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Impersonating the Standard Model Higgs Boson: Alignment without Decoupling

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Abstract

In models with an extended Higgs sector there exists an alignment limit, in which the lightest CP-even Higgs boson mimics the Standard Model Higgs. The alignment limit is commonly associated with the decoupling limit, where all non-standard scalars are significantly heavier than the Z boson. However, alignment can occur irrespective of the mass scale of the rest of the Higgs sector. In this work we discuss the general conditions that lead to "alignment without decoupling", therefore allowing for the existence of additional non-standard Higgs bosons at the weak scale. The values of $\tan\beta$ for which this happens are derived in terms of the effective Higgs quartic couplings in general two-Higgs-doublet models as well as in supersymmetric theories, including the MSSM and the NMSSM. Moreover, we study the information encoded in the variations of the SM Higgs-fermion couplings to explore regions in the $m_A - \tan\beta$ parameter space.

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I. INTRODUCTION

The Standard Model (SM) with one Higgs doublet is the simplest realization of electroweak symmetry breaking and provides a very good description of all data collected so far at hadron and lepton colliders. This includes measurements associated with the recently discovered 125 GeV Higgs boson at the CERN LHC [1, 2]. In this model, the Higgs field receives a vacuum expectation value (VEV), $v \approx 246$ GeV, which breaks the electroweak gauge symmetry and gives masses to the fundamental fermions and gauge bosons. The couplings of these particles to the Higgs boson are fixed by their masses and v. On the other hand, Higgs self interactions are controlled by the quartic coupling in the Higgs potential, which in turn is given by the Higgs mass and v. Therefore, interactions of the SM Higgs boson with fermions, gauge bosons and with itself are completely determined.

Extensions of the SM commonly lead to modifications of the Higgs couplings, especially if there exist new particles interacting with the Higgs or if there is an extended Higgs sector. The size of these modifications may be naively estimated in the decoupling limit to be:

$$\mathcal{O}\left(\frac{v^2}{m_{\rm new}^2}\right) \approx \mathcal{O}(5\%) \times \left(\frac{1\ \text{TeV}}{m_{\rm new}}\right)^2 , \tag{1}$$

where $m_{\rm new}$ is the scale of new particles. Therefore, for new particles below the TeV scale, changes in the Higgs couplings from the SM expectations are quite small. Such an estimate supports the fact that due to the large uncertainties in present measurements, no significant deviations from the SM Higgs properties should be identifiable in present data, if all new particles are at or above the TeV scale. At the same time, it stresses the need for precision Higgs measurements to uncover possible signs of new physics.

Conversely, Eq. (1) implies that, if in the future, refined measurements of the properties of the 125 GeV Higgs boson continue to be consistent with those of a SM Higgs boson, no new light particles interacting with the SM-like Higgs are to be expected. However, this estimate is only valid in the so called decoupling limit, where all non-standard Higgs bosons are significantly heavier than the Z gauge boson. On the other hand, current searches for these particles do not exclude the possibility of additional Higgs bosons in the hundred to several hundred GeV mass range. Given that initial data appears to disfavor large deviations with respect to the SM Higgs description [3], it is of special interest to consider models of extended Higgs sectors containing a CP-even Higgs that has properties mimicking quite

precisely the SM ones, even if the non-standard Higgs bosons are light.

A well-known example is that of general two-Higgs-doublet models (2HDMs) [4, 5], in which the heavy CP-even Higgs could be the SM-like Higgs boson. However, in this case the 2HDM parameter space becomes very restrictive, with masses of the non-standard scalars of the order of the W and Z boson masses, and is severely constrained by data [6]. On the other hand, the possibility of the lightest CP-even Higgs mimicking the SM Higgs, referred to as "alignment" in Ref. [7], is much less constrained and usually associated with the decoupling limit. The less known and more interesting case of alignment without recourse to decoupling deserves further study.

A few examples of "alignment without decoupling" have been considered in the literature. The first one was presented over a decade ago by Gunion and Haber in Ref. [8]. Their main focus was to emphasize the SM-like behavior of the lightest CP-even Higgs of a 2HDM in the decoupling limit. However, they also demonstrated that it can behave like a SM Higgs without decoupling the non-SM-like scalars. Much more recently, a similar situation was discussed in Ref. [9], where an extension of the Minimal Supersymmetric Standard Model (MSSM) with a triplet scalar was studied. It was found that after integrating out the triplet scalar, a SM-like Higgs boson and additional light scalars are left in the spectrum for low values of $\tan\beta \lesssim 10$. Another recent study, Ref. [7], presented a scanning over the parameter space of general 2HDMs. Solutions were found fulfilling alignment without decoupling and the phenomenological implications were investigated.

It is obvious that the possibility of alignment without decoupling would have far-reaching implications for physics beyond the SM searches. However, its existence has remained obscure and has sometimes been attributed to accidental cancellations in the scalar potential. A simple way to understand how one of the CP-even Higgs bosons in a 2HDM mimics the SM Higgs is to realize that the alignment limit occurs whenever the mass eigenbasis in the CP-even sector aligns with the basis in which the electroweak gauge bosons receive all of their masses from only one of the Higgs doublets [1]. From this perspective, it is clear that the alignment limit does not require the non-standard Higgs bosons to be heavy. After presenting the general conditions for the alignment limit in 2HDMs, we analyze in detail the

[1] This would imply that the other, non-standard CP-even Higgs has no tree-level couplings to the gauge bosons. However, there are still couplings to SM fermions in general. Therefore the non-standard Higgs boson is not inert.

possible implications for well motivated models containing two Higgs doublets. In particular, we consider the MSSM as well as its generalization to the next-to-minimal supersymmetric standard model (NMSSM), where an extra singlet is added. Along the way, we analyze the extent to which precision measurements of Higgs-fermion couplings could be useful in probing regions of parameters that are difficult to access through direct non-standard Higgs boson searches.

This article is organized as follows. In the next section we define the notation and briefly review the scalar potential and the Higgs couplings in general, renormalizable 2HDMs. In Section III we derive the alignment condition in the decoupling regime in terms of the eigenvectors of the CP-even Higgs mass matrix, which provides a simple analytical understanding of alignment. We then write down the general conditions for alignment without decoupling. In Section IV we study the alignment limit in general 2HDMs and provide new perspectives on previous works. Detailed studies on the parameter space of the MSSM and beyond are presented in Section V, which is followed by the conclusion in Section VI.

II. OVERVIEW OF 2HDM

A. Scalar Potential

We follow the notation in Ref. [10] for the scalar potential of the most general two-Higgs-doublet extension of the SM:

$$\begin{aligned} V \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} \;, \end{aligned} \tag{2}$$

where

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(\phi_i^0 + i a_i^0) \end{pmatrix} \;. \tag{3}$$

We will assume CP conservation and that the minimum of the potential is at

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \end{pmatrix} \;, \tag{4}$$

where

$$v \equiv \sqrt{v_1^2 + v_2^2} \approx 246 \text{ GeV} \ , \quad t_\beta \equiv \tan\beta = \frac{v_2}{v_1} \ . \tag{5}$$

We choose $0 \leq \beta \leq \pi/2$ so that $t_\beta \geq 0$ and write $v_1 = v\cos\beta \equiv vc_\beta$ and $v_2 = v\sin\beta \equiv vs_\beta$. The five mass eigenstates are: two CP-even scalars, H and h, with $m_h \leq m_H$, one CP-odd scalar, A, and a charged pair, $H^\pm$. The mass parameters, m_{11}^2 and m_{22}^2, can be eliminated by imposing the minimization condition [10]:

$$m_{11}^2 - t_\beta m_{12}^2 + \frac{1}{2}v^2 c_\beta^2(\lambda_1 + 3\lambda_6 t_\beta + \tilde{\lambda}_3 t_\beta^2 + \lambda_7 t_\beta^3) = 0 \ , \tag{6}$$

$$m_{22}^2 - t_\beta^{-1} m_{12}^2 + \frac{1}{2}v^2 s_\beta^2(\lambda_2 + 3\lambda_7 t_\beta^{-1} + \tilde{\lambda}_3 t_\beta^{-2} + \lambda_6 t_\beta^{-3}) = 0 \ , \tag{7}$$

where $\tilde{\lambda}_3 = \lambda_3 + \lambda_4 + \lambda_5$. It then follows that [10]

$$m_A^2 = \frac{2m_{12}^2}{s_{2\beta}} - \frac{1}{2}v^2(2\lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) \ , \tag{8}$$

and the mass-squared matrix for the CP-even scalars can be expressed as

$$\mathcal{M}^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \equiv m_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + v^2 \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \ , \tag{9}$$

where

$$L_{11} = \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 \ , \tag{10}$$

$$L_{12} = (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \ , \tag{11}$$

$$L_{22} = \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \ . \tag{12}$$

The mixing angle, α, in the CP-even sector is defined as

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \equiv R(\alpha) \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \ , \tag{13}$$

where $s_\alpha \equiv \sin\alpha$ and $c_\alpha \equiv \cos\alpha$. This leads to

$$R^T(\alpha) \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} R(\alpha) = \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \ . \tag{14}$$

From the $(1,2)$ component in the above equation we see

$$(m_H^2 - m_h^2) s_\alpha c_\alpha = \mathcal{M}_{12}^2 \ , \tag{15}$$

which implies $s_\alpha c_\alpha$ has the same sign as $\mathcal{M}_{12}^2$. There are two possible sign choices:

$$\text{(I)} \quad -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}: \quad c_\alpha \geq 0 \;\text{ and }\; \text{Sign}(s_\alpha) = \text{Sign}(\mathcal{M}_{12}^2) \;, \tag{16}$$

$$\text{(II)} \quad 0 \leq \alpha \leq \pi: \quad s_\alpha \geq 0 \;\text{ and }\; \text{Sign}(c_\alpha) = \text{Sign}(\mathcal{M}_{12}^2) \;. \tag{17}$$

We will discuss the implications of these two sign choices in some detail below.

The eigenvector associated with the eigenvalue m_h^2 corresponds to the second row in $R(\alpha)$, Eq. (13), and satisfies

$$\begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = m_h^2 \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} \;, \tag{18}$$

giving rise to two equivalent representations for $t_\alpha \equiv \tan\alpha$:

$$t_\alpha = \frac{\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2 - m_h^2} = \frac{\mathcal{M}_{22}^2 - m_h^2}{\mathcal{M}_{12}^2} \;. \tag{19}$$

The equivalence of the two representations is guaranteed by the characteristic equation, $\text{Det}(\mathcal{M}^2 - m_h^2\, I) = 0$, where I is the 2×2 identity matrix. Moreover, since

$$m_h^2 \leq \mathcal{M}_{ii}^2 \leq m_H^2 \;, \qquad \text{for} \quad i = 1, 2 \;, \tag{20}$$

due to the "level repulsion" of eigenvalues of symmetric matrices, in both representations $\text{Sign}(t_\alpha) = \text{Sign}(\mathcal{M}_{12}^2)$, consistent with the sign choices specified above.

Eq. (19) allows us to solve for the mixing angle, α, in terms of $\{\mathcal{M}_{11}^2, \mathcal{M}_{12}^2, m_h^2\}$ or $\{\mathcal{M}_{22}^2, \mathcal{M}_{12}^2, m_h^2\}$, depending on one's preference. For example, in the sign choice (I) we have the following two representations:

$$s_\alpha = \frac{\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{12}^2)^2 + (\mathcal{M}_{11}^2 - m_h^2)^2}} \;, \qquad m_H^2 = \frac{\mathcal{M}_{11}^2(\mathcal{M}_{11}^2 - m_h^2) + (\mathcal{M}_{12}^2)^2}{\mathcal{M}_{11}^2 - m_h^2}, \tag{21}$$

$$s_\alpha = \text{Sign}(\mathcal{M}_{12}^2)\frac{\mathcal{M}_{22}^2 - m_h^2}{\sqrt{(\mathcal{M}_{12}^2)^2 + (\mathcal{M}_{22}^2 - m_h^2)^2}} \;, \quad m_H^2 = \frac{\mathcal{M}_{22}^2(\mathcal{M}_{22}^2 - m_h^2) + (\mathcal{M}_{12}^2)^2}{\mathcal{M}_{22}^2 - m_h^2}, \tag{22}$$

where the expression for m_H^2 follows from solving for the corresponding eigenvalue equation for m_H^2.

One can verify that Eqs. (21) and (22) lead to the expected limiting behavior when $\mathcal{M}_{12}^2 \to 0$. For example, for Eq. (21), if $\mathcal{M}_{11}^2 > \mathcal{M}_{22}^2$, the smaller mass eigenvalue, m_h^2, is given by $\mathcal{M}_{22}^2$. Then in Eq. (21) we have $s_\alpha \to 0$ and $m_H^2 \to \mathcal{M}_{11}^2$. As expected the lightest

CP-even Higgs is mostly Φ_2 in this case. On the other hand, if $\mathcal{M}_{11}^2 < \mathcal{M}_{22}^2$ then h is mostly Φ_1 and $s_\alpha \to 1$, since

$$(\mathcal{M}_{11}^2 - m_h^2) = \frac{(\mathcal{M}_{12}^2)^2}{|\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2|} + \mathcal{O}\left((\mathcal{M}_{12}^2)^4\right) \ , \tag{23}$$

which also implies $m_H^2 \to \mathcal{M}_{22}^2$ in this case. The behavior of Eq. (22) can be verified in a similar fashion.

B. Higgs Couplings

The Higgs boson couplings to gauge bosons in 2HDMs follow from gauge invariance and have the same parametric dependence on the CP-even mixing angle, α, and the angle β in any 2HDM, namely,

$$g_{hVV} = s_{\beta-\alpha}\, g_V \ , \qquad g_{HVV} = c_{\beta-\alpha}\, g_V \ , \tag{24}$$

where $g_V = 2im_V^2/v$ is the SM value for $V = W, Z$ bosons.

