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Higgs mass from compositeness at a multi-TeV scale

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Abstract

Within composite Higgs models based on the top seesaw mechanism, we show that the Higgs field can arise as the pseudo Nambu-Goldstone boson of the broken $U(3)_L$ chiral symmetry associated with a vector-like quark and the t-b doublet. As a result, the lightest CP-even neutral state of the composite scalar sector is lighter than the top quark, and can be identified as the newly discovered Higgs boson. Constraints on weak isospin violation push the chiral symmetry breaking scale above a few TeV, implying that other composite scalars are probably too heavy to be probed at the LHC, but may be within reach at a future hadron collider with center-of-mass energy of about 100 TeV.

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1 Introduction

The discovery of the Higgs boson at the Large Hadron Collider (LHC) [1] represents an important step towards understanding the origin of electroweak symmetry breaking. Its mass around 126 GeV has major implications for different models or mechanisms of electroweak symmetry breaking.

A pressing question is whether the Higgs boson is an elementary particle or a composite one. Current data cannot distinguish between an elementary Higgs boson (up to very high energies) and a composite Higgs boson arising from a theory with a decoupling limit identical to the Standard Model (SM). Nonetheless, the requirement that the Higgs boson mass M_h is near 126 GeV poses model building challenges on SM extensions.

A composite Higgs boson has generically a large quartic coupling and hence is expected to be heavy, unless its mass is protected by some symmetry. One possibility is that the Higgs field arises as a pseudo-Nambu-Goldstone boson (pNGB) of some broken global symmetry [2]. Many types of models which realize the Higgs multiplet as a pNGB have been proposed, including Little Higgs [3] and models that invoke the AdS/CFT correspondence [4]. In these effective theories, M_h is quite model-dependent. The quartic coupling arises at tree level in some models, and from loops in others. The relatively large top quark mass, m_t, can be accommodated through partial compositeness [5], *i.e.*, mixing of the elementary top fields with composite vector-like fermions. In a class of pNGB composite Higgs models where the dominant explicit breaking of the global symmetry comes from these mixings [4], there are strong correlations between M_h and the masses of composite vector-like fermions. Typically, $M_h \approx 126$ GeV requires at least one vector-like top partner well below 1 TeV [6] which is in some tension with the 8 TeV LHC data, and will be thoroughly tested by LHC searches in the near future. An additional prediction of these models that will be soon tested is that higher dimensional operators modify the Higgs couplings [7].

A more direct way of obtaining a heavy top quark is the top condensation mechanism, where the Higgs field is a top-antitop bound state [8, 9]. The top quark, however, would have in this case a mass around 600 GeV if the compositeness scale is not much above the electroweak scale. Furthermore, the leading number of colors (N_c) approximation gives $M_h \approx 2m_t$. Both results are in conflict with the data. The top and Higgs masses can be reduced, at the expense of fine-tuning, if the compositeness scale is raised; but even with compositeness at the Planck scale, m_t and M_h would still be above 200 GeV [9].

An attractive solution to the top mass problem in top-condensation models is through the top-seesaw mechanism [10–13]. The heavy top given by top condensation mixes with a massive vector-like top partner, resulting in a light mass eigenstate of $m_t \approx 173$ GeV, identified as the observed top quark. Although the composite Higgs boson is likely to be heavier than the weak scale, for some range of parameters it is possible to obtain a relatively light Higgs boson [11, 12, 14]. Generically, the composite Higgs sector includes two doublets and two singlets [12], and a light Higgs boson could arise due to the mixing among the four CP-even scalars.

In this paper we show that a composite Higgs boson can naturally have $M_h \approx 126$ GeV (once the weak scale v is fixed at 246 GeV) in a top-seesaw model where the strong dynamics preserves a global $U(3)_L$ symmetry acting on the top-bottom doublet (t_L, b_L) and the left-handed component of the vector-like quark, χ_L. The composite Higgs doublet is the collection of pNGB's of the spontaneous $U(3)_L \to U(2)_L$ breaking. The explicit symmetry breaking required for the seesaw mechanism also controls M_h. As a consequence, the mass of the composite Higgs boson is correlated with m_t, and has an upper bound around m_t. The electroweak interactions further reduce M_h so that generically it is substantially below m_t.

The low energy effective theory is given by the SM plus the heavy states arising from two Higgs doublets and two complex singlet scalars, together with the vector-like top partner. The strongest constraint on this model is due to weak isospin violation, which requires the vector-like quark and consequently most of the scalars other than the SM Higgs boson to have masses of order 10 TeV. This implies some tuning to obtain the weak scale at $v \approx 246$ GeV. Nevertheless, given that no new physics has been found at the LHC so far, some tuning seems inevitable in any theory which attempts to explain electroweak symmetry breaking.

The strong dynamics responsible for compositeness and chiral symmetry breaking may be an asymptotically-free gauge interaction that is spontaneously broken near the scale Λ where it becomes strong. Various theories of these type have been proposed [15,16]. It may even be possible that a strongly coupled gauge interaction breaks its own gauge symmetry [17, 18]. The consequence would be a non-confining strong interaction whose bound states are present together with their constituents at energies below the compositeness scale Λ. Its effects at the scale Λ may be parametrized by certain 4-fermion interactions. Such a theory is viable because in the decoupling limit, where the scale of chiral symmetry breaking f is much larger than v, the deviations from the SM are $O(v^2/f^2)$. In practice,

the hierarchy of scales $v < f < \Lambda$ should not be too large because both f/Λ and v/f require fine-tuning.

In Section 2 we present the effective theory of the composite Higgs model below the compositeness scale. We discuss the $U(3)_L \times U(2)_R$ symmetric terms and also their explicit breaking terms. In Section 2.1 we find the vacuum which breaks the electroweak symmetry and produces the top-seesaw mechanism. In Section 2.2 we derive an approximate analytic formula for M_h, and find that its maximum value is near m_t. In Section 3 we discuss the effects of electroweak gauge loops and show that they always reduce M_h. In Section 4 we perform a detailed numerical study of M_h as a function of the parameters in this model. We show that the 126 GeV Higgs boson can be easily accommodated as a composite field in our model. After fixing $M_h \approx 126$ GeV, we calculate the spectrum of the heavy states, and then in Section 5 we comment on phenomenological implications. The conclusions are drawn in Section 6.

2 Composite Higgs from $U(3)_L \times U(2)_R$ symmetric dynamics

We consider an effective theory at a scale $\Lambda \gg 1$ TeV that includes the SM gauge group and fermions, an $SU(2)_W$-singlet vector-like quark, χ, of electric charge $+2/3$, and some 4-fermion interactions suppressed by Λ (which presumably arise due to an asymptotically-free gauge interaction that is spontaneously broken near the scale Λ where it becomes strong). This theory does not include a Higgs doublet nor any elementary scalars. We assume that some of the 4-fermion interactions involving third generation quarks and χ are attractive and sufficiently strong to form quark-antiquark bound states [12]. These strong interactions are not confining, because at distances longer than $1/\Lambda$ the effects of 4-fermion interactions (other than the presence of bound states) are exponentially suppressed.

The low-energy theory includes the composite fields that are deeply bound such that their masses are less than the compositeness scale Λ. As in the Nambu-Jona-Lasinio (NJL) model [19], if the coefficient of a 4-fermion interaction is larger than a certain critical value, then the squared mass of the scalar bound state becomes negative, and the scalar acquires a vacuum expectation value (VEV). Furthermore, as the coefficient crosses the critical value, there is a second order phase transition so that the VEV can in principle be much smaller than Λ [16].

Concretely, we assume the constituents of the deeply bound states to be only χ_L, χ_R, the right-handed top quark t_R, and the left-handed top-bottom doublet $\psi_L^3 = (t_L, b_L)$. In the limit where the electroweak interactions are ignored, the kinetic terms of these quarks have an $U(3)_L \times U(2)_R$ chiral symmetry, which we assume to be approximately preserved by the 4-fermion interactions. The $U(3)_L \times U(2)_R$ symmetric interactions give rise to the following Yukawa couplings of the composite scalars (collectively labelled by Φ) to their constituents:

$$\mathcal{L}_{\text{Yukawa}} = -\xi \left(\bar{\psi}_L^3, \bar{\chi}_L\right) \Phi \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.} \tag{2.1}$$

Here ξ is a dimensionless coupling whose value at scale Λ, upon integrating out Φ, matches the coefficient of the 4-fermion interactions. The scalar field Φ is a 3×2 complex matrix

$$\Phi = (\Phi_t \ , \ \Phi_\chi) \quad , \tag{2.2}$$

where the scalar fields Φ_t and Φ_χ are the bound states of the $U(3)_L$ triplet (t_L, b_L, χ_L) with t_R and χ_R, respectively:

$$\Phi_t \sim \bar{t}_R \begin{pmatrix} \psi_L^3 \\ \chi_L \end{pmatrix} \qquad , \qquad \Phi_\chi \sim \bar{\chi}_R \begin{pmatrix} \psi_L^3 \\ \chi_L \end{pmatrix} \quad . \tag{2.3}$$

At scales $\mu < \Lambda$, the Yukawa couplings (2.1) give rise to the following potential for Φ:

$$V_\Phi = \frac{\lambda_1}{2} \text{Tr}\big[(\Phi^\dagger \Phi)^2\big] + \frac{\lambda_2}{2} \left(\text{Tr}[\Phi^\dagger \Phi]\right)^2 + M_\Phi^2 \Phi^\dagger \Phi \quad . \tag{2.4}$$

The quartic couplings λ_1 and λ_2 depend on the scale μ; if the kinetic term for Φ is canonically normalized, then λ_1 becomes non-perturbative near Λ. In the large N_c limit, λ_1 is generated by a fermion loop, while λ_2 vanishes. Scalar loops, however, generate a non-zero value for λ_2, so that $\lambda_1 \gg |\lambda_2|$. In the Appendix we use 1-loop renormalization group (RG) equations to estimate the λ_1/λ_2 ratio.

