Lattice QCD simulation of the $Z_c^+$ channel

Sasa Prelovsek**,+, C. B. Lang †, Luka Leskovec** and Daniel Mohler‡

** Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia
† Institute of Physics, University of Graz, A–8010 Graz, Austria
‡ Jozef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia
‡ Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510-5011, USA

Abstract. We discuss the lattice QCD simulations that search for the $Z_c^+$ with the unconventional quark content $car{c}du$ in the channel $I^G(J^{PC}) = 1^+(1^{-+})$. The major challenge is presented by the two-meson states $J/\psi \pi$, $\psi_2\pi$, $\psi_3\pi$, $D^*\pi$, $D^*\bar{D}^*\pi$, $\eta_\rho$ that are also inevitably present in this channel. The available lattice simulations find expected two-meson eigenstates, but no additional eigenstate as a candidate for $Z_c^+$. This is in a striking contrast to the lattice results in the flavour non-exotic channels, where additional states are found in relation to the most of known resonances and bound states.

Keywords: charmonium-like states, lattice QCD, tetraquark

PACS: 11.15.Ha, 12.38.Gc, 14.40.Pq, 13.25.Gv

INTRODUCTION

Several charged-charmonia with quark content $car{c}du$ were discovered recently in experiment, as reviewed in [1, 2]. The first of these states was the $Z_c^+$ (4430), discovered in 2007 by Belle [3], remained unconfirmed by BaBar [4], and was recently confirmed by LHCB [5]. The $Z_c^+(3900) \to J/\psi \pi^+$ was discovered in 2013 slightly above $D\bar{D}^*$ threshold by BESIII [6] and was confirmed by Belle [7] as well as using CLEO-c data [8]. The spin and parity of $Z_c^+(3900)$ are unclear and it may correspond to the same state as $Z_c^0$ by BESIII [6], and was confirmed by Belle [7] as well as using CLEO-c data [8]. The spin and parity of $Z_c^+$ are unclear and $J^P = 1^+$ is preferred. Finally, $Z_c^+(4200) \to J/\psi \pi^+$ was reported in 2014 by Belle [12] favoring $J^P = 1^+$. All these states have $G$-parity $G = +1$ while their neutral partners have charge conjugation $C = −1$. Therefore we focus on the channel with $I^G(J^{PC}) = 1^+(1^{-+})$.

We note that $Z_c^0(3900)$ was found in $J/\psi \pi$ invariant mass only through $e^+e^− \to Y \to (J/\psi \pi^+)\pi^−$. No resonant structure in $J/\psi \pi^+$ was seen in $\bar{B}^0 \to (J/\psi \pi^+)K^−$ by BELLE [12], in $\bar{B}^0 \to (J/\psi \pi^+)\pi^−$ by LHCB [13] or in $\gamma p \to (J/\psi \pi^+)n$ by COMPASS [14]. This might indicate that the $Z_c^+$ peak might not be of dynamical origin [15, 16], which was recently questioned in [17].

It is an important theoretical task to establish whether QCD supports the presence of an exotic state with quark content $car{c}du$ using ab-initio lattice QCD. So far lattice QCD found evidence for most of the observed flavour non-exotic states: those include for example all charmonia below open charm threshold, shallow bound states $X(3872)$ with $I = 0$, $D_{s0}(2317)$, $D_{s1}(2460)$, and meson resonances $\rho$, $a_1$, $b_1$, $K^*(892)$, $K_0^*(1430)$, $D_0^*(2400)$, $D_1(2430)$. All these manifest themselves via an additional energy level in the discrete spectrum, as discussed bellow. On the other hand, lattice QCD has not found yet reliable evidence for the mesons with manifestly exotic flavour, for example $X(3872)$ with $I = 1$, $Z_c^+ \simeq car{c}du$ or $car{c}d\bar{d}$.

Here we report on the search for $Z_c^+$ in the energy region below 4.2 GeV. In a lattice QCD simulation, the states are identified from discrete energy-levels $E_n$ and in principle all physical eigenstates with the given quantum number $I^G(J^{PC}) = 1^+(1^{-+})$ appear. So the two-meson states $J/\psi \pi$, $\psi_2\pi$, $\psi_3\pi$, $D^*\pi$, $D^*\bar{D}^*\pi$, $\eta_\rho$ are eigenstates and their presence is the major challenge in searching for $Z_c^+$ candidates.

