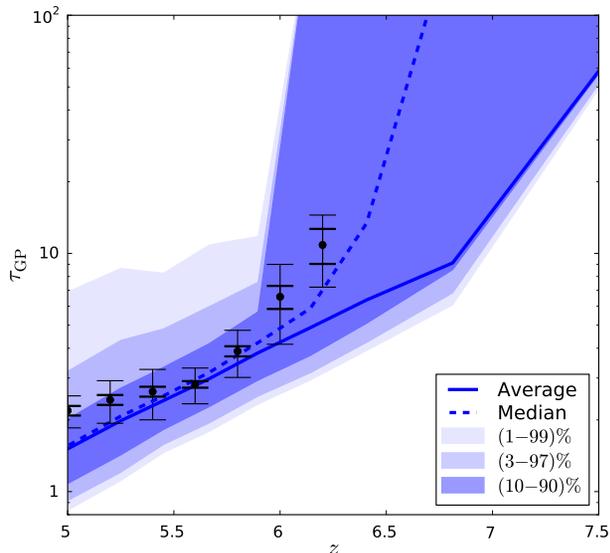


FIG. 2.— Probability Density Function (PDF) of the Gunn-Peterson optical depth τ_{GP} along individual lines of sight as a function of redshift for the B20.uv2 simulation set. PDF is shown with 3 progressively more opaque bands that mark narrow percentile ranges around the mean (the dashed line). A solid line shows the average, that at $z > 6.7$ falls outside (10–90)% percentile range due to the extreme non-linearity in the relationship between τ_{GP} and the gas properties. Data points are from Fan et al. (2006).

alizations allows to explore it well. Our (still unconverged) simulations sample a much larger number of independent sightlines than is available observationally, hence the lack of agreement between the simulations and observations at $z > 6$ cannot yet be taken seriously.

At $z < 6$ the situation is, however, completely different: simulations with the same value of the ϵ_{UV} parameter do converge, the cosmic variance is small (since the radiation field is dominated by the cosmic background - this is apparent from Figure 4 of Paper I), and the correct simulations should match the observational data. The mismatch between the simulations and the observations at $z \lesssim 5.3$ is, therefore, real. In Paper I we also showed that, similarly, our simulations fail to match the galaxy UV luminosity function at $z = 5$ well enough. Both these discrepancies indicate that our simulations become inaccurate after $z \approx 5.3$, most likely because our spatial resolution, being kept fixed in comoving units, degrades too much by $z \approx 5$.

Another illustration of the role of cosmic variance is shown in Figure 2, where we plot the distribution of the Gunn-Peterson optical depth τ_{GP} as a function of redshift for individual lines of sight. Lines of are generated randomly (i.e. starting at random locations and going in random directions) throughout each of the simulation boxes, and synthetic Ly α spectra are generated along each of them. The optical depth along each line of sight is computed as an average over a distance of $40h^{-1}$ Mpc ($\Delta z \approx 0.15$), which is well-matched to the redshift interval used in observational studies Fan et al. (2006). At $z > 6$ the distribution becomes exceptionally wide, and not just in the tails. At lower redshift, past overlap, the distribution narrows significantly, but still maintains a relatively long

separate regions of the universe; such difference is caused both by the variation of densities and by variation in the distribution of ionized bubbles, and the latter almost always dominates.