The squared mass of Φ is assumed to satisfy $|M_\Phi^2| \ll \Lambda^2$, and if the 4-fermion interactions are super-critical, then $M_\Phi^2 < 0$ triggering spontaneous symmetry breaking of $U(3)_L \times U(2)_R$.

We assume that there are additional explicit $U(2)_R$ breaking effects which distinguish t_R and χ_R. Given that $|M_\Phi| \ll \Lambda$, such effects could induce a large relative splitting of the masses for Φ_t and Φ_χ. We parametrize these effects by

$$V_{\cancel{U(2)}} = \delta M_{tt}^2 \ \Phi_t^\dagger \Phi_t + \delta M_{\chi\chi}^2 \ \Phi_\chi^\dagger \Phi_\chi + (M_{\chi t}^2 \Phi_\chi^\dagger \Phi_t + \text{H.c.}) \tag{2.5}$$

These $U(2)_R$ breaking masses can be diagonalized by a $U(2)$ rotation. As we will see later, it is convenient to work in a different basis from the one that diagonalizes these mass terms so we will keep Eq. (2.5) general.

The $U(2)_R$ breaking effects may also split the quartic couplings in (2.4), but this does not make any qualitative difference (unless the changes in couplings are larger than order one), because our discussion mostly relies on the $U(3)_L$ symmetry. For simplicity we do not include such effects.

Gauge invariant masses for the $SU(2)_W$-singlet quarks can be present at the scale Λ:

$$\mathcal{L}_{\rm mass} = -\mu_{\chi t}\bar{\chi}_L t_R - \mu_{\chi\chi}\bar{\chi}_L\chi_R + \text{H.c.} \tag{2.6}$$

We assume that $\mu_{\chi t}, \mu_{\chi\chi} \ll \Lambda$ (which is technically natural because there is an enhanced chiral symmetry in the $\mu_{\chi t}, \mu_{\chi\chi} \to 0$ limit), so that the tree-level quark masses do not disrupt the formation of bound states. The above mass terms break $U(3)_L \times U(2)_R$ down to $U(2)_L \times U(1)_R$. Below Λ, these fermion masses map to tadpole terms for the $SU(2)_W$-singlet scalars:

$$V_{\rm tadpole} = -(0,0,C_{\chi t})\Phi_t - (0,0,C_{\chi\chi})\Phi_\chi + \text{H.c.} \tag{2.7}$$

Matching at the scale Λ, we have

$$C_{\chi t} \simeq \frac{\mu_{\chi t}}{\xi}\Lambda^2 \quad , \quad C_{\chi\chi} \simeq \frac{\mu_{\chi\chi}}{\xi}\Lambda^2 \,. \tag{2.8}$$

Note that when the scalars are integrated out at the cutoff scale (where $M_\Phi^2 \sim \Lambda^2$), the fermion mass terms (2.6) are recovered.

The effective potential of the scalar sector below the compositeness scale is given by

$$\begin{aligned} V_{\rm scalar} \;=\; & \frac{\lambda_1+\lambda_2}{2}\left[(\Phi_t^\dagger\Phi_t)^2 + (\Phi_\chi^\dagger\Phi_\chi)^2\right] + \lambda_1|\Phi_t^\dagger\Phi_\chi|^2 + \lambda_2(\Phi_t^\dagger\Phi_t)(\Phi_\chi^\dagger\Phi_\chi) \\ & + M_{tt}^2\Phi_t^\dagger\Phi_t + M_{\chi\chi}^2\Phi_\chi^\dagger\Phi_\chi + (M_{\chi t}^2\Phi_\chi^\dagger\Phi_t + \text{H.c.}) \\ & - (0,0,2C_{\chi t})\,\text{Re}\,\Phi_t - (0,0,2C_{\chi\chi})\,\text{Re}\,\Phi_\chi \quad , \end{aligned} \tag{2.9}$$

where M_{tt}^2 and $M_{\chi\chi}^2$ are the sums of the mass terms in Eqs. (2.4) and (2.5). The parameters $M_{\chi t}^2$, $C_{\chi t}$, and $C_{\chi\chi}$ can be chosen to be real without loss of generality. As mentioned earlier, $M_{\chi t}^2$ can be removed by a $U(2)$ rotation. Then $C_{\chi t}$ and $C_{\chi\chi}$ can be made real by phase redefinitions of Φ_t and Φ_χ respectively.

The $SU(2)_W \times U(1)_Y$ gauge symmetry is a subgroup of the $U(3)_L \times U(2)_R$ chiral symmetry. Thus, the electroweak interactions explicitly break the $U(3)_L$ symmetry. The

$U(3)_L$ triplets Φ_t and Φ_χ can be written in terms of fields belonging to electroweak representations:

$$\Phi_t = \begin{pmatrix} H_t \\ \phi_t \end{pmatrix} \quad , \quad \Phi_\chi = \begin{pmatrix} H_\chi \\ \phi_\chi \end{pmatrix} \quad . \tag{2.10}$$

H_t and H_χ transform under $SU(2)_W \times U(1)_Y$ as the SM Higgs doublet, while ϕ_t and ϕ_χ are $SU(2)_W \times U(1)_Y$ singlets. The effects of the electroweak interactions will be discussed in Sec. 3. $H_{\chi(t)}$, $\phi_{\chi(t)}$ will get VEVs due to negative squared masses and tadpole terms. Expanding $H_{\chi(t)}$, $\phi_{\chi(t)}$ around their VEVs in terms of fields of definite electric charges, we can write

$$H_t = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_t + h_t + iA_t\right) \\ H_t^- \end{pmatrix} \quad , \quad H_\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_\chi + h_\chi + iA_\chi\right) \\ H_\chi^- \end{pmatrix} \quad ,$$

$$\phi_t = \frac{1}{\sqrt{2}}(u_t + \varphi_t + i\pi_t) \quad , \quad \phi_\chi = \frac{1}{\sqrt{2}}(u_\chi + \varphi_\chi + i\pi_\chi) \quad . \tag{2.11}$$

The VEVs v_t, v_χ, u_t and u_χ are real, and some of them may vanish, depending on the parameters of the effective potential. We use the notation $v_t^2 + v_\chi^2 = v^2$, $u_t^2 + u_\chi^2 = u^2$, and

$$f = \sqrt{u^2 + v^2} \tag{2.12}$$

is the scale of $U(3)_L$ breaking. The measured Fermi constant requires $v \approx 246$ GeV.

2.1 Electroweak symmetry breaking

It is convenient to perform an $U(2)_R$ transformation (which rotates t_R and χ_R, as well as Φ_t and Φ_χ) to go to a basis where $v_t = 0$ and $v_\chi = v$. For simplicity, we will use the same notation in this basis as in Eqs. (2.9)-(2.11). In this basis we define $u_t = u \sin\gamma$ and $u_\chi = u\cos\gamma$, and the short-hand notation $s_\gamma = \sin\gamma$ and $c_\gamma = \cos\gamma$.

The extremization conditions for V_{scalar} relate the parameters from the effective potential to the VEVs:

$$\begin{aligned} & v\left(M_{\chi t}^2 + \frac{\lambda_1}{2}u^2 s_\gamma c_\gamma\right) = 0 \ , \\ & v\left(M_{\chi\chi}^2 + \frac{\lambda_1}{2}\left(u^2 c_\gamma^2 + v^2\right) + \frac{\lambda_2}{2}\left(u^2 + v^2\right)\right) = 0 \ , \end{aligned} \tag{2.13}$$

for the derivatives with respect to h_t and h_χ, and

$$C_{\chi t} = \frac{u}{\sqrt{2}} \left[M_{\chi t}^2 \, c_\gamma + \left(M_{tt}^2 + \frac{\lambda_1}{2} u^2 + \frac{\lambda_2}{2} \left(u^2 + v^2 \right) \right) s_\gamma \right] \ ,$$

$$C_{\chi\chi} = \frac{u}{\sqrt{2}} \left[M_{\chi t}^2 \, s_\gamma + \left(M_{\chi\chi}^2 + \frac{\lambda_1 + \lambda_2}{2} \left(u^2 + v^2 \right) \right) c_\gamma \right] \ , \tag{2.14}$$

for the derivatives with respect to φ_t and φ_χ. Eqs. (2.13) and (2.14) have a solution for $v = 0$, and a different solution for $v > 0$. When the latter is a minimum (*i.e.*, the squared masses of all spin-0 states are positive), we find that it is also the global minimum of the effective potential while $v = 0$ is a saddle point.

