FERMILAB-CONF-14-365-AD

Alternative Shapes and Shaping Techniques for Enhanced Transformer Ratios in Beam Driven Techniques

F. Lemery* and P. Piot*,†

*Northern Illinois Center for Accelerator & Detector Development and Department of Physics, Northern Illinois University, DeKalb IL 60115, USA
†Accelerator Physics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

Abstract. The transformer ratio of collinear beam-driven acceleration techniques can be significantly improved by shaping the current profile of the drive bunch. To date, several current shapes have been proposed to increase the transformer ratio and produce quasi-uniform energy loss within the drive bunch. Some of these tailoring techniques are possible as a result of recent beam-dynamics advances, e.g., transverse-to-longitudinal emittance exchanger. In this paper, we propose an alternative class of longitudinal shapes that enable high transformer ratio and uniform energy loss across the drive bunch. We also suggest a simple method based on photocathode-laser shaping and passive shaping in wakefield structure to realize shape close to the theoretically optimized current profiles.

Keywords: beam driven dielectric wakefield acceleration
PACS: 29.27.-a, 41.85.-p, 41.75.F

INTRODUCTION

In beam driven techniques, a "drive" bunch passes through a high-impedance medium and experiences a decelerating field. The resulting energy loss can be transferred to a properly delayed "witness" bunch trailing the drive bunch in the medium. The transformer ratio, defined as $\mathscr{R} = E_+/E_-$ [where E_+ (reps. E_-) is the maximum accelerating (reps. minimum decelerating) field behind (reps. within) the drive bunch] is an important figure of merit as it describes the energy transfer within the two bunches. For symmetric bunches, $\mathscr{R} \leq 2$. Going beyond $\mathscr{R} = 2$ requires asymmetric bunches and can be maximized by generating a flat decelerating field over the drive bunch; however, increasing $\mathscr{R}$ generally reduces E_+ and the relationship between them is left to a compromise [1].

In practice however, the acceleration gradients and acceleration frequencies depend on the interaction medium, which for e.g. dielectric wakefield acceleration, depend on the dimensions of the structure. Therefore altogether, the structure dimensions are also limited by the beam properties such as the beam energy and beta functions. At low energy for example, due to the difficulty of fitting into relatively long structures, it is more valuable to excite high-gradient wakes with shorter bunches with poor transformer ratios. On the other hand, high energy beams with relatively smaller betatron functions can fit into much longer, and smaller structures. In this perspective it is more useful to drive wakes efficiently with high transformer ratios.

To date, several current profiles capable of generating optimal transformer ratios have been proposed. These include linearly ramped profile combined with a door-step or exponential initial distribution [2], and more recently the double-triangle current profile [3]. In general, these shapes present discontinuities and rely on complicated beam manipulation techniques which have limited applications [4, 5]. .

In this paper we introduce a new class of smooth current profiles which also lead to constant decelerating fields across the drive bunch to lead to optimal transformer ratios. We also discuss a possible scheme for realizing these shapes employing a shaped photocathode-laser pulse.

For simplicity we consider a wakefield structure that supports an axial wakefield described by the Green's function [6]

$$G(z) = 2\kappa\cos(\omega_0 z/c), \tag{1}$$

where κ is the loss factor and $\omega_0 \equiv 2\pi c/\lambda$ with λ being the wavelength of the considered mode. Here z is the distance behind the source particle responsible for the wakefield. We do not specialized to any wakefield mechanism

Operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy

and depending on the structure used many modes might be excited so that the Green's function would consists of a summation over these modes.

SMOOTH CURRENT PROFILES FOR ENHANCED TRANSFORMER RATIOS

Optimal-$\mathscr{R}$ shapes are useful for generating flat decelerating fields over the bunch; this inherently reduces the energy spread incurred over the bunch. Additionally, these shapes lead to the most efficient energy transfer between a drive and witness bunch which could lead to overall higher witness bunch energies.

Based on the work presented in Ref. [2], we consider a bunch-current profile described as a piecewise function $I(t)$. We take $I(t)$ to be piecewise on two intervals $[0,\tau]$ and $[\tau,T]$ and zero elsewhere. Also we constrain our search to functions such that $I(t)$ and $\dot{I}(t) \equiv dI/dt$ are continuous at $t=\tau$.