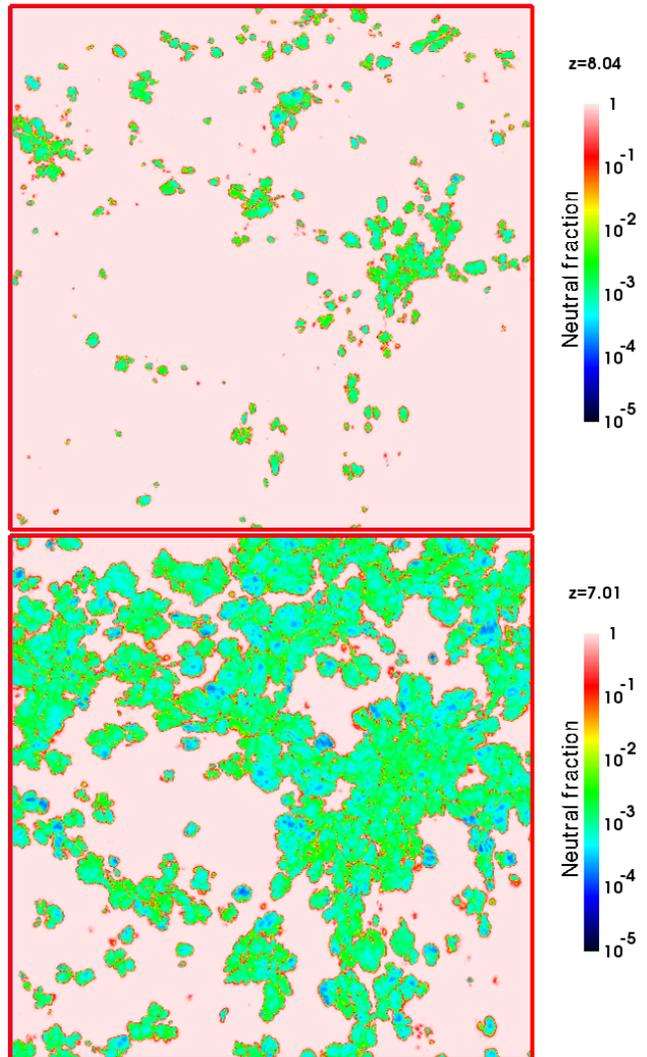


FIG. 3.— Slices through the computational domain for the first realization of B40.uv1 set (run B40.uv1.A in the notation of Paper I) at $z = 8$ and $z = 7$. While at $z = 8$ individual ionized bubbles can still be identified reasonably well, by $z = 7$ the concept of a “bubble” becomes ill-defined.

tail towards large values of τ_{GP} . Hence, we predict that in a small fraction of all quasar sightlines segments with τ_{GP} as high as 10 can be observed all the way down to $z \approx 5.5$.

3.2. Ionized Bubbles

Time evolution of the distribution of ionized bubbles is one of the most important characteristics of the reionization process. Unfortunately, in a realistic cosmological simulation the concept of an “ionized bubble” is not well defined mathematically, especially at late times, as can be easily seen from Figure 3. Hence, in order to have a working definition that can also be compared with other studies, we closely follow the procedure from Zahn et al. (2007) to compute the probability that a given point in the simulation is located inside an ionized bubble of size R . The scale R is defined as the largest radius of a sphere in which the volume-weighted average ionized fraction is higher than the threshold value, which is chosen to be 90%. The only difference with Zahn et al. (2007) in our definition of an ionized bubbles is normalization: we normalize the bubble size distribution to the total volume (an integral over the distribution is equal to the volume-weighted average ionized fraction), whereas Zahn et al. (2007) normalize their