TWO-MESON STATES IN $Z_c$ CHANNEL

All the states with given quantum numbers $I^G(J^{PC}) = 1^+(1^{-+})$ appear as eigenstates of lattice QCD in principle. The eigenstate of interest, $Z_c^+$, gives an energy level at $E_n \simeq m_{Z_c}$ if it exists. However, various two-meson states $M_1(p)M_2(−p)$ have the same quantum numbers and give also rise to physical eigenstates, which presents a major
The appearance of the two-meson decay products with a continuous energy spectrum. If the two mesons do not interact, then

\[ E^{n,i} = E_1(k) + E_2(k), \quad E_{1,2}(k) = \sqrt{m_{1,2}^2 + k(k)} \]  

(1)

with \( k \equiv k^2 \). These values are slightly shifted in presence of the interaction. In experiment, these states correspond to the two-meson decay products with a continuous energy spectrum.

Our simulation employs dynamical \( u \) and \( d \) quarks that correspond to the pion mass \( m_\pi \sim 266 \text{ MeV} \) [18]. The lattice spacing is \( a = 0.1239(13) \text{ fm} \). The rather small box \( V = 16^3 \times 32 \) with \( L \sim 2 \text{ fm} \) may lead to sizable finite volume corrections, but it is responsible for a crucial practical advantage. It makes the \( \mathcal{Z}_c \) search tractable since it reduces the number of \( M_1(k)M_2(-k) \) states in the considered energy range.

On this lattice the two-particle states with \( P^F(J^P) = 1^- (1^-) \) and total momentum zero in the energy region of interest \( E \leq 4.3 \text{ GeV} \) are

\[
\begin{align*}
J(0)\pi(0), \quad \eta_c(0), \quad \eta_c(1)J(1), \quad D(0)\bar{D}^*(0), \quad \psi_{2S}(0)\pi(0), \quad D^+(0)\bar{D}^*(0), \quad \psi_{1D}(0)\pi(0), \\
\eta_c(1)\rho(1), \quad D(1)\bar{D}^*(1), \quad \psi_3(0)\pi(0), \quad J/\psi(2)\pi(2), \quad D^+(1)\bar{D}^*(1), \quad D(2)\bar{D}^*(2)
\end{align*}
\]

(2)

in order of increasing energy. Their lattice energies \( E^{n,i} \) in the non-interacting limit are denoted by the horizontal lines in Fig. 1b and the values follow from the masses and single-meson energies determined on the same set of gauge configurations [19, 20]. Establishing two-meson states up to \( 4.3 \text{ GeV} \) at \( m_\pi = 266 \text{ MeV} \) should suffice for searching fairly narrow exotic candidates with mass below \( 4.07 \text{ GeV} \) for physical pion mass.

Our aim is to extract and identify all two-particle energy-levels (2) from the full, coupled correlator matrix of hadron operators and establish whether QCD predicts additional states related to the exotic \( \mathcal{Z}_c \) hadron.

This goal presents a considerable challenge by itself. Note that a rigorous treatment (via a Lüscher-type finite volume formalism) would require the determination of the scattering matrix for all two-particle channels that couple, and a subsequent determination of the mass and the width for any \( \mathcal{Z}_c \) resonance(s). The elastic scattering within a single channel has been rigorously treated by a number of lattice simulations recently. The first lattice simulation aimed at determining scattering matrix for two-coupled channels [21] also shows promise in this respect, while the rigorous treatment of seven coupled channels is still beyond the capabilities of any lattice simulation at present.

Therefore we take a simplified approach where the existence of \( \mathcal{Z}_c \) is investigated by analyzing the number of energy levels, their positions and overlaps with the considered lattice operators \( \Omega \langle \bar{O}_j | n \rangle \). The formalism does predict an appearance of a level in addition to the (shifted) two-particle levels if there is a relatively narrow resonance in one channel. Additional levels have been, for example, found for resonances \( \rho \) [20], \( K^*(892) \) [22, 21], \( D^*_0(2400) \) [19], and the bound state \( D^*_0(2317) \) [23]. Additional levels related to \( K^0(1430) \) [21] and \( X(3872) \) [24] have been found in the simulations of two coupled channels. Based on this experience, we expect an additional energy level if \( \mathcal{Z}_c \) is of similar origin.

**TOWARDS THE LATTICE ENERGY SPECTRUM**

The energies \( E_n \) and the overlaps \( Z_j \) of the physical eigenstates \( n \) are extracted from the correlator matrix

\[
C_{jk}(t) = \langle \Omega | \bar{O}_j (t_{\text{src}} + t) | \bar{O}_k (t_{\text{src}}) | \Omega \rangle = \sum_n Z_j^n Z_k^n \langle \bar{O}_j | n \rangle \langle \bar{O}_k | n \rangle e^{-E_n t_{\text{src}}}, \quad Z_j^n \equiv \langle \Omega | \bar{O}_j | n \rangle.
\]

(3)

The physical system for given quantum numbers is created from the vacuum \( |\Omega\rangle \) using creation operators \( \bar{O}_j \) at time \( t_{\text{src}} \) and the system propagates for time \( t \) before being annihilated at \( t_{\text{sink}} = t_{\text{src}} + t \) by \( O_j \). The creation/annihilation operators are called interpolators. Our correlation matrix is averaged over every second \( t_{\text{src}} \).