The fermion couplings, on the other hand, take different forms in different 2HDMs. However, it is common to require the absence of tree-level flavor-changing neutral currents (FCNC) by imposing the Glashow-Weinberg condition [11]. This condition requires fermions with the same quantum numbers to couple to a single Higgs doublet and leads to four different types of 2HDMs [4].[2] Amongst them the most popular ones are: the type I model, where all SM fermions couple to one doublet, and the type II model, where the up-type fermions couple to one doublet and down-type fermions couple to the other. In one of the other two models, up-type quarks and leptons couple to the same doublet, while down-type quarks couple to the other. The remaining one has all the quarks coupled to one Higgs doublet while the leptons couple to the other one. In what follows we base the discussion on the type II model, although our analysis can be easily adapted to all four types of 2HDMs.

In type II models, where at tree-level Φ_1 and Φ_2 only couple to down-type and up-type fermions, respectively, the tree-level Higgs couplings to fermions are

$$g_{hdd} = -\frac{s_\alpha}{c_\beta}\, g_f = (s_{\beta-\alpha} - t_\beta\, c_{\beta-\alpha})\, g_f \ , \quad g_{huu} = \frac{c_\alpha}{s_\beta}\, g_f = (s_{\beta-\alpha} + t_\beta^{-1}\, c_{\beta-\alpha})\, g_f \ , \tag{25}$$

$$g_{Hdd} = \frac{c_\alpha}{c_\beta}\, g_f = (c_{\beta-\alpha} + t_\beta\, s_{\beta-\alpha})\, g_f \ , \quad g_{Huu} = \frac{s_\alpha}{s_\beta}\, g_f = (c_{\beta-\alpha} - t_\beta^{-1}\, s_{\beta-\alpha})\, g_f \ , \tag{26}$$

[2] Typically the Glashow-Weinberg condition requires a discrete symmetry: $\Phi_1 \to -\Phi_1$, which demands $\lambda_6 = \lambda_7 = 0$ in the general scalar potential given in Eq. (2).

where $g_f = im_f/v$ is the coupling of the Higgs to the corresponding fermions in the SM.

We are interested in the alignment limit, where the lightest CP-even Higgs mimics the SM one. We will begin by solving for the conditions for which the Higgs couplings to fermions have the same magnitude as in the SM: $|g_{huu}/g_f| = |g_{hdd}/g_f| = 1$. There are four possibilities, which can be divided in two cases:

$$\begin{aligned} &\text{i)} \qquad g_{hdd} = g_{huu} = \pm g_f, \\ &\text{ii)} \qquad g_{hdd} = -g_{huu} = \pm g_f. \end{aligned}$$

Demanding case i) leads to

$$s_\alpha = \mp c_\beta \ , \qquad c_\alpha = \pm s_\beta \ , \tag{27}$$

which then implies

$$c_{\beta-\alpha} = 0 \qquad \text{and} \qquad s_{\beta-\alpha} = \pm 1 \ . \tag{28}$$

Couplings of the CP-even Higgs bosons now become

$$g_{hVV} \to \pm g_V \ , \ g_{hff} \to \pm g_f \ , \ g_{HVV} \to 0 \ , \ g_{Hdd} \to \pm t_\beta \, g_f \ , \ g_{Huu} \to \mp t_\beta^{-1} \, g_f \ , \tag{29}$$

where the upper and lower signs correspond to $s_{\beta-\alpha} = 1$ and -1, respectively. This is the alignment limit. The heavy CP-even Higgs couplings to SM gauge bosons vanish in this limit since it does not acquire a VEV. In other words, the alignment limit is the limit where the mass eigenbasis in the CP-even sector coincides with the basis where the gauge bosons receive all of their masses from one of the doublets. As such, the non-SM-like CP-even Higgs does not couple to the gauge bosons at the tree-level. However, in this basis H still has non-vanishing couplings to SM fermions. This feature remains true in all four types of 2HDMs, as can be seen, for example, by inspecting Table 2 in Ref. [7]. It is important to observe that $s_{\beta-\alpha} = \pm 1$ results in an overall sign difference in the couplings of the SM-like Higgs and, hence, has no physical consequences.

On the other hand, fulfillment of case ii) requires

$$s_\alpha = \mp c_\beta \ , \qquad c_\alpha = \mp s_\beta \ , \tag{30}$$

which gives

$$c_{\beta-\alpha} = \mp s_{2\beta} \ , \qquad s_{\beta-\alpha} = \pm c_{2\beta} \ . \tag{31}$$

We see that the hVV coupling does not tend to the SM value in this case and alignment is not reached. However, in the limit $t_\beta \gg 1$,

$$s_{2\beta} = \frac{2t_\beta}{1+t_\beta^2} \approx \frac{2}{t_\beta} \ , \quad c_{2\beta} = \frac{1-t_\beta^2}{1+t_\beta^2} \approx -1 \ , \tag{32}$$

we observe that the CP-even Higgs couplings become, to linear order in t_β^{-1},

$$g_{hVV} = \mp g_V \ , \quad g_{hdd} = \pm g_f \ , \quad g_{huu} = \mp g_f \ , \tag{33}$$

$$g_{HVV} = \mp 2t_\beta^{-1} g_V \ , \quad g_{Hdd} = \mp t_\beta g_f \ , \quad g_{Huu} = \mp t_\beta^{-1} g_f \ . \tag{34}$$

Hence, if Eq. (30) is required, one obtains that the lightest CP-even Higgs couplings to down-type fermions have the opposite sign as compared to its couplings to both the vector bosons and up-type fermions, although all couplings have the same strength as in the SM. If, instead of the large t_β limit, one takes $t_\beta \ll 1$, then it is straightforward to check that g_{huu} has the opposite sign to g_{hdd} and g_{hVV}. It is worth noting that, in type II 2HDMs, $t_\beta \ll 1$ leads to an unacceptably large top Yukawa coupling and should be avoided. However, the scenario of "wrong-sign" down-type fermion couplings of the SM-like Higgs in the large t_β limit is clearly of phenomenological importance. A detailed study of this scenario is beyond the scope of the present work.

Similar arguments can be made in the case in which it is the heavy Higgs that behaves as the SM Higgs. For this to occur,

$$s_{\beta-\alpha} = 0 \tag{35}$$

and therefore $c_{\beta-\alpha} = \pm 1$. In the following, we shall concentrate on the most likely case in which the lightest CP-even Higgs satisfies the alignment condition. The heavy Higgs case can be treated in an analogous way.

We also comment on the Hhh coupling since it may have a significant impact on strategies in direct searches [7]. The coupling of the heavy Higgs to the lightest Higgs is given by

$$\begin{aligned} g_{Hhh} = \frac{v}{4} \Big[& -12\ \lambda_1 c_\beta c_\alpha s_\alpha^2 - 12\ \lambda_2 s_\beta s_\alpha c_\alpha^2 + \tilde{\lambda}_3(-4c_{\alpha-\beta} + 6s_{2\alpha}s_{\alpha+\beta}) \\ & + 3\lambda_6(-4s_\alpha^2 s_{\alpha+\beta} + 8s_\alpha c_\alpha^2 c_\beta) + 3\lambda_7(8s_\alpha^2 c_\alpha s_\beta - 4c_\alpha^2 s_{\alpha+\beta}) \Big] \ . \end{aligned} \tag{36}$$

One can rewrite Eq. (36) as

$$\begin{aligned} g_{Hhh} = & -3vs_\beta c_\beta^3 \Big\{ \Big[s_{\alpha\beta}c_{\alpha\beta} \Big(\lambda_1 s_{\alpha\beta} + \tilde{\lambda}_3 t_\beta^2 c_{\alpha\beta} + \lambda_6 t_\beta (2c_{\alpha\beta} + s_{\alpha\beta}) \Big) + \lambda_7 t_\beta^3 c_{\alpha\beta}^3 \Big] \\ & - \Big[s_{\alpha\beta}c_{\alpha\beta} \Big(\lambda_2 t_\beta^2 c_{\alpha\beta} + \tilde{\lambda}_3 s_{\alpha\beta} + \lambda_7 t_\beta (2s_{\alpha\beta} + c_{\alpha\beta}) \Big) + \lambda_6 t_\beta^{-1} s_{\alpha\beta}^3 \Big] \Big\} - \tilde{\lambda}_3 c_{\alpha-\beta}, \end{aligned} \tag{37}$$

where $s_{\alpha\beta} \equiv (-s_\alpha/c_\beta)$ and $c_{\alpha\beta} \equiv (c_\alpha/s_\beta)$ tend to 1 in the alignment limit. We shall demonstrate in the next section that the alignment conditions in general 2HDMs imply that the Hhh coupling vanishes.

III. ALIGNMENT WITHOUT DECOUPLING

A. Derivation of the Conditions for Alignment

One of the main results of this work is to find the generic conditions for obtaining alignment without decoupling. The decoupling limit, where the low-energy spectrum contains only the SM and no new light scalars, is only a subset of the more general alignment limit in Eq. (28). In particular, quite generically, there exist regions of parameter space where one attains the alignment limit with new light scalars not far above $m_h = 125$ GeV.

It is instructive to first derive the alignment limit in the usual decoupling regime but in a slightly different manner. Consider the eigenvalue equation of the CP-even Higgs mass matrix, Eq. (18), which, using Eq. (9), becomes

$$\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = -\frac{v^2}{m_A^2} \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} + \frac{m_h^2}{m_A^2} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} . \tag{38}$$

Decoupling is defined by taking all non-SM-like scalar masses to be much heavier than the SM-like Higgs mass, $m_A^2 \gg v^2, m_h^2$. Then we see that at leading order in v^2/m_A^2 and m_h^2/m_A^2, the right-hand side of Eq. (38) can be ignored, and the eigenvalue equation reduces to

$$\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} \approx 0 , \tag{39}$$

leading to the well-known decoupling limit [8]: $c_{\beta-\alpha} = 0$. This is also exactly the alignment limit.

Here we make the key observation that while decoupling achieves alignment by neglecting the right-hand side of Eq. (38), alignment can also be obtained if the right-hand side of Eq. (38) vanishes *identically*, independent of m_A:

$$v^2 \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = m_h^2 \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} . \tag{40}$$

More explicitly, since $s_\alpha = -c_\beta$ in the alignment limit, we can re-write the above matrix equation as two algebraic equations: [3]

$$\text{(C1)} \; : \; m_h^2 = v^2 L_{11} + t_\beta v^2 L_{12} = v^2 \left(\lambda_1 c_\beta^2 + 3\lambda_6 s_\beta c_\beta + \tilde{\lambda}_3 s_\beta^2 + \lambda_7 t_\beta s_\beta^2 \right) \; , \tag{41}$$

$$\text{(C2)} \; : \; m_h^2 = v^2 L_{22} + \frac{1}{t_\beta} v^2 L_{12} = v^2 \left(\lambda_2 s_\beta^2 + 3\lambda_7 s_\beta c_\beta + \tilde{\lambda}_3 c_\beta^2 + \lambda_6 t_\beta^{-1} c_\beta^2 \right) \; . \tag{42}$$

Recall that $\tilde{\lambda}_3 = (\lambda_3 + \lambda_4 + \lambda_5)$. In the above m_h is the SM-like Higgs mass, measured to be about 125 GeV, and L_{ij} is known once a model is specified. Notice that (C1) depends on all the quartic couplings in the scalar potential except λ_2, while (C2) depends on all the quartics but λ_1. If there exists a t_β satisfying the above equations, then the alignment limit would occur for arbitrary values of m_A and *does not* require non-SM-like scalars to be heavy!

Henceforth we will consider the coupled equations given in Eqs. (41) and (42) as required conditions for alignment. When the model parameters satisfy them, the lightest CP-even Higgs boson behaves exactly like a SM Higgs boson even if the non-SM-like scalars are light. A detailed analysis of the physical solutions will be presented in the next Section.

B. Departure from Alignment

Phenomenologically it seems likely that alignment will only be realized approximately, rather than exactly. Therefore it is important to consider small departures from the alignment limit, which we do in this subsection.

Since the alignment limit is characterized by $c_{\beta-\alpha} = 0$, it is customary to parametrize the departure from alignment by considering a Taylor-expansions in $c_{\beta-\alpha}$ [7, 8], which defines the deviation of the g_{hVV} couplings from the SM values. However, this parametrization has the drawback that deviations in the Higgs coupling to down-type fermions are really controlled by $t_\beta \, c_{\beta-\alpha}$, which could be $\mathcal{O}(1)$ when t_β is large. Therefore, we choose to parametrize the departure from the alignment limit by a parameter η which is related to $c_{\beta-\alpha}$ by

$$c_{\beta-\alpha} = t_\beta^{-1} \eta \; , \qquad s_{\beta-\alpha} = \sqrt{1 - t_\beta^{-2} \eta^2} \; . \tag{43}$$

[3] The same conditions can also be derived using results presented in Ref. [8].

Then at leading order in η, the Higgs couplings become

$$g_{hVV} \approx \left(1 - \frac{1}{2} t_\beta^{-2} \eta^2\right) g_V \ , \qquad g_{HVV} \approx t_\beta^{-1} \eta \ g_V \ , \tag{44}$$

$$g_{hdd} \approx (1-\eta)\, g_f \ , \qquad g_{Hdd} \approx t_\beta (1 + t_\beta^{-2} \eta) g_f \ , \tag{45}$$

$$g_{huu} \approx (1 + t_\beta^{-2} \eta)\, g_f \ , \qquad g_{Huu} \approx -t_\beta^{-1} (1-\eta) g_f \ . \tag{46}$$

We see η characterizes the departure from the alignment limit of not only g_{hdd} but also g_{Huu}. On the other hand, the deviation in the g_{huu} and g_{Hdd} are given by $t_\beta^{-2}\eta$, which is doubly suppressed in the large t_β regime. Moreover, terms neglected above are of order η^2 and are never multiplied by positive powers of t_β, which could invalidate the expansion in η when t_β is large.