For $v > 0$, Eqs. (2.13) imply

$$\begin{aligned} M_{\chi t}^2 &= -\frac{\lambda_1}{2} u^2 s_\gamma c_\gamma \ , \\ M_{\chi\chi}^2 &= -\frac{\lambda_1}{2} \left(u^2 c_\gamma^2 + v^2 \right) - \frac{\lambda_2}{2} \left(u^2 + v^2 \right) \ . \end{aligned} \tag{2.15}$$

Substituting these into Eqs. (2.14) gives

$$\begin{aligned} C_{\chi t} &= \frac{u \, s_\gamma}{\sqrt{2}} \left[M_{tt}^2 + \frac{\lambda_1}{2} u^2 s_\gamma^2 + \frac{\lambda_2}{2} \left(u^2 + v^2 \right) \right] \ , \\ C_{\chi\chi} &= 0 \ . \end{aligned} \tag{2.16}$$

Thus, the basis where $\langle H_t \rangle = 0$ and $\langle H_\chi \rangle \neq 0$ is the one where $C_{\chi\chi} = 0$ (or equivalently, where t_R and χ_R are defined such that $\mu_{\chi\chi} = 0$). Since the electroweak symmetry is broken only by the VEV of H_χ, the eaten Nambu-Goldstone bosons are contained in H_χ only. The charged Higgs boson resides entirely within H_t, and its mass squared is

$$M_{H^\pm}^2 = M_{tt}^2 + \frac{\lambda_1}{2} u^2 s_\gamma^2 + \frac{\lambda_2}{2} \left(u^2 + v^2 \right) \ . \tag{2.17}$$

The nonzero tadpole coefficient is then related to $M_{H^\pm}^2$ by $C_{\chi t} = u \, s_\gamma M_{H^\pm}^2 / \sqrt{2}$. Note that $v \neq 0$ requires $M_{\chi\chi}^2 < 0$, but does not restrict the sign of M_{tt}^2. In Section 4, however, we will show that the Higgs boson would be lighter than about 100 GeV unless $M_{H^\pm} > u$, which in turn requires $M_{tt}^2 > 0$.

The constraint from weak isospin violation (discussed in Section 4) requires $v \ll u$. As a result, the $U(3)_L$ breaking scale, defined in Eq. (2.12), is given by $f \simeq u$. Eqs. (2.15) and (2.16) have solutions for v, u and s_γ that satisfy $v/u \ll 1$ provided there is a fine-tuned relation among the $M_{\chi\chi}$, $M_{\chi t}$, M_{tt} and $C_{\chi t}$ parameters.

Neglecting the mixing of the charm and up quarks with t and χ, the mass terms of the heavy charge-2/3 quarks, arising from Eq. (2.1), are given by

$$-\frac{\xi}{\sqrt{2}}\left(\bar{t}_L, \overline{\chi}_L\right)\begin{pmatrix} 0 & v \\ us_\gamma & uc_\gamma \end{pmatrix}\begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.} \tag{2.18}$$

Diagonalizing this matrix gives the masses of the top quark t and the new quark, which we label by t' in the mass eigenstate basis. Keeping only the leading nonvanishing term in v^2/f^2, we find the mass of the top quark,

$$m_t \simeq \frac{\xi}{\sqrt{2}} v s_\gamma \ . \tag{2.19}$$

Thus, ξ and s_γ can be related to the top Yukawa coupling y_t by

$$s_\gamma \simeq \frac{y_t}{\xi} \ . \tag{2.20}$$

ξ is expected to have a value around 3 or 4 (see Section 4) and $y_t \sim 1$, so $s_\gamma^2 \sim O(0.1)$. Again to leading order in v^2/f^2, the mass of the new quark is given by

$$m_{t'} \simeq \frac{\xi}{\sqrt{2}} f \ . \tag{2.21}$$

while the mixing angle θ_L, which rotates the t_L and χ_L gauge eigenstates into the mass eigenstate quarks, is given by

$$\sin\theta_L \equiv s_L \simeq \frac{v}{f} \ . \tag{2.22}$$

2.2 Analytical expression for the Higgs mass

Substituting Eqs. (2.15)-(2.17) back into the scalar potential (2.9), we find that the 4×4 mass-squared matrix of the CP-even neutral scalars (h_t, h_χ, φ_t, φ_χ) is given by

$$\begin{pmatrix} M_{H^\pm}^2 + \frac{\lambda_1}{2}v^2 & 0 & -\frac{\lambda_1}{2}uvc_\gamma & -\frac{\lambda_1}{2}uvs_\gamma \\ 0 & (\lambda_1+\lambda_2)v^2 & \lambda_2 uvs_\gamma & (\lambda_1+\lambda_2)uvc_\gamma \\ -\frac{\lambda_1}{2}uvc_\gamma & \lambda_2 uvs_\gamma & M_{H^\pm}^2 + \left[\lambda_1\left(1-\frac{c_\gamma^2}{2}\right)+\lambda_2 s_\gamma^2\right]u^2 & \left(\frac{\lambda_1}{2}+\lambda_2\right)u^2 s_\gamma c_\gamma \\ -\frac{\lambda_1}{2}uvs_\gamma & (\lambda_1+\lambda_2)uvc_\gamma & \left(\frac{\lambda_1}{2}+\lambda_2\right)u^2 s_\gamma c_\gamma & \left[\lambda_1\left(1-\frac{s_\gamma^2}{2}\right)+\lambda_2 c_\gamma^2\right]u^2 \end{pmatrix} . \tag{2.23}$$

The lightest Higgs boson mass-squared (M_h^2) is given by the smallest eigenvalue of the mass matrix (2.23). Using Eq. (2.20), keeping the leading order in v^2/f^2, and expanding in s_γ, we find

$$M_h^2 = \frac{\lambda_1 v^2 s_\gamma^2 M_{H^\pm}^2}{2M_{H^\pm}^2 + \lambda_1 u^2}\left[1 + \frac{(\lambda_1 + 2\lambda_2)(M_{H^\pm}^2 + \lambda_1 u^2)^2}{(\lambda_1 + \lambda_2) M_{H^\pm}^2 \left(2M_{H^\pm}^2 + \lambda_1 u^2\right)} s_\gamma^2 + \mathcal{O}(s_\gamma^4)\right] \quad . \tag{2.24}$$

The Higgs boson mass is suppressed by s_γ, because in the limit of $\xi \to \infty$ or $m_t \to 0$ the explicit $U(3)_L$ breaking tadpole terms $C_{\chi t}$, $C_{\chi\chi}$ vanish and the H_χ and π_χ fields become Nambu-Goldstone bosons. Keeping only the leading term in s_γ^2,

$$\begin{aligned} M_h^2 &\simeq \frac{\lambda_1}{2\xi^2}\left(1 + \frac{\lambda_1 m_{t'}^2}{\xi^2 M_{H^\pm}^2}\right)^{-1} y_t^2\, v^2 \\ &= \lambda_h\, v^2 \quad , \end{aligned} \tag{2.25}$$

where λ_h is the Higgs quartic coupling. In the fermion-loop approximation of NJL, the ratio of couplings $\lambda_1/(2\xi^2)$ is equal to 1. Scalar and gauge boson loops reduce this ratio. In the Appendix we show that it has a quasi-infrared fixed point value ~ 0.4, so we expect $\lambda_1/(2\xi^2)$ between 0.4 and 1.

Since both y_t and λ_h are obtained from integrating out the heavy quark, Eq. (2.25) relates the quartic Higgs coupling and the top Yukawa coupling at the scale $m_{t'}$, implying $\lambda_h < y_t^2$ at that scale. For $m_{t'} \sim 10$ TeV, we find $y_t^2 \sim 0.6$. Evolving λ_h down to the scale v we obtain an upper limit on the Higgs mass,

$$M_h \lesssim 185 \text{ GeV} \quad . \tag{2.26}$$

This is an interesting result, in contrast to the naïve expectation in many composite Higgs models that the Higgs boson is heavier than the weak scale. We see that the Higgs boson is light because it is a pNGB of the $U(3)_L \to U(2)_L$ symmetry breaking. In Section 4 we perform a more refined analysis, and as a result the above upper limit is further reduced.

There is one additional pNGB, A_1, mostly given by the π_χ field, with small admixtures of the other neutral CP-odd scalars in Eq. (2.11). Its squared mass is

$$M_{A_1}^2 = \frac{1}{4}\left(2M_{H^\pm}^2 + \lambda_1 f^2\right)\left[1 - \sqrt{1 - \frac{8\lambda_1 M_{H^\pm}^2 f^2 s_\gamma^2}{\left(2M_{H^\pm}^2 + \lambda_1 f^2\right)^2}}\,\right] \quad , \tag{2.27}$$

where we neglected terms suppressed by v^2/f^2. Expanding in s_γ^2, and using Eq. (2.25), we find

$$M_{A_1} \simeq \frac{f}{v} M_h \quad . \tag{2.28}$$

Even though A_1 is much heavier than the Higgs boson, it is substantially lighter than the other composite scalars.

3 $U(3)_L$ breaking from electroweak interactions

In Section 2 we have assumed that the mass and quartic terms in the potential respect the $U(3)_L$ symmetry, and the only explicit $U(3)_L$ breaking comes from tadpole terms. Other explicit $U(3)_L$ breaking effects, such as the $SU(2)_W \times U(1)_Y$ gauge interactions, can feed into the mass and quartic terms through loops. In this section we study the effect of these additional $U(3)_L$ breaking effects on M_h.

