

The future of the Higgs boson

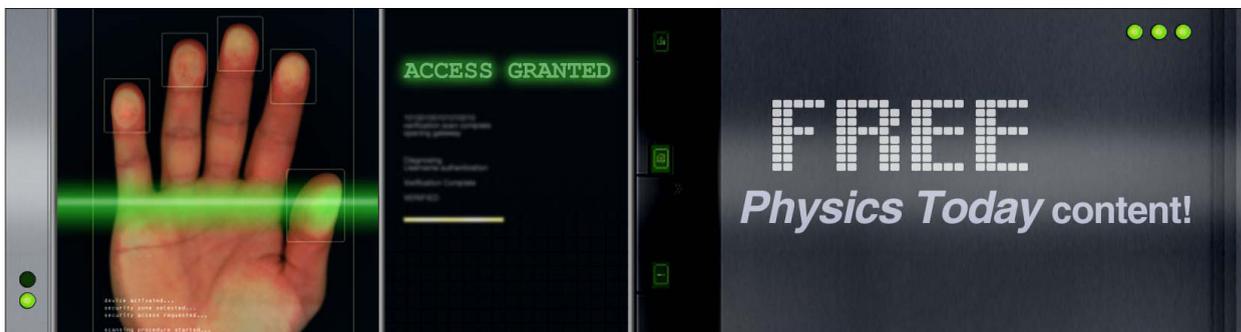
Joseph Lykken and Maria Spiropulu

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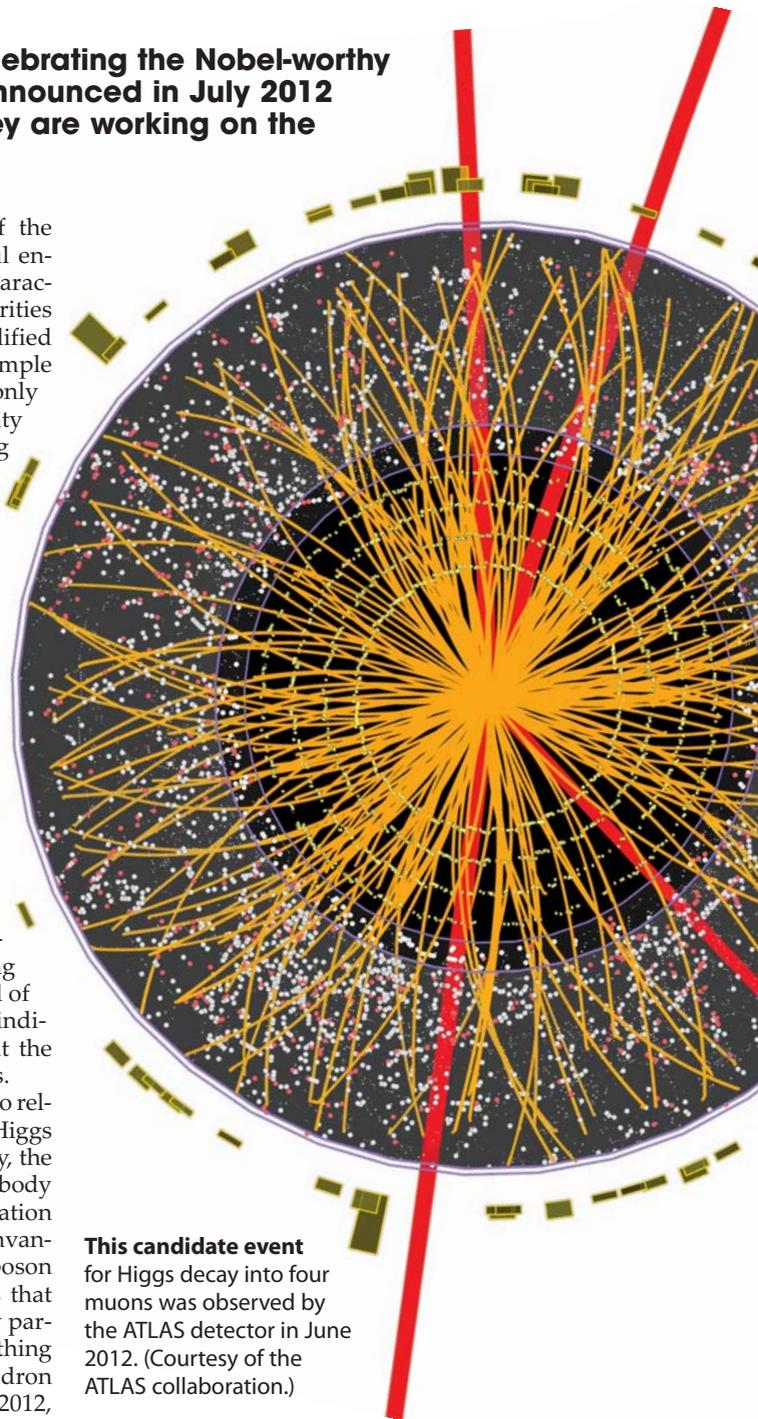
Higgs boson

