

Directional Entanglement of Quantum Fields with Quantum Geometry

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It is shown that Planck scale limits on directional information could entangle states of quantum fields at much lower energies and on macroscopic scales, and significantly reduce the total number of degrees of freedom of large systems. Transversely localized solutions of the relativistic wave equation are used to show that the path of a massless particle with wavelength λ that travels a distance z has a wave function with indeterminacy in direction given by the diffraction scale, $\langle \Delta\theta^2 \rangle > \sqrt{2}\lambda/\pi z$. It is conjectured that the spatial structure of all quantum field states is influenced by a new kind of directional indeterminacy of quantum geometry set by the Planck length, l_P , that does not occur in the usual classical background geometry. Entanglement of field and geometry states is described in the small angle approximation, using paraxial wave solutions instead of the usual plane waves. It is shown to have almost no effect on local measurements, microscopic particle interactions, or measurements of propagating states that depend only on longitudinal coordinates, but to significantly alter properties of field states in systems larger than $\approx \lambda^2/l_P$ that depend on transverse coordinates or direction. It reduces the information content of fields in large systems, consistent with holographic bounds from gravitation theory, and may lead to quantum-geometrical directional fluctuations of massive bodies detectable with interferometers. Possible connections are discussed with field vacuum energy, black hole information, and inflationary fluctuations.

INTRODUCTION

Quantum field theory[1] completely accounts for measured microscopic behavior of matter and its interactions, apart from gravity. It blends quantum matter with classical space-time: field amplitudes are quantized, but field eigenmodes have a determinate, continuous spatial structure, embedded in a classical geometry. It is widely acknowledged that this approximation is not a complete description of nature, because the quantum theory does not include dynamical degrees of freedom of the geometry. Those are described by a classical theory, general relativity.

In general relativity, geometry is not a quantum system, but it is a dynamical physical system: it carries energy and information, and interacts with matter. Geometry couples to an idealized classical model of matter, represented by an energy-momentum tensor, which is not a quantum system. Although this approximate description of how matter relates to geometry works well where effects of gravity have been tested, it also cannot be complete, because real quantum matter cannot be localized in a way that couples unambiguously to a classical metric.

It is well known that the hybrid of quantum field theory and classical geometry breaks down in the regime of strong gravity and indeterminate geometry at the Planck scale, $l_P \equiv ct_P \equiv \sqrt{\hbar G/c^3} = 1.616 \times 10^{-35}\text{m}$, where G denotes Newton's constant, \hbar denotes Planck's constant, and c denotes the speed of light— that is, at very small lengths and high energies. At such small scales, the gravity of even a single spatially-localized quantum particle is enough to create a black hole, so geometry is indeterminate, and the classical approximation to dynamical geometry becomes self-inconsistent. Many theoretical efforts to reconcile geometry with quantum physics have concentrated on resolving inconsistencies at the Planck scale[2].

However, the essential incompatibility of quantum mechanics with dynamical geometry is not confined to small scales. Physical quantum states of radiation generally have indeterminate spatial distributions— including superpositions of very different space-time histories— even on macroscopic scales. It is mathematically impossible to couple a classical space-time to matter in such a state, since there is no scale where the matter has a determinate, classical distribution in space.

For example, a photon may be radiated from a distant body in a flat space-time in state encompassing a wide angle, and its wave function can expand to a transverse size of many light years before it is detected; after it is detected, the wave function over its entire path instantly and retroactively becomes much better localized, to the width of the telescope aperture that collected it. General relativity can ignore this kind of indeterminacy in practice because the gravitational effect of such grossly indeterminate states is usually small, but at some level there must be macroscopic quantum properties of geometry that are not included in the standard picture. Somehow, the quantum

state of a space-time must be able to consistently couple to indeterminate quantum states of matter, while consistently preserving causal structure, approximate locality, and other classical attributes.

One proposed solution is that matter and geometry are really a single quantum system[3–6]. There are many examples of physical systems where a deeper level of theory reveals completely new degrees of freedom (e.g., [7, 8]). It could be that matter and geometry seem so different from each other because they emerge with very different kinds of quantum degrees of freedom in systems much larger than the Planck scale. The signature properties of classical geometry, such as locality and directionality, could emerge as a large scale approximate behavior of certain geometrical degrees of freedom. Although it is known that general relativity can be derived from a statistical or thermodynamic theory [9–12], there is no standard theory for the relationship between quantum matter and quantum geometrical degrees of freedom.

A specific new hypothesis concerning this relationship on scales much larger than the Planck length is proposed here. It is argued that the usual, seemingly benign assumption of spatially-classical field eigenmodes, whose amplitudes are quantized in field theory, must break down in a subtle way on large scales, because it imposes an unphysical degree of directional independence between quantum field states: standard field theory implies higher angular resolution between events than could be achieved by any actual measurement, even with Planck wavelength radiation. The proposal here is that fields and geometry behave like subsystems of a single quantum system, whose directional degrees of freedom are entangled. Field states propagating over a sufficient macroscopic distance are affected by Planck scale limits on directional information, and thereby become entangled with geometrical degrees of freedom originating at the Planck scale. Quantum geometry thus affects the behavior of field states in a specific and possibly measurable way, not only at Planck energy and curvature scales, but also in large systems in the limit of vanishing gravity.