We parametrize the $U(3)_L$ breaking mass and quartic terms as

$$\begin{aligned}\Delta V_{\text{breaking}} &= \frac{\kappa_1}{2}\left[(H_t^\dagger H_t)^2 + (H_\chi^\dagger H_\chi)^2 + 2(H_t^\dagger H_\chi)(H_\chi^\dagger H_t)\right] + \frac{\kappa_2}{2}\left(H_t^\dagger H_t + H_\chi^\dagger H_\chi\right)^2 \\ &\quad + \kappa_1'\left[H_t^\dagger H_t \phi_t^\dagger \phi_t + H_\chi^\dagger H_\chi \phi_\chi^\dagger \phi_\chi + \left(H_t^\dagger H_\chi \phi_\chi^\dagger \phi_t + \text{H.c.}\right)\right] \\ &\quad + \kappa_2'\left(H_t^\dagger H_t + H_\chi^\dagger H_\chi\right)\left(\phi_t^\dagger \phi_t + \phi_\chi^\dagger \phi_\chi\right) \\ &\quad + \Delta M_{tt}^2 H_t^\dagger H_t + \Delta M_{\chi\chi}^2 H_\chi^\dagger H_\chi + \left(\Delta M_{\chi t}^2 H_\chi^\dagger H_t + \text{H.c.}\right) \quad , \end{aligned} \tag{3.1}$$

where we assumed again, for simplicity, that the quartic terms are $U(2)_R$ symmetric. It is straightforward to repeat the analysis of Section 2 by including Eq. (3.1). To leading order in s_γ, v^2/f^2, and $U(3)_L$ breaking, the correction to M_h^2 is

$$\Delta M_h^2 \simeq \left(\kappa_{12} - \frac{5}{2}\kappa_{12}' - \frac{\Delta M_{\chi\chi}^2}{f^2}\right) v^2 \quad , \tag{3.2}$$

where $\kappa_{12} \equiv \kappa_1 + \kappa_2$, $\kappa_{12}' \equiv \kappa_1' + \kappa_2'$. The corrections from ΔM_{tt}^2 and $\Delta M_{\chi t}^2$ are suppressed because the Higgs boson resides mostly in H_χ. The correction of Eq. (3.2) can be most easily seen in the limit $s_\gamma \to 0$ and $\Delta M_{\chi t}^2 = 0$, where φ_t and h_t decouple and the mass matrix (2.23) becomes block-diagonal; we then only need to diagonalize the 2×2 mass matrix:

$$\begin{pmatrix} (\lambda_{12} + \kappa_{12})\, v^2 & (\lambda_{12} + \kappa_{12}')\, uv \\ (\lambda_{12} + \kappa_{12}')\, uv & -\Delta M_{\chi\chi}^2 + \left(\lambda_{12} + \frac{1}{2}\kappa_{12}'\right) u^2 - \frac{1}{2}\left(\kappa_{12} - \kappa_{12}'\right) v^2 \end{pmatrix} \quad , \tag{3.3}$$

where $\lambda_{12} \equiv \lambda_1 + \lambda_2$. In order to keep the Higgs boson light, and to have the correct vacuum, we need

$$\left|\kappa_{12} - \frac{5}{2}\kappa_{12}' - \frac{\Delta M_{\chi\chi}^2}{f^2}\right| < \frac{\lambda_1}{2\xi^2} \quad . \tag{3.4}$$

The contribution due to the mass-squared splitting between ϕ_χ and H_χ was presented in Refs. [11, 12].

In our model, the additional $U(3)_L$ breaking effects (besides the tadpole terms) come from the $SU(2)_W \times U(1)_Y$ gauge interactions. They contribute to both the mass and quartic terms. In the case where the gauge loops are the dominant source of $U(3)_L$ breaking, we have $\Delta M^2_{tt} = \Delta M^2_{\chi\chi}$ and $\Delta M^2_{\chi t} = 0$ because the gauge interactions are $U(2)_R$ symmetric. Only H_χ and H_t transform under $SU(2)_W \times U(1)_Y$ while ϕ_χ and ϕ_t are singlets. The $SU(2)_W \times U(1)_Y$ gauge loops split the masses of $H_{\chi(t)}$ and $\phi_{\chi(t)}$, analogously to the mass splitting between the charged and neutral pions due to the electromagnetic interaction. This contribution is quadratically divergent and needs to be cut off. In the case of $\pi^+ - \pi^0$ mass difference, the cutoff is effectively provided by the ρ meson mass [20] from the Weinberg sum rules [21]. Based on this analogy, we denote the cutoff by M_ρ, which is set by the mass of some (presumably vector) state in this theory. The 1-loop splitting is then given by

$$\Delta M^2_{\chi\chi} = \frac{3}{64\pi^2}\left(3g_2^2 + g_1^2\right) M_\rho^2 \quad , \tag{3.5}$$

where g_2 and g_1 are the $SU(2)_W \times U(1)_Y$ gauge couplings. This mass splitting implies that M_h^2 receives a correction (at the chiral symmetry breaking scale f) of

$$\Delta M^2_{h\,(\mathrm{mass})} \approx -\left(0.079\, v\, \frac{M_\rho}{f}\right)^2 , \tag{3.6}$$

where the gauge couplings are evaluated at 10 TeV. This effect reduces the Higgs boson mass and can be quite significant if $M_\rho \gg f$. For example, for $M_\rho = 5f$ this reduces the effective quartic Higgs coupling, λ_h, by 0.16 at the chiral symmetry breaking scale f.

The $SU(2)_W \times U(1)_Y$ gauge interactions also generate the additional quartic interactions involving H_t and H_χ. The dominant 1-loop contribution can be estimated to be

$$\frac{\kappa_{1(2)}}{\lambda_{1(2)}} \simeq 2\,\frac{\kappa'_{1(2)}}{\lambda_{1(2)}} \simeq \frac{3}{16\pi^2}\left(3g_2^2 + g_1^2\right) \ln\left(\frac{M_\rho}{\mu}\right) \quad , \tag{3.7}$$

where we have assumed the same cutoff as in Eq. (3.5), and the renormalization scale μ should be taken around the heavy scalar states in the spectrum. We have also neglected the small $g^4_{1,2}$ contributions. Note that Eq. (3.7) is valid only for $\mu < M_\rho$. The corrections to the quartic couplings give a correction to the Higgs squared mass:

$$\begin{aligned} \Delta M^2_{h\,(\mathrm{quartic})} &\simeq -\frac{3}{64\pi^2}\,\lambda_1\left(3g_2^2 + g_1^2\right) v^2 \ln\left(\frac{M_\rho}{\mu}\right) \\ &\approx -0.16\, v^2\, \frac{\lambda_1}{2\xi^2}\left(\frac{\xi}{3.6}\right)^2 \ln\left(\frac{M_\rho}{\mu}\right) \quad . \end{aligned} \tag{3.8}$$

This contribution is also negative, so that the electroweak interactions only reduce the Higgs boson mass. Thus, the Higgs mass formula obtained in the previous section [see Eqs. (2.24)-(2.26)] provides an upper bound on the Higgs mass in the absence of other $U(3)$ breaking effects. If the cutoff M_ρ is too large, the effective quartic coupling of the light Higgs can turn negative which implies that we are expanding around the wrong vacuum. This puts an upper limit on M_ρ, which depends on other parameters, as discussed in the next Section.

4 Higgs mass and the heavy state spectrum

In the previous sections we derived the approximate expression for the Higgs mass and the corrections from the electroweak gauge loops. In this section we perform a numerical study of the Higgs mass and show that the exact numerical result agrees well with the analytic approximations. After fixing $M_h = 126$ GeV, we calculate the masses of the other composite scalars.

We start with an enumeration of the parameters of this model. In Eqs. (2.1) and (2.9), our model contains the following parameters:

$$\xi, \quad \lambda_1, \quad \lambda_2, \quad M_{tt}^2, \quad M_{\chi\chi}^2, \quad M_{\chi t}^2, \quad C_{\chi t}, \quad C_{\chi\chi}. \tag{4.1}$$

One of the parameters ($M_{\chi t}^2$ or $C_{\chi\chi}$) is not independent due to the freedom of the $U(2)_R$ rotation. After minimizing the potential, the mass and tadpole terms can be written in terms of the VEVs, quartic couplings and the charged Higgs mass $M_{H^\pm}$ through Eqs. (2.15)-(2.17). The explicit $U(3)_L$ breaking from the electroweak gauge loops discussed in Section 3 introduces one more parameter M_ρ, the cutoff of the electroweak gauge loop. As a result, the spectrum is fully determined by the following eight parameters:

$$\xi, \quad \lambda_1, \quad \lambda_2, \quad M_{H^\pm}, \quad v, \quad f, \quad s_\gamma, \quad M_\rho. \tag{4.2}$$

Two of these eight parameters are fixed by the weak scale and the top mass. To produce the correct m_t, we use the SM 1-loop RG equations to evolve the top Yukawa coupling y_t to the scale of the heavy fermion mass $m_{t'}$, and use it to solve for s_γ, which in the lowest order is given by Eq. (2.20). The running top Yukawa coupling in the $\overline{\text{MS}}$ scheme at the scale m_t corresponds to $m_t(\mu = m_t) \approx 160$ GeV [22]. For a set of input values of the other six parameters, which are taken to be

$$\xi, \quad \lambda_1/(2\xi^2), \quad \lambda_2/\lambda_1, \quad f, \quad M_{H^\pm}/f, \quad M_\rho/f, \tag{4.3}$$

one can calculate the masses and couplings at scale $m_{t'}$. We choose the ratios of couplings or mass scales as the independent parameters because they are more convenient and better constrained. To calculate M_h, we match the theory to the SM at scale $m_{t'}$, compute the quartic Higgs coupling λ_h, and then evolve λ_h down to the weak scale.