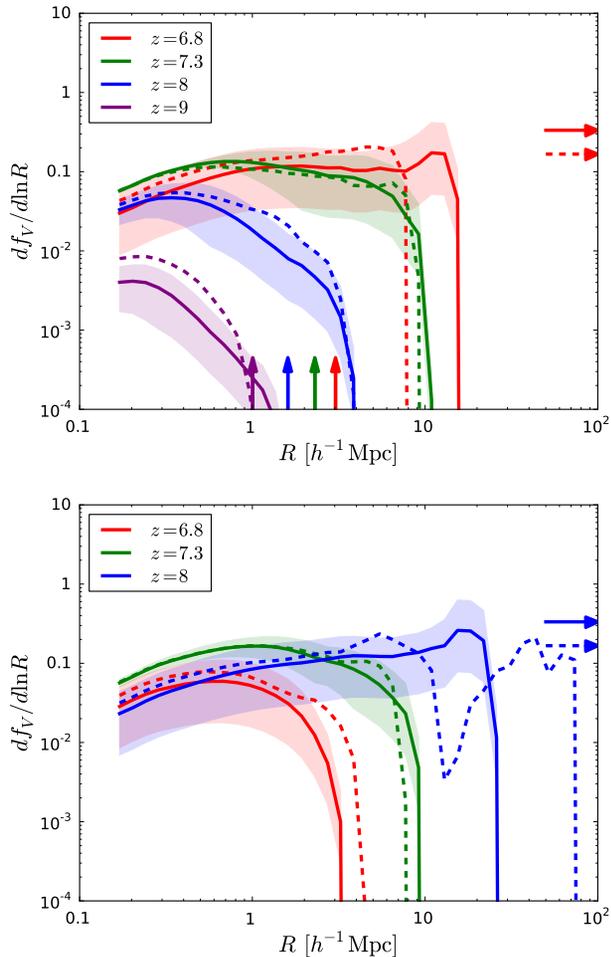


FIG. 4.— Differential volume function of ionized (top) and neutral (bottom) bubbles in our simulations at several values of redshift. Solid lines with semi-transparent bands show the average and the rms for our fiducial $40h^{-1}$ Mpc set B40.uv1; dashed lines show averages for the $20h^{-1}$ Mpc set B20.uv2. Horizontal errors show the volume fraction in individual boxes that are fully ionized (top panel) or fully neutral (bottom panel) at a given redshift. We find good convergence of sizes of ionized bubbles in boxes as small as $20h^{-1}$ Mpc at $z > 7$, and at $z < 9$ for neutral bubbles. For comparison, vertical arrows in the top panel mark the values of the mean free path of ionizing photons due to Lyman Limit system at the corresponding redshifts.

distributions to the total ionized volume (an integral over the distribution is equal to unity).

Distributions of sizes of ionized bubbles (shown as a differential volume function) are presented in Figure 4 for three values of redshift ($z = 7.3$ being the last value where the Zahn et al. (2007) definition of an “ionized bubble” is applicable) for both $20h^{-1}$ Mpc and $40h^{-1}$ Mpc simulation sets. Somewhat unexpectedly, we find good convergence in the bubble size distribution even for the $20h^{-1}$ Mpc box size all the way to $z \sim 7$. At lower redshifts the convergence does break down, simply because some of the smaller boxes are going to be completely ionized before larger boxes (and the same applies to neutral “bubbles” at high redshifts). The volume fraction in such boxes is shown with horizontal arrows on both panels, and it also is reasonably consistent between the $20h^{-1}$ Mpc and $40h^{-1}$ Mpc simulation sets.

This result is inconsistent with the recent simulations of Iliev et al. (2014), who found incomplete convergence in simulation volumes as large as $114h^{-1}$ Mpc at all redshifts. With-

out detailed comparison, it is difficult to isolate the source of the discrepancy. We notice, however, that the spatial resolution of the radiative transfer solver in Iliev et al. (2014) simulations is only about $200h^{-1}$ kpc in comoving units, which is inadequate for resolving absorptions in galactic halos and Lyman Limit systems, while our spatial resolution ($0.6h^{-1}$ comoving kpc) is better suited for properly accounting for all absorptions of ionizing radiation.

3.3. Dumping Wing of Ly α Absorption

Before the universe is completely reionized, patches of the still neutral IGM can significantly absorb Ly α emission from high-redshift galaxies, damping wings of Ly α absorption lines may extend far redward of the galaxy systemic velocity (Miralda-Escude 1998). In order to model that effect, we also generate synthetic Ly α spectra that originate at galaxy locations. The “sky” of each galaxy is sampled uniformly with 12 directions, corresponding to 12 zero level cells of the HEALPix⁵ tessellation of a sphere (Górski et al. 2005). In order to exclude the local absorption from the galactic ISM, we start the line of sight 10 kpc away from the center of the galaxy.

An exact calculation of the effect of the damping wing on the Ly α emission line of a galaxy requires complex Ly α radiative transfer in the galactic ISM and surrounding IGM. Such a calculation is a separate research project in itself, and in any case our simulations do not have enough spatial resolution to perform such a calculation with sufficient accuracy. Instead, we approximate the effect of the damping wing by computing the absorption equivalent width of the red part of the synthetic spectrum,

$$D_{EW} = \int_{\lambda_0}^{\infty} e^{-\tau_{\lambda}} d\lambda,$$

where λ_0 is the systemic velocity of each model galaxy. Hence, we compute 12 values of D_{EW} for each galaxy, achieving dense sampling of the full distribution function for D_{EW} .

From comparison between Figures 1 and 5 it is clear that beginning of the overlap of ionized bubbles corresponds to a rapid decrease in the equivalent width of the damping wing - in our fiducial B20.uv2 run D_{EW} drops by an order of magnitude between $z = 7$ and $z = 6.5$. Such behavior is consistent with the observed sharp decline in the fraction of Ly α emitters between $z = 6$ and $z = 7$ (Pentericci et al. 2011; Schenker et al. 2012; Caruana et al. 2013), although, as we mentioned above, a more quantitative comparison would require a better model of Ly α emitters in the simulations.