The proposed model does not directly address the character of Planck scale microstates, or systems where gravity is important: it applies to fields propagating on large scales in a nearly-flat space-time, and bodies nearly at rest. Still, the model implements nonlocality similar to that inferred for gravitational systems (e.g. [13–17]), and matches their maximum information content[9–12, 18]: the information in a region is given by the bounding area of a causal diamond in Planck units, instead of the volume of phase space as in field theory.

Nonlocal and holographic quantum states of extended systems are thoroughly studied in some particular highly curved space-times, for example black holes and anti-de Sitter space (e.g., [19–27]), where precise dualities relate system states in a curved bulk space to those of a conformal field theory on its boundary. However, these techniques do not address field states in a nearly-flat space-time, which is the subject here, and is also the regime most accessible to direct experimental tests.

The view here is that the relationship between field-like and geometry-like degrees of freedom can be approximately understood from the way a nearly-classical geometry emerges on large scales from a quantum system: the Planck limit appears, in the emergent space, in the form of a limit on the amount of directional information. As a result, a field mode in a large system is not a completely separate subsystem from the geometry. A theory based on paraxial wave modes is used to show quantitatively how Planck-limited directional indeterminacy entangles quantum fields with geometry in large systems. The spatial distribution of information in large systems differs substantially from quantum field theory, in a subtle but measurable way.

A similar limitation on the spatial extent of field states was considered by Cohen et al.(CKN, ref. [28]), who showed that the effects would be unobservable in small-scale particle experiments, but still greatly reduce the amount of fine tuning needed on cosmic scales to avoid unphysically dense states. The entanglement proposed here also introduces predominantly macroscopic effects of a similar magnitude in response to Planck scale physics, but is more specific about the nature of the effects. It predicts new observable consequences in some macroscopic systems, since the limit on directional information degrades the fidelity of angular relationships on large scales in a specific way. It opens up the possibility of specific new behaviors of systems that depend on nonlocalized states of matter fields, connected with fluctuations in interferometers, cosmic vacuum energy density, black hole information, and cosmic inflation.

SPATIAL WAVE FUNCTIONS OF FIELD STATES

A field can be decomposed into different kinds of states that correspond to different kinds of preparation and measurement. The most commonly used basis modes are plane waves, which have no uncertainty in orientation but are completely delocalized in space. Spatial localization requires a superposition of plane wave modes. A wave packet of modes in a single direction can create longitudinal localization, but any transverse localization is associated with transverse momentum, and necessarily includes some admixture of modes with different orientations. The extreme case is to prepare a state as a nearly point like event, by specifying where a particle is at a particular place and time.

In this case, the subsequent wave function spreads like a spherical wave with an indeterminate direction.

It is possible to define and quantize modes that have some degree of transverse localization, and also some directionality. The spatial character of these states is described not by plane waves, but by paraxial solutions, described below. These states resemble particles that travel from one somewhat localized place to another, along a somewhat localized path, albeit with some quantum indeterminacy. Frequency eigenmodes have spatial wave functions that are confined to a narrow tube out to a certain radial distance, within which they resemble classical paths, and beyond which they spread to a larger angle. There is a minimum transverse width at a given distance, the diffraction scale, approximately the geometric mean of wavelength and propagation distance. A quantum state of a particle with a direction requires preparation over some finite transverse patch, the size of which determines a propagation distance over which directional information is preserved. At a given distance and frequency, the lowest order paraxial wave function gives a minimum intrinsic uncertainty in the direction of a particle's path, which is derived here.

Paraxial Field States and Directional Uncertainty of Paths

Techniques of field quantization can be applied to field modes that are not plane waves. The appropriate choice of modes depends on boundary conditions that define and prepare the field system. Real physical field states are prepared and measured in a certain way, with spatially localized interactions. They can resemble plane waves locally, but have small transverse phase gradients that lead to curved wavefronts at large separations. One way to discuss this kind of state is to quantize paraxial modes, which split directional from longitudinal or propagation degrees of freedom in the small angle approximation. Paraxial solutions to the wave equation are familiar from applications in laser cavities[29, 30].

In three dimensions, consider the amplitude of a field component with a sinusoidal time dependence, $A \propto e^{-i\omega t}$, where $\omega = ck = 2\pi c/\lambda$. Express the spatial dependence of the field as the spatial modulation of a complex carrier wave propagating on the z axis, in the form

$$A(\vec{x}) = e^{ikz}\psi(x, y, z). \quad (1)$$

Here $A(\vec{x})$ is a complex phasor representing the amplitude and phase at each point, and Euclidean coordinates $\vec{x} = x, y, z$ denote position. The longitudinal coordinate z corresponds to position in a particular direction that defines the orientation of the reference wave, and x, y to positions in transverse dimensions. The field ψ describes deviations of amplitude from a plane wave normal to the z axis.

Starting with the wave equation for A ,

$$(\nabla^2 + k^2)A(\vec{x}) = 0, \quad (2)$$

the wave equation for ψ becomes

$$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 - 2ik\partial\psi/\partial z = 0. \quad (3)$$

To describe the deviation of the field from a plane wave in the small-angle approximation, assume that the third term is negligible compared with the others:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - 2ik\frac{\partial\psi}{\partial z} = 0. \quad (4)$$

The paraxial wave equation (Eq. 4) has the same form as the time-dependent nonrelativistic Schrödinger wave equation in two transverse dimensions, with z replacing time and $-k$ replacing m/\hbar .

