POLARIZATION ISSUES IN THE $e\pm$ FCC

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Abstract

After the Higgs boson discovery at LHC, the international physics community is considering the next energy frontier circular collider (FCC). A pp collider of 100 km with a center of mass energy of about 100 TeV is believed to have the necessary discovery potential. The same tunnel could host first a e^+e^- collider with beam energy ranging between 45 and 175 GeV. In this paper preliminary considerations on the possibility of self-polarization for the $e^\pm$ beams are presented.

INTRODUCTION

$e^\pm$ beams in a ring accelerator may become vertically polarized through the Sokolov-Ternov effect [1]. A small part of the radiation emitted by particles moving in a constant homogeneous field is accompanied by spin flip wrt the field direction. The probability of spin flip in the direction parallel to anti-parallel and from anti-paralle to parallel to the field are slightly different and this results in a polarization of 92.4 %, independently of energy. The polarization rate is given by

$$\frac{1}{\tau_{ST}} = \frac{5\sqrt{3}}{8}\frac{r_0 h}{2\pi m_0}\frac{\gamma^5}{|\rho|^3}$$

which strongly depends upon energy and radius. In actual ring accelerators there are not only dipoles. Quadrupoles for instance are needed for beam focusing. When a particle emits a photon it starts to perform synchro-betatron oscillations around the machine actual closed orbit experiencing extra possibly non vertical fields. The expectation value $\vec{S}$ of the spin operator moves according to the Thomas-Bargmann-Michel-Telegdi (Thomas-BMT) equation [2] [3]

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S}$$

$\vec{\Omega}$ depends on machine azimuth and phase space position, $\vec{u}$. In the laboratory frame and MKS units it is given by

$$\vec{\Omega}(\vec{u};s) = -\frac{e}{m_0}\Big[\Big(a+\frac{1}{\gamma}\Big)\vec{B}-\frac{a\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta}-\Big(a+\frac{1}{\gamma+1}\Big)\vec{\beta}\times\vec{E}\Big]$$

with $\vec{\beta} \equiv \vec{v}/c$ and $a = (g-2)/2$=0.0011597 ($e^\pm$). The T-BMT equation describes a precession of $\vec{S}$ around $\vec{\Omega}$. In a planar machine the *periodic* solution, $\hat{n}_0$, is vertical. The number of precessions per turn, the "naive" spin tune, in the rotating frame is $a\gamma$. Photon emission results in a randomization of the particle spin directions (*spin diffusion*). Polarization will be therefore the result of the competing process, the Sokolov-Ternov effect and the spin diffusion caused by stochastic photon emission. The problem has been studied and solved in a semiclassical approximation by Derbenev and Kondratenko [4]. They found that the polarization is oriented along the *periodic* solution, $\hat{n}_0$, of the Thomas-BMT equation along the *closed orbit* and its value is

$$P_{DK} = P_{ST}\frac{\oint ds < \frac{1}{|\rho|^3}\hat{b}\cdot(\hat{n}-\frac{\partial\hat{n}}{\partial\delta}) >}{\oint ds < \frac{1}{|\rho|^3}\Big[1-\frac{2}{9}(\hat{n}\cdot\hat{s})^2+\frac{11}{18}(\frac{\partial\hat{n}}{\partial\delta})^2\Big] >}$$

with

$$\hat{b} \equiv \vec{v}\times\dot{\vec{v}}/|\vec{v}\times\dot{\vec{v}}|$$

The <> brackets indicate averages over the phase space. The term $\partial\hat{n}/\partial\delta$, with $\delta \equiv \delta E/E$ quantifies the depolarizing effects resulting from the trajectory perturbations due to photon emission.

The corresponding polarization rate is

$$\tau_{DK}^{-1} = P_{ST}\frac{r_e\gamma^5\hbar}{m_0 C}\oint < \frac{1}{|\rho|^3}\Big[1-\frac{2}{9}(\hat{n}\cdot\hat{s})^2+\frac{11}{18}(\frac{\partial\hat{n}}{\partial\gamma})^2\Big] >$$

In a perfectly planar machine $\partial\hat{n}/\partial\delta$=0. In presence of quadrupole vertical misalignments (and/or spin rotator) $\partial\hat{n}/\partial\delta \neq 0$ and large when

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer}$$

Polarization in an actual ring accelerator has been observed for the first time at ACO in Orsay in 1968. The self polarization mechanism has been exploited more recently in large accelerators, namely HERA-e and LEP. While in LEP beam polarization was used for precise energy measurement through RF resonant depolarization, at HERA the provision of beam polarization was an integral part of the physics program and 3 pairs of spin rotators were build-in for turning the direction of polarization of the lepton beams from vertical to longitudinal at the HERMES, H1 and ZEUS experiments. HERA-e was operating at 27.5 GeV and the dipole bending radius was about 600 m, corresponding to a polarization time of the order of 30 minutes. The maximum transverse polarization achieved at HERA-e was about 75%. LEP dipole bending radius was about 3000 m and energy ranged between 40 and 100 GeV. The polarization level strongly decreased with energy and above 65 GeV no polarization was detected [5]. Qualitatively this can be explained by the increasing of spin diffusion with energy.