We also notice that in our simulations there is little dependence of the dumping wing equivalent width D_{EW} with galaxy luminosity - dotted and dashed lines in Figure 5 show the evolution of D_{EW} for two luminosity bins, but both lines trace similar behavior. The luminosity dependence of the fraction of Ly α emitters has been seen in some observational studies (c.f. Schenker et al. 2012) but not in others (c.f. Pentericci et al. 2011). If such dependence is confirmed by further observational studies, it would imply that brighter galaxies have intrinsically higher probability to become a Ly α emitter than fainter ones.

The observed measurements of the fraction of galaxies that remain strong Ly α emitters have been also used to constraint the mean volume-weighted neutral fraction. The bottom panel of Figure 5 replaces the redshift axis with the (monotonically

⁵ <http://healpix.sourceforge.net>

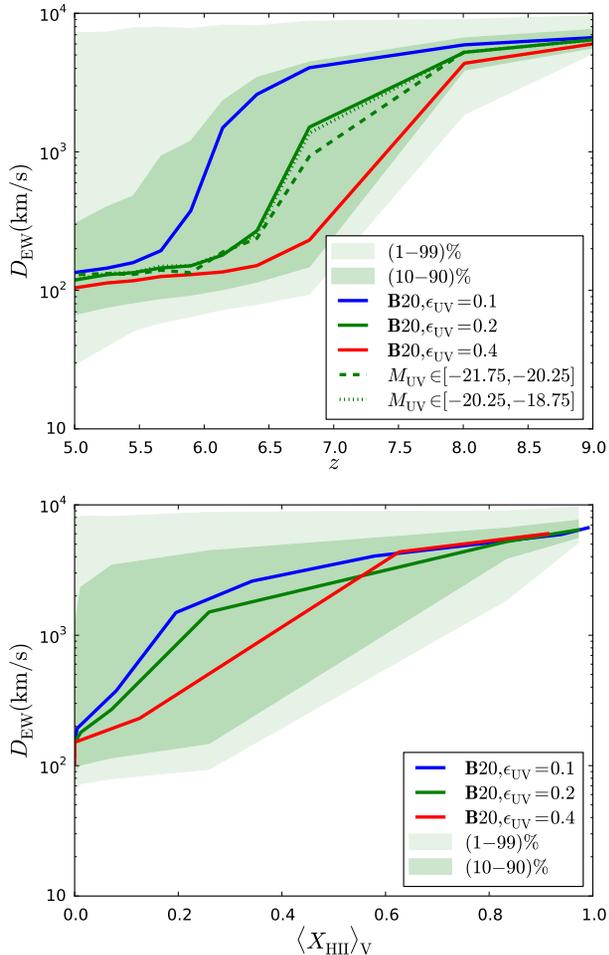


Fig. 5.— Equivalent width of Ly α absorption D_{EW} as a function of redshift (top) or volume-weighted neutral hydrogen fraction (bottom). Colored lines show averages over all realization of simulation sets B20.uv4 (red), B20.uv2 (green), and B20.uv1 (blue) for all galaxies with UV magnitudes between -22 and -18. Green semi-transparent bands give the distribution of D_{EW} around the mean (as (1-99) and (10-90) percentiles). Green dashed and dotted lines show D_{EW} for the subsets of galaxies with magnitudes in bins $[-21.75, -20.25]$ and $[-20.25, -18.75]$ respectively.

decreasing with time) volume-weighted HI fraction. The sensitivity of $D_{EW}(\langle X_{HI} \rangle_V)$ to the ϵ_{UV} parameter is not much less than when D_{EW} is treated as a function of z , confirming the validity of the assumption that the decrease in the observed fraction of Ly α emitter at higher redshifts indicates a change of the volume-weighted average neutral fraction. In fact, it appears that a condition $D_{EW} < 1,000$ km/s corresponds to $\langle X_{HI} \rangle_V \lesssim 0.2$, while a condition $D_{EW} < 500$ km/s corresponds to $\langle X_{HI} \rangle_V \lesssim 0.1$. This conclusion is in good agreement with other recent simulation studies (c.f. Taylor & Lidz 2014; Hutter et al. 2014).

