Double parton interactions in $\gamma + 3$ jet and $\gamma + b/c$ jet + 2 jet events in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV

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I. INTRODUCTION

The study of deep inelastic hadron-hadron collisions is one of the main sources of knowledge about hadronic structure. We describe such a collision as the process in which a single parton (quark or gluon) from one nucleon undergoes a hard scattering off a single parton from the other nucleon. The other “spectator” partons, which do not take part in this hard 2 \rightarrow 2 parton collision, contribute to the so-called “underlying event.” However, the probability of other partons in each nucleon to also undergo a hard scattering is not zero. The rate of multiple parton interactions (MPI) in pp collisions is directly related to the transverse spatial distribution of partons within the proton, and has been the subject of extensive theoretical studies (see e.g., [1–10]).

Relevant measurements have been performed by the AFS [11], UA2 [12], CDF [13, 14], D0 [15, 16], ATLAS [17], and CMS [18] collaborations. The first three measurements are based on samples of events having a 4-jet final state, while the CDF and D0 measurements in Refs. [14–16] use \gamma + 3 jet events produced by double parton (DP) scattering with \gamma + jet and dijet final states. The \gamma + jet production originates mainly via quark-gluon scattering in a Compton-like process, qg \rightarrow q\gamma, and an annihilation process, q\bar{q} \rightarrow q\gamma. As was shown experimentally in Refs. [13–16] and theoretically described in Ref. [19], the use of \gamma + 3 jet events leads to a greater sensitivity to the DP fraction as compared to 4-jet events mainly because of the better energy and angular resolutions for photon as compared with jets.

The total DP cross section for the events caused by two parton scatterings with \gamma + jet and dijet final states is defined as [15]

$$\sigma_{DP} = \frac{\sigma^{\gamma\gamma in\gamma\gamma\gamma}}{\sigma_{eff}}. \quad (1)$$

Here, \sigma^{\gamma\gamma in\gamma\gamma\gamma} (\sigma^{\gamma\gamma}) is the total \gamma + jet (dijet) production cross section. The parameter \sigma_{eff} in Eq. 1 is related to the distance in the transverse plane between partons in the nucleon [2, 3, 5, 11–15]:

$$\sigma_{eff} = \left[ \int (F(\beta)) d^2 \beta \right]^{-1}, \quad (2)$$

where $F(\beta) = \int \rho(r) \rho(r - \beta) d^2 r$ is the overlap function between the parton spatial distributions $\rho(r)$ in the nucleons colliding with impact parameter $\beta$ (for example, see [5–7]). Here $r$ is a distance from the center of the nucleon in transverse plane. The overlap function is normalized to unity, $\int F(\beta) d^2 \beta = 1$. In case of a Gaussian spatial density $\rho(r)$, the overlap function $F(\beta) = (4\pi a^2)^{-1} \exp(-\beta^2/2a^2)$, and thus $\sigma_{eff} = 8\pi a^2,$
where $a$ is the Gaussian width $[7, 15]$. The overlap function characterizes the transverse area occupied by the interacting partons. The larger the overlap (i.e., smaller $\beta$), the more probable it is to have one or more hard parton interactions in the colliding nucleons.

Table I summarizes the currently available measurements of the value of $\sigma_{\text{eff}}$. Within uncertainties, existing measurements of $\sigma_{\text{eff}}$ for final states with jets and photons or $W$ bosons are consistent. They are more precise than those with 4-jet final state. The dependence of $\sigma_{\text{eff}}$ on $\sqrt{s}$ is expected to be small $[5]$. In this paper we present the first measurement of the DP rates and $\sigma_{\text{eff}}$ involving heavy flavor leading jet using the $\gamma + b/c$ jet $+ 2$ jet final state and compare this measurement to the results obtained with $\gamma + 3$ jet events. The $\gamma + b/c$-jet production is mainly caused by $b(c)g \rightarrow b(c)\gamma$ and $q\bar{q} \rightarrow q\gamma$ with $g \rightarrow Q\bar{Q}$, where $Q = b(c) [20]$. Figure 1 shows the fractions of $qg$ and $gb$ subprocesses in events with $\gamma +$ jet and $\gamma + b$-jet final states, calculated using default pythia 6.4 $[21]$ settings and the CTEQ 6.1L parton distribution function $[22]$. At $p_T^\gamma \approx 30$ GeV, Compton-like scattering dominates over the annihilation process, contributing about 85%–88% of events. Since the initial quarks in the Compton-like scattering for inclusive $\gamma +$ jet and $\gamma + b/c$-jet production are typically light ($\approx 92\%$, according to the estimates done with PYTHIA) and $b/c$ quarks, respectively, the difference between effective cross sections measured in the two processes should be sensitive to difference between light quark and heavy quark transverse spatial distributions (see Eq. 2).

![Figure 1](color online) Fractional contribution of the Compton-like $qg \rightarrow q\gamma$ ($q$ is any quark type) and $bg \rightarrow b\gamma$ subprocesses to the associated production of inclusive $\gamma +$ jet and $\gamma + b$-jet final states as a function of $p_T^\gamma$.

The outline of the paper is as follows. Section II briefly describes the technique for extracting the effective cross section $\sigma_{\text{eff}}$. Section III includes the description of the D0 detector and the data and Monte Carlo simulation (MC) samples used in the measurement. Section IV presents signal and background models. Section V describes the discriminating variable used to determine the DP fractions. The DP fractions are estimated in Section VI. Section VII describes the determination of other parameters needed to calculate $\sigma_{\text{eff}}$. In Section VIII, we calculate the effective cross section $\sigma_{\text{eff}}$ for $\gamma + 3$ jet and $\gamma + b/c$ jet $+ 2$ jet events, and discuss the effects related to parton distribution functions (PDF) in Section IX. The results are summarized in Section X.

II. TECHNIQUE FOR EXTRACTING $\sigma_{\text{eff}}$ FROM DATA

To extract $\sigma_{\text{eff}}$, we use the same technique as in earlier measurements $[14, 15]$, which requires only quantities determined from data, minimizing the impact of theoretical assumptions. We avoid using theoretical predictions of the $\gamma +$ jet and dijet cross sections by comparing the number of $\gamma + 3$ jet events produced in DP interactions in single $p\bar{p}$ collisions to the number of $\gamma + 3$ jet events produced in two distinct hard parton interactions occurring in two separate $p\bar{p}$ collisions in the same beam crossing. The latter class of events is referred to as double interaction (DI) events. Assuming uncorrelated parton scatterings in the DP process $[1–5]$, DP and DI events should be kinematically identical, and only differ by the presence of one (two) $p\bar{p}$ collision vertex in the case of DP (DI) events. This assumption has been tested in Ref. $[15]$ and is discussed further in Section VIII. Both DP and DI interactions provide a source of events with two instances of parton scattering. It is necessary to measure both DP and DI rates to extract $\sigma_{\text{eff}}$. Background processes include single hard interactions producing similar final states with or without the presence of additional soft $p\bar{p}$ interactions.