To make a generic prediction for M_h, we first examine the expected ranges of input parameters listed in Eq. (4.3). The $U(3)_L$ symmetry breaking scale must satisfy $f > v$. In fact, as we will see later, f is constrained to be much larger than v by the precision measurement of weak isospin violating effects. On the other hand, larger f also means more fine tuning for the weak scale. We will consider f up to 10 TeV to avoid excessive fine-tuning. For the effective theory to be a valid description, the states in the theory should have masses below the cutoff scale. Therefore we will take $M_{H^\pm} < 4\pi f \sim \Lambda$. Similarly, the cutoff of electroweak gauge loops also satisfies $M_\rho < 4\pi f$. The ranges of the coupling ratios are discussed in the Appendix and they are expected to be $0.4 \lesssim \lambda_1/(2\xi^2) \lesssim 1$ and $-0.2 \lesssim \lambda_2/\lambda_1 \lesssim 0$. The Yukawa coupling ξ is expected to be $\sim 3-4$ in a strongly coupled theory. The 1-loop estimate from the compositeness condition $Z_\Phi(\Lambda) = 0$ gives

$$\xi^2 \simeq \frac{8\pi^2}{\overline{N}\ln(\Lambda/m_{t'})} \quad , \tag{4.4}$$

where $\overline{N} = N_c = 3$ if only fermion loops are included, and $\overline{N} = N_c + 5/2 = 11/2$ if both fermion and scalar loops are included. For $\Lambda/m_{t'} = 3\,(10)$, $\xi \approx 4.9\,(3.4)$ for $\overline{N} = 3$, and $\xi \approx 3.6\,(2.5)$ for $\overline{N} = 11/2$. In our numerical study, we use $\xi = 2\pi/\sqrt{3} \approx 3.6$ [23] as the standard reference value. For comparison, in QCD a constituent mass of the up and down quarks $m_p/3 \approx 313$ MeV corresponds to a Yukawa coupling ~ 3.4, which is close to our estimate.

In Fig. 1 we show the Higgs boson mass as a function of the dimensionful parameters $M_{H^\pm}$, f and M_ρ by fixing the dimensionless couplings to some typical values, $\xi = 3.6$, $\lambda_1/(2\xi^2) = 0.7$ and $\lambda_2/\lambda_1 = 0$. We see that $M_h = 126$ GeV can be obtained with reasonable parameters of our model. Higgs mass increases as $M_{H^\pm}$ increases, as expected from the approximate analytic formula Eq. (2.25). The dependence on f is mild because it only enters the higher order corrections for fixed $M_{H^\pm}/f$ and affects the starting point of RG running of the couplings. On the other hand, M_h decreases as the cutoff M_ρ of the electroweak gauge loops increases due to its negative contribution to λ_h. For this set of $\{\xi,\, \lambda_1,\, \lambda_2\}$, we see that the correct Higgs boson mass is close to the upper bound for $M_\rho \approx 5f$ and hence we cannot have $M_\rho > 5f$. This upper bound depends on the values of the coupling parameters, which will be examined later.

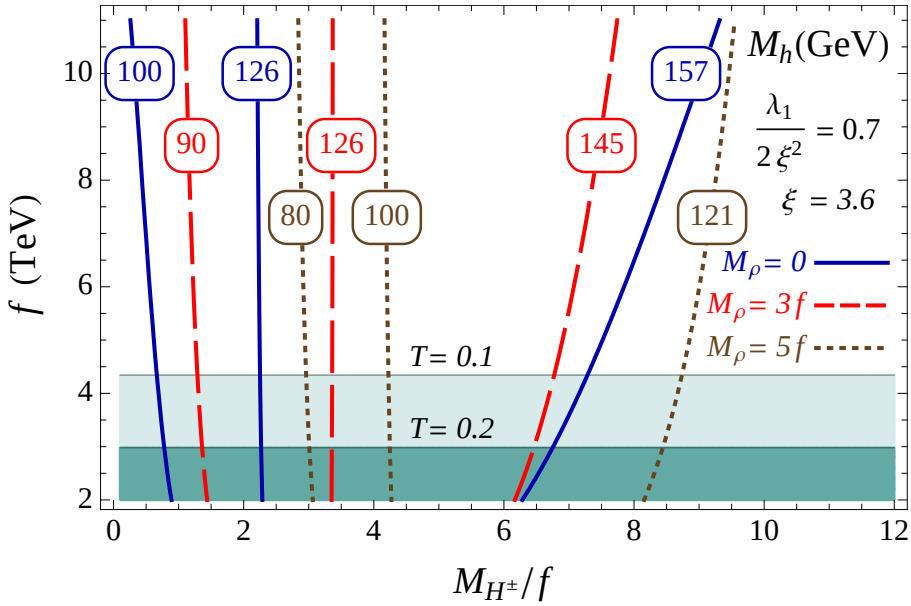

Figure 1: Contours of Higgs boson mass (labelled in GeV), for $\xi = 3.6$, $\lambda_1/2\xi^2 = 0.7$ and $\lambda_2 = 0$. The solid (blue) lines correspond to no electroweak corrections ($M_\rho = 0$, see Section 3), while the dashed (red) and dotted (brown) lines correspond to $M_\rho/f = 3$ and 5, respectively. The dark and light shaded regions correspond to $T > 0.2$ and $0.1 < T < 0.2$, respectively, with the T-parameter given by Eq. (4.5).

In Fig. 1 we also include contours of the T parameter [24], which measures the weak isospin violation and constitutes the strongest constraint on this model. As shown in Ref. [10–12], the heavy fermion t' can give a large contribution to the T parameter. In our model this is related to the fact that $U(3)_L$ does not contain a custodial $SU(2)$ symmetry.[1] The contribution from t' is given by [11]

$$T = \frac{3s_L^2}{16\pi^2\alpha v^2}\left[s_L^2 m_{t'}^2 + 4(1-s_L^2)\frac{m_{t'}^2 m_t^2}{m_{t'}^2 - m_t^2}\ln\left(\frac{m_{t'}}{m_t}\right) - (2-s_L^2)m_t^2\right], \tag{4.5}$$

where s_L is the sine of the left-handed mixing angle, given in Eq. (2.22). Other contributions to the T parameter, such as loops with heavier scalars, are very small compared to the contribution from t'. The correction to the S parameter is also negligible because we only add vector-like quarks to the SM.

Eq. (4.5) only depends on ξ, f, v and m_t, where the last two are fixed by their experimental values. With $m_{t'} \approx \xi f/\sqrt{2}$, $s_L \approx v/f$ [Eqs. (2.21), (2.22)], one can see that all three terms in Eq. (4.5) are roughly proportional to $1/f^2$, while they have different

[1]The dimension-6 operator $(H_\chi^\dagger D_\mu H_\chi)^2$, which represents the T parameter, is generated by the heavy fermion loop, by the same diagram which produces the quartic coupling.

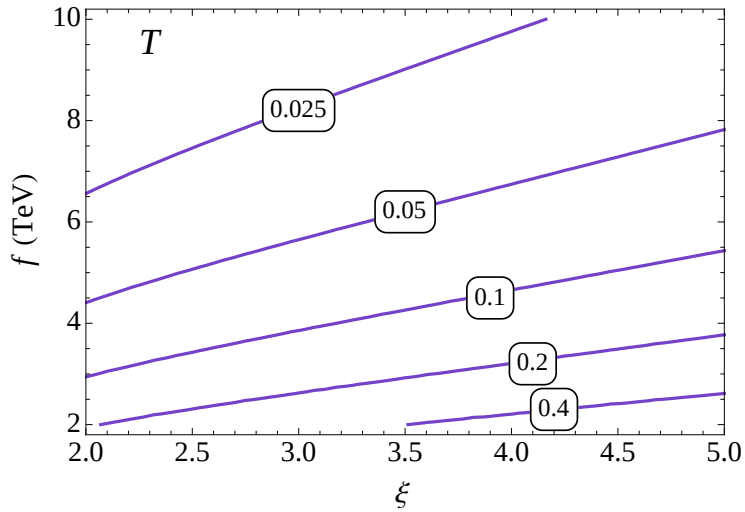

Figure 2: Contour plot of the T parameter in the $\xi - f$ plane, given by Eq. (4.5). The T parameter is roughly proportional to $1/f^2$. The constraint from the electroweak fit ($T < 0.15$ at the 95% CL) implies $f \gg v$.

dependence on ξ. Eq. (4.5) can be rewritten in terms of ξ and f as

$$T \approx \frac{3}{16\pi^2 \alpha f^2} \left[\frac{v^2 \xi^2}{2} + 4m_t^2 \ln\left(\frac{\xi f}{\sqrt{2} m_t}\right) - 2m_t^2 \right] \quad . \tag{4.6}$$

Contours of T in the (f, ξ) plane are shown in Fig. 2. From the current electroweak fit [25], the 68% and 95% bound roughly correspond to $T = 0.1$ and $T = 0.15$ (for $S = 0$). For $\xi = 3.6$, these bounds translate to $f \gtrsim 4.3$ TeV at 68% CL, and $f \gtrsim 3.5$ TeV at 95% CL.

Among the six parameters of Eq. (4.3), the Higgs boson mass in the leading order is only sensitive to $\lambda_1/(2\xi^2)$, $M_{H^\pm}/f$ and M_ρ/f. The dependence on the other parameters are expected to be mild. To study the M_h dependence on the more sensitive parameters, we fix $\xi = 3.6$, $\lambda_2 = 0$, and $f = 4$ TeV (which corresponds to T=0.12, close to its lower bound), and make the contour plot of the Higgs boson mass in the $\lambda_1/(2\xi^2)$-versus-$M_{H^\pm}/f$ plane for several different values of M_ρ/f (Fig 3). As expected, a larger M_h occurs for larger $M_{H^\pm}/f$, $\lambda_1/(2\xi^2)$ and smaller M_ρ/f. There is an upper limit $M_h \lesssim 175$ GeV even for extreme values of these parameters. This is close to our estimate from the analytic formula, Eq. (2.26).