4. CONCLUSIONS

We present reionization history and global characteristics of the reionization process from a suite of recent numerical simulations performed as part of the Cosmic Reionization On Computers (CROC) project. CROC simulations reproduce the observed evolution of the galaxy UV luminosity function between $z = 10$ and $z = 6$ well, and, hence, include realistic treatment of the dominant class of ionizing sources. We do not find anything unexpected, all results presented in this paper are in good agreement with previous work.

We find that, in order to match the observational constraints on the post-reionization Ly α forest at $5 < z < 6$, we need to set the ionizing emissivity parameter ϵ_{UV} (that measures the escape fraction up to the resolution limit of our simulations) to just under $\epsilon_{UV} = 0.2$. However, as we also emphasized in Paper I, cosmic variance increases sharply with redshift, and at $z > 6$ our simulations do not yet converge on the global properties of the IGM, such as the mean Gunn-Peterson optical depth or the volume weighted HI fraction. Since the statistical power of our simulations is much higher than the statistical reach of the existing absorption spectra of high-redshift quasars, we conclude that, unfortunately, the observations are unlikely have reached the convergence either.

In a further illustration of this, we showed that the distribution of the Gunn-Peterson optical depth over the redshift intervals $\Delta z \approx 0.15$ is extraordinary wide at $z > 6$, but even at $z < 6$ the high τ_{GP} tail extends to values as high as $\tau_{GP} \approx 10$ at $z = 5.7$ in 1% of all sightlines and to $\tau_{GP} \approx 8$ in 3% of all sightlines.

The distributions of ionized and neutral bubbles during most of cosmic reionization is approximately flat, meaning that it is roughly equally likely for a random place of the universe to be in a large or a small bubble. We find good numerical convergence in bubble sizes down to $z \sim 7$, at which point the finite sizes of our simulation boxes start biasing the distribution of ionized bubbles. That result illustrates the importance of achieving consistent numerical resolution between the gas dynamic solver and the radiative transfer solver - the mis-match between the two resolution likely results in erroneous over-propagation of ionizing radiation beyond the few mean free path lengths.

Finally, we find that the equivalent width of the damping wing of Ly α absorption increases rapidly from mellow values of $D_{EW} \sim 100$ km/s at $z = 6$ to whopping $D_{EW} \sim 2000$ km/s by $z = 7$. While D_{EW} serves only as a rough proxy for the suppression of galaxy Ly α emission line by the neutral IGM in front of it, this result is generally consistent with the observed sharp decline in the fraction of Ly α emitting galaxies at $z = 7$ as compared to $z = 6$. We also confirm conclusions from the previous simulation and analytical work that such suppression corresponds to substantial, but not dominant volume weighted neutral fraction of about 0.2.

One observational constraint that we ignores so far is the optical depth to Thompson scattering from the CMB observations by the *WMAP* mission. While the history of *WMAP* measurements of the optical depth to Thompson scattering is rocky, the latest value of the Thompson optical depth from the 9-year *WMAP* data is 0.089 ± 0.014 (or 0.081 ± 0.012 if other data are included in a joint fit Bennett et al. 2013; Hinshaw et al. 2013). The value we get for fiducial sets B20.uv2 and B40.uv1 is 0.052 ± 0.003 , and the reionizing earlier set B40.uv2 the value for the Thompson optical depth only rises to 0.057 ± 0.004 , which are only marginally (at $2 - \sigma$ level) consistent with the *WMAP* values. This is not new, the mild tension between the astronomical data and the value for the Thompson optical depth from *WMAP* has been noticed before (e.g. Mitra et al. 2011, 2012; Kuhlen & Faucher-Giguère 2012; Robertson et al. 2013b; Kurov & Gnedin 2013) and has also been widely known informally among the practitioners in this field.

A simple resolution of that tension would be a lower value for the optical depth from the Planck polarization data. Even if the high value of the optical depth withstands further observational verification, it will bear little relevance to our present

results, but will indicate new interesting physics happening in the beginning of reionization (which, of course, will have to be taken into account in the subsequent simulation work). For example, Gnedin (2004) showed how an arbitrary value of the Thompson optical depth can be made consistent with the observed evolution of the Ly α forest at $z \lesssim 6$.