Experimentalists and theorists are still celebrating the Nobel-worthy discovery of the Higgs boson that was announced in July 2012 at CERN's Large Hadron Collider. Now they are working on the profound implications of that discovery.

Symmetries and other regularities of the physical world make science a useful endeavor, yet the world around us is characterized by complex mixtures of regularities with individual differences, as exemplified by the words on this page. The dialectic of simple laws accounting for a complex world was only sharpened with the development of relativity and quantum mechanics and the understanding of the subatomic laws of physics. A mathematical encapsulation of the standard model of particle physics can be written on a cocktail napkin, an economy made possible because the basic phenomena are tightly controlled by powerful symmetry principles, most especially Lorentz and gauge invariance.

How does our complex world come forth from symmetrical underpinnings? The answer is in the title of Philip Anderson's seminal article "More is different."¹ Many-body systems exhibit emergent phenomena that are not in any meaningful sense encoded in the laws that govern their constituents. One reason those emergent behaviors arise is that many-body systems result from symmetries being broken. Consider, for example, a glucose molecule: It will have a particular orientation even though the equations governing its atoms are rotationally symmetric. That kind of symmetry breaking is called spontaneous, to indicate that the physical system does not exhibit the symmetry present in the underlying dynamics.

It may seem that the above discussion has no relevance to particle physics in general or to the Higgs boson in particular. But in quantum field theory, the ground state, or vacuum, behaves like a many-body system. And just as a particular glucose orientation breaks an underlying rotation symmetry, a nonvanishing vacuum expectation value of the Higgs boson field, as we will describe, breaks symmetries that would otherwise forbid masses for elementary particles. Now that the Higgs boson (or something much like it) has been found at the Large Hadron Collider (LHC; see PHYSICS TODAY, September 2012, page 12), particle experimentalists are searching for more kinds of Higgs bosons and working to find out if the Higgs boson interacts with the dark matter that holds the universe together. Cosmologists are trying to understand the symmetry-breaking Higgs phase transition, which took place early in the his-



This candidate event for Higgs decay into four muons was observed by the ATLAS detector in June 2012. (Courtesy of the ATLAS collaboration.)

Joe Lykken is a research scientist at the Fermi National Accelerator Laboratory in Batavia, Illinois. **Maria Spiropulu** is a professor of physics at the California Institute of Technology in Pasadena.

tory of the universe, and whether that event explains the excess of matter over antimatter. The measured mass of the Higgs boson implies that the symmetry-breaking vacuum is metastable. If no new physics intervenes, an unlucky quantum fluctuation will eventually spark a cosmic catastrophe.

Symmetry breaking and the vacuum

Since symmetry breaking is a step on the road to complexity, it is only natural that condensed-matter physics abounds with important examples: crystals, ferromagnets, superfluids, superconductors, and many more. When the symmetry is continuous, the broken state is just one of an infinite number of equivalent ground states. For example, the electron spins in a particular magnetic domain of a ferromagnet are all aligned in the same direction, breaking rotational symmetry. But the direction itself is arbitrary—it varies from domain to domain according to tiny details in the history of the material.

A characteristic feature of the spontaneous breaking of a continuous symmetry is the presence of Goldstone modes—also known as Nambu–Goldstone modes or, in condensed-matter physics, as Anderson–Bogoliubov modes. They are long-wavelength excitations that deform a system from one broken state toward another. Because of the underlying continuous symmetry, it costs little energy to excite a Goldstone mode. A familiar example is an acoustic phonon in a crystal, described further in the box on this page.