Unlike plane waves, normal modes of this system, called paraxial modes, are spatially confined in the transverse directions. This leads to a transverse momentum and therefore a transverse spread of propagation direction, described by ψ . The lowest order axially symmetric mode is given by[29, 30]

$$\psi(r, z) = \exp[-i(P + kr^2/2q)], \quad (5)$$

where $r^2 \equiv x^2 + y^2$, $dq/dz = 1$, and $dP/dz = -i/q$. The complex beam parameter q can be expressed in terms of two real beam parameters that depend on z : the width w of the gaussian profile, and the radius of curvature R of the wave fronts of constant phase, related by:

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}. \quad (6)$$

In this family of solutions, the gaussian has a minimum width w_0 or “waist” at $z = 0$, where the wave fronts lie in a plane. As a function of z , the beam width is

$$w^2(z) = w_0^2[1 + (\lambda z/\pi w_0)^2], \quad (7)$$

and the wavefront curvature radius is

$$R(z) = z[1 + (\pi w_0^2/\lambda z)^2]. \quad (8)$$

The width increases, that is, the beam diverges at larger z . A smaller waist—that is, a better transverse localization at the origin—makes for a more rapid divergence, and less localization far away:

$$\frac{\lambda z}{\pi w_0^2} = \frac{\pi w^2}{\lambda R}. \quad (9)$$

The wavefronts are nearly flat near the origin ($R = \infty$), and become curved far away, with $R \rightarrow z$ at large z . The transverse width maintains localization—suppresses the spreading—until z is of order R . For $z \ll w_0$, a paraxial solution resembles a plane wave; for $w_0 < z < w_0^2/\lambda$, it resembles a wave confined to a tube of width $\approx w_0$; and for $z \gg w_0^2/\lambda$, it resembles a spherical wave emanating from a point source.

Higher order modes of propagation form a complete and orthogonal set of functions, into which any arbitrary distribution of monochromatic radiation can be decomposed. Their wave functions have an overall Gaussian envelope that follows the fundamental mode, so that mode represents a lower bound on the overall transverse width. They have finer-scale 2D structure in x and y that scales with w .

For any z , there is thus a unique solution that minimizes the width $w(z)$, for which $w/w_0 = R/z = \sqrt{2}$. The variance of $\psi(r, z)$ has a minimum value,

$$w_{min}^2 = \langle \Delta r_{min}^2 \rangle = \sqrt{2}\lambda z/\pi, \quad (10)$$

and introduces a new transverse length scale that grows with system size. The corresponding wave function of direction $\theta = r/z$ for a particle path,

$$\psi(\theta, z) = \exp[-i\theta^2/2\langle \Delta\theta^2 \rangle] \quad (11)$$

has a minimum uncertainty,

$$\langle \Delta\theta_{min}^2 \rangle = w_{min}^2/z^2 = \sqrt{2}\lambda/\pi z. \quad (12)$$

These states display the minimal diffractive uncertainty inherent in any wave state in a finite system. Physically, it may be interpreted as *a directional uncertainty of particle paths, or directional relationships between events, defined by any field at a given wavelength*. States with more precise directionality are possible—for example, plane waves—but they have a larger transverse width, so their entire wave functions actually subtend a larger angle, and create a larger uncertainty for the orientation of a particle path from the origin to z . It is worth emphasizing once more that this uncertainty is a general property of frequency eigenstates of any field. Although we are calling it a diffractive uncertainty, it has nothing to do with any additional physical effect of propagation, such as dispersion, scattering or scintillation.

The paraxial modes are a better approximation than plane waves to states resembling wave functions of real-world particles that travel from one place to another, since the modes themselves (as opposed to superpositions of modes) include the maximal amount of directional localization consistent with a system’s size. Their properties thus interpolate between particle and wave. They show how light does not travel in a straight line, or indeed any kind of definite path: it is a wave, and even in flat space travels in a state that is a superposition of straight lines with different orientations.

Paraxial Modes as a Basis for Field Quantization

The paraxial formulation lets us quantize the amplitude of the field in the usual way as a simple harmonic oscillator, with the usual dependence on classical space-time coordinates of time and propagation distance. The usual machinery of quantum field theory still applies, such as raising and lowering operators for occupation number. This quantization can be applied to the harmonic radial z component, while the directional components are represented by

a transverse wave function $|\psi\rangle$ that includes explicit geometrical localization and uncertainty, and is time-independent in the paraxial approximation. The transverse spatial dimensions can then be quantized and entangled geometrically, separately from the quantum field amplitude.

Usually, field modes are quantized by introducing a quantum operator for the amplitude of plane waves. The wave function is

$$\exp[i(\vec{k} \cdot \vec{x} - \omega t - \phi)]|A(\vec{k}, n)\rangle, \quad (13)$$

where $|A(\vec{k}, n)\rangle$ represents a simple quantum harmonic oscillator with frequency $\omega = c|k|$ and the occupation quantum number n corresponds to particle number. As far as quantum mechanics is concerned, the wave vectors \vec{k} are also just quantum numbers: they label states of fields in a given volume. The \vec{k} 's depends on the volume of the system.

For paraxial field modes in the basis aligned with the z axis, we have instead a wave function

$$\exp[i(kz - \omega t - \phi)]|A(n)\rangle|\psi(x, y, z)\rangle, \quad (14)$$

where $|\psi(x, y, z)\rangle$ represents the transverse wave function whose eigenstates have the structure just described. The number of modes gives the usual answer for total field degrees of freedom in a volume, but unlike the plane wave decomposition, the individual modes are localized transversely on the diffraction scale. An excitation of a mode corresponds to a particle whose spatial wavefunction is spread spatially over a diffraction-width tube—the quantum state of a coherent “beam of light”.