Simulations used in this work have been performed on the Joint Fermilab - KICP cluster “Fulla” at Fermilab, on the University of Chicago Research Computing Center cluster “Midway”, and on National Energy Research Supercomputing Center (NERSC) supercomputers “Hopper” and “Edison”.

REFERENCES

- Ahn, K., Iliiev, I. T., Shapiro, P. R., Mellema, G., Koda, J., & Mao, Y. 2012, *ApJ*, 756, L16
- Alvarez, M. A. & Abel, T. 2007, *MNRAS*, 380, L30
- 2012, *ApJ*, 747, 126
- Aubert, D. & Teysier, R. 2010, *ApJ*, 724, 244
- Battaglia, N., Trac, H., Cen, R., & Loeb, A. 2013, *ApJ*, 776, 81
- Bennett, C. L., Larson, D., Weiland, J. L., Jarosik, N., Hinshaw, G., Odegard, N., Smith, K. M., Hill, R. S., Gold, B., Halpern, M., Komatsu, E., Nolte, M. R., Page, L., Spergel, D. N., Wollack, E., Dunkley, J., Kogut, A., Limon, M., Meyer, S. S., Tucker, G. S., & Wright, E. L. 2013, *ApJS*, 208, 20
- Bouwens, R. J., Illingworth, G. D., Franx, M., & Ford, H. 2007, *ApJ*, 670, 928
- Bouwens, R. J., Illingworth, G. D., Oesch, P. A., Labbé, I., Trenti, M., van Dokkum, P., Franx, M., Stiavelli, M., Carollo, C. M., Magee, D., & Gonzalez, V. 2011, *ApJ*, 737, 90
- Bowler, R. A. A., Dunlop, J. S., McLure, R. J., Rogers, A. B., McCracken, H. J., Milvang-Jensen, B., Furusawa, H., Fynbo, J. P. U., Taniguchi, Y., Afonso, J., Bremer, M. N., & Le Fevre, O. 2013, *ArXiv e-prints*
- Bradley, L. D., Trenti, M., Oesch, P. A., Stiavelli, M., Treu, T., Bouwens, R. J., Shull, J. M., Holwerda, B. W., & Pirzkal, N. 2012, *ApJ*, 760, 108
- Caruana, J., Bunker, A. J., Wilkins, S. M., Stanway, E. R., Lacy, M., Jarvis, M. J., Lorenzoni, S., & Hickey, S. 2012, *MNRAS*, 427, 3055
- Caruana, J., Bunker, A. J., Wilkins, S. M., Stanway, E. R., Lorenzoni, S., Jarvis, M. J., & Elbert, H. 2013, *ArXiv e-prints*
- Choudhury, T. R. & Ferrara, A. 2005, *MNRAS*, 361, 577
- 2006, *MNRAS*, 371, L55
- Croft, R. A. C. & Altay, G. 2008, *MNRAS*, 388, 1501
- Dillon, J. S., Liu, A., Williams, C. L., Hewitt, J. N., Tegmark, M., Morgan, E. H., Levine, A. M., Morales, M. F., Tingay, S. J., Bernardi, G., Bowman, J. D., Briggs, F. H., Emrich, D., Mitchell, D. A., Oberoi, D., Prabu, T., Wayth, R., & Webster, R. L. 2013, *ArXiv e-prints*
- Fan, X., Strauss, M. A., Becker, R. H., White, R. L., Gunn, J. E., Knapp, G. R., Richards, G. T., Schneider, D. P., Brinkmann, J., & Fukugita, M. 2006, *AJ*, 132, 117
- Friedrich, M. M., Mellema, G., Alvarez, M. A., Shapiro, P. R., & Iliiev, I. T. 2011, *MNRAS*, 413, 1353
- Furlanetto, S. R., Hernquist, L., & Zaldarriaga, M. 2004, *MNRAS*, 354, 695
- Furlanetto, S. R., McQuinn, M., & Hernquist, L. 2006, *MNRAS*, 365, 115
- Furlanetto, S. R. & Oh, S. P. 2005, *MNRAS*, 363, 1031
- Gnedin, N. Y. 2000, *ApJ*, 535, 530
- 2004, *ApJ*, 610, 9
- 2014, submitted