There are some interesting features regarding the pattern of deviations. First, whether the coupling to fermions is suppressed or enhanced relative to the SM values, is determined by the sign of η: g_{hdd} and g_{Huu} are suppressed (enhanced) for positive (negative) η, while the trend in g_{huu} and g_{Hdd} is the opposite. In addition, as $\eta \to 0$, the approach to the SM values is the fastest in g_{hVV} and the slowest in g_{hdd}. This is especially true in the large t_β regime, which motivates focusing on precise measurements of g_{hdd} in type II 2HDMs.

Our parametrization of $c_{\beta-\alpha} = t_\beta^{-1}\eta$ can also be obtained by modifying Eq. (39), which defines the alignment limit, as follows:

$$\begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = t_\beta^{-1} \eta \begin{pmatrix} -s_\beta \\ c_\beta \end{pmatrix} \ . \tag{47}$$

The eignevalue equation for m_h in Eq. (40) is modified accordingly,

$$v^2 \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} = m_h^2 \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix} - m_A^2\, t_\beta^{-1} \eta \begin{pmatrix} -s_\beta \\ c_\beta \end{pmatrix} \ . \tag{48}$$

From the above, taking $\eta \ll 1$ and expanding to first order in η, we obtain the "near-alignment conditions",

$$(\mathrm{C1}') \ : \ m_h^2 = v^2 L_{11} + t_\beta v^2 L_{12} + \eta \left[t_\beta (1 + t_\beta^{-2}) v^2 L_{12} - m_A^2 \right] \ , \tag{49}$$

$$(\mathrm{C2}') \ : \ m_h^2 = v^2 L_{22} + {t_\beta}^{-1} v^2 L_{12} - \eta \left[t_\beta^{-1} (1 + t_\beta^{-2}) v^2 L_{12} - m_A^2 \right] \ . \tag{50}$$

We will return to study these two conditions in the next section, after first analyzing solutions for alignment without decoupling in general 2HDMs.

IV. ALIGNMENT IN GENERAL 2HDM

In what follows we solve for the alignment conditions (C1) and (C2), assuming all the scalar couplings are independent of t_β. This is not true in general, as radiative corrections to the scalar potential often introduce a t_β dependence in the quartic couplings that are not present at the tree-level. However, this assumption allows us to analyze the solutions analytically and obtain the necessary intuition to understand more complicated situations.

When all the quartics are independent of t_β, the conditions (C1) and (C2) may be rewritten as cubic equations in t_β, with coefficients that depend on m_h and the quartic couplings in the scalar potential,

$$\text{(C1)} \ : \ (m_h^2 - \lambda_1 v^2) + (m_h^2 - \tilde{\lambda}_3 v^2) t_\beta^2 = v^2 (3\lambda_6 t_\beta + \lambda_7 t_\beta^3) \ , \tag{51}$$

$$\text{(C2)} \ : \ (m_h^2 - \lambda_2 v^2) + (m_h^2 - \tilde{\lambda}_3 v^2) t_\beta^{-2} = v^2 (3\lambda_7 t_\beta^{-1} + \lambda_6 t_\beta^{-3}) \ . \tag{52}$$

Alignment without decoupling occurs only if there is (at least) a common physical solution for t_β between the two cubic equations.[4] From this perspective it may appear that alignment without decoupling is a rare and fine-tuned phenomenon. However, as we will show below, there are situations where a common physical solution would exist between (C1) and (C2) without fine-tuning.

Regarding the coupling of the heaviest CP-even Higgs to the lightest one, it is now easy to see from Eqs. (51) and (52) that each term inside the square brackets in Eq. (37) tends to $m_h^2(1 + t_\beta^2)/v^2$ in the alignment limit, and hence, as stated in Ref. [7], g_{Hhh} vanishes.

A. Alignment for Vanishing Values of $\lambda_{6,7}$

It is useful to consider solutions to the alignment conditions (C1) and (C2) when $\lambda_6 = \lambda_7 = 0$ and $\lambda_1 = \lambda_2$, which can be enforced by the symmetries $\Phi_1 \to -\Phi_2$ and $\Phi_1 \to \Phi_2$. Then (C1) and (C2) collapse into quadratic equations:

$$\text{(C1)} \ \to \ (m_h^2 - \lambda_1 v^2) + (m_h^2 - \tilde{\lambda}_3 v^2) t_\beta^2 = 0 \ , \tag{53}$$

$$\text{(C2)} \ \to \ (m_h^2 - \tilde{\lambda}_3 v^2) + (m_h^2 - \lambda_1 v^2) t_\beta^2 = 0 \ . \tag{54}$$

[4] Since $t_\beta > 0$ in our convention, a physical solution means a real positive root of the cubic equation.

We see that a solution exists for $t_\beta = 1$ whenever

$$\lambda_{\rm SM} = \frac{\lambda_1 + \tilde{\lambda}_3}{2} \,, \tag{55}$$

where we have expressed the SM-like Higgs mass as

$$m_h^2 = \lambda_{\rm SM} v^2 \ . \tag{56}$$

From Eq. (55) we see that the above solution, $t_\beta = 1$, is obviously special, since it demands $\lambda_{\rm SM}$ to be the average of λ_1 and $\tilde{\lambda}_3$.

We next relax the $\lambda_1 = \lambda_2$ condition while still keeping $\lambda_6 = \lambda_7 = 0$. Recall that the Glashow-Weinberg condition [11] on the absence of tree-level FCNC requires a discrete symmetry, $\Phi_1 \to -\Phi_1$, which enforces $\lambda_6 = \lambda_7 = 0$ at the tree-level. The two quadratic equations have a common root if and only if the determinant of the Coefficient Matrix of the two quadratic equations vanishes,

$$\text{Det} \begin{pmatrix} m_h^2 - \tilde{\lambda}_3 v^2 & m_h^2 - \lambda_1 v^2 \\ m_h^2 - \lambda_2 v^2 & m_h^2 - \tilde{\lambda}_3 v^2 \end{pmatrix} = (m_h^2 - \tilde{\lambda}_3 v^2)^2 - (m_h^2 - \lambda_1 v^2)(m_h^2 - \lambda_2 v^2) = 0 \ . \tag{57}$$

Then the positive root can be expressed in terms of $(\lambda_1, \tilde{\lambda}_3)$,

$$t_\beta^{(0)} = \sqrt{\frac{\lambda_1 - \lambda_{\rm SM}}{\lambda_{\rm SM} - \tilde{\lambda}_3}} \ . \tag{58}$$

We see from Eqs. (57) and (58), that a real value of $t_\beta^{(0)}$ can exist only if the set of parameters $\{\lambda_{\rm SM}, \lambda_1, \lambda_2, \tilde{\lambda}_3\}$ has one of the two orderings

$$\lambda_1, \lambda_2 \ \geq \ \lambda_{\rm SM} \ \geq \ \tilde{\lambda}_3 \ , \tag{59}$$

or

$$\lambda_1, \lambda_2 \ \leq \ \lambda_{\rm SM} \ \leq \ \tilde{\lambda}_3 \ . \tag{60}$$

A solution for $t_\beta^{(0)}$ can be found using the following procedure: once one of the conditions in Eqs. (59) or (60) is satisfied, Eq. (58) leads to the alignment solution $t_\beta^{(0)}$ for a given $(\lambda_1, \tilde{\lambda}_3)$. However, Eq. (57) must also be satisfied, which is then used to solve for the desired λ_2 so that $t_\beta^{(0)}$ is a root of (C2) as well. More specifically, the relations

$$\lambda_2 - \lambda_{\rm SM} \ = \ \frac{\lambda_{\rm SM} - \tilde{\lambda}_3}{\left(t_\beta^{(0)}\right)^2} \ = \ \frac{\lambda_1 - \lambda_{\rm SM}}{\left(t_\beta^{(0)}\right)^4} \tag{61}$$

must be fulfilled. Therefore, the alignment solution demands a specific relationship between the quartic couplings of the 2HDM. In addition, it is clear from Eqs. (58) and (61) that if all the quartic couplings are $\mathcal{O}(1)$, $t_\beta^{(0)} \sim \mathcal{O}(1)$ as well, unless $\tilde{\lambda}_3$ and λ_2 are very close to $\lambda_{\rm SM}$, or λ_1 is taken to be much larger than $\lambda_{\rm SM}$. For examples, $t_\beta^{(0)} \sim 5$ could be achieved for $(\lambda_1, \tilde{\lambda}_3, \lambda_2) \sim (1., 0.23, 0.261)$, or for $(\lambda_1, \tilde{\lambda}_3, \lambda_2) \sim (5., 0.07, 0.263)$. Our discussion so far applies to alignment limit scenarios studied, for instance, in Refs. [7, 9], both of which set $\lambda_6 = \lambda_7 = 0$.

B. Alignment for Non-Zero $\lambda_{6,7}$

The symmetry $\Phi_1 \to -\Phi_1$ leading to $\lambda_6 = \lambda_7 = 0$ is broken softly by m_{12}. Thus a phenomenologically interesting scenario is to consider small but non-zero $\lambda_{6,7}$. Therefore, in this subsection we study solutions to the alignment conditions (C1) and (C2) under the assumptions

$$\lambda_{6,7} \ll 1 \ . \tag{62}$$

Although general solutions of cubic algebraic equations exist, much insight can be gained by first solving for the cubic roots of (C1) as a perturbation to the quadratic solution $t_\beta^{(0)}$,

$$t_\beta^{(\pm)} = t_\beta^{(0)} \pm \frac{3}{2}\frac{\lambda_6}{\lambda_{\rm SM} - \tilde{\lambda}_3} \pm \frac{\lambda_7(\lambda_1 - \lambda_{\rm SM})}{(\lambda_{\rm SM} - \tilde{\lambda}_3)^2} + \mathcal{O}(\lambda_6^2, \lambda_7^2) \ . \tag{63}$$

The solutions $t_\beta^{(\pm)}$ lie in the same branch as $t_\beta^{(0)}$, to which they reduce in the limit $\lambda_{6,7} \to 0$. In addition, both solutions are again $\mathcal{O}(1)$ given our assumptions. More importantly, similar to $t_\beta^{(0)}$, specific fine-tuned relations between the quartic couplings are required to ensure $t_\beta^{(\pm)}$ are also cubic roots of (C2).

However, a new solution also appears,

$$t_\beta^{(1)} = \frac{\lambda_{\rm SM} - \tilde{\lambda}_3}{\lambda_7} - \frac{3\lambda_6}{\lambda_{\rm SM} - \tilde{\lambda}_3} - \frac{\lambda_7(\lambda_1 - \lambda_{\rm SM})}{(\lambda_{\rm SM} - \tilde{\lambda}_3)^2} + \mathcal{O}(\lambda_6^2, \lambda_7^2) \ . \tag{64}$$

The solution $t_\beta^{(1)}$ belongs to a new branch that disappears when $\lambda_7 \to 0$ and exists provided the condition

$$\text{Sign}(\lambda_{\rm SM} - \tilde{\lambda}_3) = \text{Sign}(\lambda_7) \tag{65}$$

is satisfied. For $|\lambda_{\rm SM} - \tilde{\lambda}_3| \gg |\lambda_7|$, as is natural due to the assumption $|\lambda_7| \ll 1$, we are led to $t_\beta^{(1)} \gg 1$. As an example, for $(\lambda_1, \tilde{\lambda}_3, \lambda_6, \lambda_7) = (0.5, -0.1, 0.01, 0.01)$, one obtains $t_\beta^{(1)} \sim 35$

by solving for the cubic root of (C1) exactly. Lower values of $t_\beta^{(1)} = \mathcal{O}(10)$ may be obtained for somewhat larger values of λ_7 and/or larger values of $\tilde{\lambda}_3$.

The $t_\beta^{(1)}$ solution is an example of alignment without decoupling that does not require fine-tuning. This is because the condition (C2), in the limits $\lambda_6, \lambda_7 \ll 1$ and $t_\beta \gg 1$, becomes insensitive to all quartic couplings but λ_2:

$$m_h^2 - \lambda_2 v^2 = \mathcal{O}\left(\frac{1}{t_\beta^2}, \frac{\lambda_7}{t_\beta}, \frac{\lambda_6}{t_\beta^3}\right) . \tag{66}$$

Unlike the fine-tuned relation in Eq. (57), in this case λ_2 is determined by the input parameter m_h, or equivalently $\lambda_{\rm SM}$, and is insensitive to other quartic couplings in the scalar potential. Therefore, provided the condition given in Eq. (65) is fulfilled, the value of the quartic couplings, $\tilde{\lambda}_3$ and λ_7, are still free parameters and thus can be varied, leading to different values of t_β for which alignment occurs.

For the purpose of demonstration, let us again use the example below Eq. (65), $(\lambda_1, \tilde{\lambda}_3, \lambda_6, \lambda_7) = (0.5, -0.1, 0.01, 0.01)$. The condition that $t_\beta^{(1)} \sim 35$ is also a root of (C2) requires

$$\lambda_2 \approx 0.26 + \left(\frac{\tilde{\lambda}_3}{-0.1}\right) \times 8 \times 10^{-5} - \left(\frac{\lambda_7}{0.01}\right) \times 8 \times 10^{-4} - \left(\frac{\lambda_6}{0.01}\right) \times 8 \times 10^{-7} . \tag{67}$$

From this we see that the required value of λ_2 is very insensitive to the values of the other quartic couplings in the potential, and is determined only by m_h.