Both at HERA-e and LEP the high level of polarization was obtained through

- Optimization of energy;
- Choice of orbital tunes: small values of the fractional part result in a larger region free from low order resonances;

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- Careful correction of the (vertical) closed orbit;

- Dedicated correction of the nominal $\hat{n}_0$ distortion, $\delta\hat{n}_0$, due to magnet misalignments.

FCC SCENARIO

For the FCC e^-e^+ collider precise energy measurement are required for Z and WW resonances at 45 and 80 GeV respectively.

Giving that the geometry is fixed by the maximum field, B_{max}, attainable for bending the 50 TeV protons and assuming B_{max}=16 T, the bending radius is $\rho_b = p/(eB)$=10423.6 m. The total length of the dipoles is $L_{bends} = 2\pi\rho_b$=65493.5 m which, for L_{tot}=100 km, gives a filling factor of L_{bends}/L_{tot}=0.655.

Chromaticity correction limits the minimum value of the dispersion in the arcs. In a FODO cell, for instance, the maximum dispersion is given by

$$\hat{D} = \frac{L_{cell}\phi_b}{2}\frac{1+0.5\sin\mu/2}{\sin^2\mu/2}$$

ϕ_b and thus ℓ_b [1] should be large for avoiding too small dispersion.

For the following computations a "toy" machine made out of FODO cells has been used with 30 m long bending dipoles and $\phi_b = \ell_b/\rho_b$=2.878 mrad bending angle and $\mu = 60^o$ phase advance in both planes. The resulting FODO optics is shown in Fig. 1. The momentum compaction is 3.2e-5.

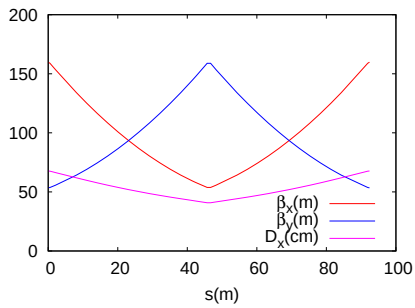

Figure 1: FODO Twiss functions (m) and dispersion (cm).

The large bending radius is may be appealing for some beam parameters (small energy loss and equilibrium emittance) but increases the damping time and the Sokolov-Ternov polarization time.

In Table 1 are shown the relevant beam parameters for a 100 km long machine with ρ_b=10423.6 m.

[1] $L_{cell} = 0.655 L_{bends}/2\pi/\phi_b$

Table 1

E (GeV)	U_0 (MeV)	$\Delta E/E$ (%)	ϵ_x (μm)	τ_x (ms)	τ_{pol} (h)
45	35	0.038	0.85e-3	868	256
80	349	0.067	0.27e-2	218	14

POLARIZATION WITH WIGGLER MAGNETS

For decreasing the polarization time the obvious recipe is increasing synchrotron radiation emission by introducing *wiggler* magnets.

The polarization rate in a perfect planar machine, with vertical fields possibly pointing in opposite directions is given by

$$\tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0C}\oint\frac{ds}{|\rho|^3} = F\Big[\int_{dip}\frac{ds}{|\rho_d|^3} + \int_{wig}\frac{ds}{|\rho_w|^3}\Big]$$

with $F \equiv 5\sqrt{3}r_e\gamma^5\hbar/8m_0C$. Therefore *any* wiggler decreases τ_p. The polarization is given by

$$\vec{P} = \hat{n}_0\, P_{ST}\frac{\oint ds\frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds\frac{1}{|\rho|^3}}$$

that is

$$P \propto \tau_p\oint ds\,\frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}$$

Lowering the polarization time may lower the polarization level. We can separate the contribution of guiding dipoles and wigglers

$$\oint ds\,\frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3} = \int_{dip} ds\,\frac{\hat{B}_d\cdot\hat{n}_0}{|\rho_d|^3} + \int_{wig} ds\,\frac{\hat{B}_w\cdot\hat{n}_0}{|\rho_w|^3}$$

The wiggler does not change $\hat{n}_0$ which in a perfectly planar ring is vertical; therefore

$$\int_{wig} ds\,\frac{\hat{B}_w\cdot\hat{n}_0}{|\rho_w|^3} = \frac{1}{ep}\int_{wig} ds\,B_w^3$$

This term must be large, and should have the same sign as the guiding field contribution, in order to maximize the level of polarization. For instance, an antisymmetric wiggler, $B(s) = -B(-s)$, would results in very small polarization.