- Gnedin, N. Y., Kravtsov, A. V., & Rudd, D. H. 2011, *ApJS*, 194, 46
- Górski, K. M., Hivon, E., Banday, A. J., Wandelt, B. D., Hansen, F. K., Reinecke, M., & Bartelmann, M. 2005, *ApJ*, 622, 759
- Hinshaw, G., Larson, D., Komatsu, E., Spergel, D. N., Bennett, C. L., Dunkley, J., Nolte, M. R., Halpern, M., Hill, R. S., Odegard, N., Page, L., Smith, K. M., Weiland, J. L., Gold, B., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Tucker, G. S., Wollack, E., & Wright, E. L. 2013, *ApJS*, 208, 19
- Hu, E. M., Cowie, L. L., Barger, A. J., Capak, P., Kakazu, Y., & Trouille, L. 2010, *ApJ*, 725, 394
- Hutter, A., Dayal, P., Partl, A. M., & Müller, V. 2014, *ArXiv e-prints*
- Iliiev, I. T., Mellema, G., Ahn, K., Shapiro, P. R., Mao, Y., & Pen, U.-L. 2014, *MNRAS*
- Iliiev, I. T., Mellema, G., Pen, U.-L., Merz, H., Shapiro, P. R., & Alvarez, M. A. 2006, *MNRAS*, 369, 1625
- Iliiev, I. T., Pen, U.-L., McDonald, P., Shapiro, P. R., Mellema, G., & Alvarez, M. A. 2009, *Ap&SS*, 320, 39
- Kashikawa, N., Shimasaku, K., Matsuda, Y., Egami, E., Jiang, L., Nagao, T., Ouchi, M., Malkan, M. A., Hattori, T., Ota, K., Taniguchi, Y., Okamura, S., Ly, C., Iye, M., Furusawa, H., Shioya, Y., Shibuya, T., Ishizaki, Y., & Tshikawa, J. 2011, *ApJ*, 734, 119
- Kaurov, A. A. & Gnedin, N. Y. 2013, *ApJ*, 771, 35
- Kravtsov, A. V. 1999, PhD thesis, New Mexico State University
- Kravtsov, A. V., Klypin, A., & Hoffman, Y. 2002, *ApJ*, 571, 563
- Kuhlen, M. & Faucher-Giguère, C.-A. 2012, *MNRAS*, 423, 862
- Lee, K.-G., Cen, R., Gott, III, J. R., & Trac, H. 2008, *ApJ*, 675, 8
- McQuinn, M., Lidz, A., Zahn, O., Dutta, S., Hernquist, L., & Zaldarriaga, M. 2007, *MNRAS*, 377, 1043
- Mesinger, A. & Furlanetto, S. 2007, *ApJ*, 669, 663
- Mesinger, A., Furlanetto, S., & Cen, R. 2011, *MNRAS*, 411, 955
- Miralda-Escude, J. 1998, *ApJ*, 501, 15
- Mitra, S., Choudhury, T. R., & Ferrara, A. 2011, *MNRAS*, 413, 1569
- 2012, *MNRAS*, 419, 1480
- Norman, M. L., Reynolds, D. R., So, G. C., & Harkness, R. P. 2013, *ArXiv e-prints*
- Oesch, P. A., Bouwens, R. J., Illingworth, G. D., Gonzalez, V., Trenti, M., van Dokkum, P. G., Franx, M., Labbé, I., Carollo, C. M., & Magee, D. 2012, *ApJ*, 759, 135
- Oesch, P. A., Bouwens, R. J., Illingworth, G. D., Labbé, I., Franx, M., van Dokkum, P. G., Trenti, M., Stiavelli, M., Gonzalez, V., & Magee, D. 2013a, *ApJ*, 773, 75
- Oesch, P. A., Bouwens, R. J., Illingworth, G. D., Labbe, I., Smit, R., van Dokkum, P. G., Momcheva, I., Ashby, M. L. N., Fazio, G. G., Huang, J., Willner, S. P., Gonzalez, V., Magee, D., Trenti, M., Brammer, G. B., Skelton, R. E., & Spitler, L. R. 2013b, *ArXiv e-prints*
- Ono, Y., Ouchi, M., Mobasher, B., Dickinson, M., Penner, K., Shimasaku, K., Weiner, B. J., Kartaltepe, J. S., Nakajima, K., Nayyeri, H., Stern, D., Kashikawa, N., & Spinrad, H. 2012, *ApJ*, 744, 83
