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Circular modes for flat beams in the LHC

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Typically x/y optical coupling is considered as unwanted and thus suppressed; particular exclusions are electron and ionization coolers. Could some special coupled modes be effectively applied for the LHC complex? Perhaps, the answer is positive: use of the circular modes in the injectors with their transformation into planar modes in the LHC allows both the space charge and beam-beam luminosity limitations to be significantly reduced, if not practically eliminated.

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I. PLANAR AND CIRCULAR MODES

Conventional x/y betatron oscillations can be referred to as planar, since in the geometrical 3D space every one of them is seen as a plane, horizontal or vertical. Planar optical modes are described by the conventional Twiss functions; Courant-Snyder invariants and betatron phases are canonical actions and phases for these modes. In the more general case, the two transverse degrees of freedom are arbitrary coupled, and their description requires a more complicated set of general 4D Twiss functions, which can be taken ether in Edwards-Teng or Mais-Ripken form (see e.g. Ref. [1]). For the coupled case, there are still two Courant-Snyder invariants, expressed as quadratic forms of the 4D phase space vectors. Provided that an optical transformation is linear and symplectic, the two Courant-Snyder invariants are preserved.

An interesting example of fully coupled optics is presented by circular modes, when particles are moving along clockwise or counterclockwise spirals [2]. These modes are eigenfunctions for rotation-invariant focusing elements, like solenoids and bending magnets with the field index 1/2:

$$-\frac{\partial B_y}{\partial x}\frac{\rho}{B_y} = \frac{1}{2}.$$

For quadrupole and skew-quadrupole based focusing, optical modes are generally elliptical, being more close to planar or circular in special cases. In principle, there is a direct analogy between light polarization and optics of particles in accelerators: the eigenfunctions are determined by symmetry of the media or focusing. For rotation-invariant matrices, the angular momentum is preserved; thus, the angular momentum $M = xp_y - yp_x$ has to be proportional to a difference of the two circular Courant-Snyder invariants $J_\pm$; in fact, it is just equal to that: $M = J_+ - J_-$. Since the beam emittances are nothing else but averages of the two actions, the beam angular momentum is given by a difference of its two circular emittances: $\langle M \rangle = \varepsilon_+ - \varepsilon_-$. An important aspect of the analogy between light and charged particle optics relates to planar-circular transformations. For charged particle optics, a possibility of this transformation was pointed out in Ref. [3] and practically demonstrated in Ref. [4], where it was shown that these *optical adapters* can be implemented by means of skew triplets. If a beam in a planar state is coming into a planar-to-circular adapter, the outgoing beam would be circular, and vice versa. If a beam with very different emittances is in a planar state, it is flat. If this beam is transformed into a circular state, it becomes a round vortex, whose angular

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momentum is equal to the larger emittance. Since adapters are based on linear optics, circular-planar transformations preserve both emittances: $\varepsilon_{\pm} \leftrightarrow \varepsilon_{x,y}$.

II. SPACE CHARGE SUPPRESSION

For planar modes, the space charge tune shift is determined by both emittances, being maximal for a plane of smaller emittance. That is why space charge limitation suggests for the two emittances to be close to each other. For circular modes, the space charge limitation works differently: when the emittance ratio is high, the space charge tune shift is determined by the larger emittance only, even if the smaller emittance goes to zero. Indeed, for the circular beam state, the beam size is determined by the larger emittance, making space charge insensitive to the smaller one. To make this tune shift of a circular beam with two very different emittances identical to the tune shift of a planar beam with two equal emittances, the larger circular emittance has to be 2 times higher than one of the planar emittances [5].

This property of the circular modes suggests an idea to use them for low-energy accelerators, with the larger emittance determined by the space charge tune shift, and the smaller one can be as small as possible. In this case, the beam brightness can be significantly increased due to reduced value of the smaller emittance. After acceleration to sufficiently high energy, the beam can be transformed into a planar state, becoming flat.

III. FLAT BEAMS IN LHC

A flat beam in the collider provides significant advantages, seen in electron machines. The first one relates to luminosity. Keeping in mind that the space charge requires the larger emittance be 2 times higher than in the conventional planar case, it can be concluded that luminosity can be increased as soon as the emittance ratio is 4 or higher, being proportional to the square root of the emittance ratio.

Another important benefit of the flat beam is suppression of beam-beam resonances. Indeed, the power of a resonance of an order (n, m) is determined by the beam-beam Hamiltonian term $\propto \Delta x^n \Delta y^m$. If one of the sizes goes to zero, this 2D net of the resonances reduces to 1D, allowing a lot more resonance-free space in the tune plane. As a consequence, the beam separation can be significantly reduced without any damage for emittances or lifetime.