As an example with consider a modulated linear ramp function over $[0,\tau]$ of the form

$$s(t) = at + b\sin(\omega t), \tag{2}$$

where a and b are constant, and ω_0 is the modulation frequency. We correspondingly introduce the current profile as the piecewise)continuous) function

$$I(t) = I_0 \begin{cases} s(t) & \text{if } 0 \le t < \tau, \\ s'(\tau)t - s'(\tau)\tau + s(\tau) & \text{if } \tau \le t \le T, \\ 0 & \text{elsewhere.} \end{cases} \tag{3}$$

where I_0 is the peak current. An example of current shape is depicted in Fig. 1 (a).

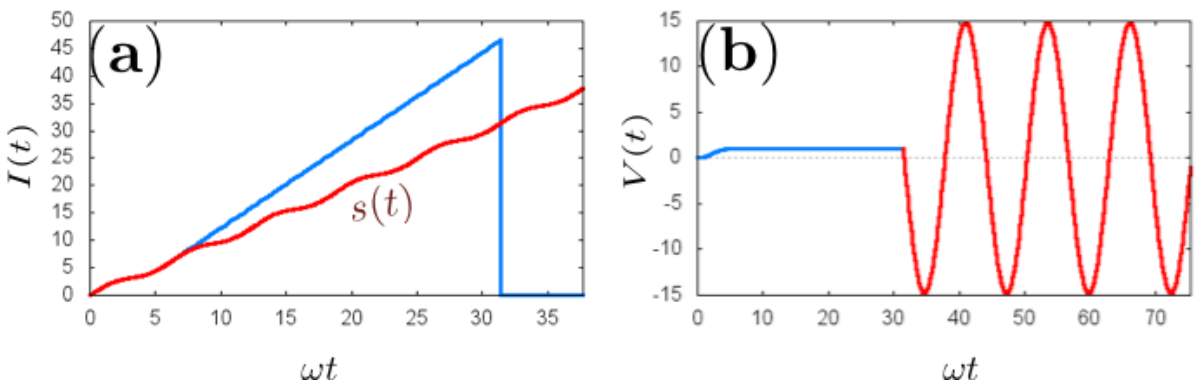

FIGURE 1. Example (a) of current profile (blue trace) described by Eq. 3 (a) and $s(t)$ function (red trace). The corresponding induced voltage (b) within (blue trace) and behind (red trace) the bunch. For these data $I_0 = 1$, $a = 1$, $b = 3a/5$, $n = 1$, $m = 5$ and $q = 2$. (the current and voltage units are arbitrary).

We further specialize, for simplicity, to the case where $\tau = 2n\pi/\omega$ (where $n \in \mathbb{N}$) and $\omega_0 = \omega/q$ (where ω_0 is defined in Eq. 1 and $m \in \mathbb{N}$)) The decelerating field (V^-) over the drive bunch is found to be

$$V^-(t) = I_0 \begin{cases} -\frac{q^2}{\omega^2(q^2-1)}\left([aq^2-(b\omega+a)]\cos\frac{\omega t}{q} + b\omega\cos\omega t - a(q^2-1)\right) & \text{if } 0 \le t < \tau, \\ -\frac{q^2}{\omega^2(q^2-1)}\left(b\omega q^2\cos\frac{\omega t - 2\pi n}{q} + [aq^2-(b\omega+a)]\cos\frac{\omega t}{q} + (b\omega+a)(1-q^2)\right) & \text{if } 0 \le t < \tau, \\ 0 & \text{elsewhere.} \end{cases} \tag{4}$$

The above decelerating voltage shows that when $\frac{n}{q} = \frac{l}{2}$ (where $l \in \mathbb{N}$) the cos can be factored and results in a constant decelerating field for $t \in [\tau,T]$ of the form

$$V^-_{t\in[\tau,T]}(t) = \frac{-I_0 q^2}{\omega^2(q^2-1)}\left([b\omega((-1)^{\frac{2n}{q}}q^2-1)+a(q^2-1)]\cos\frac{\omega t}{q} + (b\omega+a)(1-q^2)\right), \tag{5}$$

so that the time dependence of the decelerating potential can be cancelled by choosing the ramp parameters a and b to satisfy the equation

$$b = \frac{a(1-q^2)}{\omega[(-1)^{\frac{2n}{q}} q^2 - 1]}. \tag{6}$$

Specializing to the case where $2n/q$ is an odd number gives $b = a(q^2-1)/[\omega(1+q^2)]$ and the decelerating voltage across the bunch takes the form

$$V^-(t) = I_0 \begin{cases} \frac{aq^2 I_0}{\omega^2(1+q^2)} \left(q^2 \cos\frac{\omega t}{q} + a\cos\omega t - (1+q^2)\right) & \text{if } 0 \le t < \tau, \\ \frac{2aI_0 q^4}{\omega^2(1+q^2)} & \text{if } \tau \le t < T, \\ 0 & \text{elsewhere.} \end{cases} \tag{7}$$