Provided we include all the higher order modes, there is no physical difference between these descriptions of the overall field system, if the geometry is (as usual) assumed to be classical. They simply quantize different decompositions of a wave field, and refer to different ways of preparing and measuring field states. In many situations, the waist size is so much bigger than a wavelength that for many purposes in local interactions, these states behave in almost the same way as plane waves. However, the paraxial modes each explicitly display the physical limit on directional resolution.

ENTANGLEMENT OF FIELD STATES WITH QUANTUM GEOMETRY

Planck Limited Directional Information

Consider a plane wave in a space-time volume defined by a causal diamond of duration $\tau = z/c$. (That is, the spacetime region invariantly defined by the future and past light cones of the two events with this proper timelike separation). The normal to the wavefronts defines a direction with an uncertainty $\approx \lambda/c\tau$, much less than the diffractive directional uncertainty $\approx \sqrt{\lambda/c\tau}$. Of course, that precision comes at the expense of complete transverse delocalization: there is no transverse position information in the plane wave. By contrast, geometrical states that resemble a classical spacetime must not have the character of plane waves, because they must appear on large scales as a nearly-classical geometry, with approximate localization in all three spatial dimensions. As we have seen, localization is accompanied by directional uncertainty.

The diffraction limit (Eq. 12) for a Planck wavelength state and a propagation distance z ,

$$\Delta\theta_P = \langle\Delta\theta_{min}^2(\lambda = l_P)\rangle^{1/2} = (\sqrt{2}l_P/\pi z)^{1/2}, \quad (15)$$

corresponds to the best precision with which directions between events can in principle be measured at separation z , using Planck wavelength radiation or particles. We propose the hypothesis that *Eq. (15) approximately represents a fundamental limit to directional resolution*—a new effect of quantum geometry. Information in geometrical degrees of freedom thus has a different character from that in quantum fields, and coherently affects the spatial structure of field states.

The total amount of directional information—the number of directional states in a volume—must not exceed the amount of directional information in the geometry. Directional entanglement of field states with geometry becomes significant when the smallest angular structure of normal field states, on the scale of the field wavelength $\Delta\theta \approx \lambda/z$, is smaller than the Planck angular resolution of the geometry, on scale $\Delta\theta_P \approx (l_P/z)^{1/2}$. That happens for systems of duration larger than a “geometrical entanglement scale” τ_g ,

$$c\tau_g(\lambda) \approx \lambda^2/l_P. \quad (16)$$

For larger systems—in the lower region shown in Figure (1)—field modes are significantly altered, due to a breakdown of the classical geometry approximation.

Model of the Entangled System

Suppose then that field modes and emergent geometry are entangled parts of a composite quantum system. The entanglement is significant if there is not enough geometrical angular resolution to resolve the structure of the field wave function, and transverse coordinates do not behave classically. The paraxial decomposition provides a way to quantitatively estimate the effect of separately quantizing the geometry in the transverse (x, y) directions, while retaining the usual field-theory quantization of longitudinal components.

Consider a wave function of the field mode (Eq. 14) entangled with geometry, in a way similar to standard entanglement of quantum subsystems[31, 32]. The overall wave function can be approximated by a product of field-like and geometry-like subsystems,

$$\exp[i(kz - \omega t - \phi)] \otimes |A(n)\rangle |\psi_f(x - x_g, y - y_g, z)\rangle \otimes |\psi_g(x_g, y_g, z)\rangle \quad (17)$$

where f and g refer to the field and geometrical wave functions, respectively. Here, x_g, y_g denote transverse geometrical position variables, and the wave function $|\psi_g(x_g, y_g, z)\rangle$ represents the amplitude for the geometry to depart from the classical one by those values. Its width is approximately given by the minimal width for a Planckian wave in Eq. (10),

$$w_g^2(z) \approx \langle \Delta r_{min}^2(\lambda = l_P) \rangle = \sqrt{2} l_P z / \pi. \quad (18)$$

The transverse width of $|\psi_g(x_g, y_g, z)\rangle$ can be estimated more precisely from an operator model of the geometry that allows a normalization of directional information from gravitational thermodynamics, as discussed below.

The product of the field and geometry structures gives an approximation to the overall transverse wave function of the combined system. For a given λ , the directional uncertainty is field-like at small z , geometry-like at large z . The geometrical part ψ_g represents a shared, coherent displacement for fields and massive bodies that only appears with reference to the large separation z ; in a small volume, there is no locally detectable deviation from usual theory.

The nature of the entanglement depends on the scale, orientation and preparation of the state. The phase of the overall state is modulated by the quantum effects of geometry by roughly an amount

$$\Delta\phi \approx (\partial\psi_f/\partial x_i) \cdot w_g, \quad (19)$$

where x_i with $i = 1, 2$ refers to the transverse coordinates x, y . If this quantity is much less than unity, then entanglement has little effect, and the field behaves ordinarily, almost as if geometry were classical; if it is much greater than unity, then the bulk of the directional information is entangled with the geometry.