We employ 22 interpolators \( O^{M_1M_2} \) that couple well to the two-meson states and the choice is expected to be complete enough to render all two-meson states listed in (2) [18]. In addition, we implement 4 diquark-antidiquark interpolators \( \bar{O}^{DQ} \) with structure \( [\bar{c}\bar{d}]_3, [\bar{c}u]_{3s} \) which is expected to couple well to possible \( \mathcal{Z}_c \) if it has a sizable Fock

---

1 The appearance of \( \psi_3 \pi \), where \( \psi_3 \) denotes the charmonium with \( J^{PC} = 3^{--} \), is an artifact due to reduced symmetry on the cubic lattice.
component of this kind. We point out that $\sigma_{4q} \simeq [\bar{c}\bar{d}]_3, [\bar{c}d]_3$ couples also to two-meson states via Fierz rearrangement. Representative examples of employed interpolators are

$$\begin{align*}
\sigma_{4}(0)x(0) &= \bar{c}\gamma c(0) \not{\bar{u}}u(0), \\
\sigma_{4}(1)x(-1) &= \sum_{\epsilon_{k} = \pm 1, s, c, \bar{c}}\bar{c}\gamma c(\epsilon_{k}) \not{\bar{u}} u(-\epsilon_{k}), \\
\sigma_{4}(2)x(-2) &= \sum_{|u_{k}|^2 = 2}\bar{c}\gamma c(u_{k}) \not{\bar{u}}u(-u_{k}), \\
\sigma_{4}(3)x(1) &= \bar{c}\gamma c(0) \not{\bar{u}} u(0), \\
\sigma_{4}^{D}(0) &= \bar{c}\gamma u(0) \not{\bar{c}} c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
\sigma_{4}^{(1)D}(0) &= \bar{c}\gamma u(0) \not{\bar{c}} c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
\sigma_{4}^{(2)D}(0) &= \epsilon_{ijk} \bar{c}\gamma u(0) \not{\bar{c}} c(0) \\
\sigma_{4}^{4q} &= \epsilon_{abcd}\epsilon_{ab'c'd'}(\bar{c}_5 c_\gamma d_\gamma d_c \not{c} u_c - \bar{c}_5 c_\gamma d_\gamma d_{\bar{c}} \not{c} u_{\bar{c}}) \\
\sigma_{4}^{4q} &= \epsilon_{abcd}\epsilon_{ab'c'd'}(\bar{c}_5 c_\gamma d_\gamma d_c \not{c} u_c - \bar{c}_5 c_\gamma d_\gamma d_{\bar{c}} \not{c} u_{\bar{c}}),
\end{align*}$$

While the full list of interpolators together with related details is provided in [18]. The momenta are projected separately for each meson in $O(M^2)$ as $M' = 22$ matrix of interpolators, (c) shows energies based on complete $22 \times 22$ matrix of interpolators, (c) is based on the $18 \times 18$ correlator matrix without diquark-antidiquark interpolating fields $\sigma_{4q}^{M}$. The thirteen lowest lattice energy levels (black circles) are interpreted as two-particle states, which are inevitably present in a dynamical lattice QCD simulation. No additional candidate for the exotic $Z_{3}^{+}$ is found below 4.2 GeV. The experimental widths of the resonances is indicated by the dashed vertical lines.
The energies $E_n$ and overlaps $Z_n^\ell$ are obtained from the $22 \times 22$ correlator matrix (3) using the generalized eigenvalue method $C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$ [26]. The energies $E_n$ are extracted from the asymptotically exponential behaviour of the eigenvalues; $\lambda^{(n)}(t) \propto e^{-E_n t}$ at large $t$.

The treatment of the charm quarks requires special care due to discretization errors. We employ the Fermilab method [27, 28], where discretization uncertainties are suppressed in the difference $E_n - m_{s,a}$ with the spin-average mass $m_{s,a} \equiv \frac{1}{2}(m_{u,d}+3m_{c,\psi})$. The same method and tuning of the charm quark mass $m_c$ lead to a good agreement with experiment for conventional charmonium [19], masses and widths of $D$ mesons [19], and the $D_s$ spectrum [23, 29] on this ensemble. In view of this, we will compare $E_n^{\text{lat}} - m_{s,a}^{\text{lat}} + m_{c,\psi}^{\text{lat}}$ to experiment where $am_{c,\psi}^{\text{lat}} = 1.47392(31)$ and $am_{c,\psi} = 1.54171(43)$.