The solution $t_\beta^{(1)}$ is perhaps the most interesting among the three branches of solutions because its existence does not require specific relationships amongst the quartic couplings, and to our knowledge has never been studied in the literature. The crucial observation to arrive at this scenario of alignment without decoupling is to turn on small but non-vanishing λ_7, which arises automatically in 2HDMs without tree-level FCNC. In this case, we see alignment without decoupling is not only a generic feature of the model, but also a "natural" phenomenon, and can occur at $t_\beta = \mathcal{O}(10)$ for which direct searches for non-standard Higgs bosons become difficult. In fact, in the next section we will see that this solution can be realized in one of the most popular models for beyond the SM physics, the MSSM.

C. Departure from Alignment

So far we have analyzed solutions for the alignment conditions (C1) and (C2) in general 2HDMs. However, it is likely that the alignment limit, if realized in Nature at all, is only approximate and the value of t_β does not need to coincide with the value at the exact alignment limit. It is therefore important to study the approach to alignment and understand patterns of deviations in the Higgs couplings in the "near-alignment limit," which was introduced in Section III B.

Although we derived the near-alignment conditions (C1′) and (C2′) in Eqs. (49) and (50) using the eigenvalue equations, it is convenient to consider the (near-)alignment limit from a slightly different perspective. Adopting the sign choice (I) in Eq. (16) and using the expression for the mixing angle, α, in Eq. (21), we can re-write the g_{hdd} and g_{huu} couplings as follows

$$g_{hdd} = -\frac{s_\alpha}{c_\beta}\, g_f = \frac{\mathcal{A}}{\sqrt{\mathcal{A}^2 c_\beta^2 + \mathcal{B}^2 s_\beta^2}}\, g_f \ , \tag{68}$$

$$g_{huu} = \frac{c_\alpha}{s_\beta}\, g_f = \frac{\mathcal{B}}{\sqrt{\mathcal{A}^2 c_\beta^2 + \mathcal{B}^2 s_\beta^2}}\, g_f \ . \tag{69}$$

where

$$\mathcal{A} = -\frac{\mathcal{M}_{12}^2}{c_\beta} = \left(m_A^2 - (\lambda_3+\lambda_4)v^2\right) s_\beta - \lambda_7 v^2 s_\beta t_\beta - \lambda_6 v^2 c_\beta \ , \tag{70}$$

$$\mathcal{B} = \frac{\mathcal{M}_{11}^2 - m_h^2}{s_\beta} = \left(m_A^2 + \lambda_5 v^2\right) s_\beta + \lambda_1 v^2 \frac{c_\beta}{t_\beta} + 2\lambda_6 v^2 c_\beta - \frac{m_h^2}{s_\beta} \ . \tag{71}$$

Again it is instructive to consider first taking the pseudo-scalar mass to be heavy: $m_A \to \infty$. In this limit we have $\mathcal{A} \to m_A^2 s_\alpha$ and $\mathcal{B} \to m_A^2 s_\alpha$, leading to $-s_\alpha/c_\beta \to 1$ and $c_\alpha/s_\beta \to 1$. We recover the familiar alignment-via-decoupling limit. On the other hand, alignment without decoupling could occur by setting directly

$$\mathcal{A} = \mathcal{B}\,, \tag{72}$$

where, explicitly,

$$\mathcal{B} - \mathcal{A} = \frac{1}{s_\beta}\left(-m_h^2 + \tilde{\lambda}_3 v^2 s_\beta^2 + \lambda_7 v^2 s_\beta^2 t_\beta + 3\lambda_6 v^2 s_\beta c_\beta + \lambda_1 v^2 c_\beta^2\right) = 0 \ , \tag{73}$$

is nothing but the alignment condition (C1) in Eq. (41). The alignment condition (C2) would be obtained if the representation in Eq. (22) is used instead, leading to $\mathcal{A} =$

$-\text{Sign}(\mathcal{M}_{12}^2)(\mathcal{M}_{22}^2 - m_h^2)/c_\beta$ and $\mathcal{B} = |\mathcal{M}_{12}^2|/s_\beta$. Further, m_h is the mass of the lightest CP-even Higgs boson and $\mathcal{M}_{ii}^2 - m_h^2 > 0,\ i = \{1,2\}$ by Eq. (20). Therefore Eq. (72) implies

$$\mathcal{A} \geq 0 \qquad \text{and} \qquad \mathcal{B} \geq 0 \tag{74}$$

at the alignment limit.

Now in the near-alignment limit, where the alignment is only approximate, one can derive

$$g_{hdd} = \frac{\mathcal{A}}{\mathcal{B}\sqrt{1-(1-\mathcal{A}^2/\mathcal{B}^2)c_\beta^2}}\, g_f \tag{75}$$

$$= \left[1 - s_\beta^2\left(1 - \frac{\mathcal{A}}{\mathcal{B}}\right) + \mathcal{O}\left((1-\mathcal{A}/\mathcal{B})^2\right)\right] g_f\ , \tag{76}$$

which, when comparing with Eq. (45), implies

$$\eta = s_\beta^2\left(1 - \frac{\mathcal{A}}{\mathcal{B}}\right) = s_\beta^2 \frac{\mathcal{B}-\mathcal{A}}{\mathcal{B}}\ . \tag{77}$$

Therefore, the g_{hdd} coupling is enhanced (suppressed) if $\mathcal{B}-\mathcal{A} < 0$ (> 0). It is easy to verify that the above equation is identical to the near-alignment condition (C1′) in Eq. (49). The condition (C2′) could again be obtained using Eq. (22).

It is useful to analyze Eq. (76) in different instances. For example, when $\lambda_6 = \lambda_7 = 0$, one obtains

$$g_{hdd} \simeq \left[1 + s_\beta \frac{\left(\lambda_{\rm SM} - \tilde{\lambda}_3 s_\beta^2 - \lambda_1 c_\beta^2\right) v^2}{\mathcal{B}}\right] g_f\ . \tag{78}$$

Hence, for $\tilde{\lambda}_3 > \lambda_{\rm SM} > \lambda_1$, a suppression of g_{hdd} will take place for values of t_β larger than the ones necessary to achieve the alignment limit. On the contrary, for $\lambda_1 > \lambda_{\rm SM} > \tilde{\lambda}_3$, larger values of t_β will lead to an enhancement of g_{hdd}.

On the other hand, for $\lambda_7 \neq 0$ and large values of t_β, one obtains

$$g_{hdd} \simeq \left[1 + s_\beta \frac{\left(\lambda_{\rm SM} - \tilde{\lambda}_3 - \lambda_7 t_\beta\right) v^2}{\mathcal{B}}\right] g_f\ , \tag{79}$$

which shows that for $\lambda_{\rm SM} > \tilde{\lambda}_3$ and λ_7 positive, g_{hdd} is suppressed at values of t_β larger than those necessary to obtain the alignment limit, and vice versa.

One can in fact push the preceding analysis further by deriving the condition giving rise to a particular deviation from alignment. More specifically, the algebraic equation dictating the contour $g_{hdd}/g_f = r$, where $r \neq 1$, can be obtained by using Eq. (75):

$$m_A^2 = \frac{1}{R(\beta)-1}\frac{\mathcal{A}-\mathcal{B}}{s_\beta} + \frac{m_h^2}{s_\beta^2} - v^2\lambda_5 - \lambda_1 v^2 t_\beta^{-2} - 2\lambda_6 v^2 t_\beta^{-1}\ , \tag{80}$$

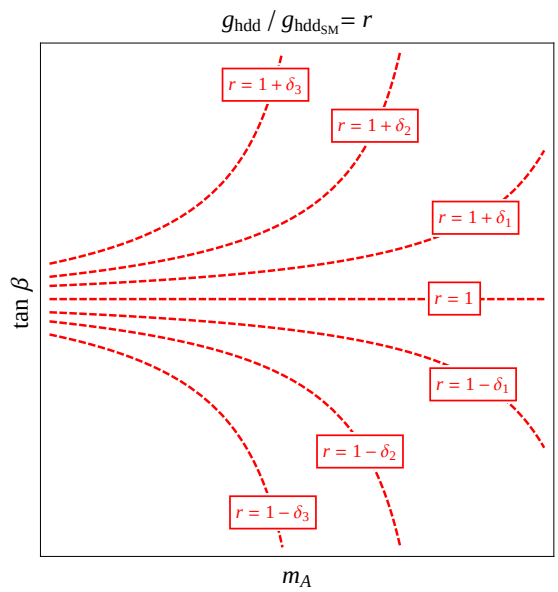

FIG. 1: General behavior of contours with constant $g_{hdd}/g_{hdd_{\rm SM}} = r$ in the $m_A - \tan\beta$ plane. $r = 1$ corresponds to the alignment limit. At constant t_β, moving toward smaller m_A results in larger deviations from $r = 1$. In the plot δ_i for $i = 1, 2, 3$ can be either positive or negative and $|\delta_3| > |\delta_2| > |\delta_1|$.

where

$$R(\beta) \;=\; \frac{t_\beta\, r}{\sqrt{1 + t_\beta^2 - r^2}} \;. \tag{81}$$

When r is close to unity, the above equation becomes

$$R(\beta) \;\approx\; 1 + \frac{r-1}{s_\beta^2} \;. \tag{82}$$

Several comments are in order. First, for $r \approx 1 - \eta$ with $\eta \ll 1$, $R(\beta) \approx 1 + \eta/s_\beta^2$. Second, once all the scalar quartic couplings are known, which in general could also depend on t_β, Eq. (80) gives the contour corresponding to $g_{hdd}/g_f = r$ in the $m_A - \tan\beta$ plane. Third, if we consider a slice of constant t_β away from the alignment limit then larger values of m_A correspond to values of $R(\beta)$, and hence r, closer to 1. Therefore, large deviations from $r = 1$ lie in regions with small m_A and t_β far from the alignment limit. These considerations allow for an understanding of the general behavior of contours with constant r in the $m_A - \tan\beta$ plane, which is shown in Fig. 1. To a large extent, the various examples we will consider later simply correspond to zooming in on Fig. 1 in different regions of parameters of interest

in representative scenarios like the MSSM and the NMSSM. However, as we shall explain in the next section, radiative corrections induce a departure of the MSSM Higgs sector from the type II 2HDM behavior. This does not change the qualitative behavior shown in Fig. 1, but leads to a modification of the contours of constant r at large values of t_β.

V. ALIGNMENT IN SUPERSYMMETRY

In this section we first give a detailed overview of the Higgs mass dependance on the general 2HDM quartics and the constraints this implies for the MSSM parameters, given $m_h \approx 125$ GeV. We then present detailed analyses of alignment without decoupling in the MSSM and in the NMSSM.

A. MSSM Higgs Mass and Quartic Couplings

The tree-level Higgs sector in the MSSM belongs to the so-called type II 2HDM, where one doublet couples to the up-type fermions, denoted by H_u, and the other doublet couples to the down-type fermions, denoted by H_d. Both the tree-level and higher-order contributions to the CP-even mass matrix are well-known. At tree-level we have

$$\mathcal{M}^2_{\rm MSSM,tree} = \begin{pmatrix} m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 & -(m_A^2 + m_Z^2) s_\beta c_\beta \\ -(m_A^2 + m_Z^2) s_\beta c_\beta & m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 \end{pmatrix} . \tag{83}$$

Typically one is interested in the region where $m_A \gtrsim m_Z$ and $t_\beta \gtrsim 1$.[5] Then $\mathcal{M}^2_{11} > \mathcal{M}^2_{22}$ in $\mathcal{M}^2_{\rm MSSM,tree}$ and it is conventional to use the sign choice (I) in Eq. (16), $-\pi/2 \leq \alpha \leq \pi/2$. In addition, $\mathcal{M}^2_{12} < 0$ at tree-level and one is further restricted to $-\pi/2 \leq \alpha \leq 0$. However, beyond tree-level one could have $\alpha > 0$ in the MSSM.

[5] Values of $t_\beta \lesssim 1$ lead to such large values of the top-quark Yukawa coupling that the perturbative consistency of the theory is lost well below the grand unification scale, $M_G \simeq 2 \times 10^{16}$ GeV. In this work we shall assume that t_β is moderate or large so that the perturbativity of the top Yukawa is not a concern.

Only four of the quartic couplings are non-zero at tree-level in the MSSM,

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) = \frac{m_Z^2}{v^2} \ , \tag{84}$$

$$\lambda_3 = \frac{1}{4}(g_1^2 - g_2^2) = -\frac{m_Z^2}{v^2} + \frac{1}{2}g_2^2 \ , \tag{85}$$

$$\lambda_4 = -\frac{1}{2}g_2^2 \ , \tag{86}$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0 \ . \tag{87}$$

Therefore we see that the Higgs sector of the MSSM at tree-level is an example of 2HDMs in which the condition $\lambda_6 = \lambda_7 = 0$ and $\lambda_1 = \lambda_2$ is fulfilled. Moreover, $\tilde{\lambda}_3 < 0$ and $\lambda_{1,2} = -\tilde{\lambda}_3 < \lambda_{\rm SM}$, so the alignment conditions, Eqs. (59) and (60) cannot be fulfilled. As a result, alignment without decoupling never happens at the tree-level in the MSSM.

At the loop level, however, λ_{1-4} are modified and, furthermore, the remaining three couplings λ_{5-7} acquire non-zero values. These radiative corrections to the quartic couplings depend relevantly on the values of t_β.