For a mode with wavefronts nearly normal to the z axis, $|(\partial\psi_f/\partial x_i)| \ll \lambda^{-1}$, so the phase of modes close to this direction are affected hardly at all by the geometry, even for $\tau \gg \tau_g$. We can say that measurement of such a field prepares the geometry in an eigenstate of this direction. However, a field mode oriented in a different direction typically has $|(\partial\psi_f/\partial x_i)| \approx \lambda^{-1}$, so entanglement changes the field phase by

$$\Delta\phi \approx \sqrt{l_P \tau} / \lambda = \sqrt{\tau / \tau_g}; \quad (20)$$

the field state is substantially affected by geometry at $\tau > \tau_g$. On the scale τ_g , the quantum geometry typically changes the phase of transverse modes by an amount of order unity, so the entanglement is substantial and the field modes no longer contain angular information approximately independent of the geometry or of each other. At small $\tau < \tau_g$, the geometry always produces small fractional changes in the transverse field phases, so field theory behaves almost as if it inhabits a classical geometry. Small quantum-geometrical effects may nevertheless be detectable in signals that measure small phase differences by using very large numbers of quanta, as discussed below for the case of laser interferometers.

The geometrical state has substantial spatial coherence. Suppose a transverse measurement is made that ‘‘collapses’’ the geometrical position state associated with a particular direction to a definite value. The geometrical wave function then spreads only slowly with time, with a width after time τ given by $w_g(z \approx c\tau)$. Thus, neighboring bodies and particles share almost the same transverse geometrical state, if measured in the same direction, with positions differing by much less than their separation. In a continuous measurement in a fixed apparatus, the difference nevertheless gives rise to a slow fluctuation of measured transverse position of massive bodies. The slow time variation on a timescale $\tau \approx z/c$ has not been explicitly included here, as it represents a deviation from the paraxial approximation.

Information content of geometry and field states

In standard field theory, the the number of independent frequencies in a causal diamond of duration τ is roughly the bandwidth divided by the resolution, $\approx c\tau/\lambda$. The number of independent directional states comes from enumerating the complete family of independent wave solutions in the volume of radius $c\tau$. The directional information grows like $(c\tau/\lambda)^2$, so the total information grows extensively, like the product $\mathcal{N} \approx (c\tau/\lambda)^3$. The same result can be derived in the usual way using rectilinear coordinates and modes.

For the quantum geometry, the radial information also grows linearly, like τ/t_P . However, in this case the directional information grows only like $\Delta\theta_P^{-2} \approx \tau/t_P$, determined by the minimal angular uncertainty $\Delta\theta_P$ in the wave function of the fundamental Planck-wavelength gaussian mode. Therefore, the total geometrical information is about $(\tau/t_P)^2$. That agrees with estimates from gravitational theory, for example with the entropy of black holes.

As shown in Figure (2), for large volumes and small λ (but still $\gg l_P$), the geometrically-limited information is much less than predicted by standard field theory. The information in fields in this regime is the product of the radial and limited directional information, $\mathcal{N} \approx (c\tau/\lambda)(\tau/t_P)$. For smaller volumes it is the other way around— there are many geometrical degrees of freedom not resolved by fields with wavelengths much longer than Planck. As a result, the angular effect of geometrical entanglement cannot be resolved by fields in small scale experiments, so the geometry looks classical, and fields behave in the usual way.

PHYSICAL EFFECTS IN SYSTEMS MUCH LARGER THAN THE PLANCK LENGTH

Entanglement with geometry affects nonlocal aspects of field behavior on scales much larger than the Planck length. Directional entanglement influences the behavior of physical systems both in near-vacuum states, where fields propagate in nearly-flat space-time, and in situations with significant gravity. The model of entanglement presented above provides a tool to estimate the nature and magnitude of new physical effects. Estimates are presented here for several different kinds of physical systems.

Particle Experiments

Microscopic Interactions

Consider interactions measured in direct particle experiments of the usual kind, such as particle collider experiments. The interactions occur in a microscopic volume that is nevertheless much larger than the Planck length. In this situation, geometry-limited paraxial modes are almost indistinguishable from standard plane waves; at TeV scale, the transverse phase gradient in the wave fronts is of the order of $10^{-8}/\lambda$. The precision of experiments, and the dynamic range of scales probed, are not sufficient to detect the geometrical constraint on directional information.

A similar limitation on the extent of normal field states was previously analyzed by CKN[28]. They posited a nonlocal infrared frequency cut off in field theory to address a problem introduced by gravity: without it, populated states ($\bar{n} \approx 1$) of independent plane-wave modes in standard field theory predict unphysical systems, such as configurations of matter more compact than black hole states. From a field theory point of view, that constraint if it exists must apply even in flat space-time. They showed that such a cutoff is consistent with tests using precision applications of field theory, based on dimensional and renormalization group arguments.

Similar estimates apply here as well: in microscopic experiments, the effects of directional entanglement are too small to appear in particle experiments with realistic precision. This point is made in Figure (3) by the large vertical gap in directional information between geometrical limits and microscopic systems such as particle collisions and atoms. The proposal here effectively introduces the same IR limit on field states as CKN, but provides a specific geometrical rationale for the limit based on geometrical directional information, and connects it with other kinds of specific macroscopic effects.

Tests of Lorentz Invariance Violation

We have adopted a set of coordinates and normal modes that introduce a preferred rest frame and a preferred axis, so Lorentz invariance is no longer manifest. However, the underlying relativistic wave equation describes the same

Lorentz-invariant physical system as usual. Without the addition of the new Planck scale directional resolution limit, there is no physical difference and no violation of Lorentz invariance.

On the other hand, Planckian directional entanglement necessarily violates Lorentz invariance. A measurement of geometry prepares a quantum state of the space-time with respect to a particular frame, fixed by the world-line that defines a system bounded by a causal diamond. However, the violation only occurs in nonlocal measurements that are sensitive to transverse components of position or phase; the longitudinal part of the Lorentz transformation is not changed, and wave propagation is not changed. In particular, no effect (such as dispersion) is predicted on particles that have traveled over cosmic distances, as constrained by current experiments [33–35].