The condition on $\int_{wig} ds\,B_w^3$ must be added to the usual constraint that the orbit outside the wiggler region should be unperturbed which translates in the conditions

$$\int_{wig} ds\,B_w = 0 \Rightarrow x' = 0 \quad \text{outside wiggler}$$

$$\int_{wig} ds\,sB_w = 0 \Rightarrow x = 0 \quad \text{outside wiggler}$$

A symmetric wiggler automatically fulfills the condition for $x = 0$. If in addition the field integral vanishes thus also $x' = 0$. A similar field arrangement as proposed for the LEP polarization wigglers [6] has been here considered (see Fig. 2) with B_+/B_-=6.

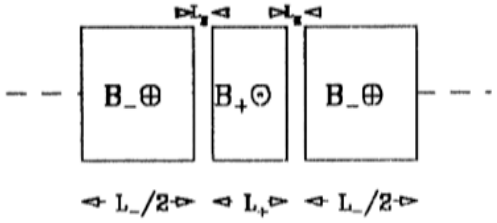

Figure 2: LEP polarization wiggler (figure from [6]).

Four dispersion free sections have been inserted in the "toy" machine for accommodating 4 of such wigglers with L_+= 8 m. The optics is almost unperturbed (see Fig. 3)

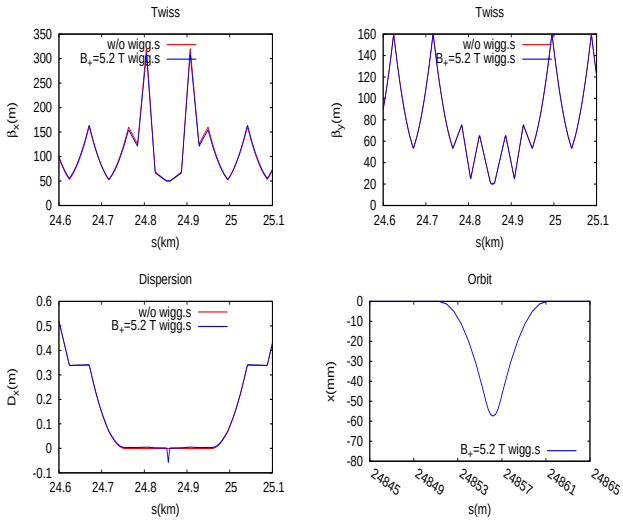

Figure 3: Twiss functions (up), horizontal dispersion and orbit (bottom) at the wiggler location with B_+=5.2 T.

Relevant beam parameters in presence of wigglers [2] are quoted in Table 2

Table 2

B_+ (T)	U_0 (MeV)	$\Delta E/E$ (%)	ϵ_x (μm)	τ_x (s)	P (%)	τ_{pol} (min)
0	37	.04	.8e-3	.82	92.4	14e3
1.3	64	.22	.5e-2	.48	87.6	247
2.6	144	.41	.070	.21	87.6	31
3.9	278	.55	.274	.11	87.6	9
5.2	466	.65	.691	.06	87.6	4

The increase of the energy spread is potentially harmful for polarization. As a comparison the beam relative energy spread was ~0.1% in HERA-e and ~0.16% in LEP at 100 GeV.

Distance between *imperfection* (or zeroth) order resonances is ΔE= 440 MeV independently of energy. How well must be corrected the closed orbit and δn_0 in order to achieve a minimum useful level of polarization for energy measurement (~ 5-10%) must be investigated by accurate simulations.

PRELIMINARY SIMULATIONS

In [7] a detailed description of available codes for evaluating radiative polarization may be found.

In Figs. 4 and Figs. 5 polarization and $\delta\hat{n}_0$ vs. energy computed by SLIM [8] (*linear* orbital and spin motion) for the FCC "toy" ring w/o and with 4 wigglers (B_+=5.2 T) respectively and in presence of random quadrupole vertical misalignment is shown. The rms value of the misalignment is 150μm and the resulting rms vertical closed orbit is 5.4 mm. No corrections have been applied. The orbital tunes are Q_x=181.185, Q_x=183.227 and Q_s=0.09. The red line indicates the polarization in the ideal machine. The curves labeled as P_z (with z=x, y, s) are the polarization levels related to the three degrees of freedom separately.