At moderate or large values of $t_\beta \equiv v_u/v_d$, H_u acquires a VEV, $v_u \lesssim v$, while $v_d \ll v$. Therefore, the SM-like Higgs is approximately identified with the real component of H_u^0, and its squared mass is approximately given by the (2,2) component of the CP-even Higgs mass matrix. More precisely, multiplying both sides of Eq. (18) from the left by the row vector $(-s_\alpha, c_\alpha)$ and using the alignment relation, $(\beta - \alpha) = \pi/2$, we obtain

$$\begin{aligned} m_h^2 &= \mathcal{M}_{22}^2 s_\beta^2 + 2\mathcal{M}_{12}^2 s_\beta c_\beta + \mathcal{M}_{11} c_\beta^2 \\ &= v^2 \left(\lambda_2 \, s_\beta^4 + 4\lambda_7 s_\beta^3 c_\beta + 2\tilde{\lambda}_3 \ s_\beta^2 c_\beta^2 + 4\lambda_6 s_\beta c_\beta^3 + \lambda_1 \, c_\beta^4 \right) \ , \\ &= m_Z^2 c_{2\beta}^2 + v^2 \left(\Delta\lambda_2 \, s_\beta^4 + 4\lambda_7 s_\beta^3 c_\beta + 2\Delta\tilde{\lambda}_3 \ s_\beta^2 c_\beta^2 + 4\lambda_6 s_\beta c_\beta^3 + \Delta\lambda_1 \, c_\beta^4 \right) \ , \end{aligned} \tag{88}$$

where the $\Delta\lambda_i$'s denote a change of the corresponding quartic coupling, λ_i, due to radiative corrections. Hence we see that the tree-level value of the SM-like Higgs mass in the MSSM is bounded above by m_Z,

$$\left(m_h^2\right)^{\rm tree} \leq m_Z^2 c_{2\beta}^2 \ . \tag{89}$$

Since we are focusing on $t_\beta \gtrsim 1$, this upper bound is maximized for large values of t_β. It is well known that in the MSSM loop corrections to the quartic couplings are necessary to raise the SM-like Higgs mass from values below m_Z to values consistent with the LHC measured value, $m_h \simeq 125$ GeV. Note that since the upper bound on the tree-level m_h is minimized

for $t_\beta = 1$, the radiative corrections required to raise the Higgs mass from its small tree-level value must be very large for $t_\beta \sim 1$. Such radiative corrections may only be obtained for very heavy scalar top-quarks, with masses far above the TeV scale.

At moderate or large values of t_β, $s_\beta \simeq 1$ and the Higgs mass is mostly governed by λ_2, as can be seen from Eq. (88), although other terms may become relevant for smaller values of t_β. The most important contributions to the quartic couplings come from the stop sector. When the two stop masses are close to each other, $(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}) < 0.5(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})$, one can approximate the Higgs mass including the most relevant two loop corrections, namely [16, 17]

$$m_h^2 \simeq m_Z^2 c_{2\beta}^2 + \frac{3}{8\pi^2} h_t^4 \, v^2 \left[\frac{1}{2}\tilde{X}_t + t + \frac{1}{16\pi^2}\left(\frac{3h_t^2}{2} - 32\pi\alpha_s\right)\left(t\,\tilde{X}_t + t^2\right)\right] , \tag{90}$$

where h_t is the top Yukawa coupling, $M^2_{\rm SUSY} = (m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})/2$ and $t = \log M^2_{\rm SUSY}/m_t^2$. The parameter $\tilde{X}_t$ is defined as

$$\begin{aligned} \tilde{X}_t &= \frac{2\tilde{A}_t^2}{M^2_{\rm SUSY}}\left(1 - \frac{1}{12}\frac{\tilde{A}_t^2}{M^2_{\rm SUSY}}\right) , \\ \tilde{A}_t &= A_t - \mu \cot\beta , \end{aligned} \tag{91}$$

where A_t is the trilinear Higgs-stop coupling, μ is the Higgsino mass parameter and the running couplings in the $\overline{\rm MS}$ scheme must be evaluated at the top quark mass scale.

In addition to the stop sector, m_h^2 also receives negative radiative corrections proportional to the fourth power of the bottom and/or tau Yukawa couplings. However, these corrections become relevant only at very large values of t_β, where the bottom and/or tau Yukawa couplings become comparable to the top Yukawa (see, for example, Ref. [18])

$$\Delta m_h^2 \simeq -\frac{v^2 h_b^4 \mu^4}{32\pi^2 M^4_{\rm SUSY}} - \frac{v^2 h_\tau^4 \mu^4}{96\pi^2 M^4_{\tilde{\tau}}} . \tag{92}$$

where μ is the higgsino mass parameter. In the above $M^2_{\tilde{\tau}}$ is the average stau mass-squared and $h_b(h_\tau)$ is the Yukawa coupling for the bottom quark (τ lepton). We have also assumed the sbottom masses to be of the same order as the stop masses and, for simplicity, neglected higher loop corrections.

B. Couplings of the Down -Type Fermions to the Higgs in the MSSM

It is also important to recall that the MSSM Higgs sector is a type II 2HDM only at the tree-level. Beyond tree-level, however, supersymmetry breaking effects induce H_u couplings

to *down*-type fermions, denoted by Δh_d. These loop-induced couplings modify the relation between the down-type fermion Yukawa couplings and their running masses, namely

$$h_{b/\tau} \simeq \frac{\sqrt{2}\ m_{b/\tau}}{v c_\beta (1+\epsilon_{b/\tau} t_\beta)}\ , \tag{93}$$

where $\epsilon_{b/\tau} = (\Delta h_{b/\tau}/h_{b/\tau})$ are the one-loop corrections whose dominant contribution depends on the sign of (μM_3) and (μM_2), respectively[19, 20]. Positive values of (μM_3) induce positive contributions to ϵ_b which reduces the bottom Yukawa coupling, h_b. Smaller values of h_b, in turn, reduce the negative sbottom effect on the Higgs mass and increase the values of t_β for which the theory remains perturbative up to the GUT scale. Negative values of (μM_3), instead, give the opposite trend. On the contrary, positive values of (μM_2) tend to induce negative values for ϵ_τ, increasing the τ-Yukawa coupling and hence the impact of the stau sector.

These effects give small negative corrections to the Higgs mass, $\mathcal{O}(\sim$ few GeV), however, they could be quite relevant to the couplings at hand. Including these loop effects, the couplings of the lightest Higgs to the bottom quarks and the tau leptons are [21],

$$g_{hdd} = -\frac{m_d\ s_\alpha}{v\ c_\beta (1+\Delta_d)} \left(1 - \frac{\Delta_d}{t_\beta\ t_\alpha}\right), \tag{94}$$

where $\Delta_d \equiv \epsilon_d t_\beta$ and $d = b$ or τ respectively. Although the down-type couplings, g_{hdd}, depend in a relevant way on ϵ_d, in the alignment limit we have $t_\alpha t_\beta \to -1$. Thus the down couplings approach the SM values independently of ϵ_d.

C. Alignment: $\mu \ll M_{\rm SUSY}$ and Moderate $\tan\beta$

When $|\mu| \ll M_{\rm SUSY}$ and $1 \ll t_\beta \ll m_t/m_b$, then both the μ induced and t_β enhanced corrections associated with the bottom and tau-Yukawa couplings are negligible [17]. Therefore, the only relevant radiative corrections affecting the Higgs sector are those coming from the top-stop sector, affecting $\Delta\lambda_2$ and leading to Eq. (90). In this case λ_6 and λ_7 remain very small and the conditions for alignment to occur are still determined to a good approximation by Eqs. (59) and (60). However, neither of these two alignment conditions are fulfilled in this corner of the MSSM, which has the following relation

$$\lambda_{\rm SM} > \lambda_1 > \tilde{\lambda}_3\ . \tag{95}$$

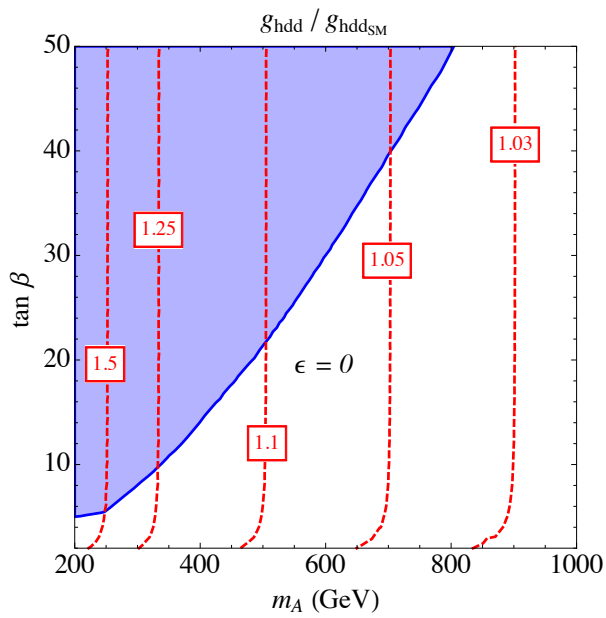

FIG. 2: Ratio of the value of the down-type fermion couplings to Higgs bosons to their SM values in the case of low μ ($L_{1j} \sim 0$), as obtained from Eq. (96), and $\epsilon_d \simeq 0$.

We can reach the same conclusion by using Eq. (21) for s_α in this regime,

$$s_\alpha = \frac{-(m_A^2 + m_Z^2) s_\beta c_\beta}{\sqrt{(m_A^2 + m_Z^2)^2 s_\beta^2 c_\beta^2 + \left(m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 - m_h^2\right)^2}} \ , \tag{96}$$

which, for $m_A \gtrsim 2m_h$ and moderate t_β implies

$$-\frac{s_\alpha}{c_\beta} \simeq \frac{m_A^2 + m_Z^2}{m_A^2 - m_h^2} \ . \tag{97}$$

This clearly demonstrates that in this case the deviation of $(-s_\alpha/c_\beta)$ from 1 depends only on m_A and is independent of t_β. In other words, alignment is only achieved in the decoupling limit, $m_A^2 \gg m_Z^2, m_h^2$.

This also agrees with our expressions regarding the approach to the alignment limit via decoupling, Eq. (77). In this regime $\lambda_{5,6,7}$ are very small implying

$$\mathcal{B} \simeq m_A^2 - m_h^2, \qquad \text{and} \qquad \mathcal{B} - \mathcal{A} \simeq -(m_Z^2 + m_h^2) \ . \tag{98}$$

In Fig. 2 we display the value of $-s_\alpha/c_\beta$ in the $m_A - \tan_\beta$ plane, for low values of μ, for which the radiative corrections to the matrix element L_{11} and L_{12} are small, Eq. (96). As expected from our discussion above, the down-type fermion couplings to the Higgs become

independent of t_β in large regions of parameter space, and the ratio to their SM values is well described by Eq. (97). We see that under these conditions, a measurement of the down-type fermion coupling to the Higgs that deviates from the SM value by 3% or less will allow us to infer that the non-standard Higgs boson masses are at or above the TeV range. The strong enhancements of the down-type couplings at low values of m_A would also imply a correlated enhancement of the Higgs width, leading to a suppression of the decay branching ratio of the Higgs into photons and weak gauge bosons. The above results imply that such suppression is only weakly dependent on t_β, which explains the numerical results of Ref. [22] where the relevant Higgs decays are studied.

D. Alignment: $\mu \sim \mathcal{O}(M_{\rm SUSY})$ and Large $\tan\beta$

The situation is quite different for $\mu \sim \mathcal{O}(M_{\rm SUSY})$, since in this case there may be important radiative corrections to quartic couplings, leading to non-vanishing values for $\lambda_{5,6,7}$. Instead of just taking the alignment conditions in terms of the quartic couplings, we shall rewrite them in terms of the radiative corrections to the matrix elements $\mathcal{M}_{11}$ and $\mathcal{M}_{12}$, since it allows us to get a clear idea of where the important contributions are coming from. We shall then make contact with previous expressions. Quite generally, from Eq. (21),

$$s_\alpha = \frac{-(m_A^2+m_Z^2)s_\beta c_\beta + v^2(s_\beta^2\,\Delta L_{12} + s_\beta c_\beta\,\Delta\tilde{L}_{12})}{\sqrt{\left[(m_A^2+m_Z^2)s_\beta c_\beta - v^2(s_\beta^2\,\Delta L_{12} + s_\beta c_\beta\,\Delta\tilde{L}_{12})\right]^2 + \left(m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 - m_h^2 + v^2\,\Delta L_{11} s_\beta^2\right)^2}} \tag{99}$$

where, as before, the ΔL_{ij} denote variation due to radiative corrections. We have further separated out the corrections to the L_{12} component into ΔL_{12} and $\Delta\tilde{L}_{12}$, which contribute with different t_β factors. In terms of the quartics these loop corrections are

$$\Delta L_{12} \simeq \lambda_7, \qquad \Delta\tilde{L}_{12} \simeq \Delta\left(\lambda_3+\lambda_4\right), \qquad \Delta L_{11} \simeq \lambda_5, \qquad \Delta L_{22} \simeq \lambda_2. \tag{100}$$

In the above, we have only kept terms which are relevant for moderate or large values of t_β, and we have included ΔL_{22} for future use. In particular, since $c_\beta^2 \ll 1$, we have dropped the λ_1 term which is proportional to c_β^2 and the $\lambda_6 c_\beta$ term, since λ_6 is already a small quantity, generated by radiative corrections. We have kept the $\Delta\tilde{L}_{12}$ contribution to the matrix element L_{12} since it has the same t_β dependance as the tree-level contribution, and

for sizable μ but small μA_f, with $f = \{t, b, \tau\}$, may also be competitive to the radiatively generated λ_7 contribution.