As was shown in Ref. $[15]$, the number of DI events with the final topology of interest, $N_{\text{DI}}$, can be obtained from the probability for a DI event, $P_{\text{DI}} = 2(\sigma^{J}/\sigma_{\text{hard}})(\sigma^{J}/\sigma_{\text{hard}})$, in a $p\bar{p}$ beam crossing with two hard collisions. Here $\sigma_{\text{hard}}$ is the total hard $p\bar{p}$ interaction cross section. This probability should be corrected for the combination of the acceptance (geometric and kinematic) and selection efficiency ($\epsilon_{\text{DI}}$), the two-vertex event selection efficiency ($\epsilon_{2\text{vtx}}$), and the number of beam crossings with two hard collisions ($N_{2\text{coll}}$):

$$N_{\text{DI}} = P_{\text{DI}} N_{2\text{coll}} \epsilon_{\text{DI}} \epsilon_{2\text{vtx}}.$$

Analogously to $N_{\text{DI}}$, the number of DP events, $N_{\text{DP}}$, can be expressed from the probability for a DP event, $P_{\text{DP}} = (\sigma^{J}/\sigma_{\text{hard}})(\sigma^{J}/\sigma_{\text{eff}})$, in a $p\bar{p}$ beam crossing with one hard collision. Similarly to the DI events, this probability is corrected for the combination of the acceptance (geometric and kinematic) and selection efficiency ($\epsilon_{\text{DP}}$), the single-vertex event selection efficiency ($\epsilon_{1\text{vtx}}$), and
the number of beam crossings with one hard collision ($N_{1\text{coll}}$):

$$N_{\text{DP}} = P_{\text{DP}}N_{1\text{coll}}\varepsilon_{\text{DP}}\varepsilon_{1\text{vtx}}.$$ (4)

The ratio of the number of DP to DI events, $N_{\text{DP}}/N_{\text{DI}}$, allows us to obtain the expression for $\sigma_{\text{eff}}$ [14, 15]:

$$\sigma_{\text{eff}} = \frac{N_{\text{DI}}\varepsilon_{\text{DP}}}{N_{\text{DP}}\varepsilon_{\text{DI}}}R_c\sigma_{\text{hard}},$$ (5)

where the factor $R_c \equiv (1/2)(N_{1\text{coll}}/N_{2\text{coll}})(\varepsilon_{1\text{vtx}}/\varepsilon_{2\text{vtx}})$. The cross sections $\sigma_{1\gamma}$ and $\sigma_{1\gamma}$ do not appear in this equation, and all efficiencies for DP and DI events enter only as ratios, resulting in a reduction of the correlated systematic uncertainties.

The background to DP events are single parton (SP) scatterings with the radiation of at least two hard gluons in the initial or final state, $gg \to q\bar{q}gg$, $gq \to g\gamma gg$, which leads to the same $\gamma + 3$ jet signature. The fraction of DP events is determined using a variable sensitive to the kinematic configurations of the two independent scatterings of parton pairs.

The largest background to DI events is two-vertex SP events with one hard $\gamma + 3$ jet interaction occurring in one $p\bar{p}$ collision and an additional soft interaction (i.e., having no reconstructed jets) occurring at the other $p\bar{p}$ vertex.

### III. D0 DETECTOR AND DATA SAMPLES

The D0 detector is described in detail in Refs. [23–25]. Photon candidates are identified as isolated clusters of energy depositions in one of three uranium and liquid argon sampling calorimeters. The central calorimeter covers the pseudorapidity $|\eta_{\text{det}}| < 1.1$, and the two end calorimeters cover up to $|\eta_{\text{det}}| \approx 4.2$. In addition, the plastic scintillator intercryostat detector covers the region $1.1 < |\eta_{\text{det}}| < 1.4$. The electromagnetic (EM) section of the calorimeter is segmented longitudinally into four layers and transversely into cells in pseudorapidity and azimuthal angle $\Delta\eta_{\text{det}} \times \Delta\phi_{\text{det}} = 0.1 \times 0.1 (0.05 \times 0.05$ in the third layer of the EM calorimeter). The hadronic portion of the calorimeter is located behind the EM section. The calorimeter surrounds a tracking system consisting of a silicon microstrip tracking (SMT) detector and scintillating fiber tracker, both located within a 2 T solenoidal magnetic field. The solenoid magnet is surrounded by the central preshower (CPS) detector located immediately before the calorimeter. The CPS consists of approximately one radiation length of lead absorber surrounded by three layers of scintillating strips.

The current measurement is based on 8.7 $fb^{-1}$ of data collected after the D0 detector upgrade in 2006 [25], while the previous measurements [15, 16] were made using data collected before this upgrade.

The events used in this analysis pass triggers designed to identify high-$p_T$ clusters in the EM calorimeter with loose shower shape requirements for photons. These triggers have $\approx 96\%$ efficiency at $p_T \gamma \approx 30$ GeV and are $100\%$ efficient for $p_T \gamma > 35$ GeV.

To select photon candidates in our data samples, we use the following criteria [27, 28]: EM objects are reconstructed using a simple cone algorithm with a cone size of $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = 0.2$. Regions with poor photon identification capability and degraded $p_T$ resolution at the boundaries between calorimeter modules and between the central and endcap calorimeters are excluded from analysis. Each photon candidate is required to deposit more than 96% of the detected energy in the EM section of the calorimeter and to be isolated in the angular region between $\Delta R = 0.2$ and $\Delta R = 0.4$ around the center of the cluster: $(E_{\text{iso}}^{\text{core}} - E_{\text{iso}}) / E_{\text{iso}} < 0.07$, where $E_{\text{iso}}^{\text{core}}$ is the total (EM+hadronic) tower energy in the $\eta, \phi$ cone of radius $\Delta R = 0.4$ and $E_{\text{iso}}$ is EM energy within a radius of $\Delta R = 0.2$. Candidate EM clusters that match to a reconstructed track are excluded from the analysis. We also require the energy-weighted EM cluster width in the finely-segmented third EM layer to be consistent with that expected for a photon-initiated electromagnetic shower. In addition to the calorimeter isolation cut, we also apply a track isolation cut, requiring the scalar sum of track transverse momenta in an annulus $0.05 \leq \Delta R \leq 0.4$ to be less than 1.5 GeV.