Interferometers

The most sensitive technique for measuring very small phase differences in macroscopic systems is Michelson interferometry. In an interferometer, modes of light propagating in different directions can be mixed by reflecting surfaces of macroscopic massive bodies— in particular, beamsplitter mirrors. The signal depends on the phase difference of light impinging on the beamsplitter from two directions, so it is sensitive to variations in transverse position. The standard quantum theory of interferometers[36, 37] includes entanglement of mirrors and laser fields as subsystems in a classical geometry, and predicts a standard quantum noise limit; Planckian directional entanglement adds a new source of noise that depends only on the spatial configuration of an interferometer. The spectral density, or variance per frequency interval of phase differences $\Delta\phi$ in Eq. (20), is given approximately by the Planck time:

$$d\langle\Delta\phi^2\rangle/df \approx d\langle\Delta\theta^2\rangle/df \approx t_P = 5.39 \times 10^{-44} \text{Hz}^{-1}, \quad (21)$$

with most of the fluctuation in an apparatus of size L_a coming at frequencies $f \approx c/L_a$. The predicted phase fluctuations may be detectable, and distinguishable from other sources of noise by their correlations in space and time[38–40].

Vacuum Energy Density

The most spectacular experimental failure of standard field theory appears in cosmology: it over-predicts the gravitational energy density of fluctuations in the physical vacuum on cosmic scale by over 120 orders of magnitude, compared with the effective cosmic mean density of “dark energy”[41, 42], even though it correctly predicts behavior of vacuum fluctuations in laboratory systems[43, 44]. The proposal here does not explain the physics of dark energy, but it does hint at as resolution of this extreme contradiction.

As discussed above, the number of field degrees of freedom \mathcal{N} is the product of radial and directional information. For a system of fields with minimum wavelength λ and size τ , it is limited by $\mathcal{N} < (\tau/\lambda)(\tau/t_P)$. The ground state vacuum energy of a system of fields is the sum of zero-point energies of all the modes, $E \approx \mathcal{N}/\lambda$, in units with $c = \hbar = 1$. The vacuum energy density is then $\rho_{vac} \approx E/\tau^3 < 1/\lambda^2\tau l_P$. If we only count states in any region for which directional entanglement is small from Eq. (16), $\tau < \lambda^2/l_P$, any choice of τ and λ then gives

$$\rho_{vac} < \tau^{-2} l_P^{-2}. \quad (22)$$

Since the latter expression is roughly the mean density of matter in a system of gravitational curvature radius τ , directional entanglement offers a partial solution of the vacuum energy problem: the energy of the field vacuum state naturally matches the gravitational energy on any scale, without additional fine tuning. CKN[28] accomplished the same effect with an infrared cutoff to field states; the main difference here is the specific directional mechanism for the cutoff.

Eliminating the fine tuning is not a solution to the dark energy problem. It does not provide any prediction for particular properties of cosmic acceleration or connections with other known properties of fields. Such a prediction would require deeper understanding of the state of emergent space on cosmic scales[17].

Black Holes

The approximations used here break down when the curvature of wave fronts is comparable to the curvature of emergent space-time, so they cannot be applied to black holes. However, the holographic information content on null surfaces matches that of black holes, and we can extrapolate to make some inferences about states far from the holes.

Matching Black Holes to Field Systems

The estimate just given for vacuum energy also implies that maximum total information and energy in a region roughly match to black hole states. For any λ , a thermally populated field state in some region, or degenerate relativistic fermionic matter with occupation number of order unity, has approximately the mass and radius of a black hole. For example, a nearly relativistic, self-gravitating degenerate system with Fermi energy on the GeV scale corresponds roughly to a neutron star. It has a size of about $\tau_g(\lambda \approx 1/\text{GeV})$, of order a few kilometers. The black hole configuration of comparable mass and size corresponds to far higher entropy, as the geometrical degrees of freedom are also not in their ground state. In the black hole state, the classical directional information is scrambled by strong gravity near the hole. The gap (in figure 2) between neutron star and black hole represents the larger number of geometrical states compared to field states. The transition to a black hole qualitatively resembles a phase transition[45–47].

Black Hole Information: Evaporation and Indeterminacy of the Event Horizon

For a black hole of mass M in asymptotically nearly-flat space, the evaporation time is about $\tau \approx M^3$, where both are expressed in Planck units. At a distance τ , the transverse geometrical position uncertainty has a value $\approx \tau^{1/2}$. The position of the hole at the distance of the evaporated particles is therefore uncertain by a displacement $\tau^{1/2} \approx M^{3/2}$, which is greater than the size of the event horizon, M . Thus, the position of the black hole is significantly entangled with the state of the entire geometrical system. On the scale of the evaporation products, the position of the hole, whether or not a particle path falls into the event horizon, are indeterminate.

Nonlocal geometrical states apply both to the hole and to the distant space-time. The position of the event horizon is defined only in the asymptotic future, which in this case is still essentially affected by directional indeterminacy. Thus, *the event horizon of even a macroscopic black hole is a quantum object: the location of the event horizon of a black hole is an indeterminate property of global quantum geometry.* That indeterminacy is not observable in any local measurement, since observers in the vicinity of the hole are entangled in the same geometrical state as the hole, relative to very distant observers. The geometrical uncertainty far from hole is relevant to apparent black hole information paradoxes, since it provides a concrete model for extremely nonlocal position information[16, 48, 49]. (Of course paradoxes may in any case not arise in the real universe, where the scale M^3 for real (stellar mass) black holes is much larger than the Hubble scale where new dark energy physics becomes relevant.)