A lower bound on M_h follows from the condition that the quartic coupling λ_h is positive at the matching scale $m_{t'}$. Otherwise of our vacuum is not a minimum of the tree-level potential, and the universe is more likely to end up in wrong vacuum. Imposing the boundary condition $\lambda_h = 0$ at the scale $m_{t'} \simeq \xi f/\sqrt{2}$, and using the SM 1-loop

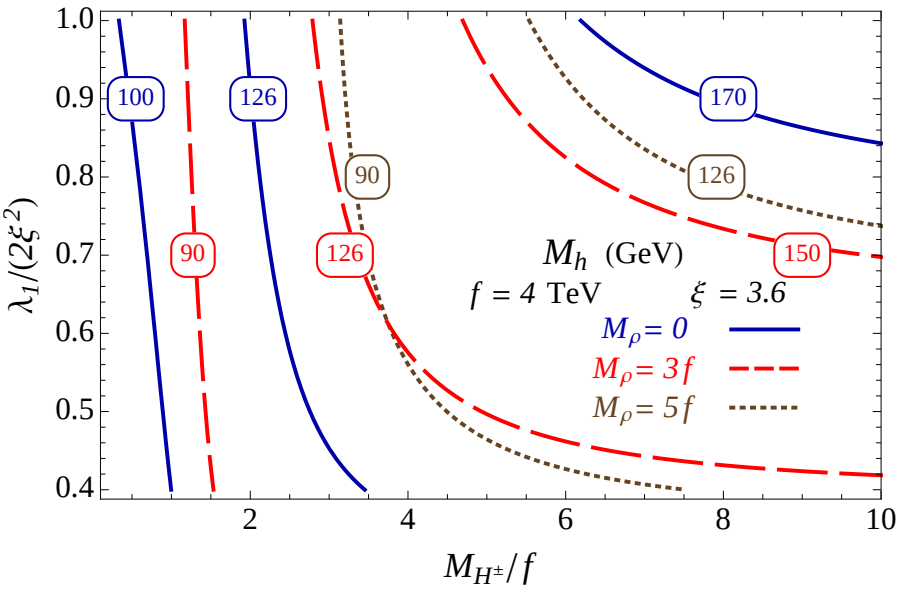

Figure 3: Contours of Higgs boson mass (labelled in GeV), for $\xi = 3.6$, $\lambda_2 = 0$, and $f = 4$ TeV. The solid (blue) lines correspond to no electroweak corrections ($M_\rho = 0$, see Section 3), while the dashed (red) and dotted (brown) lines correspond to $M_\rho/f = 3$ and 5, respectively.

RG equations to evolve λ_h down to the weak scale, we find that the physical M_h grows monotonically from 80 GeV for $m_{t'} = 6$ TeV (corresponding, *e.g.*, to $f = 3.5$ TeV, $\xi = 2.5$) to 90 GeV for $m_{t'} = 25$ TeV. Thus, the lower bound on M_h is around 80 GeV. We conclude that, in our composite Higgs model, the Higgs boson mass is constrained to be in the

$$80 \text{ GeV} < M_h < 175 \text{ GeV} \tag{4.7}$$

range, with the upper limit significantly tighter for most of the parameter space (as shown in Figs. 1 and 3); the measured 126 GeV Higgs mass sits comfortably in the middle of this range.

So far we have considered the Higgs boson mass dependences on the more sensitive parameters. To check how the Higgs mass varies with the less sensitive parameters, we show in Fig. 4 the Higgs mass as a function of $M_{H^\pm}/f$ for several different sets of $\{\xi,\ f,\ \lambda_2\}$, by fixing $\lambda_1/(2\xi^2) = 0.7$ and $M_\rho = 3f$. We see that indeed the dependences on these parameters are rather mild. Among them, the Yukawa coupling ξ, which enters both the higher order correction in Eq. (2.25) (through $m_{t'}$) and the electroweak gauge loop correction in Eq. (3.8) [for a fixed value of $\lambda_1/(2\xi^2)$], has a slightly larger effect. The λ_2 dependence is almost negligible.

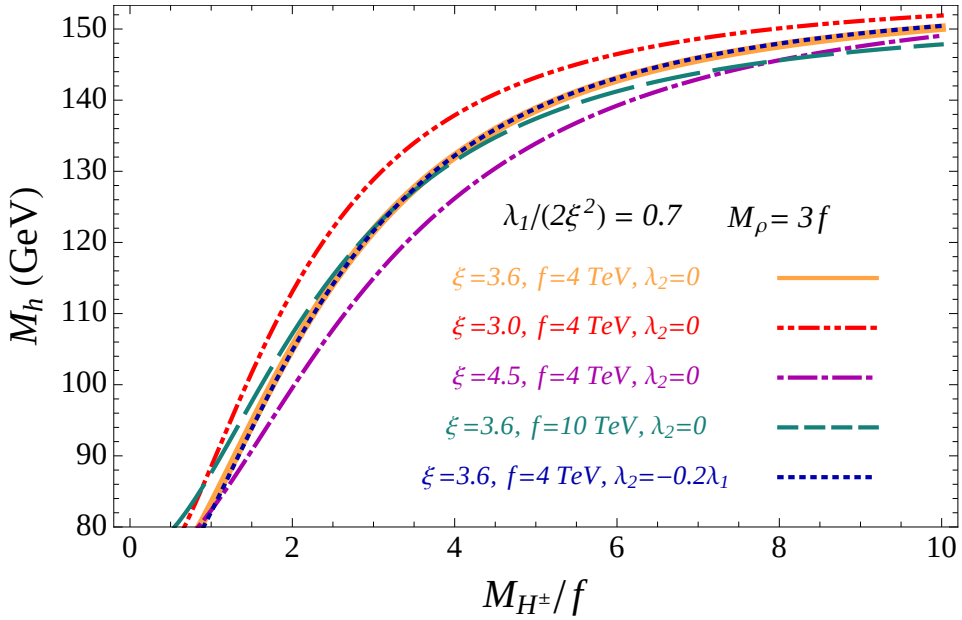

Figure 4: Higgs boson mass as a function of $M_{H^\pm}/f$ for various values of ξ, f, λ_2, when $\lambda_1/2\xi^2 = 0.7$ and $M_\rho/f = 3$. The different curves are obtained by varying one parameter at a time with respect to the solid (orange) curve.

In addition to M_h, we are also interested in other predictions of this model, such as the spectrum of heavy states. This model contains two doublet and two singlet scalars in the effective theory. After electroweak symmetry breaking, apart from the eaten Nambu-Goldstone modes and the SM-like Higgs boson, there are 3 neutral CP-even scalars (denoted by $H_{1,2,3}$ according to the ascending order of their masses), 3 neutral CP-odd scalars (denoted by $A_{1,2,3}$), and a complex charged Higgs boson ($H^\pm$). One CP-odd scalar (A_1) is lighter than other heavy states because it is also the pNGB of the broken $U(3)_L$ symmetry and hence its mass is also suppressed by s_γ. However, its mass is controlled by f rather than v, so it is still quite heavy compared to the Higgs boson. The masses of the states coming from Φ_t receive contribution from both M_{tt}^2 and $\lambda_1 f^2$. In the limit of $M_{tt}^2 \gg \lambda_1 f^2$, all these states are expected to be around $M_{H^\pm}$ and decouple from low energy physics.

We use the mass of the discovered Higgs boson to fix one more parameter. We choose it to be M_ρ and plot its required values to obtain the correct Higgs boson mass in the $M_{H^\pm}/f - \lambda_1/(2\xi^2)$ plane in the first panel of Fig. 5. The contour $M_\rho/f = 0$ represents the case where the explicit $U(3)_L$ breaking from the electroweak gauge loops is absent. $M_h = 126$ GeV cannot be obtained in the region to the left of that contour, unless there is additional explicit $U(3)_L$ breaking from the cutoff scale that has positive contributions