- Ouchi, M., Shimasaku, K., Furusawa, H., Saito, T., Yoshida, M., Akiyama, M., Ono, Y., Yamada, T., Ota, K., Kashikawa, N., Iye, M., Kodama, T., Okamura, S., Simpson, C., & Yoshida, M. 2010, *ApJ*, 723, 869
- Parsons, A. R., Liu, A., Aguirre, J. E., Ali, Z. S., Bradley, R. F., Carilli, C. L., DeBoer, D. R., Dexter, M. R., Gugliucci, N. E., Jacobs, D. C., Klima, P., MacMahon, D. H. E., Manley, J. R., Moore, D. F., Pober, J. C., Stefan, I. I., & Walbrugh, W. P. 2013, *ArXiv e-prints*
- Pentericci, L., Fontana, A., Vanzella, E., Castellano, M., Grazian, A., Dijkstra, M., Boutsia, K., Cristiani, S., Dickinson, M., Giallongo, E., Giallisco, M., Maiolino, R., Moorwood, A., Paris, D., & Santini, P. 2011, *ApJ*, 743, 132
- Robertson, B. E., Furlanetto, S. R., Schneider, E., Charlot, S., Ellis, R. S., Stark, D. P., McLure, R. J., Dunlop, J. S., Koekemoer, A., Schenker, M. A., Ouchi, M., Ono, Y., Curtis-Lake, E., Rogers, A. B., Bowler, R. A. A., & Cirasuolo, M. 2013a, *ApJ*, 768, 71
- 2013b, *ArXiv e-prints*
- Rudd, D. H., Zentner, A. R., & Kravtsov, A. V. 2008, *ApJ*, 672, 19
- Schenker, M. A., Robertson, B. E., Ellis, R. S., Ono, Y., McLure, R. J., Dunlop, J. S., Koekemoer, A., Bowler, R. A. A., Ouchi, M., Curtis-Lake, E., Rogers, A. B., Schneider, E., Charlot, S., Stark, D. P., Furlanetto, S. R., & Cirasuolo, M. 2013, *ApJ*, 768, 196
- Schenker, M. A., Stark, D. P., Ellis, R. S., Robertson, B. E., Dunlop, J. S., McLure, R. J., Kneib, J.-P., & Richard, J. 2012, *ApJ*, 744, 179
- Shapiro, P. R., Iliiev, I. T., Mellema, G., Ahn, K., Mao, Y., Friedrich, M., Datta, K., Park, H., Komatsu, E., Fernandez, E., Koda, J., Bovill, M., & Pen, U.-L. 2012, in *American Institute of Physics Conference Series*, Vol. 1480, American Institute of Physics Conference Series, ed. M. Uememura & K. Omukai, 248–260
- Shin, M.-S., Trac, H., & Cen, R. 2008, *ApJ*, 681, 756
- Shull, J. M. & Venkatesan, A. 2008, *ApJ*, 685, 1
- So, G. C., Norman, M. L., Reynolds, D. R., & Harkness, R. P. 2013, *ArXiv e-prints*
- Sobacchi, E. & Mesinger, A. 2014, *ArXiv e-prints*
- Taylor, J. & Lidz, A. 2014, *MNRAS*, 437, 2542
- Trac, H., Cen, R., & Loeb, A. 2008, *ApJ*, 689, L81
- Trac, H. & Gnedin, N. Y. 2009, *ArXiv e-prints*
- Venkatesan, A. & Benson, A. 2011, *MNRAS*, 417, 2264
- Willott, C. J., McLure, R. J., Hibon, P., Bielby, R., McCracken, H. J., Kneib, J.-P., Ilbert, O., Bonfield, D. G., Bruce, V. A., & Jarvis, M. J. 2013, *AJ*, 145, 4

Zahn, O., Lidz, A., McQuinn, M., Dutta, S., Hernquist, L., Zaldarriaga, M., & Furlanetto, S. R. 2007, *ApJ*, 654, 12
Zahn, O., Mesinger, A., McQuinn, M., Trac, H., Cen, R., & Hernquist, L. E. 2011, *MNRAS*, 414, 727

Zhou, J., Guo, Q., Liu, G.-C., Yue, B., Xu, Y.-D., & Chen, X.-L. 2013, *Research in Astronomy and Astrophysics*, 13, 373