Fluctuations during Cosmic Inflation

In standard inflation models, cosmic large scale structure is generated by quantum fluctuations of field modes expanding in a classical space-time. Zero point fluctuations of an effective inflaton field are frozen when their frequency matches the expansion rate H_{inf} , and persist until today as classical perturbations in the metric. Their properties are determined by the effective potential and interactions that govern the inflaton. Standard theory also predicts that quantum metric fluctuations imprint tensor perturbations with mean square amplitude $\Delta_{+, \times}^2 = H_{inf}^2 t_P^2 / 2\pi^2$. Both kinds of modes directly imprint observable large scale structure in the cosmic microwave background, as well as the matter distribution today (e.g., [50, 51]).

Trajectories of field or metric modes in a typical model are shown in Figure (4). The relevant system size during inflation is the apparent horizon scale $\tau \approx 1/H_{inf}$, which is almost constant during most of inflation. The trajectory of a comoving mode moves vertically as it expands. Initial states of modes are prepared when the wavelength is much smaller than c/H_{inf} . Usually, the initial conditions are independent vacuum states $|n = 0, \vec{k}\rangle$ for each comoving wavenumber \vec{k} . Here, initial states are significantly entangled with geometry on a sub-horizon scale, as shown in Figure (4). Directional entanglement reduces the information content, and may create new kinds of observable, nongaussian correlations in the preparation of modes[52–54].

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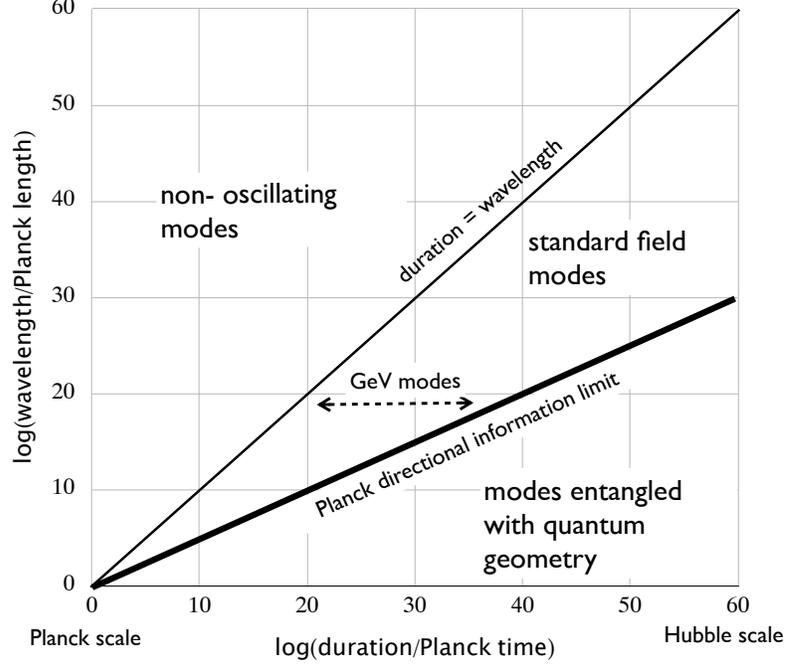


FIG. 1: Mode wavelengths and system durations for field modes, showing the scales affected in a geometry that has the directional fidelity of Planck scale waves. Wavelength λ is plotted as function of causal diamond duration τ , both expressed as decimal logarithms in Planck units. Horizontal scale extends approximately from the Planck time t_P to almost the Hubble scale, $H_0^{-1} \approx 8 \times 10^{60} t_P$. Field theory normally inhabits the entire right/lower half, but it is proposed here that in the lowest region, at large separations or small wavelengths, field modes are significantly entangled with geometry due to the Planck limit on directional information. GeV field modes are shown as an example: states are geometrically entangled beyond separations of a few kilometers.