Jets are reconstructed using an iterative midpoint cone

| $\sqrt{s}$ (GeV) | final state | $p_T^{\gamma\gamma}$ (GeV) | $|\eta| < 1$ | $|\eta| < 2$ | $|\eta| < 3.5$ | $|\eta| < 9$ | $\sigma_{\text{eff}}$ |
|-----------------|-------------|---------------------------|-------------|-------------|-------------|-------------|----------------|
| AFS [11]        | 63          | 4 jets $p_T^{\gamma\gamma} > 4$ | 1960        | 4 jets $p_T^{\gamma\gamma} > 15$ | 7000        | $|\eta| < 1$ | $\approx 5$ mb |
| UA2 [12]        | 630         | 4 jets $p_T^{\gamma\gamma} > 15$ | 630         | 4 jets $p_T^{\gamma\gamma} > 25$ | 1800        | $|\eta| < 2$ | $> 8.3$ mb (95% C.L.) |
| CDF [13]        | 1800        | 4 jets $p_T^{\gamma\gamma} > 25$ | 1800        | 4 jets $p_T^{\gamma\gamma} > 6$ | 1800        | $|\eta| < 3.5$ | $12.1 \pm 5.4$ mb |
| CDF [14]        | 1800        | $\gamma + 3$ jets $p_T^{\gamma\gamma} > 6$ | $|\eta| < 3.5$ | $14.5 \pm 1.7$ (stat) $\pm 2.8$ (syst) mb |
| D0 [15]         | 1960        | $\gamma + 3$ jets $60 < p_T < 80$ | $|\eta| < 2.8$ | $16.4 \pm 0.3$ (stat) $\pm 2.3$ (syst) mb |
| ATLAS [17]      | 7000        | $W + 2$ jets $p_T^{\gamma\gamma} > 20$ | $|\eta| < 2.0$ | $20.7 \pm 0.8$ (stat) $\pm 6.6$ (syst) mb |
| CMS [18]        | 7000        | $W + 2$ jets $p_T^{\gamma\gamma} > 20$ | $|\eta| < 2.0$ | $20.7 \pm 0.8$ (stat) $\pm 6.6$ (syst) mb |
To reject background from cosmic rays and \( W \rightarrow e\nu \) decay [27], the missing transverse momentum in the event is required to be less than 0.7\( p_T \). All photon-jet pairs must be separated by \( \Delta R > 0.7 \) and all jet-jet pairs must be separated by \( \Delta R > 1.0 \). Each event must contain at least one photon in the pseudorapidity region \( |\eta^\gamma| < 1.0 \) or \( 1.5 < |\eta^\gamma| < 2.5 \) and at least three jets with \( |\eta^{\text{jet}}| < 2.5 \). The jet with the highest \( p_T \) is termed the “leading jet” or first jet, and the jets with the second and third highest \( p_T \) are denoted as the second and third jets in the following. Events are selected with photon transverse momentum \( p_T^\gamma > 26 \) GeV, leading jet \( p_T^{\text{jet}} > 15 \) GeV, while the next-to-leading (second) and third jets must have \( 15 < p_T^{\text{jet}} < 35 \) GeV. The upper limit on the \( p_T \) of the second and third jets increases the fraction of DP events in the sample [15].

To select the sample of \( \gamma + b/c \) jet + 2 jet candidate events, the leading jet is required to have at least two associated tracks with \( p_T > 0.5 \) GeV and each track must have at least one hit in the SMT detector. At least one track must have \( p_T > 1.0 \) GeV. These requirements ensure that there is sufficient information to identify the leading jet as a heavy flavor candidate and have a typical efficiency of about 90%. To enrich the sample with heavy flavor jets, a neural network based \( b \)-tagging algorithm (\( b \)-NN) [31] is used. It exploits long decay lengths of \( b \)-flavored hadrons. The leading jet is required to pass a tight \( b \)-NN cut \( > 0.225 \) [31].

Data events with a single \( p\bar{p} \) collision vertex (“1vtx” sample), which contain DP candidates, are selected separately from events with two vertices (“2vtx” sample), which contain DI candidates. The collision vertices in both samples are required to have at least three associated tracks and to be within 60 cm of the center of the detector along the beam (\( z \)) axis. The total number of \( \gamma + 3 \) jet and \( \gamma + b/c \) jet + 2 jet candidate events, referred to below as inclusive and heavy flavor (HF) samples, in each of the 1vtx or 2vtx categories after all selection criteria have been applied are given in Table II. No requirement on the origin vertex for the photon or jets is imposed here for the 2vtx events.

### Table II: The numbers of selected 1vtx and 2vtx candidate events, \( N_{1\text{vtx}} \) and \( N_{2\text{vtx}} \), and their ratio in the \( \gamma + 3 \) jet (inclusive) and \( \gamma + b/c \) jet + 2 jet (HF) samples.

<table>
<thead>
<tr>
<th>Data Sample</th>
<th>( N_{1\text{vtx}} )</th>
<th>( N_{2\text{vtx}} )</th>
<th>( N_{2\text{vtx}}/N_{1\text{vtx}} )</th>
<th>( \gamma + 3 ) jet (inclusive)</th>
<th>( \gamma + b/c ) jet + 2 jet (HF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive</td>
<td>218686</td>
<td>269445</td>
<td>1.23 ± 0.01</td>
<td>1.23 ± 0.01</td>
<td>1.16 ± 0.02</td>
</tr>
<tr>
<td>HF</td>
<td>5004</td>
<td>5811</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## IV. DATA, SIGNAL, AND BACKGROUND EVENT MODELS

This section gives an overview of the DP and DI models built using data and MC samples, to estimate the number of DP and DI events in data, \( N_{\text{DP}} \) and \( N_{\text{DI}} \). These models are also used to calculate selection efficiencies and geometric and kinematic acceptances for DP and DI events.

### A. Signal models

- **DP data event model (mixdp):**
  The DP signal event model exploits the fact that two parton-parton scatterings can occur in the same \( p\bar{p} \) collision. Therefore, an individual signal DP event is constructed by overlaying one event from an inclusive data sample of \( \gamma + \geq 1 \) jet data events with another event from a sample of inelastic non-diffractive events selected with a minimum bias trigger and a requirement of at least one reconstructed jet (“MB” sample) [15, 30]. Both input samples contain only events with a single \( p\bar{p} \) collision vertex. The \( p_T \) values of the jets from the MB event are recalculated relative to the vertex of the \( \gamma + \) jet event. The resulting mixed event is required to satisfy the same selection criteria as applied to \( \gamma + 3 \) jet data events with a single \( p\bar{p} \) collision. The MINDP sample provides independent parton scatterings with \( \gamma + \) jet and dijet final states, by construction. In particular, since the \( \gamma + \) jet process is dominated by small parton momentum fractions (\( x \)), the \( x \) values in the dijet production process remaining after the first parton interaction occurred is generally unaffected, i.e. the two interactions have negligible correlation in the momentum space. The mixing procedure is shown schematically in Fig. 2. The MINDP events shown in Fig. 2(b) comprise about 60% of both inclusive and HF samples.

- **DI data event model (mixdi):**
  The DI signal event model assures that the \( \gamma + 3 \) jet DI events originate from two separate \( p\bar{p} \) collisions by preparing a mixture of \( \gamma + \geq 1 \) jet events from the \( \gamma + \) jet data and of MB events with requirements of \( \geq 1 \) selected jets and two \( p\bar{p} \) collision vertices for both data samples. Thus, the second \( p\bar{p} \) collision contains only soft underlying energy that can contribute energy to a jet cone, or a photon isolation cone. In addition, in the case of \( \geq 2 \) jets in either component of the MINDI mixture (i.e., in \( \gamma + \) jet or MB events), the two leading jets are required to originate from the same vertex, using jet track information, as discussed in Appendix B of Ref. [15]. Since the \( p_T \) of all reconstructed objects is calculated with respect to the primary \( p\bar{p} \) collision vertex.
Using the $\gamma+\text{jet}$ and dijet MC samples, we create $\gamma+3\text{jet}$ and dijet MC models, similar to those constructed for MIXDP and MIXDI data samples, by examining information for jets and the photon at both the reconstructed and particle level. These samples are used to calculate efficiencies and acceptances for DP and DI events. As a cross check, we have compared $p_T$ and $\eta$ distributions for the jets and the photon at the reconstruction level in these models with those in the MIXDP and MIXDI data samples. Small discrepancies have been resolved by reweighting the MC spectra and creating models denoted as data-like MCDP and MCDI.