The Nambu–Goldstone modes are named after Yoichiro Nambu and Jeffrey Goldstone (shown in figure 1), who in 1960 took a grand intellectual leap: They began to apply condensed-matter ideas about spontaneous symmetry breaking to particle physics. Nambu was attempting to get insight about the then-mysterious properties of baryons, such as the proton and neutron, and the lightest mesons—the pions. And he succeeded, in a fashion that won him the 2008 Nobel Prize in Physics.

Goldstone took a more general approach; in his paper “Field theories with ‘superconductor’ solutions,” he began with the disclaimer that “the present work merely considers models and has no direct physical applications.”² He discussed a complex (that is, having real and imaginary parts) self-interacting spinless boson field. (A boson field, in general, corresponds to a particle with integer spin.) The self-interactions are encoded in the shape of the potential energy density of the field, which, in Goldstone’s formulation, had the Mexican-hat shape shown in figure 2. One might expect that the vacuum of the theory is the state for which the expectation value of the field vanishes. But as the figure shows, the lowest-energy states of the theory correspond to the boson field having a nonvanishing value dependant on an arbitrary phase.

For a large but finite volume, quantum tunneling processes connect all the different ground states, but even a tiny perturbation of the system will over-

whelm that effect and select just one ground state at random. The vacuum expectation value of the boson field spontaneously breaks the phase invariance of the dynamics. Starting from any of the broken vacua, quantum excitations up the brim correspond to a massive particle—the analog of the Higgs boson—with the steepness of the brim being directly connected to its mass. Excitations along the trough correspond to a massless particle, called a Goldstone boson.

The loophole

To understand Goldstone’s almost apologetic preface to his paper, consider the known subatomic particles circa 1960. Nambu had already correctly identified the relatively light pions as approximate Goldstone bosons, but if spontaneous symmetry breaking occurs generally in particle physics, where were all the other Goldstones? More discouraging, in 1962 Goldstone, Steven Weinberg, and Abdus Salam proved a seemingly general theorem saying that in a relativistic quantum field theory, spontaneous breaking of any continuous symmetry will produce massless bosons.³

A similar embarrassment involving massless particles had already been festering in the particle-physics community for some years. In 1954 C. N. Yang and Robert Mills produced a mathematically elegant generalization of electromagnetism.⁴ In their theory, new forces are mediated by new particles called gauge bosons in a way similar to the way electromagnetic forces are mediated by photons. Yang gave a seminar on his new idea at the Institute for Advanced Study in Princeton, New Jersey, where Wolfgang Pauli verbally attacked him. As it turned out, Pauli had developed the same construction on his own, but he had abandoned it when he realized that the symmetry of the theory, called a gauge symmetry, would force the gauge bosons of such models to be exactly massless, just as the photon is. If nature employed gauge theories beyond electromagnetism, then where were all the massless cousins of the photon?

In 1962 Anderson (shown in the right-hand panel of figure 1) realized that the twin problems of massless Goldstone bosons and massless gauge bosons were related.⁵ Consider the Bardeen-Cooper-Schrieffer superconductor that was Nambu and Goldstone’s original inspiration. It has a symmetry-breaking condensate—the Cooper pairs—but no

Acoustic phonons and order parameters

The dynamics underlying crystal formation is rotationally and translationally invariant, but the crystal has an orientation and the atoms in its lattice have particular locations: The crystal spontaneously breaks the underlying dynamical symmetries. An acoustic phonon corresponds to coherent microscopic displacements of the atoms in the lattice—a move from one spontaneously broken state to another.

Symmetry breaking itself can usually be quantified as the value of some observable called an order parameter. For a ferromagnet, the order parameter is the magnetization. For a Bardeen-Cooper-Schrieffer superconductor, the order parameter measures the density of Cooper pairs—loosely bound pairs of electrons that condense at low temperatures and give rise to the superconductivity.¹³ In all cases the possibility of spontaneous symmetry breaking is related to long-range order in the material system.