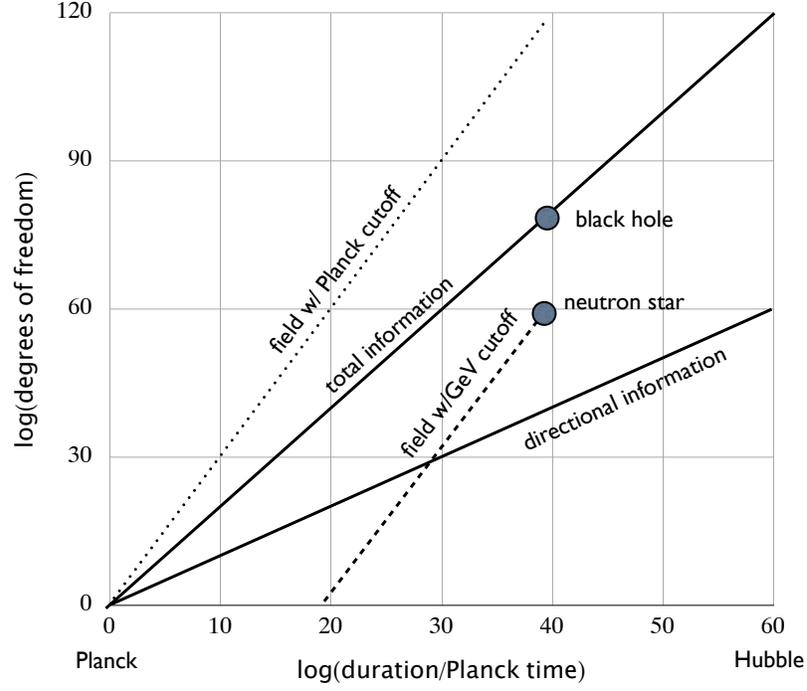


FIG. 2: Information, or number of degrees of freedom, as a function of the duration of a space-time causal diamond in Planck units. Solid lines show total information and directional information. Dotted line shows standard field theory in a classical geometry, for a UV cutoff at the Planck scale. Dashed line shows standard fields with a cutoff at the GeV scale, but terminated on the scale of directional entanglement with geometry. A large system is needed before the geometrical entanglement significantly constrains standard field degrees of freedom. Large dots indicate the geometrical information in a stellar mass black hole, and the (far smaller) GeV-scale field information in a neutron star. The gap between them corresponds to the large increase in entropy that occurs when an event horizon forms.

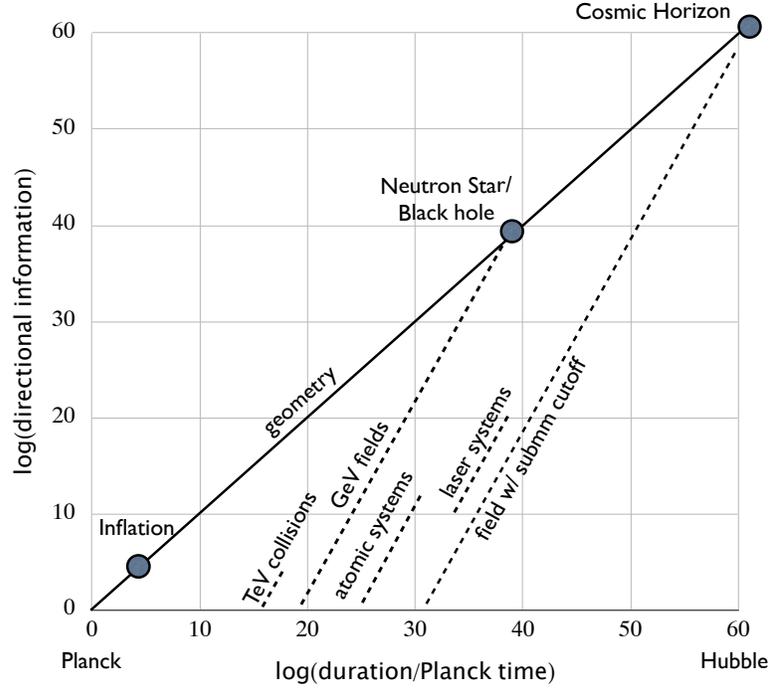


FIG. 3: Examples of directional information content of physical systems of various sizes. A saturated system of excited relativistic fields approximates the energy content and directional information of a black hole (albeit with less total information); these are represented by the neutron star/black hole dot. On the cosmological scale, the field vacuum, with a sub-millimeter cutoff enforced by directional entanglement, similarly matches dark energy density and information. Microscopic systems, such as hadronic collisions or assemblies of atoms, are far from being limited by geometrical directional bounds, so field states act almost as if they are in a classical space-time. Very small transverse field phase displacements caused by directional entanglement may nevertheless be detectable in large systems with very large numbers of coherent quanta, such as laser interferometers.

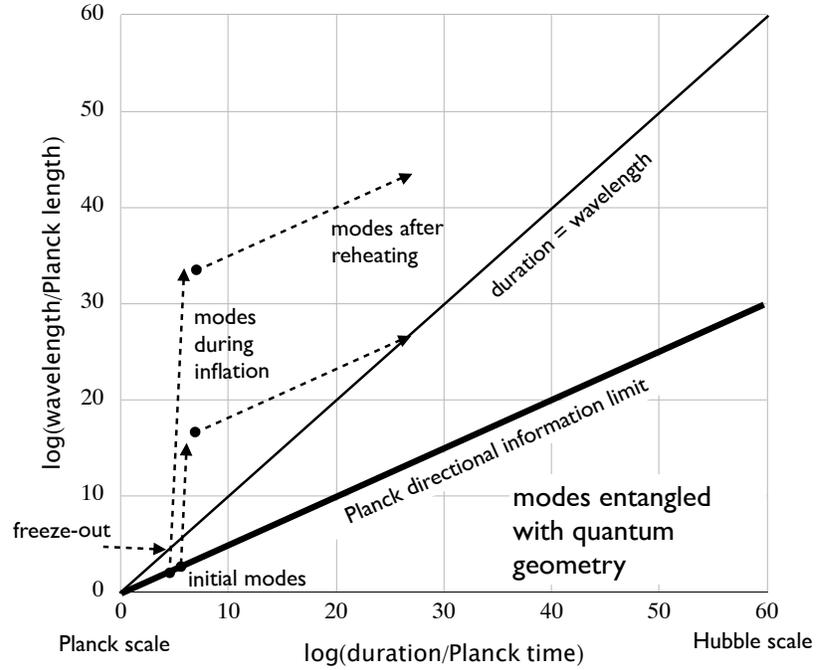


FIG. 4: As in Fig (1), with trajectories added for field modes during inflation and reheating. In the standard description of fields during cosmic inflation, a comoving mode moves almost vertically, so the initial vacuum state is set at the boundary of the entangled regime.