### B. Background models

To extract fractions of DP and DI events from data, we need to build SP background models.

- **SP one-vertex event model (SP1VTX):**

  A background to the DP events are single parton-parton scatterings with two additional bremsstrahlung jets resulting in a $\gamma+3$ jet final state in a single $p\bar{p}$ collision event. To model this background, we consider a sample of MC $\gamma+3$ jet events generated with MPI modeling removed. The SP1VTX sample contains the final state with a photon, leading jet, and two additional bremsstrahlung jets with the same selection criteria as applied to the data sample with a single $p\bar{p}$ collision vertex. The SHERPA SP model is taken as the default.

- **SP two-vertex event model (SP2VTX):**

  The background to DI events differs from the SP1VTX model in that the $\gamma+3$ jet MC events are selected with two reconstructed $p\bar{p}$ collision vertices. Events with no jet activity in the second vertex are selected by requiring the three jets to originate from the primary $p\bar{p}$ collision vertex.

To model the background to the $\gamma+b/c$ jet + 2 jet DP and DI processes, the SP1VTX and SP2VTX samples are constructed using the same techniques, but using $\gamma+b/c$-jet events generated with the PYTHIA and SHERPA MCs with MPI modeling removed.

### V. DISCRIMINATING VARIABLE

Unlike the SP scattering $2 \rightarrow 4$ process, which produces a $\gamma+\text{jet}$ final state and two bremsstrahlung jets, the DP mechanism has two independent $2 \rightarrow 2$ parton-parton scatterings within the same $p\bar{p}$ collision, resulting in substantially different kinematic distributions in the final state. Discrimination between these processes is obtained by examining the azimuthal angle between the $p_T$...
vectors of two object pairs in $\gamma + 3$ jet events,

$$\Delta S \equiv \Delta \phi \left( \vec{P}_T^1, \vec{P}_T^2 \right),$$

where $\vec{P}_T^1 = \vec{p}_T^\gamma + \vec{p}_T^{jet1}$ and $\vec{P}_T^2 = \vec{p}_T^{jet2} + \vec{p}_T^{jet3}$. Figure 3 illustrates a possible orientation of photon and jets transverse momentum vectors in $\gamma + 3$ jet events, as well as the vectors $\vec{P}_T^1$ and $\vec{P}_T^2$.

The differential cross section as a function of $\Delta S$ was measured in Ref. [16] and compared with various SP and MPI models. Momentum conservation causes $\Delta S$ to peak near $\pi$, and this is particularly visible in SP, although detector resolution effects and additional gluon radiation produce a significant number of events at smaller angles. For DP events, where the photon and leading jet usually come from one parton-parton scattering and the two other jets usually come from another parton-parton scattering, the pairwise balance $\Delta S$ angle has no pronounced peak at any particular value, although some residual bias remains towards $\Delta S = \pi$ caused by the DP events shown in Fig. 2(b).

**VI. FRACTIONS OF DP AND DI EVENTS**

**A. Fractions of DP events**

To calculate $\sigma_{\text{eff}}$, we need the number of DP events ($N_{\text{DP}}$) in Eq. (5), given by the product of the fraction of DP events ($f_{\text{DP}}$) and the size of the 1vtx sample. The fraction $f_{\text{DP}}$ is estimated in the $\gamma + 3$ jet 1vtx data sample using the DP (MIXDP) and SP (SP1vtx) models. The DP fractions (and $\sigma_{\text{eff}}$) are measured in the inclusive and HF samples separately.

The fraction $f_{\text{DP}}$ is found using a maximum likelihood fit [34] of the $\Delta S$ distribution of the data to signal and background templates that are taken to be the shapes of the $\Delta S$ distribution in the MIXDP and SP1vtx models, respectively. Signal and background samples used as templates, described in Section IV, satisfy all the selection criteria applied to the data sample.

**FIG. 4:** (color online) The $\Delta S$ distribution in the data, DP and SP models, and the sum of the DP and SP contributions weighted with their fractions (“Total”). The plots (a) and (b) correspond to the inclusive and HF samples, respectively. The lower subplots show the relative difference of the data points with respect to the fitted sum, along with the total uncertainties, i.e. DP fraction and statistical uncertainties from data and MC added in quadrature.

A first approximation to the fractions can be obtained from the fits to inclusive and HF data shown in Fig. 4.
The measured DP fractions are:

\[ f_{\text{DP}}^{\text{inc}} = 0.202 \pm 0.007 \]  

and

\[ f_{\text{DP}}^{\text{HF}} = 0.171 \pm 0.020, \]  

respectively. If it is not stated otherwise, the uncertainties shown in Eqs. 7, 8, and in the text below are only statistical. The sum of DP and SP models weighted with their fractions describes the data with \( \chi^2/\text{ndf} = 0.45 \) for the inclusive case and \( \chi^2/\text{ndf} = 0.26 \) (with the number of degrees of freedom, \( \text{ndf} = 7 \)) for the HF sample, i.e. \( \approx 87\% \) and \( \approx 97\% \) \( \chi^2 \)-probability, respectively.

While the default SP model obtained with sherpa, provides a reasonable description of the \( \Delta S \) distribution in data, it might not be perfect for other related kinematic variables, which may affect the DP fractions as well. For this reason we examine two alternative models. Since the fraction of events with the leading jet coming from the second parton interaction is small (\( \lesssim 10\% \)), the \( \Delta\phi(\gamma, \text{jet1}) \) distribution (the azimuthal angle between the photon and leading jet \( p_T \) vectors) in the inclusive \( \gamma + 3 \) jet events should be sensitive to initial and final state radiation effects in the \( \gamma + \text{jet} \) events. We construct a modified \( \gamma + 3 \) jet SP model in which the MC \( \Delta\phi(\gamma, \text{jet1}) \) distribution is reweighted to agree with data, as discussed in the Appendix. The \( f_{\text{DP}}^{\text{inc, rew1}} \) fraction obtained with the \( \Delta\phi(\gamma, \text{jet1}) \) reweighted SP model is 0.216 \( \pm 0.007 \). The shapes of the \( p_T \) spectra of the second and third jets are important for the \( \Delta S \) calculation. To estimate the effects of possible mismodeling of the jet \( p_T \) spectra, we create an alternative SP model by reweighting the jet \( p_T \) distributions in the default MC SP model in two dimensions (\( p_T \) of the second and third jet) to SP data. After reweighting, the DP fraction is recalculated and found to be \( f_{\text{DP}}^{\text{inc, rew2}} = 0.195 \pm 0.007 \). The sum of DP and the \( \Delta\phi(\gamma, \text{jet1}) \) (jet \( p_T \) reweighted SP models weighted with their fractions describes the data with \( \chi^2/\text{ndf} = 0.43 \) (\( \chi^2/\text{ndf} = 0.43 \)), \( \text{ndf} = 7 \).