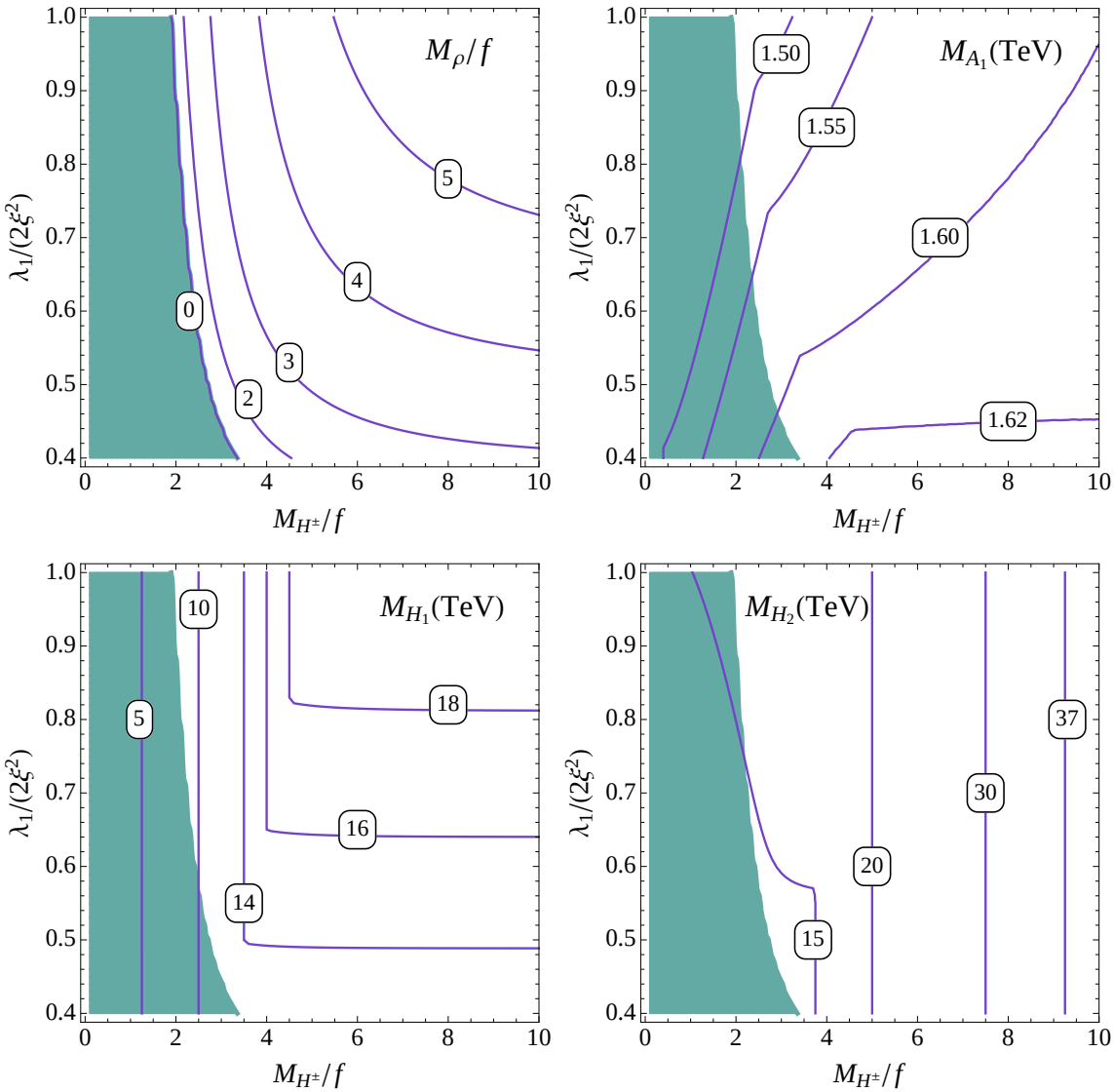

Figure 5: Contour plots for M_ρ/f, the mass of the lightest CP-odd neutral scalar (M_{A_1}) and the masses of the second and third lightest CP-even neutral scalars (M_{H_1} and M_{H_2}). We have fixed $f = 4$ TeV, $\xi = 3.6$ and $\lambda_2 = 0$, which corresponds to $m_{t'} = 10.2$ TeV and $T = 0.12$. The shaded regions are not consistent with $M_h = 126$ GeV (unless there is additional $U(3)_L$ breaking with effects opposite to the electroweak corrections).

to λ_h. For small values of $M_{H^\pm}$ or $\lambda_1/(2\xi^2)$, M_ρ/f also needs to be small, which means that additional states responsible for cutting off the electroweak gauge loop contribution are probably required to appear near or even below the scales of our heavy scalars.

Assuming that M_ρ takes the value which produces $M_h \approx 126$ GeV, we can calculate the masses of the heavy scalars in this theory. We show the masses of the lightest CP-odd

scalar (A_1) and the first two heavy CP-even neutral scalars (H_1, H_2) in the other three panels of Fig. 5, for the same parameters as in Fig. 3. For fixed $M_{H^\pm}/f$, the heavy states scale proportionally to f. Since $f = 4$ TeV is close to the lower bound allowed by the T parameter, the masses in these plots are close to their lower limits in this model. We see that A_1 is indeed much lighter compared to other heavy states, though it is still above 1 TeV. For small values of $M_{H^\pm}/f$, H_1 mostly comes from Φ_t and its mass is close to $M_{H^\pm}$, so the contours run vertically. On the other hand for large $M_{H^\pm}/f$, H_1 dominantly consists of φ_χ and its mass is proportional to λ_1, and is independent of $M_{H^\pm}$. The second pseudoscalar A_2 is closely degenerate with $H^\pm$, and the masses of H_3, A_3 are a little bit higher but are also close.

5 Phenomenology

Within this composite Higgs model, the lightest particle beyond the SM is the CP-odd scalar A_1. Its mass is larger than M_h by a factor of f/v, as shown in Eq. (2.28). Taking into account the scale dependence of M_h, the A_1 mass M_{A_1} is slightly reduced (see second panel of Fig. 5). The smallest value of f allowed by the electroweak data, $f = 3.5$ TeV, gives $M_{A_1} = 1.3$ TeV. A_1 may be singly produced at hadron colliders through gluon fusion. Given that A_1 is mostly part of the ϕ_χ singlet, its coupling to the top quark is suppressed. The dominant contribution to gluon fusion is a t' loop, relying on the $(\xi/\sqrt{2})\, A_1 \bar{t}' \gamma_5 t'$ coupling. For $f = 3.5$ TeV, the A_1 production cross section is ~ 0.6 fb at the 14 TeV LHC. We obtained this result using FeynRules [26] and MadGraph 5 [27] and then multiplying by a factor of 2 to account for large higher order effects. A_1 may decay into $t\bar{t}$ or hZ, but in either case the cross section for $pp \to A_1 X$ is too small, even for a luminosity of 3000 fb^{-1} [28].

The other composite scalars as well as the vector-like quark are too heavy to be probed at the LHC. A hadron collider at a center-of-mass energy of $O(100)$ TeV would be needed in order to produce them. For a typical $m_{t'} = 9$ TeV, the $t'\bar{t}'$ production cross section at a 100 TeV pp collider (we refer to it as the VLHC) is 0.12 fb, based on MadGraph 5, with a 50% increase to account for higher order effects. The VLHC cross section can be as large as 1.8 fb, for $m_{t'} = 6$ TeV. Besides the usual $t' \to Wb, th, tZ$ decays [29], there is an interesting $t' \to tA_1$ decay, followed by $A_1 \to t\bar{t}$ or hZ. Given that the backgrounds relevant at an invariant mass $\gtrsim 6$ TeV are small, it is possible to discover the t' quark at the VLHC even with the initial luminosity of 2×10^{34} cm^{-2}s^{-1} [30].

Single A_1 production at the VLHC through gluon fusion would have a cross section around 100 fb, for $M_{A_1} = 1.3$ TeV. Detailed studies of the backgrounds are necessary before deciding whether single A_1 production leads to promising channels.

The heaviness of the particles beyond the SM puts this model close to the decoupling limit, so the Higgs couplings are close to the SM predictions. The main correction comes from mixing with the singlet scalar fields, which can be seen in Eq. (2.23). We have verified numerically that throughout the allowed parameter space, the couplings of the Higgs boson to SM particles are given, to a very good approximation, by their SM values times the factor $\cos(v/f) \simeq 1 - v^2/(2f^2)$, which is the fraction of the doublet component in the Higgs boson. The deviation from the SM couplings is only 0.2% for $f = 4$ TeV and is inversely proportional to f^2. The modifications to the branching fractions are even smaller because the dominant correction is universal. Such small deviations can not be tested at the LHC and are probably even beyond the reach of a future e^+e^- collider. Nevertheless, a precise determination of the custodial $SU(2)$ breaking T parameter can help to probe or constrain this model further.

Since the SM-like Higgs field is composite and made of the top quark and the new quark χ, the light SM fermion masses presumably come from some 4-fermion interactions generated above the cutoff scale. Constraints on flavor changing neutral currents (FCNC) limit the coefficients and patterns of these 4-fermion interactions [12]. Below the cutoff, our model has two Higgs doublets. General couplings of SM fermions to the two Higgs doublets lead to tree-level FCNCs. However, it is reasonable to assume that the couplings of the SM fermions to heavy Higgs states have a hierarchical structure correlated with the corresponding Yukawa couplings of the SM Higgs boson, due to some approximate flavor symmetries [31]. In that case, the FCNC constraints require the scalars from the other Higgs doublet to be heavier than a few hundred GeV to ~ 1 TeV, with the strongest bounds coming from the neutral meson-anti-meson mixings and $\mu \to e\gamma$. (See Ref. [32] for a comprehensive study and review.) With the heavy states around 10 TeV in our model, these FCNC constraints are easily satisfied. Only A_1 is relatively lighter, but it is mostly part of the ϕ_χ singlet, so that it does not induce any significant FCNC effects.

6 Conclusions

We have studied a composite Higgs model based on non-confining dynamics, in which the newly discovered Higgs boson is a bound state of a vector-like quark and the left-handed

top quark. The strongly coupled 4-quark interactions that describe the non-confining dynamics at the compositeness scale Λ produce scalar bound states which consist of two $SU(2)_W$-doublets and two gauge singlets [12]. We have shown that if the underlying strong dynamics preserves an approximate $U(3)_L$ chiral symmetry, a SM-like Higgs doublet arises naturally as the pNGB of $U(3)_L \to U(2)_L$ breaking.

Explicit $U(3)_L$ breaking terms produce the correct m_t through the top-seesaw mechanism. They also give the mass to the SM-like Higgs boson. As a result, the Higgs and top masses are tightly correlated, and satisfy $M_h \lesssim m_t$. Electroweak effects further reduce M_h, so that it is easily compatible with the measured Higgs mass within the natural range of parameter space.

The strongest constraint on this model comes from weak isospin violation due to heavy quark loops. Requiring $T \lesssim 0.15$ pushes the $U(3)_L$ symmetry breaking scale f above 3.5 TeV, so that some fine-tuning is needed to obtain the weak scale $v \approx 246$ GeV $\ll f$. It also means that most of the new states beyond the SM (except the lightest CP-odd scalar) will have masses around or above 10 TeV, beyond the reach of the LHC. The corrections to the SM Higgs couplings are tiny as the new sector is close to the decoupling limit. Nevertheless, the fact that no new particles or any deviation from the SM has been discovered at the LHC so far suggests that the SM Higgs sector is somewhat tuned, and the scale of new physics may be higher than previously thought. Our model is certainly consistent with the current experimental observations. It would require a collider beyond the LHC with a center-of-mass energy $O(100)$ TeV to probe the heavy states in this model directly.