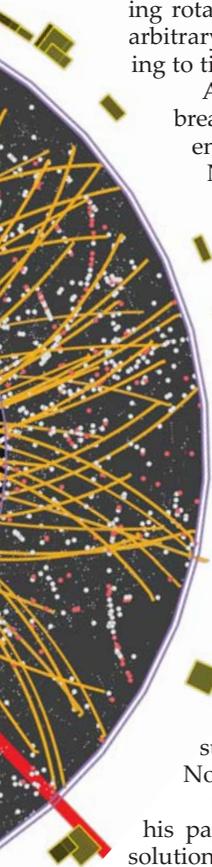




Figure 1. Yoichiro Nambu, Jeffrey Goldstone, and Philip Anderson penned important early chapters in the story of the Higgs boson. Beginning in 1960, particle physicists Nambu (**left**) and Goldstone (**center**) adapted ideas from condensed-matter physics to explore the relationship of symmetry breaking to the generation of massive particles. Two years later, condensed-matter physicist Anderson (**right**) argued that two types of troubling massless particles—Goldstone bosons and gauge bosons—could together yield a massive particle. (Nambu photo courtesy of the AIP Emilio Segrè Visual Archives, Marshak Collection. Goldstone and Anderson photos courtesy of the AIP Emilio Segrè Visual Archives, PHYSICS TODAY Collection.)

Goldstone boson. Superconductors also exhibit the famous Meissner effect, the expulsion of external magnetic fields that makes possible magnetic levitation. Anderson started with the simple London theory of the Meissner effect, rewrote the equations in a relativistic form more palatable to particle physicists, and showed that they describe what is, in effect, a massive photon; he called it a plasmon. Being a massive spin-1 boson, the plasmon has an extra longitudinal polarization compared with a propagating photon, which also is a spin-1 boson but with only two transverse polarizations. Where did the extra degree of freedom come from? It's the Goldstone mode! Anderson concluded that "these two types of bosons [the massless Goldstone boson and the massless gauge boson] seem capable of 'cancelling each other out' and leaving finite mass bosons only." Anderson's work made it clear that gauge theories and symmetry breaking have a special relationship; what remained was to understand it.

In his first paper of 1964, Peter Higgs pointed out a loophole in the Goldstone, Salam, and Weinberg theorem: The proof assumes explicit Lorentz invariance in relating broken symmetries to particles.⁶ Due to the necessity of fixing a gauge, that assumption is violated when electromagnetism or any other gauge theory is quantized.

Gauge symmetries, in fact, are not symmetries in the usual sense of relating seemingly distinct physical processes or configurations. Instead, gauge symmetry is a redundancy; you can formulate the quantum theory in a way that is manifestly Lorentz invariant and manifestly gauge invariant, but that formulation actually represents an infinite number of copies of the same physical system. In quantizing the theory, you need to choose arbitrarily one of the equivalent quantum descriptions of the physics; that's what is meant by fixing a gauge. The Lorentz

and gauge symmetries that are built into the classical theory still control the quantum physics, but they do so through a delicate choreography. Just how gauge fixing invalidates the Goldstone, Salam, and Weinberg theorem was shown in a paper by Gerald Guralnik, Carl Hagen, and Tom Kibble,⁷ published shortly after the Higgs loophole paper.

Mass effects

Even with the theorem by Goldstone and company evaded, the question remains as to how a massless gauge boson can obtain mass. To answer the question, in his second paper of 1964, Higgs reconsidered Goldstone's Mexican-hat model and added a photon.⁸ Once the boson field, also called the Higgs field, acquires a vacuum expectation value (particle physicists refer to "turning on the Higgs field") and the theory is quantized, the gauge symmetry of electromagnetism imposes two new interactions not present in ordinary electrodynamics. One of those is a mass term for the photon; the other is a coupling of the photon to the would-be Goldstone boson (see PHYSICS TODAY, September 2012, page 14).

Early in 1964 Robert Brout and François Englert showed that those two interactions work together to allow a gauge invariant quantum theory with a massive gauge boson.⁹ The gauge invariance, already explicitly broken by gauge fixing in quantization and seemingly again by the new mass term, is restored in the quantum theory by the coupling between the photon and the Goldstone boson. For one particular choice of gauge fixing called the Coulomb gauge, the physical degrees of freedom are manifest and the two interactions show that the would-be Goldstone boson becomes the longitudinal polarization needed to turn a massless gauge boson into a massive one. It has become customary to say that the massless Goldstone boson is "eaten" to give the gauge boson

mass. The “cancellation” of massless bosons to give a massive boson, as anticipated by Anderson and developed in the 1964 papers, is the famous Higgs mechanism; for their contributions to its discovery, Englert and Higgs received this year’s Nobel Prize in Physics. (For more, see page 10 of this issue.)