One more important dimension opened by the flat beams is related to luminosity leveling. To do that, the beta function in the horizontal (larger emittance) plane can be increased as much as needed, gradually squeezed later with the brightness reduction. A large horizontal beta function allows increasing the crossing angle without any luminosity reduction, similar to electron colliders. As a result, neither crab cavity nor high aperture in the focusing triplet [6] is needed any more, and the long-range beam-beam collisions can be excluded.

Exclusion of the long-range collisions entails a valuable benefit for the coherent stability. As it is already clear from LHC operations, the octupole currents required for the stabilization are 2–3 times higher for two beams seeing each other compared with the single beam case. Getting rid of the long-range collisions would allow working at negative octupole polarity with sufficiently low stabilizing octupole current.

IV. HOW TO PREPARE CIRCULAR MODES

In some details, this problem was considered in Ref. [7]. Obviously, the beam in a circular state requires accelerator optics to be circular or reasonably close to that. To have a high ratio of the emittances, the beam has to be injected predominantly into one of the circular modes, keeping another emittance as small as possible. This could be done by means of multiturn painting with a small-emittance beam from a linac: the emittance ratio is determined then by a number of injection turns. Most likely, emittance ratio >10 would require the space charge to be taken into account at injection: otherwise a space charge related mismatch would heat the smaller emittance preventing having it that small. For a not so high emittance ratio, like 10 or so, this mismatch, probably can be ether neglected or compensated on average only. To inject into a truly space charge self-consistent state [8], allowing emittance ratio ~ 20 or higher, the inductive synchrotron can be used [9]. The application of circular modes for the LHC complex definitely requires circular optics for the booster and PS. Whether this optics is required for the SPS as well, or if the SPS could work with flat planar beams, is one of the open questions.

V. OPEN QUESTIONS

The described proposal of circular modes and flat beams entails a whole number of questions for the LHC complex. Their preliminary list is suggested as follows: (i) What is the minimal lower emittance justifying application of the circular modes? (ii) What is the optimal scenario for LHC for that value of the lower emittance? (iii) What is the luminosity lifetime for the flat beams, taking into account intrabeam scattering, gas scattering, synchrotron radiation, and direct burning out in collisions? (iv) What can be the benefits from even smaller values of the lower emittance? (v) How can the lower emittance be minimized at injection from the linac? (vi) Can the space charge tune shift be increased by injection into a self-consistent state, as foreseen in Ref. [8]? (vii) How can lower emittance growth be minimized at acceleration? How can optics be optimized for that? (viii) Which optics has to be used at the SPS—planar or circular? (ix) Can there be any use of the circular optics in the LHC (for instance, everywhere outside IRs)?

These and related questions open a wide sphere of research required for any practical decision about the suggested proposal of circular modes—flat beams for the LHC.

VI. SUMMARY

Circular optics in the injector complex makes space charge tune shift independent of the smaller emittance, thus suggesting having it much smaller than the larger one, with significant increase of the beam brightness. This scheme requires proper multiturn injection from the linac. Having a beam with emittance ratio ~ 10 or higher allows one to get rid of long-range collisions in the LHC, to increase its luminosity, to reduce the aperture in the IR triplet, eliminate the need in the crab cavity, and make the beam more stable.

Hopefully, the promising points of this concept are sufficiently attractive to start a new research program for the possible future of both the LHC complex and post-LHC era, with a goal for the luminosity being as high as possible for the detectors.

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[1] V. Lebedev and S. Bogacz, JINST **5** P10010 (2010).
[2] A. Burov, S. Nagaitsev, and Ya. Derbenev, Phys. Rev. E **66**, 016503 (2002).
[3] Ya. Derbenev, University of Michigan Report No. UM HE 93-20, 1993.
[4] A. Burov and V. Danilov, Report No. FERMILAB-TM-2043, 1998.

[5] A. Burov and Ya. Derbenev, Report No. FERMILAB-PUB-09-392-AD, 2009.
[6] A. Valishev (private communication).
[7] V. Danilov, S. Cousineau, S. Henderson, J. Holmes, and M. Plum, in *Proceedings of the 9th European Particle Accelerator Conference, Lucerne, 2004* (EPS-AG, Lucerne, 2004), p. 2227.
[8] J. Holmes, V. Danilov, and S. Cousineau, in *Proceedings of the 21st Particle Accelerator Conference, Knoxville, 2005* (IEEE, Piscataway, NJ, 2005), 2194 (2005).
[9] K. Takayama, Y. Arakida, T. Dixit, T. Iwashita, T. Kono, E. Nakamura, K. Otsuka, Y. Shimosaki, K. Torikai, and M. Wake, Phys. Rev. Lett. **98**, 054801 (2007).
[10] A. Burov, A. Valishev, ''Circular optics for flat beams in LHC,'' CERN AP Forum talk, 2012.