Since $t_\beta \gg 1$, and hence $s_\beta \simeq 1$, the condition in Eqs. (41) and (42) now read

$$m_h^2 = -m_Z^2 + v^2 \left(\Delta L_{11} + \Delta \tilde{L}_{12} + t_\beta \Delta L_{12} \right) , \tag{101}$$

$$m_h^2 = m_Z^2 + v^2 \left(\Delta L_{22} + c_\beta^2 \Delta \tilde{L}_{12} + c_\beta \Delta L_{12} \right) . \tag{102}$$

Observe that for moderate or large values of t_β the second expression above just shows that the Higgs mass is strongly governed by λ_2, while the first expression shows that one reaches the alignment limit for values of t_β given by

$$t_\beta \simeq \frac{m_h^2 + m_Z^2 - v^2(\Delta L_{11} + \Delta \tilde{L}_{12})}{v^2 \Delta L_{12}} = \frac{m_h^2 - v^2 \tilde{\lambda}_3}{v^2 \lambda_7} \tag{103}$$

in agreement with Eq. (65) derived in the previous section.

1. *Radiative Loop Corrections*

The radiative corrections to the quartic couplings $\tilde{\lambda}_3$ and λ_7, for small differences between the values of the two stops, sbottoms and stau masses, have been computed previously in Refs. [10], [17], [18]. Using these expressions one obtains

$$\Delta L_{12} \simeq \frac{1}{32\pi^2} \left[h_t^4 \frac{\mu A_t}{M_{\rm SUSY}^2} \left(\frac{A_t^2}{M_{\rm SUSY}^2} - 6 \right) + h_b^4 \frac{\mu^3 A_b}{M_{\rm SUSY}^4} + \frac{h_\tau^4}{3} \frac{\mu^3 A_\tau}{M_{\tilde{\tau}}^4} \right] , \tag{104}$$

$$\begin{aligned} \left(\Delta \tilde{L}_{12} + \Delta L_{11} \right) \simeq & \frac{3\,\mu^2}{16\pi^2 M_{\rm SUSY}^2} \left[h_t^4 \left(1 - \frac{A_t^2}{2M_{\rm SUSY}^2} \right) + h_b^4 \left(1 - \frac{A_b^2}{2M_{\rm SUSY}^2} \right) \right. \\ & \left. + h_\tau^4 \frac{M_{\rm SUSY}^2}{3M_{\tilde{\tau}}^2} \left(1 - \frac{A_\tau^2}{2M_{\tilde{\tau}}^2} \right) \right] , \end{aligned} \tag{105}$$

where, for simplicity, we have ignored two-loop corrections.

Observe that for moderate values of $|A_t| < \sqrt{6} M_{\rm SUSY}$, the top contributions to ΔL_{12}, (λ_7), become positive for negative values of A_t and negative for positive ones. On the other hand, for $|A_t| > \sqrt{6} M_{\rm SUSY}$, the sign of ΔL_{12} is given by the sign of A_t. Interestingly enough, the radiative corrections to λ_2 (and therefore to m_h) are maximized at $|A_t| \simeq \sqrt{6} M_{\rm SUSY}$, leading to a Higgs mass of order 130 GeV for stop masses of the order of 1 TeV (see, for example, Refs. [16, 17]). Therefore, one can get consistency with the measured Higgs mass for values of $|A_t|$ larger or smaller than $\sqrt{6} M_{\rm SUSY}$. The sbottom and stau contributions to λ_7 become relevant only at large values of t_β and are positive for $(\mu A_{b,\tau}) > 0$. Regarding, $(\Delta \tilde{L}_{12} + \Delta L_{11})$,

$\Delta\tilde{\lambda}_3$, again we see that the sbottom and stau contributions will only become relevant for large values of t_β. However, the sign of all of the corrections, including the stops, does not depend on the sign of A_f, but rather would be positive (negative) if $|A_f| < (>)\sqrt{2}M_{\rm SUSY}$. Further, noting the different coefficients in Eqs. (104) and (105), observe that values of $|\Delta\tilde{L}_{12} + \Delta L_{11}|$ may be pushed to larger values compared to values of $|\Delta L_{12}|$.

Keeping these considerations in mind, in our numerical work we will take representative values of these loop corrections to be $32\pi^2\Delta L_{12} = \{-1, 5\}$ and $32\pi^2(\Delta\tilde{L}_{12} + \Delta L_{11}) = \pm 25$. Such values can be naturally obtained for non-extreme values of $(\mu/M_{\rm SUSY})$ and $(A_f/M_{\rm SUSY})$ and lead to either no alignment or alignment at $t_\beta \sim 20$, respectively.

2. *Values of* $\tan\beta$ *at Alignment*

One can write the large t_β alignment condition in the MSSM as

$$t_\beta \simeq \frac{120 - 32\pi^2\left(\Delta L_{11} + \Delta\tilde{L}_{12}\right)}{32\pi^2\Delta L_{12}} \tag{106}$$

where we have made use of the fact that all contributions to λ_7 in Eq. (104) are proportional to $1/(32\pi^2) \sim \mathcal{O}(1/300)$ and rescaled both the denominator and numerator by a factor of $32\pi^2$. The numerator of Eq. (106) tends to remain positive and large after the inclusion of the radiative corrections. Therefore, in order to obtain sensible values of t_β consistent with a perturbative description of the theory, $0 < t_\beta \lesssim 100$, it is necessary that $32\pi^2\Delta L_{12}$ be positive and larger than one. This can only be achieved for large values of $|\mu|$ and of some of the trilinear couplings, A_f.

One important implication that can be inferred from Eqs. (104) and (105) is that the values of ΔL_{12} depend in a relevant way on the values of the bottom and tau Yukawa couplings. Since these couplings grow with t_β, it is clear that ΔL_{12} (λ_7) is not independent of t_β. This leads to new solutions for the alignment condition at very large values of t_β, that would not exist if λ_7 were independent of t_β.

Fig. 3 shows contour plots of the values of t_β where alignment is achieved for different values for the ratio $(\mu/M_{\rm SUSY})$ and for positive/negative values of $(A_t/M_{\rm SUSY})$, keeping the t_β dependance in the Yukawas explicit. The value of t_β is obtained by solving the corresponding algebraic equation exactly, without the approximations done in Eq. (106). In Fig. 3 we chose equal values of the $(A_f/M_{\rm SUSY})$ parameters. We find two different roots for the algebraic

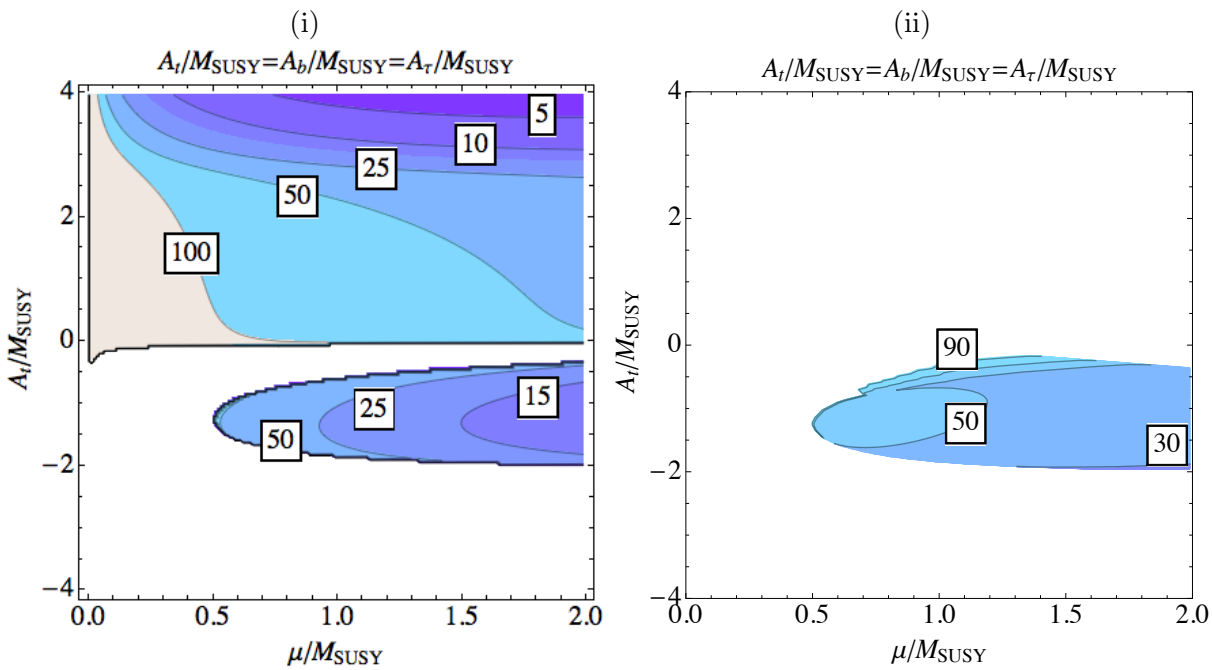

FIG. 3: Values of t_β, at which alignment without decoupling occurs, using Eqs. (104)-(105) . We assume common masses, $M_{\rm SUSY}$ and include the contribution from the bottom and tau Yukawas, fixing $A_b = A_\tau = A_t$. The branch of solutions displayed in (i) (left panel) would exist even if one neglected the bottom and tau Yukawa couplings. Those in (ii) (right panel) appear due to the extra t_β dependence associated with the down-type fermion Yukawa couplings.

equation and display them in Figs. 3(i) and 3(ii). The second root, displayed in Fig. 3(ii) appears due to the t_β dependence of the bottom and tau Yukawa couplings.

In Fig. 4 we show similar results, but for $A_t = -A_b = -A_\tau$. Moderate values of t_β may be obtained for either large values of $(A_t/M_{\rm SUSY})$ or for negative values of this parameter. The second root, displayed in Fig. 4(ii), moves now to positive values of A_t.

Finally, in Fig. 5 we show the effect of setting the A_b and A_τ parameters to zero or to large values. Only one root appears in both cases, and the difference between the results in the left and right panel of Fig. 5 is only visible for very large values of t_β, which is when the bottom and tau Yukawa couplings become relevant.

Looking at the results presented in Figs. 3-5, we see that in general, one obtains a wide range in both the values of t_β at alignment and the associated parameter space where this would occur. However note that values of $t_\beta \lesssim 10$ at alignment are only obtained for

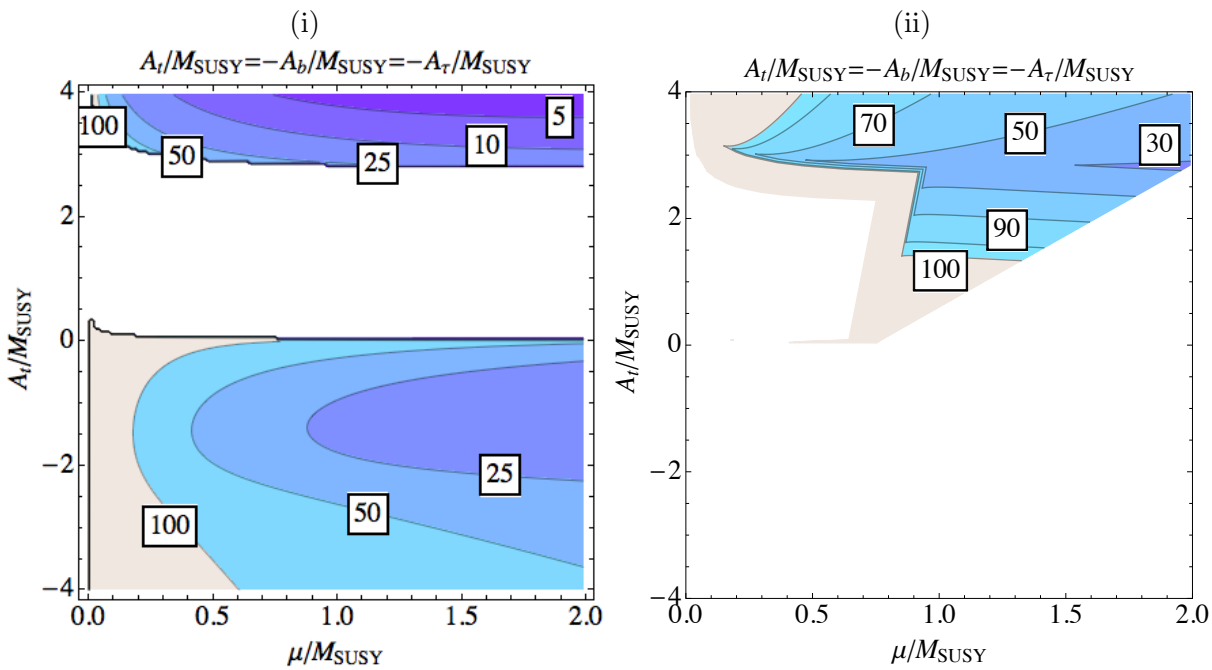

FIG. 4: Same as Fig. 3 but now showing alignment values of t_β under the assumptions $A_b = A_\tau = -A_t$.