The fraction obtained by averaging \( f_{\text{DP}} \) values after reweighting the \( \Delta\phi(\gamma, \text{jet1}) \) and second and third jet \( p_T \) spectra is used as a central value, and the difference between this and the value obtained with the default SP model (Eqs. 7 and 8) is taken as a systematic uncertainty. The final DP event fraction in the inclusive sample is

\[ f_{\text{DP}}^{\text{inc, avg}} = 0.206 \pm 0.007 \text{ (stat)} \pm 0.004 \text{ (syst)}. \]  

A similar reweighting procedure and determination of central value and the assignment of uncertainties is applied for the SP model in the HF sample, and the DP fraction is found to be

\[ f_{\text{DP}}^{\text{HF, avg}} = 0.173 \pm 0.020 \text{ (stat)} \pm 0.002 \text{ (syst)}. \]  

All the results on DP fractions are summarized in Table III.

The measured DP fraction is lower than that measured in the earlier D0 analysis [15]. This is primarily because of the smaller jet cone radius used in the current measurement (\( R = 0.5 \) vs. \( R = 0.7 \) in [15]), what leads to a smaller probability to pass the jet reconstruction threshold (6 GeV for the uncorrected jet \( p_T \)). The use of a smaller jet cone also significantly reduces the dijet cross section (a factor of \( 1.5 - 2 \)) in the \( p_T \) region of interest. Because the second parton interaction produces mostly a dijet final state, the measured DP fraction drops.

In addition to the SP events produced in single \( p\bar{p} \) collisions, another source of possible background to the single-vertex \( \gamma + 3 \) jet DP events are two \( p\bar{p} \) collisions produced very close to each other along the beam direction, so that a single vertex is reconstructed. This contribution is estimated using the instantaneous luminosity, the bunch size, the time between bunch crossings, and the vertex resolution, and found to be negligible at a level of \( \lesssim 0.2\% \).

### B. Fractions of DI events

In addition to \( f_{\text{DP}} \), the fraction of DI events (\( f_{\text{DI}} \)) occurring in events with two \( p\bar{p} \) collisions within the same bunch crossing must be determined to measure \( \sigma_{BB} \). A discriminant is constructed using the track information of a jet and of the assignment of tracks to the two \( p\bar{p} \) collision vertices (PV0 and PV1). We use the \( p_T \)-weighted position along the beam (\( z \)) axis of all tracks associated to the jet and the fraction of charged particles in the jet (CPF). The CPF discriminant is based on the fraction of total charged particles’ transverse momentum (i.e., total track \( p_T \)) in each jet \( i \) originating from each identified vertex \( j \) in the event:

\[ \text{CPF}(\text{jet}_i, \text{vtx}_j) = \frac{\sum_k p_T(\text{trk}_k^{\text{jet}_i, \text{vtx}_j})}{\sum_n \sum_l p_T(\text{trk}_l^{\text{jet}_i}, \text{vtx}_n)}. \]  

Each jet is required to have CPF > 0.5 and at least two tracks.

In events with two \( p\bar{p} \) collisions, jets in \( \gamma + 3 \) jet events may originate either from PV0 or PV1. The leading jet is required to originate from PV0. Four classes of events are defined:

I: All three jets originate from PV0.

II: Jet 1 and jet 2 originate from PV0 while jet 3 originates from PV1.

III: Jet 1 and jet 3 originate from PV0 while jet 2 originates from PV1.
IV: Jet 1 originates from PV0 while jet 2 and jet 3 originate from PV1.

Class I corresponds to a type of $\gamma + 3$ jet event that has all three jets originating from the same $p\bar{p}$ collision with no reconstructed jets in the other, i.e., background (non-DI) events, while classes II, III and IV correspond to three types of signal (DI) events.

To assign a jet to a vertex and extract $f_{\text{DI}}$ using the jet track information, we need the $z$ resolution of the jet-to-vertex assignment algorithm, $\sigma_z$. This resolution can be calculated in the $\gamma + 3$ jet data event sample with a single $p\bar{p}$ collision. Since these events have only one reconstructed $p\bar{p}$ collision vertex, all the jets should originate from this vertex. To find the $z$ position of a jet's origin, we consider all tracks inside a jet cone and calculate the $p_T$-weighted position in $z$ of all the tracks ($z_{\text{jet}}$). The track $z$ position is calculated at the point of closest approach of each track to the beam axis. For each jet in the 1vtx data sample, we estimate the distance between the $z_{\text{jet}}$ and the $z$-vertex position, $\Delta z_{\text{(vtx, jet)}}$. We find $\sigma_z \approx 1.2$ cm and that 98%–99% of jets in 1vtx events have $\Delta z_{\text{(vtx, jet)}} < 3 \sigma_z$. We consider a jet to originate from a vertex if $|z - z_{\text{jet}}| < 3 \sigma_z$. If the jet is located within $3 \sigma_z$ of both vertices it is assigned to the closest vertex.

Table IV shows the fractions of 2vtx data events in each class. From this table, one can see that the single interaction events (Class I) dominate over DI events (sum of classes II, III, and IV). The DI event fraction is $f_{\text{DI}} = 0.135 \pm 0.002$ for the inclusive sample and $f_{\text{DI}}^{\text{HF}} = 0.131 \pm 0.010$ for the sample with a heavy flavor leading jet.

The distance in $z$ between two vertices $\Delta z_{\text{(PV0, PV1)}}$ may affect the measured DI fraction, since about 5% of events have $\Delta z_{\text{(PV0, PV1)}} < 3 \sigma_z$. No requirement is placed on this distance in the analysis. To quantify the dependence of the DI fraction on this distance, we have also measured the DI fraction with the requirement that the two vertices are separated by $\Delta z_{\text{(PV0, PV1)}} > 5 \sigma_z$. Table V shows $f_{\text{DI}}$ for the two cases: no cut and $\Delta z_{\text{(PV0, PV1)}} > 5 \sigma_z$. The difference between them is taken as a systematic uncertainty.

An additional uncertainty is due to the determination of the photon vertex. This uncertainty has been estimated using events with a photon EM cluster in the central region ($|\eta_{\text{jet}}| < 1.0$) with a matched CPS cluster. These events allow us to extrapolate the photon direction along the $z$ axis and determine the vertex position on the $z$ axis [28]. Using the $\gamma + 3$ jet data, we estimate the photon pointing resolution in $z$ to be about 4.5 cm. Using this resolution and the distribution of the distance in $z$ between the first and second vertices in 2vtx events, we find that the photon origin vertex may be misidentified in about 4% of events, which is taken as a systematic uncertainty.