The latter case is illustrated in Fig. 1 (b) with $l \equiv n/q = 1/2$. Likewise the induced oscillating voltage behind the bunch can be obtained from

$$V^+(t) = \frac{aq^2 I_0}{\omega^2(1+q^2)} \left(\pi[q^2(1-2m)-1]\sin\left(\frac{\omega t}{q} - \pi m\right) + 2q^2\cos\left(\frac{\omega t}{q} - \pi m\right)\right) \tag{8}$$

The second term can be dropped to good approximation to find a lower estimate on the transformer ratio, without loss of generality

$$\mathscr{R} = \left(m + \frac{1-q^2}{2q^2}\right)\pi, \tag{9}$$

where m is the number of the fundamental-mode wavelengths within the total bunch length. We note that for short bunch durations (i.e. as needed to produce high accelerating fields), the latter transformer ratio is higher than other proposed distributions.

BEAM SHAPING USING A TAILORED PHOCATHODE LASER

To date, longitudinal shaping for enhanced transformer ratios has only been demonstrated using a mask in a dispersive section. Another proposed technique relies on using a mask to shape a beam transversely, and use an emittance exchanger to rotate the shape longitudinally. In the former case, the scheme is limited to specific shapes in the mask, and inherits beam losses. Additionally shapes which require sharp edges (e.g. double-triangle) are limited by the dispersion and emittance of the beam at the mask. The latter scheme requires a deflection cavity in a dispersive section which complicates the overall procedure; moreover, this scheme also inherits beam losses from a mask upstream, and is also limited by beam emittances.

An alternative approach is to longitudinally tailor the laser profile in a high-gradient photoinjector. In such a scenario, electrons emitted from the photocathode will have an initial longitudinal distribution similar to the injected laser pulse. We consider an example of an S-Band gun [7] operating at 140 MV/m to reduce the space-charge effects. Generally higher accelerating fields in a gun lead to higher charge densities which could drive larger gradient wakefields inside DLWs. We use the particle-tracking program *astra* which takes into account space charge effects using a cylindrical-symmetric algorithm [11]. We model the 1-nC electron bunch considered in our studies with 50,000 macroparticles. Additionally the laser is chosen to have a transverse rms spot size of $\sigma_c = 0.8$ mm and rms duration of $\sigma_t = 1$ ps. A solenoidal lens is placed downstream of the gun to maintain the transverse size below 1 cm over a drift length of 1 m.

In general, photoemission is a intricate process which depends on many parameters including the amplitude and phase (with respect to the laser) of the applied accelerating field on the photocathode surface, the bunch charge, and the cathode material properties. It is challenging to analytically find an optimum laser shape that provide a given current distribution at a given location. Work toward the production of ramped current profile was investigated in the context of seeded free-electron laser research [8] and experimentally demonstrated at the FERMI facility [9]. Additionally, we demonstrated in Ref. [10] that a passive technique employing a wakefield structure combined with a non-ultra-relativistic bunch could produce ramped bunch.

Therefore to explore the production of tailored electron bunches using temporally-shaped photocathode laser pulses, we carry numerical simulation using the ASTRA particle tracking program [11]. and explored the outcome of various shapes. In this section we report on the performance of the exponential distribution given by:

$$f(t) = f_0(e^{\mu t} - 1)H(\theta - t), \tag{10}$$

where μ is a growth rate and θ is the ending time of the pulse $H(t)$ is the Heaviside function. The laser intensity f_0 has to be normalized to produce the required bunch charge after the photo-emission process. We consider three case of exponentially-shaped laser distributions. All have a total pulse duration of $\theta = 5$ ps but different steepness characterized by a growth rate $\mu = 3.90 \times 10^5$, 6.92×10^5, 1.00×10^6; see Fig. 2 (upper-right plot).

The growth rate μ primarily influences the electrostatic field inside the bunch which affects the final longitudinal phase space. The case of large growth rate yield the formation of ellipsoidal distribution via the blow-out regime. In the limit of small growth rates, the laser profile become linearly ramped but the ramp quickly dissipates. Figure 2 (upper right, and lower row plots) displays snapshots of the the longitudinal phase spaces and associated current distribution for the three case of growth rate mentioned.