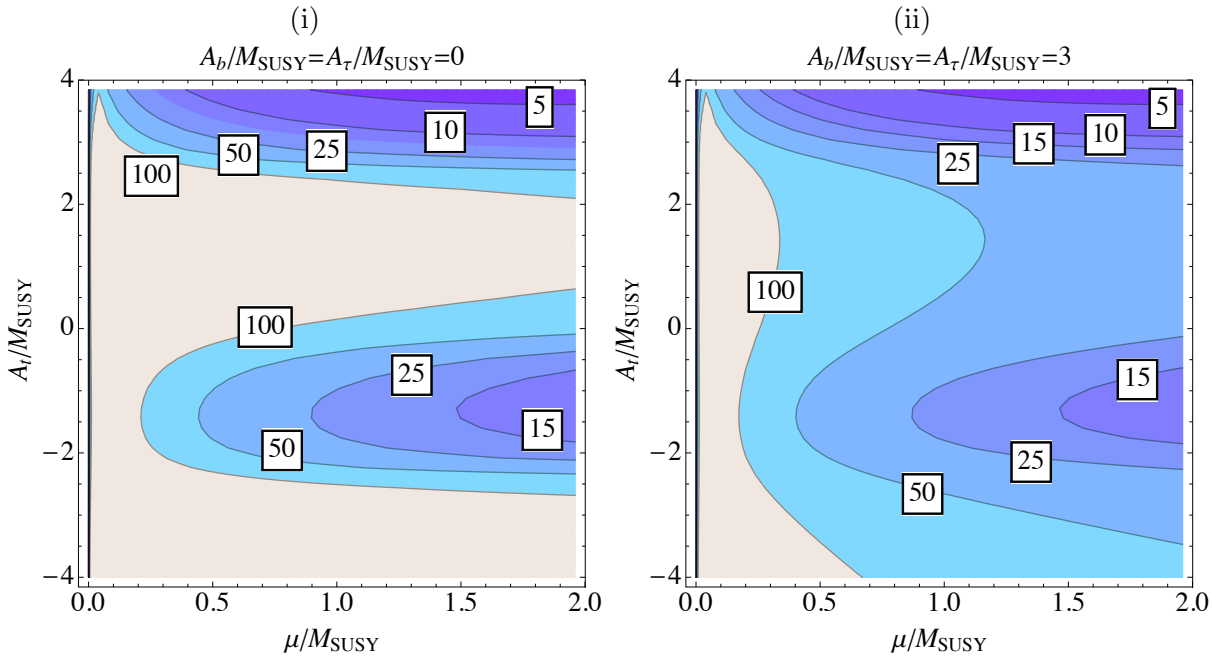

FIG. 5: Same as Fig. 3 but now showing alignment values of t_β under the assumptions: (i) $A_b = A_\tau = 0$ and (ii) $A_b/M_{\rm SUSY} = A_\tau/M_{\rm SUSY} = 3$. Only one set of solutions appear in these cases.

very large values of $(A_f/M_{\rm SUSY})$, which could be significantly constrained from both the Higgs mass and the stability of the vacuum. A detailed analysis of the phenomenological implications for such a scenario is beyond the scope of this work.

3. Departure from Alignment

As explained previously section, it is important to study the departure from the alignment condition. The effect of this will be most readily visible in the couplings of the lightest SM-like Higgs boson to down-type fermions. Re-writing Eq. (94), close to alignment for small values of ϵ_d, this coupling reads, approximately

$$\frac{g_{hdd}}{g_f} \simeq -\frac{s_\alpha}{c_\beta} + \epsilon_d t_\beta \left(1 + \frac{s_\alpha}{c_\beta}\right). \tag{107}$$

This means that positive (negative) values of ϵ_d tend to suppress (enhance) the coupling departures from the SM values, a tendency that is enforced for larger values of t_β. On the other hand, at values of t_β larger than the ones leading to alignment, positive values of ΔL_{12} tend to suppress the departure of g_{hdd} from SM values. Hence, the effect of negative values of ϵ_d may be partially compensated (enhanced) for positive (negative) values of ΔL_{12}, and vice versa.

In Figs. 6 and 7 we display deviations with respect to the SM of the Higgs-to-down-type-fermion couplings in the $m_A - \tan\beta$ plane for fixed representative values of $32\pi^2\Delta L_{ij}$ and for two different signs of the one-loop contribution to the Yukawa couplings, $\epsilon_d = -0.01$ and $\epsilon_d = 0.01$, which could be considered to be associated with the τ and bottom couplings, respectively (see Eq. (93)). We are neglecting the t_β dependence of the quartic couplings which eliminates additional alignment solutions that may appear at very large values of t_β, but makes the interpretation of the results more transparent. Moreover, we show regions that are excluded by current direct searches at the LHC [23] [6].

For negative values of $32\pi^2\Delta L_{12}$, as shown in the lower panels, there is no alignment solution and the deviation of the couplings from the SM values depend mostly on m_A and not on t_β. The value of the down-type fermion couplings to the Higgs is always enhanced with respect to the SM value, as happens whenever t_β is below the one associated with the

[6] Larger values of μ than those assumed in the m_h^{max} scenario would lead to slighter stronger bounds [12].

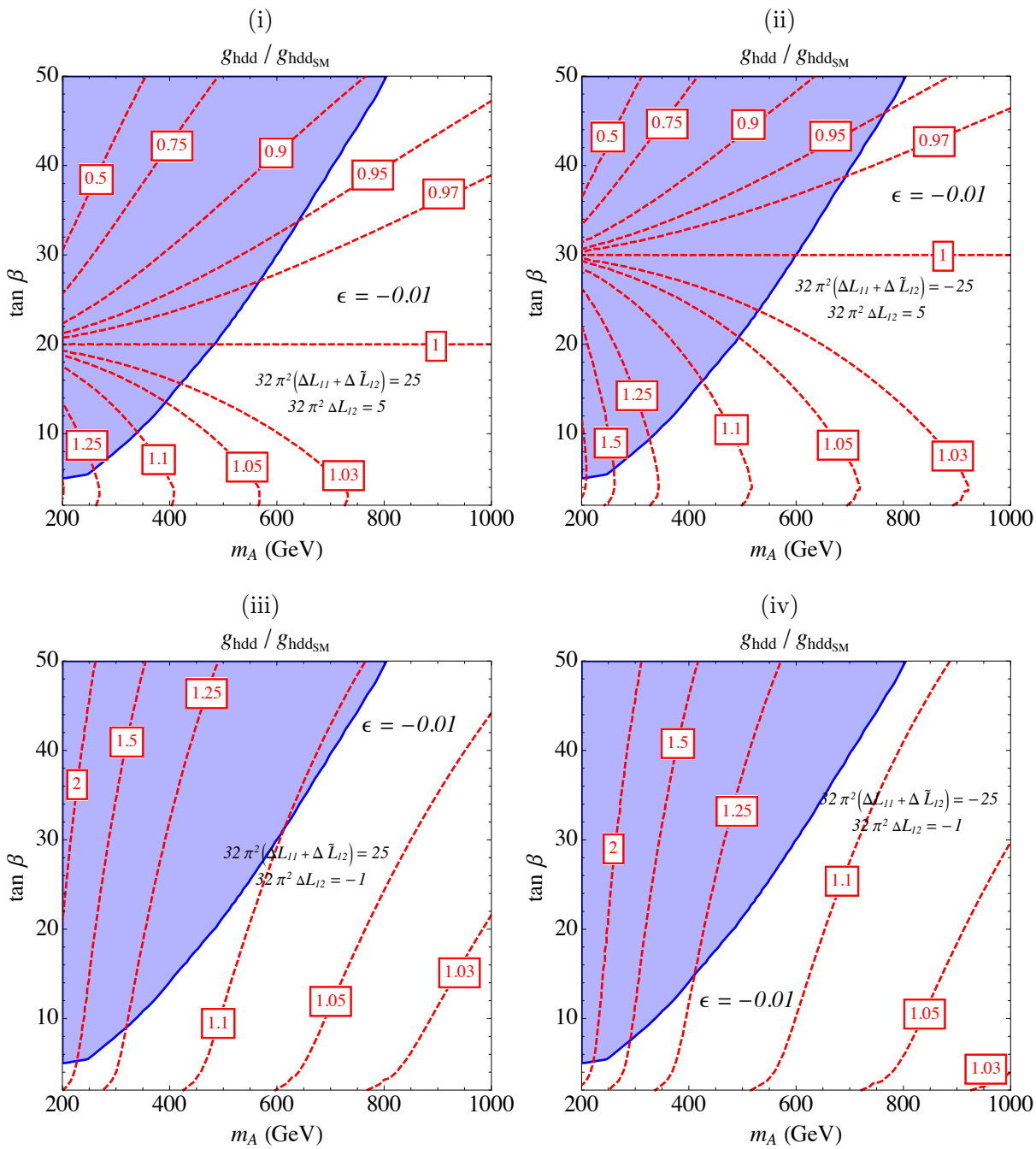

FIG. 6: Blue shaded region denotes current LHC limits. The ratio of the Higgs coupling to down-type quarks to the SM limit is shown by the red dashed contours for $\epsilon_d = -0.01$. The top two panels have $(32/\pi^2)\Delta L_{12} = 5$ and the lower ones $(32/\pi^2)\Delta L_{12} = -1$. The left panels are for $(32/\pi^2)\Delta L_{11} = 25$ and right for $(32/\pi^2)\Delta L_{11} = -25$. $\Delta\tilde{L}_{12} = 0$ in these figures since its contribution in the mixing angle is suppressed by c_β and is effectively negligible.

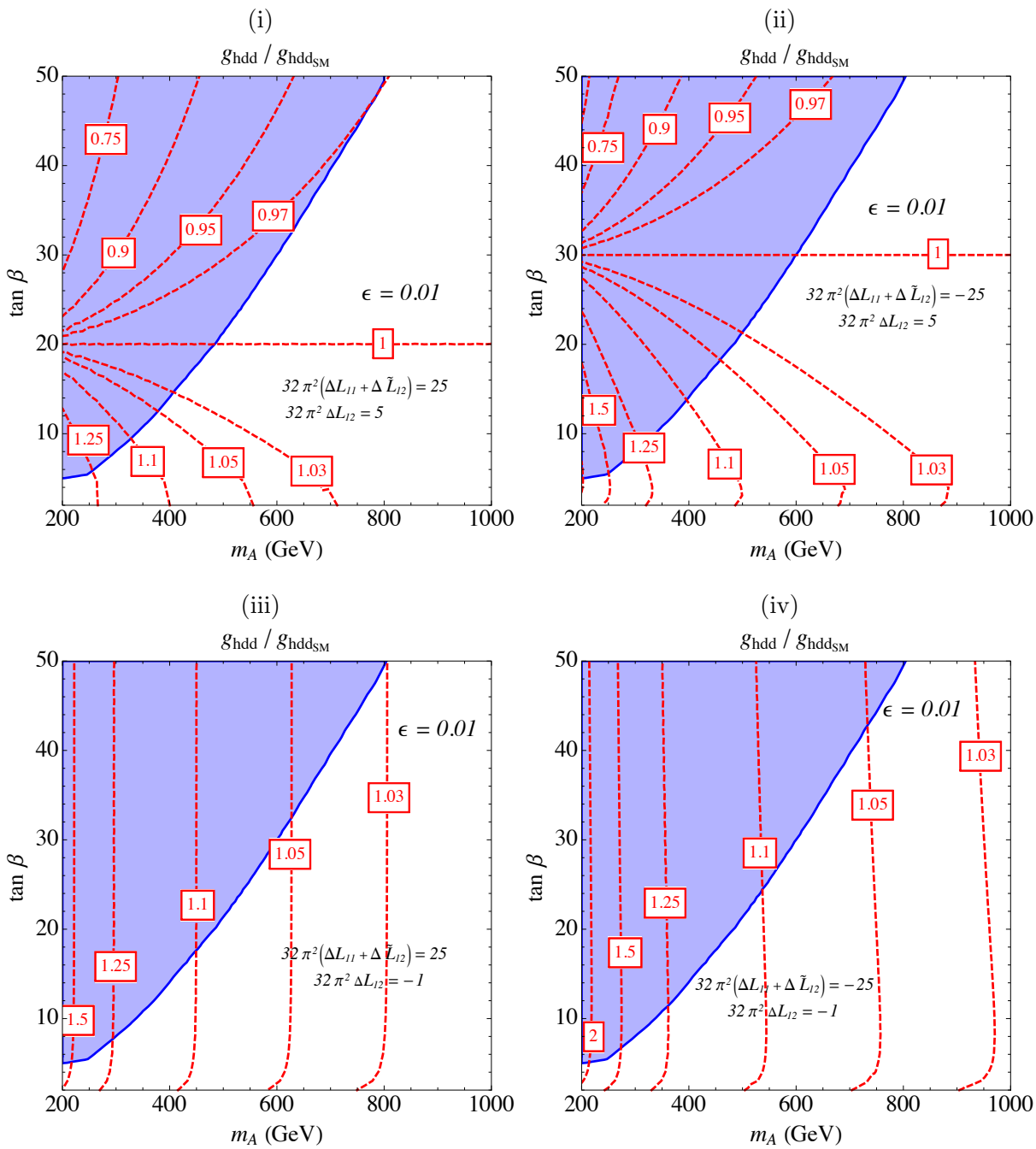

FIG. 7: Blue shaded region denotes current LHC limits. The ratio of the Higgs coupling to down-type quarks to the SM limit is shown by the red dashed contours for $\epsilon_d = 0.01$. The top two panels have $(32/\pi^2)\Delta L_{12} = 5$ and the lower ones $(32/\pi^2)\Delta L_{12} = -1$. The left panels are for $(32/\pi^2)\Delta L_{11} = 25$ and right for $(32/\pi^2)\Delta L_{11} = -25$. $\Delta\tilde{L}_{12}$ is again chosen to be 0.

alignment solution. On the other hand, in the presence of alignment, as is the case in the upper panels of Figs. 6 and 7, suppression of the Higgs couplings to down fermions may be obtained for t_β larger than the ones leading to alignment (see Eq. (79)). Additionally, even when the t_β at alignment is large, there can be significant variations in the g_{hdd} couplings at much smaller values of t_β, as can be seen from the top panels in both Figs. 6 and 7.