The DI fractions extracted for the inclusive and heavy flavor samples are:

$$f_{\text{DI}} = 0.135 \pm 0.002 \text{(stat)} \pm 0.008 \text{(syst)},$$

$$f_{\text{DI}}^{\text{HF}} = 0.131 \pm 0.010 \text{(stat)} \pm 0.011 \text{(syst)}.$$

A cross check of the measured DI fractions is performed by fitting the $\Delta S$ templates for signal and background models to data as was done to extract the DP fraction in Section VI A. We use the MIXDI sample for the signal template and the SP2vtx sample for the background template (see Section IV). The measured fractions $f_{\text{DI}} = 0.127 \pm 0.021$ (with SP2vtx model taken from SHERPA) and $f_{\text{DI}} = 0.124 \pm 0.056$ (PYTHIA) are in good agreement with each other and with $f_{\text{DI}}$ obtained by the jet-track method. The results for the heavy flavor jet sample are $f_{\text{DI}}^{\text{HF}} = 0.153 \pm 0.044$ with the SP model from SHERPA and $f_{\text{DI}}^{\text{HF}} = 0.143 \pm 0.056$ using PYTHIA, which are also in agreement with the jet-track method. Since the results of this cross check agree with the values obtained using the jet track method, we do not assign an additional systematic uncertainty.

VII. DP AND DI EFFICIENCIES, $R_c$ AND $\sigma_{\text{hard}}$

A. Ratio of signal fractions in DP and DI events

A fraction of events with jets containing energetic $\pi^0$ or $\eta$ mesons may satisfy the photon selection criteria. The photon fraction in the selected data is estimated using the maximum likelihood fit of templates from the output of the photon identification neural network ($O_{\Sigma_N}$) in signal and background events to that in data, as described in detail in Ref. [27]. The photon fractions in DP and DI events are found to be similar. For example, for a photon in the central calorimeter (CC) region, $f_{\text{DP}}^{\Sigma_N} = 0.432 \pm 0.002$ and $f_{\text{DI}}^{\Sigma_N} = 0.437 \pm 0.004$ for DP and DI events, respectively. The photon fractions are slightly higher in the forward region due to tighter photon selections.

The fractions of events with $b$ or $c$ jets in the 1vtx and 2vtx data samples are estimated using templates for the
invariant mass of charged particle tracks associated with the secondary vertex, $M_{SV}$ (see Ref. [20]) for $\gamma + b/c$-jet and $\gamma +$ jet MC samples. The resulting HF fractions are dominated by $c$ quarks, $f_{\gamma c}^{DP} = 0.352 \pm 0.025$, $f_{\gamma c}^{DI} = 0.551 \pm 0.041$, and $f_{\gamma b}^{DP} = 0.327 \pm 0.019$, $f_{\gamma b}^{DI} = 0.573 \pm 0.043$. The HF fractions in DP and DI samples are in good agreement. Approximately 10% of the jets tagged as HF come from mistagged light quark jets.

The overall signal fractions in DP and DI samples and their ratio in the inclusive, $(f_{\gamma c}^{DP}/f_{\gamma c}^{DI})$ and HF samples, $(f_{\gamma c}^{HF}/f_{\gamma b}^{HF})$, are summarized in Table VI. The systematic uncertainties on the signal fraction are caused by the uncertainties on the photon and heavy flavor fractions from $O_{NN}$ and $M_{SV}$ template fitting.

### TABLE VI: The overall signal fractions in DP and DI samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>DP</th>
<th>DI</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive</td>
<td>$0.445 \pm 0.005$</td>
<td>$0.456 \pm 0.008$</td>
<td>$0.976 \pm 0.019$</td>
</tr>
<tr>
<td>HF</td>
<td>$0.402 \pm 0.030$</td>
<td>$0.405 \pm 0.030$</td>
<td>$0.993 \pm 0.104$</td>
</tr>
</tbody>
</table>

#### B. Ratio of signal efficiencies in DP and DI events

The selection efficiencies for DP and DI events enter Eq. (5) only as ratios, substantially canceling correlated systematic uncertainties. The DP and DI events differ from each other by the number of $p\bar{p}$ collision vertices (one vs. two), and therefore their selection efficiencies $\varepsilon_{DI}$ and $\varepsilon_{DP}$ may differ due to different amounts of soft unclustered energy in the single and double $p\bar{p}$ collision events. This could lead to a difference in the jet reconstruction efficiencies because of the different probabilities for jets to pass the $p_T > 6$ GeV requirement applied during jet reconstruction. It could also lead to different photon selection efficiencies because of different amounts of energy in the track and calorimeter isolation cones around the photon. To estimate these efficiencies, we use the data-like MC$D_P$ and MC$D_I$ samples described in Section IV.

Using these models, we find the ratio of the geometric and kinematic acceptances for DP and DI events to be $A_{DP}/A_{DI} = 0.551 \pm 0.010$ (stat) $\pm 0.030$ (syst) for the inclusive sample and $A_{BF}^{HF}/A_{BF}^{HI} = 0.567 \pm 0.021$ (stat) $\pm 0.052$ (syst) for the HF sample. The difference between the $A_{DP}$ and $A_{DI}$ acceptances is caused by an average difference of 0.5 GeV in jet $p_T$ due to the offset energy entering the jet cone from the second vertex [30]. This significantly increases the reconstruction efficiency of jets (mainly for second and third jets) in DI events. The differences between the acceptances obtained with data-like and default MC$D_P$ and MC$D_I$ models are taken as systematic uncertainty. An additional systematic uncertainty (about 1%) is caused by the difference between photon identification efficiencies obtained with SHERPA and PYTHIA. For the HF sample, we also correct for the $b$-tagging selection efficiency. The ratio of the HF jet selection efficiencies is $\varepsilon_{\gamma c}^{HF}/\varepsilon_{\gamma b}^{HF} = 1.085 \pm 0.019$. This number is obtained by weighting $b$- and $c$-jet efficiencies with their fractions found in Section VII A. The typical HF jet selection efficiency is 60% (10%) for the tight $b(c)$ jet selection. Only about 0.5% of the light jets are misidentified as heavy flavor jets [20, 31]. The $b$-tagging efficiency decreases with increasing number of $p\bar{p}$ collision vertices due to a larger hit density in the SMT detector and a decrease in the track reconstruction efficiency. This also explains the lower $N_{2vtx}/N_{1vtx}$ ratio for the HF sample compared to the inclusive sample in Table II.

#### C. Vertex efficiencies

The vertex efficiency $\varepsilon_{1vtx} \ (\varepsilon_{2vtx})$ corrects for single (double) collision events that are lost in the DP (DI) candidate sample because of the single (double) vertex requirements ($|z_{vtx}| < 60$ cm and $\geq 3$ tracks). The ratio $\varepsilon_{1vtx}/\varepsilon_{2vtx}$ is calculated from the data and found to be $1.05 \pm 0.01$. The probability to miss a hard interaction event having at least one jet with $p_T > 15$ GeV due to a non-reconstructed vertex is < 0.5% and is ignored.

We might also have an additional fake reconstructed vertex that passes the vertex requirement. This probability is estimated using $\gamma +$ jet events and $\gamma + 3$ jet events simulated in MC without zero-bias events overlay, as these events should contain only one vertex. We find that the probability to have a second (fake) vertex is < 0.1% and is ignored.