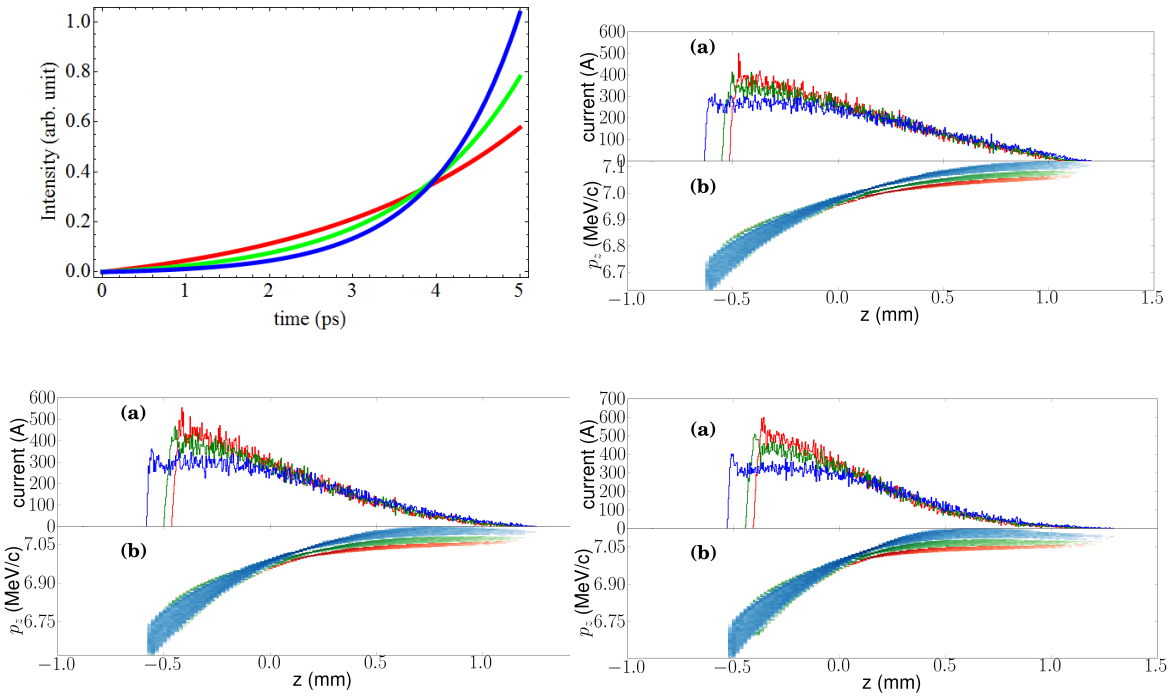

FIGURE 2. Initial photocathode laser pulse temporal distributions (top left) and snapshots of the current (a) and longitudinal phase space (b) at 25 cm (red), 50 cm (green), 100 cm (blue) from the photocathode. The top-right, lower-left and lower-right plots correspond respectively to the initial laser distributions appearing as a red, green, and blue trace (with increasing steepness) in the top-left plot.

Finally, we quantify the capability of the formed electron bunches to serve as a wakefield drive by considering their applications to drive wakefield in a dielectric structure. We therefore convolve the shapes with the one-dimensional Green's function associated to a cylindrically-symmetric dielectric-line waveguide (DLW) [12]. We choose a DLW with inner radius $a = 165$ μm, outer radius $b = 195$ μm, and relative dielectric permittivity $\varepsilon = 5.7$; here we note that in order to fit the beam into a structure with such dimensions would require a significant increase in beam energy which could also "freeze" the evolution of the beam. For the three cases, we record E_+ and $\mathscr{R}$ along the drift following the photocathode, the results are shown in Fig. 3. The results indicate that both large E_+ and $\mathscr{R}$ are achievable for the different laser distribution along the beamline. The variation parameter μ adds some flexibility between large E_+ and R and generally larger μ lead to larger E_+. Although in our examples, the bunch shape is still evolving, it could in