On the other hand, the scale of $U(3)_L$ chiral symmetry breaking may be significantly lowered if the contribution to T from the heavy quark can be cancelled by some additional contribution. This would make the model less tuned and the new states lighter and more accessible. If the model can be extended to include an approximate custodial $SU(2)$ symmetry, for example, by adding a new vector-like quark to mix with the bottom quark, then f can be lowered to $\lesssim 1$ TeV, and the new quarks could have masses below 2 TeV and be within the reach of the 14 TeV LHC.

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A Renormalization group running of couplings

In Sec. 2.2 the light Higgs boson mass is shown to be proportional to the ratio of the couplings, $\sqrt{\lambda_1/(2\xi^2)}$, in the absence of $SU(2)_W \times U(1)_Y$ gauge interactions. In the fermion-loop (bubble) approximation, this ratio is predicted to be 1, which corresponds to the well-known result of $m_\phi = 2m_f$ in the NJL type model [19], where m_ϕ is the mass of the composite scalar and m_f is the constituent fermion mass after chiral symmetry breaking. The fermion-loop approximation neglects the gauge loop corrections and the back reaction of the scalar self-interactions. It can be viewed as the leading N_c result if the gauge interactions are ignored. The presence of the other interactions will modify this ratio. If the chiral symmetry breaking scale f is tuned to be much smaller than the compositeness scale Λ, this ratio can also be well-determined due to the infrared fixed point structure of the RG equations [9]. Quasi-infrared fixed points have been used to predict top quark and Higgs boson masses in some theoretical models with large couplings at high scales [18, 33]. For f not much smaller than Λ as in the case we are interested, one cannot trust the RG analysis because the couplings are strong and the logarithms are only $\mathcal{O}(1)$. Nevertheless, finding the infrared fixed point of the RG equations may still provide us some ideas of the possible range of the relevant coupling ratio $\lambda_1/(2\xi^2)$. In the far infrared the couplings become perturbative, and the fixed point can be determined by 1-loop RG equations presented below.

To be general and to identify the fermion and scalar loops, we write down the coupled RG equations of the couplings ξ, λ_1, λ_2, and QCD strong coupling g_3 for an $U(N_L)_L \times U(N_R)_R$ theory:

$$16\pi^2 \frac{dg_3}{dt} = -\left(11 - \frac{2}{3}N_f\right) g_3^3, \tag{A.1}$$

$$16\pi^2 \frac{d\xi}{dt} = \left(\frac{N_L + N_R}{2} + N_c\right)\xi^3 - 3\,\frac{N_c^2 - 1}{N_c} g_3^2\,\xi, \tag{A.2}$$

$$16\pi^2 \frac{d\lambda_1}{dt} = 2(N_L + N_R)\lambda_1^2 + 4\lambda_1\lambda_2 + 4N_c(\xi^2\lambda_1 - \xi^4), \tag{A.3}$$

$$16\pi^2 \frac{d\lambda_2}{dt} = 4\lambda_1^2 + 4(N_L + N_R)\lambda_1\lambda_2 + 2N_L N_R \lambda_2^2 + 4N_c\xi^2\lambda_2, \tag{A.4}$$

where we have ignored the electroweak couplings g_1, g_2, and the light fermion Yukawa

couplings. N_f is the number of quark flavors. These equations can be inferred from Ref. [34].

These RG equations are in the mass-independent scheme. Near the composite scale where the composite scalars dissolve, the scalar masses are large and the scalar loops should decouple [35]. This justifies the fermion-loop approximation near the compositeness scale if the gauge couplings are relatively small. If we drop the scalar loop contributions (*i.e.*, terms without the N_c factor) and ignore the gauge couplings, we obtain

$$16\pi^2 \frac{d}{dt} \ln\left(\frac{\lambda_1}{\xi^2}\right) = 2N_c\xi^2\left(1 - \frac{2\xi^2}{\lambda_1}\right) . \tag{A.5}$$

We see that the infrared fixed point corresponds to $\lambda_1 = 2\xi^2$, agreeing with the result of the fermion-loop approximation. On the other hand, λ_2 is not generated by the fermion loops. Note that the fermion-loop approximation sums fermion loops to all orders so it applies even to large couplings. As a result, one may treat $\lambda_1 = 2\xi^2$, $\lambda_2 = 0$ as the initial condition when the scalar loops become relevant.

It is instructive to derive the approximate IR fixed point analytically for the ratios of couplings. For simplicity we first neglect the QCD coupling g_3 because it is much smaller than the other couplings near the cutoff scale. We obtain the RG equations for $r \equiv \lambda_2/\lambda_1$ and $s \equiv \lambda_1/\xi^2$ by combining Eqs. (A.2)-(A.4):

$$16\pi^2 \frac{d\ln r}{dt} = 2\lambda_1\left[\frac{2}{r} + N_L + N_R + (N_L N_R - 2)\, r + \frac{2N_c}{s^2}\right] , \tag{A.6}$$

$$16\pi^2 \frac{d\ln s}{dt} = \xi^2\left[2(N_L + N_R + 2r)\, s + 2N_c - N_L - N_R - \frac{4N_c}{s}\right] . \tag{A.7}$$

The infrared fixed point is reached when the right-hand side of the equations vanishes:

$$\begin{aligned} &\frac{2}{r} + N_L + N_R + (N_L N_R - 2)\, r + \frac{2N_c}{s^2} = 0 , \\ &2(N_L + N_R + 2r)\, s + 2N_c - N_L - N_R - \frac{4N_c}{s} = 0 . \end{aligned} \tag{A.8}$$

There are multiple solutions to these polynomial equations. The actual IR fixed point has $|r| \ll 1$, so we can further simplify the equations by ignoring the terms proportional to positive powers of r, then Eq. (A.8) gives

$$s_* \simeq \frac{1}{4}\left(1 - x + \sqrt{1 + 14x + x^2}\right) , \tag{A.9}$$

where $x \equiv 2N_c/(N_L + N_R)$. We have chosen the positive solution because both ξ^2 and λ_1 stay positive. Substituting it into Eq. (A.8), we obtain

$$r_* \simeq -\frac{x}{N_c\,(1 + x/s_*^2)} . \tag{A.10}$$

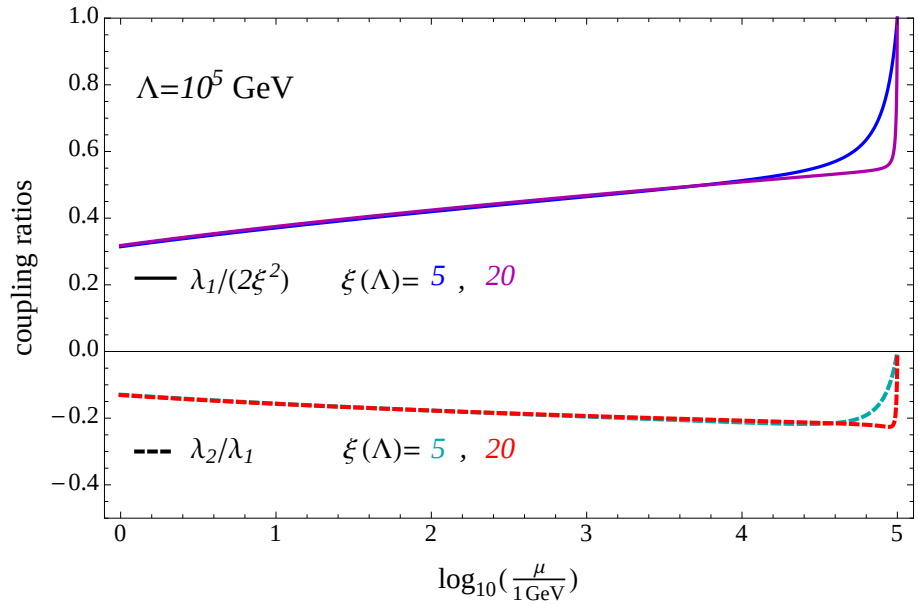

Figure 6: One-loop RG evolutions of the coupling ratios $\lambda_1/(2\xi^2)$ and λ_2/λ_1 for initial values $\lambda_1/(2\xi^2) = 1$, $\lambda_2/\lambda_1 = 0$ and $\xi = 5$ or 20. The horizontal axis is the logarithm of the energy scale.

For $N_L = 3$, $N_R = 2$ and $N_c = 3$, we have

$$s_* \approx 1 \quad \text{and} \quad r_* \approx -0.2 \ . \tag{A.11}$$

To check the accuracy of the analytical approximation of the IR fixed point solution, we solve the 1-loop RG equations (A.1)–(A.4) numerically. We set the initial condition $\lambda_1 = 2\xi^2$, $\lambda_2 = 0$ and choose several different initial values for ξ. The results of 1-loop RG running are shown in Fig. 6. We see that the ratios of couplings are quickly driven to the approximate fixed point values given by Eq. (A.11), though we should not trust the exact evolution in the beginning due to potentially large higher loop contributions. The infrared value of r is a bit smaller than the approximate result in Eq. (A.11) due to the gauge loop contribution from g_3.

If the chiral symmetry breaking scale is not far below the compositeness scale, we can not trust the 1-loop RG results. However, if we assume a smooth evolution, the ratios of couplings are expected to lie in between their initial values and the infrared fixed point values:

$$0.4 \lesssim \frac{\lambda_1}{2\xi^2} \lesssim 1, \quad -0.2 \lesssim \frac{\lambda_2}{\lambda_1} \lesssim 0 \, . \tag{A.12}$$

We use these ranges in Sections 2 and 4.

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