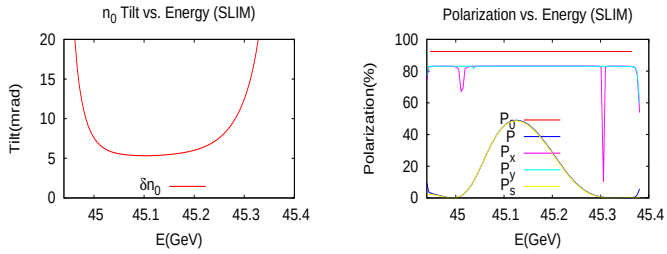

Figure 4: $\delta\hat{n}_0$ (left) and polarization(right) vs. energy w/o wigglers with δ_y^Q=150 μm.

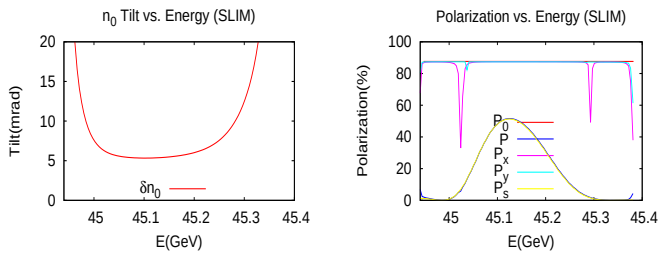

Figure 5: $\delta\hat{n}_0$ (left) and polarization(right) vs. energy with 4 wigglers (B_+=5.2 T) with δ_y^Q=150 μm.

In presence of errors first order resonances appear, the strongest being the ν_s=$integer \pm Q_s$ ones. Taking into account that no corrections have been applied, the situation seems at a first sight not hopeless.

[2] Implications on luminosity, beam-beam etc not investigated!

Actually the spin motion is *not* linear and non-linear calculations are mandatory. Codes treating non-linearized spin motion in a semi-analytical approach have either convergence problems at high energy or requires very large computing power. Two tracking codes are available, SITROS [9] and SLICKTRACK [10]. The first one, the only currently at hand to the author, has been used here.

In Figs. 6 and 7 the SITROS calculations for the machine with random errors and w/o wigglers are shown for δ_y^Q=10 and 50 μm respectively. No corrections have been applied. The orbital tunes are Q_x=181.124, Q_x=183.207 and Q_s=0.1.

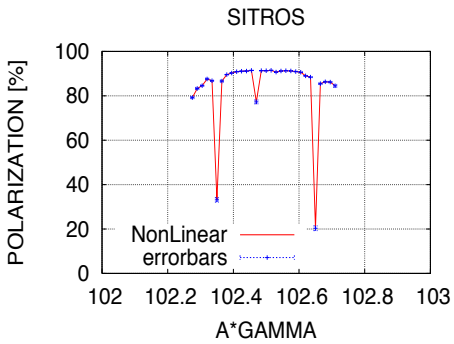

Figure 6: Polarization vs. spin tune w/o wigglers with δ_y^Q=10 μm.

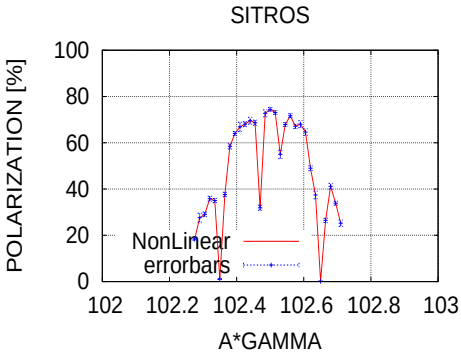

Figure 7: Polarization vs. spin tune w/o wigglers with δ_y^Q=50 μm. The corresponding rms vertical orbit is 2 mm.

Much needed computations in the presence of wigglers are going on.

SUMMARY

A preliminary study of polarization in the FCC $e^{\pm}$ ring at the Z resonance energy (45 GeV) has been presented. It has been shown that owing to the unfavorable dimension of the machine the polarization time can be reduced to reasonable values only by inserting wigglers. The resulting large energy spread is expected to lead to depolarization in presence of magnet misalignments, in particular the vertical ones. The sensitivity of orbit and $\delta\hat{n}_0$ to errors increases with the strengths of the quadrupoles, the Twiss functions at their location and their *absolute* number.

At 80 GeV weaker wigglers will be needed; however the natural energy spread will be higher too so that the same issues as for the 45 GeV case will be likely encountered.

Finally it is worth noting that in [11] a resurrection of polarization is predicted at high energy (or in presence of a large energy spread) when the condition

$$\frac{\nu_{spin} T_{rev}}{\tau_p Q_s^3} \ll 1$$

is satisfied. This means that, unlike the case when the energy spread is small, large values of the synchrotron tune are preferable. This prediction should be tested by simulations.

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