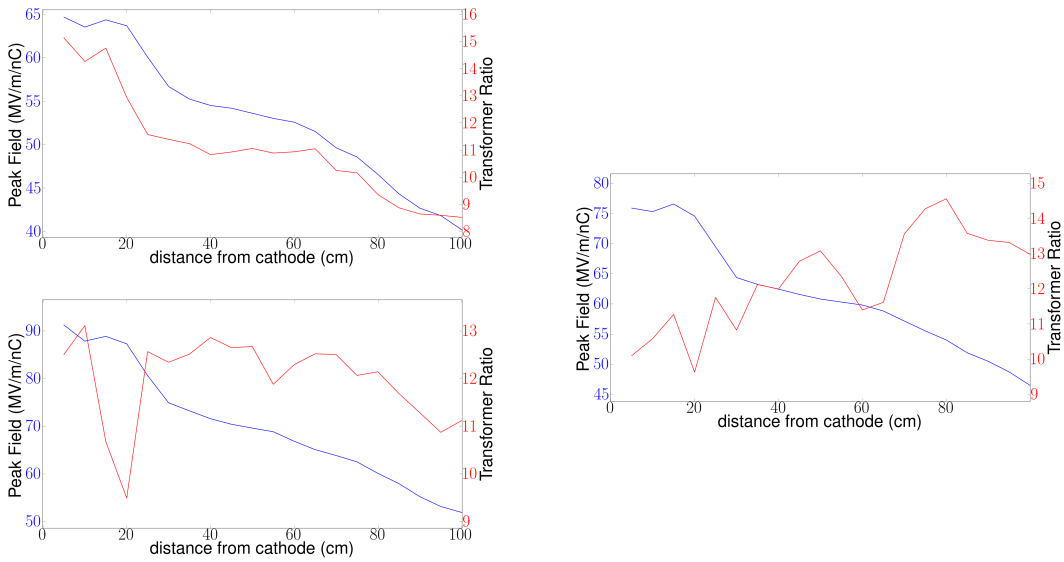

FIGURE 3. The calculated E_+ (blue trace) and $\mathscr{R}$ (red trace) as a function of drift length from the photocathode in a DLW with dimensions described above. The top left, right, and and bottom left plots correspond to initial exponential distributions red, green, blue shown in Fig. 2 (top left) respectively.

principle be possible to add a booster cavity and optimize the setup to ensure the optimum shape is formed downstream of the booster cavity at a sufficiently high energy to freeze the longitudinal motion within the bunch.

ACKNOWLEDGMENTS

This work was supported by the Defense Threat Reduction Agency, Basic Research Award # HDTRA1-10-1-0051, to Northern Illinois University, by US DOE contracts No. DE-SC0011831 to Northern Illinois University and DE-AC02-07CH11359 to the Fermi research alliance LLC.

REFERENCES

1. F. Lemery, P. Piot, International Particle Accelerator Conference (IPAC). WEPPR035 (2012).
2. K. L. F. Bane, P. Chen, P. B. Wilson, “On collinear wakefield acceleration", SLAC-PUB-3662 (1985).
3. B. Jiang, C. Jing, P. Schoessow, J. Power, and W. Gai, Phys. Rev. ST Accel. Beams **15**, 011301 (2012).
4. P. Muggli, V. Yakimenko, M. Babzien, E. Kallos, and K. P. Kusche, Phy. Rev. Lett. **101**, 054801 (2008).
5. P. Piot, Y.-E Sun, J. G. Power, and M. Rihaoui, Phys. Rev. ST Accel. Beams **14**, 022801 (2011).
6. A. Chao, *Physics of Collective Instabilities in High-Energy Accelerators*, Wiley Series in Beams & Accelerator Technologies, John Wiley and Sons (1993).
7. D. T. Palmer, R. H. Miller, H. Winick, X.J. Wang, K. Batchelor, M. Woodle, and I. Ben-Zvi, “Microwave measurements of the BNL/SLAC/UCLA 1.6-cell photocathode RF gun", in Proceedings of the 1995 Particle Accelerator Conference, PAC’95 (Dallas, TX, 1995), 982 (1996).
8. G. Penco, M. Trovøand S. M. Lidia, Proceedings of FEL 2006, BESSY, Berlin, Germany, 621 (2006).
9. G. Penco, M. Danailov, A. Demidovich, E. Allaria, G. De Ninno, S. Di Mitri, W. M. Fawley, E. Ferrari, L. Giannessi, and M. Trovó, Phys. Rev. Lett. **112**, 044801 (2014).
10. F. Lemery, and P. Piot, these proceedings.
11. K. Flöttmann, ASTRA*: A space charge algorithm, User’s Manual*, available from the world wide web at http://www.desy.de/∼mpyflo/AstraDokumentation (unpublished).
12. M. Rosing, and W. Gai, Phys. Rev. D **42**, 1829 (1990).