#### D. Calculating $R_c$, $\sigma_{\text{hard}}$, $N_{1\text{coll}}$ and $N_{2\text{coll}}$

We calculate the numbers of expected events with one ($N_{1\text{coll}}$) and two ($N_{2\text{coll}}$) $p\bar{p}$ collisions resulting in hard interactions following the procedure of Ref. [15], which uses the hard $p\bar{p}$ interaction cross section $\sigma_{\text{hard}} = 44.76 \pm 2.80$ mb. The values of $N_{1\text{coll}}$ and $N_{2\text{coll}}$ are obtained from a Poisson distribution parametrized with the average number of hard interactions in each bin of the instantaneous luminosity $L_{\text{inst}}$, distribution, $(n) = (L_{\text{inst}}/f_{\text{cross}})\sigma_{\text{hard}}$, where $f_{\text{cross}}$ is the frequency of beam crossings for the Tevatron [23]. Summing over all $L_{\text{inst}}$ bins, weighted with their fractions, we get $R_c = (1/2)\langle N_{1\text{coll}}/N_{2\text{coll}}\rangle(\varepsilon_{1vtx}/\varepsilon_{2vtx}) = 0.45$. This number is smaller by approximately a factor of two compared to that for the data collected earlier as reported in Ref. [15]. Since $R_c$ and $\sigma_{\text{hard}}$ enter Eq. 5 for $\sigma_{\text{eff}}$ as a product, any increase of $\sigma_{\text{hard}}$ leads to an increase of $(n)$ and, as a consequence, to a decrease in $R_c$ and vice versa. Due to this partial cancellation of uncertainties, although the measured value of $\sigma_{\text{hard}}$ has a 6% relative uncertainty, the product $R_c\sigma_{\text{hard}}$ only has a 2.6% uncertainty, $R_c\sigma_{\text{hard}} = 18.92 \pm 0.49$ mb.


VIII. RESULTS

Using Eq. 5, we obtain the following effective cross sections:

\[
\sigma_{\text{eff}}^{\text{incl}} = 12.7 \pm 0.2 \text{ (stat)} \pm 1.3 \text{ (syst)} \text{ mb},
\]

\[
\sigma_{\text{eff}}^{\text{HF}} = 14.6 \pm 0.6 \text{ (stat)} \pm 3.2 \text{ (syst)} \text{ mb}.
\]

Within uncertainties, the effective cross section in the inclusive event sample is consistent with that in the event sample with identified heavy flavor jets.

The main sources of systematic uncertainties are summarized in Table VII. They are caused by uncertainties in the DP and DI fractions, the ratio of efficiencies and acceptances in DP and DI events ("\(\varepsilon_{\text{DP}}/\varepsilon_{\text{DI}}\)"), signal fractions ("sig. frac."), the uncertainty in the ratio of the number of hard interactions with single and double \(p\bar{p}\) hard collisions times \(\sigma_{\text{hard}}^{\gamma}(\frac{R_T}{\sigma_{\text{hard}}})\), and jet energy scale ("JES"). The latter is obtained from the variation of JES uncertainties up and down by one standard deviation for all three jets [30].

Figure 5 shows all existing measurements of \(\sigma_{\text{eff}}\). The \(\sigma_{\text{eff}}^{\text{incl}}\) and \(\sigma_{\text{eff}}^{\text{HF}}\) from this measurement agree both with the previous D0 measurement [15] and with those obtained by other experiments. These new measurements of \(\sigma_{\text{eff}}\) are the most accurate to date, and also provide the first measurement involving heavy quarks.

\[
\text{Experiment, Final state (Year)}
\]

\[
\begin{array}{|c|}
\hline
\text{AFS, 4j (1986)} \quad \bullet \\
\text{UA2, 4j (1991)} \quad \bullet \\
\text{CDF, 4j (1993)} \quad \Delta \\
\text{CDF, 3j (1997)} \quad \Delta \\
\text{D0, 3j (2009)} \quad \bullet \\
\text{ATLAS, W2j (2013)} \quad \Delta \\
\text{CMS, W2j (2013)} \quad \bullet \\
\text{D0, yb(c)2j (2013)} \quad \Delta \\
\hline
\end{array}
\]

![Figure 5](image_url)

FIG. 5: (color online) Existing measurements of effective cross section, \(\sigma_{\text{eff}}\), compared with result presented here (AFS: no uncertainty is reported; UA2: only a lower limit is provided).

IX. DISCUSSION OF PDF EFFECTS

The experimentally measured effective cross section \(\sigma_{\text{eff}}\), presented in Eqs. 14 and 15, should be corrected for the effect of double parton PDF (dPDF) evolution [35–37]. The dPDF evolution starts at a small scale \(Q_0\), \(\mathcal{O}(1 \text{ GeV})\), where the two PDFs corresponding to partons participating in DP scattering can be factorized. The dPDF evolution results in a correlation term at a larger energy scale \(Q\), which necessitates the following correction:

\[
[\sigma_{\text{eff}}]^{-1} = [\sigma_{\text{eff}}^0]^{-1}(1 + \Delta(Q)) \quad [35],
\]

where \(\Delta(Q)\) is a contribution induced by the dPDF correlation term, and \(\sigma_{\text{eff}}^0\) depends only on the spatial distribution of parton flavors. To estimate this correction factor, we have employed software, provided by the authors of Ref. [36]. It uses a numerical integration of the leading order DGLAP [38] equation for the dPDFs, and which may be used to evolve the input dPDFs to any other scale. To get access to the kinematics of the first and second parton interactions, the relevant part of the PYTHIA code was modified for us by the PYTHIA authors. The evolution effect has been evaluated by examining the ratio

\[
R_p(x_1, x_2; Q) = \frac{D_p(x_1, x_2; Q)}{D_p(x_1; Q)D_p(x_2; Q)}
\]

where \(D_p(x_1, x_2; Q)\) is the dPDF with the parton momentum fractions \(x_1, x_2\) of the two partons participating in the first and second parton interactions on the proton side at scale \(Q\), and \(D_p(x_1; Q)\) is a single parton MSTW2008LO PDF [39]. A similar equation can be written for the partons on the antiproton side. Using the simulated \(\gamma + 3\) jet and \(\gamma + b/c + 2\) jet events, and applying our kinematic cuts, we have found the product of the two ratios \(R_pR_p = 1.01\) for \(\gamma + 3\) jet and 1.02 for \(\gamma + b/c + 2\) jet events. This correction is expected to have a larger deviation from unity for higher \(Q\) (e.g., it was found to be 0.93 for \(\gamma + 3\) jet at \(p_T = 70\text{ GeV}\) that corresponds to the previous D0 measurement [15]). In general, it should be calculated for each set of final states and kinematic selections. Currently, the dPDF evolution implemented in Ref. [36] is available at leading order accuracy, while having it at next-to-leading order would be preferable. Due to the smallness of the found correction (1.01–1.02), and uncertainties related with the leading order approximation, this correction is not applied to the measured \(\sigma_{\text{eff}}\).