As recounted in his 2010 talk “My Life as a Boson,” Higgs submitted his second paper of 1964 to *Physics Letters*, which promptly rejected it.¹⁰ Shocked at that setback, he revised and expanded the manuscript, adding the key observation that when applied to a charged spinless boson, the Higgs mechanism leaves behind a neutral spinless boson. That neutral particle—the Higgs boson—has a mass determined by the shape of the Mexican-hat potential energy density, but that mass cannot be expressed in terms of the mass generated for the gauge boson. Higgs sent the improved revision to a different journal, *Physical Review Letters*, and it was promptly accepted.

At first, theorists thought that the most suitable application of spontaneous symmetry breaking to particle physics was in the arena of the strong interactions. Only in 1967 did Weinberg and, independently, Salam realize that the Higgs mechanism offered an elegant explanation of the weak interactions. In their model, which is now the electroweak portion of the standard model, four Higgs fields are related by a gauge symmetry of the type introduced by Yang and Mills. Three Goldstone bosons are eaten to give large masses to the W^+ , W^- , and Z bosons that mediate the weak interactions. An added bonus, not foreseen by Higgs and the rest, is that the Higgs field also gives mass to quarks and leptons, the elementary fermions that make up matter.

The mass of the Higgs boson left behind is not predicted, but the interactions of the Higgs with other elementary particles can be precisely computed as a function of its mass and the masses of the other particles. Furthermore, the exchange of virtual Higgs bosons generates an attractive short-range force. If the Higgs boson is an elementary particle, as so far appears to be the case, then that force is every bit as fundamental as the gauge-boson-mediated forces of the standard model. In that case, the Higgs would be the first fundamental force mediator ever detected that is not a gauge boson.

The discovery

The ATLAS and CMS (Compact Muon Solenoid) experiments at the LHC were built to probe the mechanisms of electroweak symmetry breaking and the particle origins of dark matter. Wired up with about a hundred million readout channels each and made up of many thousands of tons of material that interacts with the particles emanating from the LHC’s high-energy proton–proton collisions, the two detectors have already managed to capture and reconstruct many rare Higgs boson candidate events.¹¹

Since Higgs bosons decay into other particles after about 100 yoctoseconds (10^{-22} seconds), the collider searches involve several different decay signatures or channels. Figure 3 illustrates the two most important channels used by ATLAS and CMS in their quest for the Higgs. One represents the Higgs

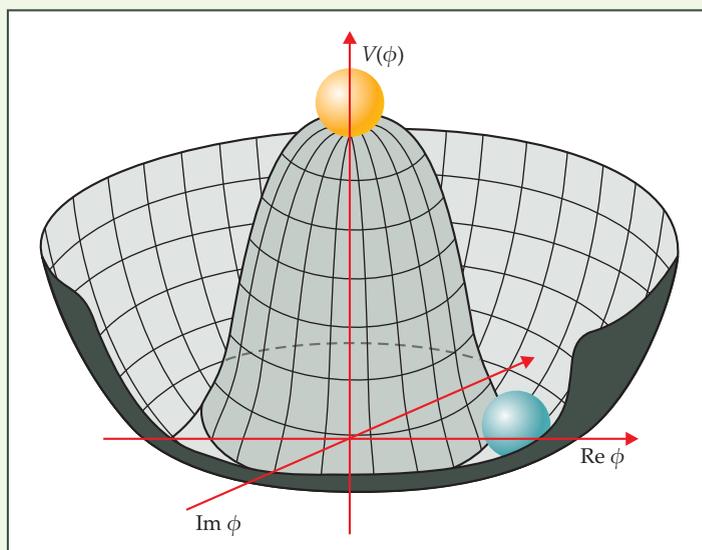


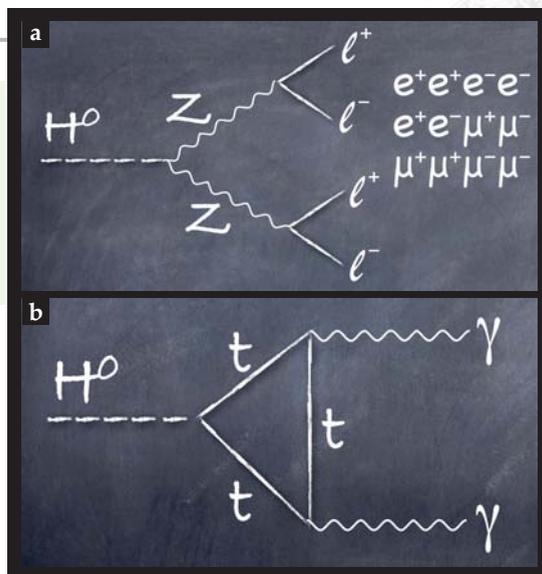
Figure 2. The Mexican-hat potential energy density considered by Jeffrey Goldstone in his seminal 1961 paper.² The energy density is a function of the real (Re) and imaginary (Im) values of a spinless field ϕ . In the context of the electroweak theory developed later in the decade, the yellow ball at the top of the hat would represent the symmetric solution for the potential, in which the photon, W bosons, and Z boson are all massless. The blue ball in the trough represents the solution after symmetry breaking. In that solution the W and Z bosons are massive and the photon remains massless. The steepness of the trough is related to the mass of the Higgs boson.