RESULTS

The central result of simulation [18] is the discrete spectrum in Fig. 1b, while experimental candidates in the same channel are collected in Fig. 1a. In the energy region below $E \leq 4.3$ GeV one expects thirteen discrete two-particle states (2) near the horizontal lines, which continue in Fig. 1a to show their relation to the continuum of scattering states in experiment.

We interpret the lowest thirteen levels (indicated by black circles) as interacting two-particle states: the levels appear near the non-interacting energies (1) of the two-particle states (2), and each of these levels $n$ has the largest overlap $\langle \Omega|\mathcal{G}_J|n \rangle$ with the corresponding $\mathcal{G}_J^{M_1 M_2}$ [18].

The main conclusion of our simulation is that we do not find any additional state below 4.2 GeV that could be related to an exotic candidate. We only find the expected two-meson states (2).

It is indeed surprising that with a basis, which contains a great variety of interpolating fields with the quantum numbers of interest $I^G(I^{PC}) = 1^+(1^{++})$, one does not, for example, induce $Z_c(3900)/Z_c^+(3885)$ that has been confirmed by several experiments [6, 7, 8, 9]. Note that our list of creation/annihilation operators (4) contains also a number of field structures $J/\Psi \pi$ and $DD^*$ which correspond to channels where these resonances have been found in experiments.

We envisage several possible reasons for the absence of an energy levels related to the exotic $Z_c^+$ candidate in our simulation. Based on the experience, discussed in Section II, we would expect an additional energy level if the $Z_c^+$ state was a resonance associated to pole near the real axis in the unphysical Riemann sheet. The absence of an additional energy level could indicate a different origin of the experimental peak like, e.g., a coupled-channel threshold effect. Even if the $Z_c^+$ resonant structure seen in experiment is due to a relatively narrow $cc\bar{d}u$ state, there might be several reasons that an additional state is absent in our simulation. It is possible that $Z_c$ exists only at physical $m_{c/d}$ and is absent at unphysical $m_{c/d}$ in our simulation. Furthermore, our set of eighteen interpolators $\mathcal{G}_J^{MM}$ may not be complete enough to render a $Z_c^+$ candidate in addition to thirteen two-meson states, even if $Z_c^+$ existed at $m_c = 266$ MeV.

Further analytical work and lattice simulations are needed to resolve the question on the existence of $Z_c^+$ from first principle QCD.

OTHER LATTICE SIMULATIONS AIMED AT $Z_c^+$

All lattice searches for $Z_c^+$ considered $I^G(I^{PC}) = 1^+(1^{++})$ channel, which is the most relevant experimentally. The first lattice search for $Z_c^+(3900)$ considered $J/\Psi \pi$ and $DD^*$ scattering and only two-meson states $J/\Psi \pi$ and $DD^*$ were found, but no additional candidate for $Z_c^+(3900)$ [32].

The $DD^*$ interpolators were used to determine the s-wave and p-wave phase shift near $DD^*$ threshold where experimental $Z_c(3900)$ is located. Partially twisted boundary conditions were applied taking into account s/p-wave mixing [33]. The authors conclude that no evidence for $Z_c^+(3900)$ is found.

The lattice simulation of the same channel based on the HISQ action gives the energies in Fig. 2 for various interpolator basis [31]. The figure illustrates that the two-meson states are seen if the corresponding interpolators are employed. No additional state that could represent $Z_c^+$ is found, in agreement with our results.

The HALQCD is also simulating this channel and their interesting preliminary results have been reported at the meetings that took place after this meeting [HALQCD].
FIGURE 2. The spectrum in the $Z^{±}$ channel with $I(J^{PC}) = 1^{±−}$ [31]. The red and blue boxes are energies obtained from the simulation for various interpolator basis indicated at the bottom. The $c$ and $cc$ denote basis including charmonium and charmonium-pion interpolators. The green levels denote non-interacting $ψπ$ states, while the light blue levels denote $DD^*$.

CONCLUSIONS

We discuss the lattice QCD simulations for the $c\bar{c}d\bar{u}$ channel with $J^{PC} = 1^{±−}$ where exotic charmonia have been found in recent experiments. In the scanned energy region these find the expected meson-meson signals (mostly close to the non-interacting levels) but no convincing signal for an extra $Z^{±}$ state. Possible physics and methodology-related reasons for the absence of the exotic candidates are mentioned. This is in striking contrast to the lattice results in the flavour non-exotic channels, where the extra states are found in relation to the most of known resonances and bound states.

ACKNOWLEDGMENTS

We thank Anna Hasenfratz for providing the gauge configurations. The computations were done on the clusters at the Theoretical Physics department of Jozef Stefan Institute, at the University of Graz, NAWI Graz, and TRIUMF. We acknowledge the support by the Slovenian Research Agency ARRS project N1-0020, by the Austrian Science Fund FWF project I1313-N27 and by DFG project SFB/TRR55. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

REFERENCES