X. SUMMARY

We have analyzed samples of \(\gamma + 3\) jet and \(\gamma + b/c + 2\) jet events collected by the D0 experiment with an integrated luminosity of about 8.7 fb\(^{-1}\) and determined the fractions of events with hard double parton scattering occurring in a single \(p\bar{p}\) collision at \(\sqrt{s} = 1.96\text{ TeV}\). In the kinematic region \(p_T > 26\text{ GeV}, p_T^{\gamma} > 15\text{ GeV}, 15 < p_T^\text{jet} < 35\text{ GeV}\), we observe that about (21 ± 1)% and (17±2)% of the events are produced in double parton interactions in the \(\gamma + 3\) jet and \(\gamma + b/c + 2\) jet final states. The effective cross section \(\sigma_{\text{eff}}\), which characterizes the spatial transverse parton distribution in a nucleon, is found to be \(\sigma_{\text{eff}}^{\text{incl}} = 12.7 \pm 0.2\text{ (stat)} \pm 1.3\text{ (syst)}\) mb in \(\gamma + 3\) jet and \(\sigma_{\text{eff}}^{\text{HF}} = 14.6 \pm 0.6\text{ (stat)} \pm 3.2\text{ (syst)}\) mb in \(\gamma + b/c + 2\) jet final states.
TABLE VII: The systematic uncertainties from measurement of DP \( f_{DP} \) and DI \( f_{DI} \) fractions, the ratio of efficiencies and acceptances in DP and DI events \( \frac{e_{DP}}{e_{DI}} \), signal fractions \( \text{sig. frac.} \), the uncertainty in the number of hard interactions with single and double \( pp \) hard collisions times \( \sigma_{\text{hard}} \) \( (R, \sigma_{\text{hard}}) \), and jet energy scale \( \text{JES} \), shown together with overall systematic \( \delta_{\text{sys}} \), statistical \( \delta_{\text{stat}} \) and total \( \delta_{\text{total}} \) uncertainties (in \%) for the \( \sigma_{\text{eff}} \) measurement. The total uncertainty \( \delta_{\text{total}} \) is calculated by adding the systematic and statistical uncertainties in quadrature.

<table>
<thead>
<tr>
<th>Data</th>
<th>Sources of systematic uncertainty</th>
<th>( f_{DP} )</th>
<th>( f_{DI} )</th>
<th>( e_{DP}/e_{DI} )</th>
<th>sig. frac.</th>
<th>( R, \sigma_{\text{hard}} )</th>
<th>JES</th>
<th>( \delta_{\text{sys}} )</th>
<th>( \delta_{\text{stat}} )</th>
<th>( \delta_{\text{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive</td>
<td></td>
<td>3.9</td>
<td>6.5</td>
<td>5.6</td>
<td>2.0</td>
<td>2.6</td>
<td>2.9</td>
<td>10.4</td>
<td>1.8</td>
<td>10.6</td>
</tr>
<tr>
<td>HF</td>
<td></td>
<td>11.6</td>
<td>11.2</td>
<td>9.4</td>
<td>10.4</td>
<td>2.6</td>
<td>1.3</td>
<td>21.6</td>
<td>4.0</td>
<td>22.0</td>
</tr>
</tbody>
</table>

Our value of \( \sigma_{\text{eff}} \) is in agreement with the results of previous measurements and has a higher precision. This is the first measurement of \( \sigma_{\text{eff}} \) with heavy flavor jets in the final state. Due to the significant dominance of the Compton-like process (see Fig. 1), we may conclude that there is no evidence for a dependence of \( \sigma_{\text{eff}} \) on the initial parton flavor.

Acknowledgements

We are grateful to J. R. Gaunt, W. J. Stirling, T. Sjöstrand and P. Z. Skands for providing their codes and many helpful discussions.

We thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); MON, NRC KI and RFBR (Russia); CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); NRF (Korea); FOM (The Netherlands); STFC and the Royal Society (United Kingdom); MSMT and GACR (Czech Republic); BMBF and DFG (Germany); SFI (Ireland); The Swedish Research Council (Sweden); and CAS and CNSF (China).

XI. APPENDIX

In Section VI A, we estimate the DP event fraction using the predictions of Monte Carlo SP models. In this appendix, we test variables that characterize the SP model and are related to the \( \Delta S \) distribution used to calculate the DP fractions in Section VI A.

The variable \( \Delta \phi(\gamma, \text{jet1}) \) is sensitive to initial and final state radiation and is strongly correlated to the \( p_T \) sum vector of the photon and leading jet system, \( \vec{P}_T = \vec{p}_T^\gamma + \vec{p}_T^{\text{jet1}} \) (see Eq. 6). We compare the distribution of \( \Delta \phi(\gamma, \text{jet1}) \) in the MC SP sample to data. The latter is obtained after subtracting the DP contribution, predicted by the DP data model \( \text{mixdp} \), according to the DP fractions in Eqs. 7 and 8.

The comparison of the \( \Delta \phi(\gamma, \text{jet1}) \) spectra for the SP model extracted from data with those in the SHERPA and PYTHIA MC generators is shown in Fig. 6. The SHERPA SP event model agrees better with the data compared to PYTHIA, where the \( \Delta \phi(\gamma, \text{jet1}) \) distribution is shifted towards \( \pi \), resulting in much worse agreement with data. For this reason, the subsequent analysis is performed using the SHERPA SP model only.

The MC SP predictions for the \( p_T \) spectra of the second and third jets are also important since, in addition to the vector \( \vec{P}_T \), they form the other imbalance vector of the \( \Delta S \) variable, \( \vec{P}_3 = \vec{p}_T^{\text{jet2}} + \vec{p}_T^{\text{jet3}} \) (see Eq. 6). Figure 7 illustrates the transverse momenta of the second and third jets of the SHERPA and data SP models. Both jet-\( p_T \) spectra in SHERPA agree well with those in data.

However to construct a better (data-like) SP model, the original default SP model from SHERPA is reweighted either in \( \Delta \phi(\gamma, \text{jet1}) \) bins, or in two dimensions of second and third jet \( p_T \). These two alternative data-like SP models are considered in Section VI A to calculate the DP fractions. The later are compared to the DP fractions obtained with the default SP model to derive related systematic uncertainties.

FIG. 6: (color online) The $\Delta\phi(\gamma,\text{jet1})$ distribution in the SP model extracted from data compared to that in (a) SHERPA, (b) PYTHIA. The uncertainties shown are statistical only.

FIG. 7: (color online) Spectra of the transverse momenta of (a) second and (b) third jets in the SHERPA and data SP models. The uncertainties shown are statistical only.

[18] S. Chatrchyan et al. (CMS Collaboration), submitted to
[26] The polar angle \( \theta \) and the azimuthal angle \( \phi \) are defined with respect to the positive \( z \) axis, which is along the proton beam direction. Pseudorapidity is defined as \( \eta = -\ln[\tan(\theta/2)] \). \( \eta_{\text{det}} \) and \( \phi_{\text{det}} \) are the pseudorapidity and the azimuthal angle measured with respect to the center of the detector.