decay process into two virtual Z bosons, each of which, in turn, decays into an electron–positron or muon–antimuon pair. The other shows the Higgs decay into two photons. The image on pages 28 and 29 shows a visualization of the data produced by a Higgs boson candidate at the LHC; the four decay products are muons or antimuons—a pair of each—whose tracks are depicted as red lines.

The experimental results so far suggest that the particle observed at the LHC is indeed a Higgs boson, though not necessarily possessing exactly the properties postulated by the standard model. The discovery itself is based on large excesses of Higgs-like events in the two decay channels described above, supported by less conclusive but compatible excesses observed in other channels. Figure 4 displays CMS data for the four-lepton channel. The measured mass is about 126 GeV/ c^2 , intermediate between the mass of the Z boson and the mass of the top quark.

The new particle cannot be a spin-1 particle because the decay of such an object into two photons is forbidden by a general result known as the Landau–Yang theorem. Its wavefunction does not change sign when operated on by CP (a product of the discrete symmetries of charge conjugation and coordinate inversion, or parity), as the pion wavefunction does. So either the new particle is unchanged by CP , as a Higgs boson is, or it could be a CP -violating admixture if there exists a new source of matter–antimatter asymmetry related to the Higgs. The production rate of the particle and the degree to which it decays into different channels appear consistent with the standard-model predictions for the Higgs boson, although the experimental uncertainties are

Figure 3. Two crucial decay channels revealed the Higgs boson in experiments conducted at CERN. **(a)** The Higgs boson decays into two virtual Z bosons, and each Z boson decays into an electron–positron pair or a muon–antimuon pair. (The four daughters are called leptons, whence the symbol ℓ .) The possible four-lepton final states are indicated to the right of the panel. **(b)** The Higgs boson decays into two photons (γ). Note that the decay proceeds through a triangle of virtual top quarks (t).



still rather large. The new particle decays roughly eight times more often into a pair of W bosons than into a pair of Z bosons, as would be expected for a Higgs with a mass of $126 \text{ GeV}/c^2$ that is related to the three Goldstone bosons eaten to give mass to the W^+ , W^- , and Z. Exotic spin-2 particles and so-called dilatons have been proposed as alternative explanations of the LHC signals, and those Higgs impostors cannot be entirely excluded. The full picture will become far clearer during the upcoming LHC Run 2, which starts in 2015; the data collected and analyzed then should yield precise values for a large number of coupling parameters.

The fate of the universe

Data from the LHC already fix one property of the Higgs boson with precision: its mass, known to better than 1% accuracy. Assuming the validity of the standard model, that determination allows for a calculation of the shape of Goldstone's Mexican hat. In the standard model, the energetics of turning on the Higgs field has calculable quantum corrections from the couplings of the Higgs to the other particles. The largest effect, which comes from the heavy top quark, mitigates the energy penalty for the Higgs field vacuum expectation value to increase to even larger values and suggests that the vacuum is unstable.

In fact, that suggestion is backed up by the most precise calculations to date. The possibility that the universe could be in a metastable vacuum has been studied since the 1970s, but only now can scientists plug in the numbers. Taken at face value, the result implies that eventually (in 10^{100} years or so) an unlucky quantum fluctuation will produce a bubble of a different vacuum that will expand at nearly the speed of light, destroying everything. Strikingly, the measured Higgs and top-quark masses put the universe right at the edge of the stability versus metastability divide; if the Higgs boson were a few percent heavier, or the top quark a few percent lighter, then the vacuum would be stable. Is our existence at the edge just a coincidence, or is Nature telling us something?

Particle theorists realize that they may be doing the wrong calculation; supersymmetry, for example, restricts the Higgs potential energy density in such a way as to ensure stability. Supersymmetry predicts the existence of superpartners for all the standard-model particles and at least four more kinds of Higgs boson. In realistic models, supersymmetry is assumed to be spontaneously broken, and thus all the superpartner particles and extra Higgs bosons may be quite heavy. If supersymmetry breaking is connected to electroweak symmetry breaking, at least some of those new particles should be discovered at the LHC. As yet, none have been detected.

The simplest supersymmetry models led theorists to predict long ago that the Higgs boson would be lighter than about $130 \text{ GeV}/c^2$. That's an impressive success, especially since alternatives to supersymmetry typically imply a much heavier Higgs. Still, the observed mass of $126 \text{ GeV}/c^2$, while compatible with supersymmetry, is uncomfortably high and has caused many theorists to question whether the simpler models are neglecting some crucial ingredient.

To address such questions, physicists need to determine experimentally if there are any new particles that interact with the Higgs boson and if there are Higgs bosons different from the one discovered last year. At the LHC, experimentalists will both search for new heavy particles and attempt to measure the Higgs properties with sufficient precision to see the effects of the particle's interaction with unknown states. The need for precision even beyond what the LHC will achieve is a strong argument for building a Higgs factory such as the proposed International Linear Collider (ILC).

A portal to dark matter?

We noted earlier that most of the matter in the universe is unaccounted for by the baryonic matter that makes up stars and planets. Instead, the cosmos is shaped and held together by dark matter consisting of one or more varieties of unknown exotic particles. The Higgs discovery has pointed physicists in an astonishing direction in their quest to solve one of the primary puzzles of cosmology: What is the nature of that dark matter?

A heavy stable particle that interacts weakly with ordinary matter can explain the observed abundance and clustering of the dark matter in the universe. That WIMP (weakly interacting massive particle) paradigm has motivated ultrasensitive "direct" detection experiments that look for interactions of WIMPs with ordinary matter, attempts to observe "indirect" signals of dark matter annihilating to standard-model particles in and around the Milky Way, and LHC searches for both WIMPs and their heavier unstable parents. No confirmed signals have been seen in any of those searches, which greatly constrains particle-physics scenarios for explaining dark matter (see PHYSICS TODAY, May 2013, page 14).

One of the basic challenges confronting particle physicists is to identify the force mediator, in addition to gravity, between dark matter and ordinary matter. The WIMP paradigm suggests the Z boson of the weak interactions is a viable candidate. For many dark-matter candidates, however, Z-boson mediation is already ruled out by experimental results over the past decade, and it is an increasingly constrained possibility for the remaining candidates.

If experimental results were to force physicists to give up on the Z boson, then the Higgs would be the only known particle that could interact directly with dark matter. The observed dark-matter abundance gives us some idea how strongly the Higgs should interact with dark particles. With that guidance, direct-detection rates of dark matter turn out to be a sensitive function of the Higgs boson mass, and a mass of $126 \text{ GeV}/c^2$ implies that some direct-detection experiments should see interaction signals in the next few years. Both the LHC and the ILC might also produce dark particles and enable a rigorous determination of their identity.

Back to the beginning

In most condensed-matter systems, simply raising the temperature will restore symmetries that had been broken. The transition from a state of broken symmetry to one in which the symmetry is restored, or vice versa, is a phase transition. Particle physicists postulate that the same phenomenon occurs for the symmetries of the basic forces of nature.

Early in the history of the cosmos, when the temperature was very high, all those symmetries were manifest. As the universe expanded and cooled, a series of phase transitions spontaneously broke many but not all of them. The electroweak phase transition denotes the cosmic event in which the Higgs field turned on to its current fixed vacuum expectation value. Depending on the detailed properties of the Higgs field, a suitable source of *CP* violation, and the existence of other heavy particles that interact with the Higgs, theorists have shown that the electroweak phase transition may have produced the slight excess of matter over antimatter that is responsible for our existence. Future data from the LHC and ILC will clarify whether that theoretical result paints a correct picture of the genesis of matter.

Given the intellectual history of the Higgs mechanism, it is perhaps no surprise that Higgs bosons are also a hot topic in condensed-matter physics. More than 30 years ago, Peter Littlewood and Chandra Varma realized that a precise analog of Higgs bosons can be seen in the fluctuations of the amplitude of the Cooper pair density in niobium selenide superconductors.¹² With the advent of ultrafast, high-intensity, and spectrally narrow sources in the UV and x-ray regimes, condensed-matter experiments are beginning to identify the components of similar Higgs boson-like modes in a variety of interesting systems, including superfluids and antiferromagnets. Physicists study those modes to gain new insights about the fundamental theories underlying exotic materials. Perhaps in the coming decade the story of the Higgs boson will come full

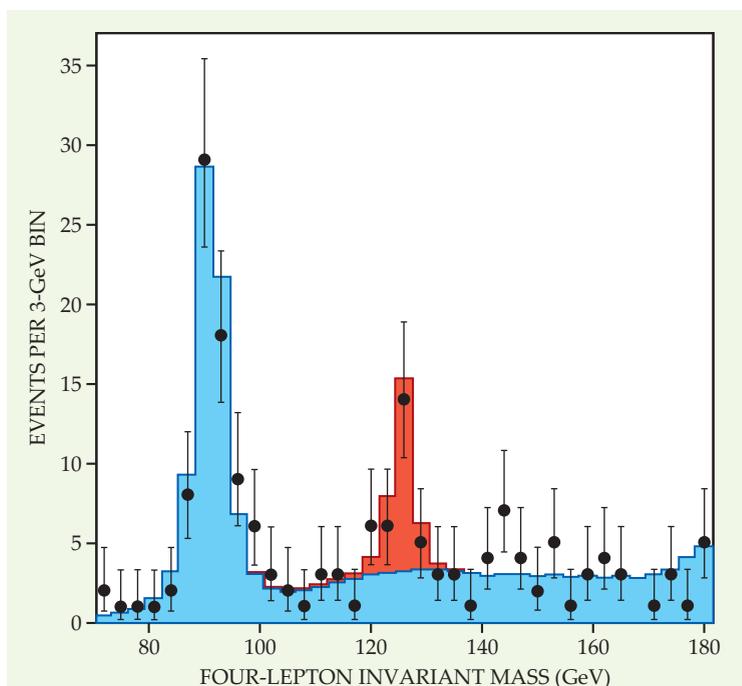


Figure 4. A Higgs boson peak at a mass of $126 \text{ GeV}/c^2$ rises above background in these data obtained by the Compact Muon Solenoid collaboration. In this plot, the horizontal axis represents the relativistically invariant mass of four leptons produced in proton-proton collisions at CERN's Large Hadron Collider. The blue histogram represents expected background, including extremely rare four-lepton decays of the Z boson, whose mass is $91 \text{ GeV}/c^2$. The red histogram shows the expected events arising from a $126\text{-GeV}/c^2$ Higgs boson. The experimental data (black points) clearly exceed background near $126 \text{ GeV}/c^2$ and agree with the Higgs-based expectations. (Adapted from ref. 11, CMS collaboration.)

circle, with condensed-matter physicists again identifying surprising features in materials that inspire new paradigms for particle theory.

We are grateful to Phil Anderson, Laura Greene, David Gross, and Peter Littlewood for insightful discussions and critical comments.

References

1. P. W. Anderson, *Science* **177**, 393 (1972).
2. J. Goldstone, *Nuovo Cimento* **19**, 154 (1961).
3. J. Goldstone, A. Salam, S. Weinberg, *Phys. Rev.* **127**, 965 (1962).
4. C. N. Yang, R. L. Mills, *Phys. Rev.* **96**, 191 (1954).
5. P. W. Anderson, *Phys. Rev.* **130**, 439 (1963).
6. P. Higgs, *Phys. Lett.* **12**, 132 (1964).
7. G. Guralnik, C. Hagen, T. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964).
8. P. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964).
9. F. Englert, R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964).
10. P. Higgs, "My Life as a Boson," talk given at King's College London, 24 November 2010. Available at <http://www.ph.ed.ac.uk/higgs/life-boson>.
11. G. Aad et al. (ATLAS collaboration), *Phys. Lett. B* **716**, 1 (2012); S. Chatrchyan et al. (CMS collaboration), *Phys. Lett. B* **716**, 30 (2012).
12. P. B. Littlewood, C. M. Varma, *Phys. Rev. Lett.* **47**, 811 (1981).
13. See, for example, C. M. Varma, <http://arxiv.org/abs/cond-mat/0109409>. ■