In the absence of any ϵ_d corrections, the deviations in g_{hdd} couplings are flavor universal. The impact of non-zero ϵ_d can be seen by comparing Figs. 6 and 7. These corrections are enhanced by t_β in g_{hdd}. Any deviation in the ratio of $g_{hbb}/g_{h\tau\tau}$ from its SM value, m_b/m_τ, should predominantly come from the ϵ_d dependance of these couplings. Further, independently of the value of ΔL_{12}, the largest deviation in $g_{hbb}/g_{h\tau\tau}$ occurs at low values of m_A and larger t_β, which are constrained by direct searches for non-standard Higgs bosons.

Figs. 6 and 7 also illustrate the so-called wedge region which is difficult to access using direct searches. It is commonly assumed that measurements of the Higgs couplings to down-type fermions could effectively constrain this wedge region. However, as can be seen from these figures and from Eq. (79), these constraints depend strongly on the precise value of t_β leading to alignment, and become weaker when this value becomes smaller.

E. Beyond the MSSM

In the previous section it was shown that in the region of MSSM parameter space where $\mu \ll M_{SUSY}$ and t_β is moderate, alignment without decoupling never occurs because the quartic coupling $\tilde{\lambda}_3$ is too small and the alignment conditions in Eqs. (59) and (60) are never fulfilled. However, one could increase the value of $\tilde{\lambda}_3$ by augmenting the MSSM with either a singlet scalar, as in the case of NMSSM [24, 25], or a triplet scalar [9]. In these models, a gauge singlet, S, in the case of NMSSM, and a triplet scalar, Σ, in the case of triplet augmented MSSM, are added to the superpotential with the following cubic couplings, among others,

$$\Delta_S \mathcal{W} = \lambda S H_u H_d \ , \qquad \Delta_T \mathcal{W} = \lambda H_u \Sigma H_d \ . \tag{108}$$

Since the Higgs with a mass of 125 GeV appears to be mostly a doublet scalar by all accounts [3], it is reasonable to consider a limit where the singlet and triplet scalars are much heavier than the doublet scalars so that the singlet/doublet or triplet/doublet mixing is small. As such, the singlet and triplet scalars can be integrated out of the low energy

spectrum. However, if the superpartners of singlet and triplet scalars are also integrated out at the same time, one would just regain the MSSM at low energies and no new insights could be obtained relative to what was already discussed previously. Therefore we shall consider a limit where only the scalar components of the singlet and triplet superfield are integrated out [26]. In this scenario new contributions to $\tilde{\lambda}_3$ in the scalar potential are generated, which are not present in the MSSM. Then a solution for alignment without decoupling could be found in the branch of $t_\beta \sim \mathcal{O}(1)$.

More explicitly, after integrating out the singlet or triplet fields, one finds a correction to the quartic coupling $\tilde{\lambda}_3$ given by,

$$\delta\tilde{\lambda}_3 = c\ \lambda^2\ , \tag{109}$$

where $c = 1$ or $c = 1/2$ for the NMSSM [27] or the triplet-augmented MSSM [9], respectively.

Alignment in this case would occur if the values of λ are such that the relations in Eqs. (57) and (60) are satisfied. Since $\lambda_1 \simeq m_Z^2/v^2 \simeq 0.137$ and thus $(\lambda_{\rm SM} - \lambda_1) \simeq 0.123$, using Eq. (61) we find

$$\tilde{\lambda}_3 - \lambda_{\rm SM} \simeq \frac{0.123}{t_\beta^2}\ , \qquad \lambda_{\rm SM} - \lambda_2 \simeq \frac{0.123}{t_\beta^4}\ . \tag{110}$$

Then, in order to obtain a solution for $t_\beta \gtrsim 2$, not only does one have to obtain large radiative corrections to λ_2 to raise its value to be very close to $\lambda_{\rm SM}$, but also $\tilde{\lambda}_3$ has to be adjusted so that Eq. (110) is satisfied simultaneously. Using the tree level value of $\tilde{\lambda}_3^{\rm MSSM} \simeq -0.137$, we see the new coupling in the superpotential must be

$$\lambda^2 = \frac{1}{c}\left(0.397 + \frac{0.123}{t_\beta^2}\right). \tag{111}$$

In the triplet-augmented MSSM ($c = 1/2$), for $t_\beta \simeq 2.7$, one gets $\lambda \simeq 0.9$, which is in excellent agreement with Eq. (2.21) in Ref. [9]. Larger values of t_β can be obtained by simultaneously adjusting λ and λ_2 so that both conditions in Eq. (110) are satisfied.

The NMSSM has been widely studied in the literature, since for small values of t_β large corrections to the SM-like Higgs mass may be obtained due to the non-decoupling F-terms described above. This tree-level correction reduces the need for large radiative corrections from the stop sector, ameliorating the fine-tuning problem. For this to happen without spoiling the perturbative consistency of the theory up to the GUT scale, one must have $\lambda \lesssim 0.7$ [24]. On the other hand, since the tree-level Higgs mass is given by

$$\left(m_h^2\right)^{\rm tree} = M_Z^2 c_{2\beta}^2 + 2\lambda^2 v^2 s_\beta^2 c_\beta^2\ , \tag{112}$$

a large tree-level contribution to the Higgs mass, beyond the one obtained in the MSSM, may only be obtained if $\lambda > \sqrt{2}M_Z/v \simeq 0.5$, and a sizable reduction of fine tuning demands even larger values of λ and $t_\beta = \mathcal{O}(1)$. Therefore, there is a small range of values of λ and t_β for which the fine-tuning is reduced without spoiling the perturbative consistency of the theory.

Interestingly enough, for $c = 1$ and $t_\beta > 1$, Eq. (111) shows that alignment can also be obtained for $0.65 \lesssim \lambda \lesssim 0.75$, with smaller values of λ corresponding to the larger values of t_β necessary to satisfy the alignment conditions. Therefore, there is an overlap between the widely studied NMSSM parameter space, $t_\beta \sim \mathcal{O}(1)$ and $\lambda \simeq 0.7$, and the ones necessary to satisfy the alignment conditions. Small departures of t_β from the alignment values lead to a variation of the bottom and tau couplings with respect to the SM values. Smaller values of t_β lead to an enhancement of these couplings and larger values to a decrease in them. This is shown in Fig. 8 where the ratio of the down fermion coupling to the Higgs with respect to the SM values is presented for different values of λ as a function of the heavy CP-odd Higgs mass, m_A. We have assumed that the singlet states are decoupled and do not affect the heavy Higgs bounds.

In general, due to the presence of alignment, values of $0.75 \geq \lambda \geq 0.6$ lead to a drastic variation of g_{hdd} with respect to the SM expectations as compared to the small μ case in the MSSM ($\lambda = 0$) represented in Fig. 2 and Eq. (97). The variations of these couplings depend very strongly on the precise value of λ. [7] As happened in the MSSM case, suppression is obtained for values of t_β larger than the one leading to alignment, while enhancement is obtained otherwise. However, note that unlike in the MSSM, alignment can occur for small values of $t_\beta \lesssim 5$. This implies that one can go to very low values of m_A without significant variations of the Higgs couplings to fermions and gauge bosons (Fig. 8 (iii)). In addition, as can be seen from Figs. 8 (i) and (ii), it is possible to have significant suppression of the down-type couplings in the wedge, unlike the MSSM case.

[7] Because there is such strong dependance on the precise value of λ, the t_β at alignment could be very sensitive to small loop corrections to $\tilde{\lambda}_3$, which enter into $\delta\tilde{\lambda}_3$ as in Eq. (108), which have been neglected in this analysis.

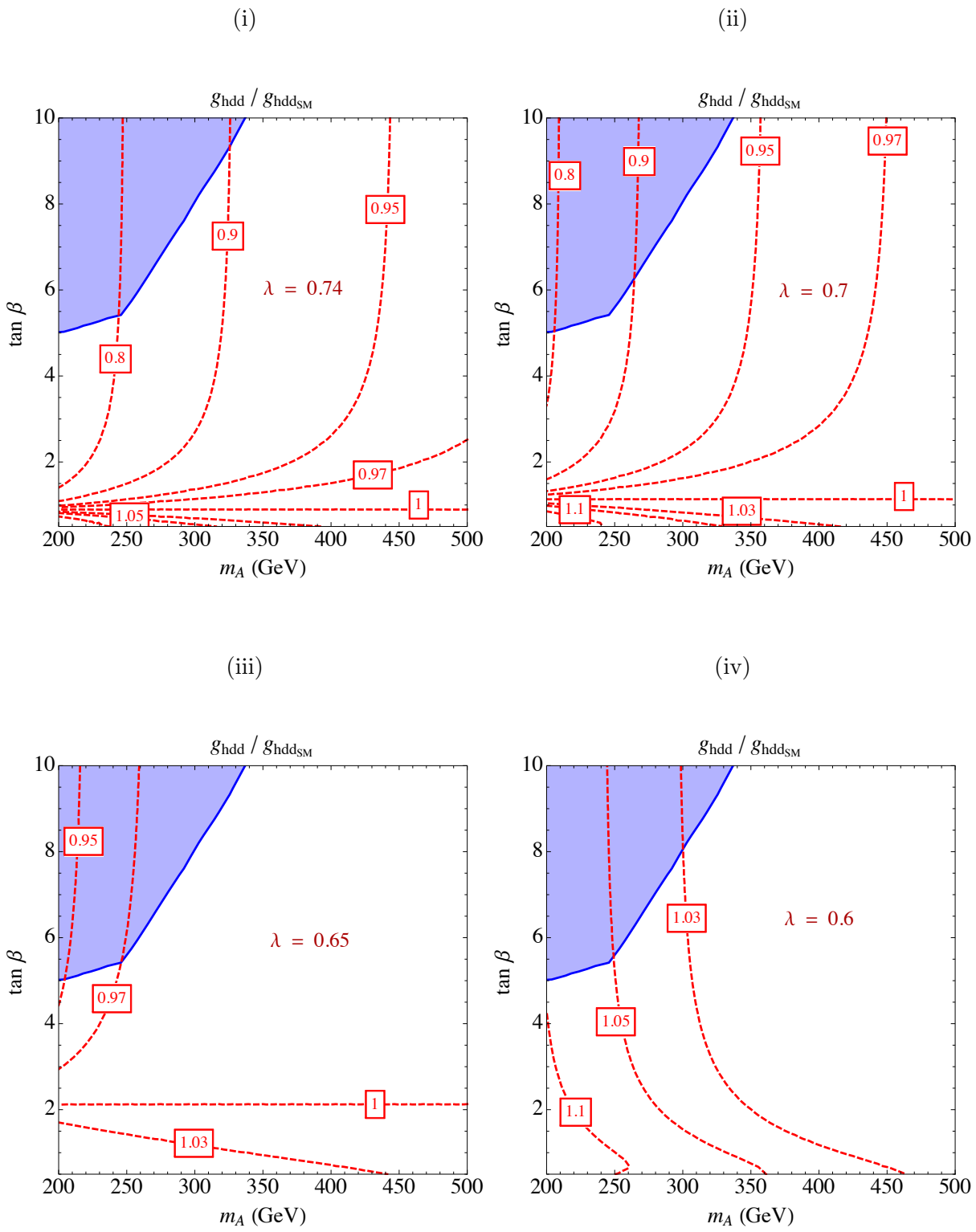

FIG. 8: Blue shaded region denotes current LHC limits. The ratio of the Higgs coupling to down-type quarks to the SM limit is shown by the red dashed contours for various values of λ.

VI. CONCLUSIONS

Models with an extended Higgs sector appear in many theories beyond the SM. Among them, 2HDMs represent some of the simplest extensions of the SM Higgs sector and are realized in both supersymmetric and non-supersymmetric theories. When masses of the non-standard scalar are much larger than the Z mass, it is well-known that the mass eigenbasis, described by the mixing angle, α, in the CP-even sector, aligns with the basis characterized by the angle β, in which the Higgs VEV resides in only one of the doublets. This leads to the alignment limit where $\cos(\alpha-\beta)=0$ and the lightest CP-even Higgs behaves as the SM one. It has also been known for a long time that alignment can be obtained without having to decouple the non-standard Higgs scalars [8]. However, to the best of our knowledge a comprehensive study of the conditions for which alignment occurs in 2HDMs has not been carried out. In this article we present such an analysis.

In general 2HDMs, absence of tree-level FCNC could be achieved by a discrete symmetry forbidding the λ_6 and λ_7 couplings, in which case we show that alignment occurs for specific values of the quartic couplings. More specifically, alignment only happens when t_β is a common solution to two independent algebraic equations and is therefore not a natural occurrence in this scenario. Furthermore, if certain orderings in the quartic couplings are not satisfied, alignment would never occur; this is the case for the MSSM at tree-level.

More interestingly, if λ_6 and λ_7 are non-zero but much smaller than the other couplings in the theory, perhaps as a result of a softly broken symmetry, we discover that alignment could exist for generic values of the quartic couplings. In this case, one can show that alignment may occur at both small and large values of t_β. In particular, the large t_β solution may be realized in a wide region of parameter space. Furthermore, this solution may be realized in the MSSM, where the coupling λ_7 is generated radiatively. A positive λ_7 is required in this case. Moreover, we obtain that small values of $(\mu/M_{\rm SUSY})$ are not compatible with alignment without decoupling. Therefore, small deviations from the SM expectations in the Higgs couplings to fermions and weak gauge bosons without any signal of non-standard Higgs bosons in direct searches, would imply either large m_A or large values of $(\mu/M_{\rm SUSY})$.

Other realizations of alignment without decoupling in supersymmetric theories can be obtained by going beyond the MSSM, which relaxes the strict relations among the tree-level quartic couplings of the MSSM Higgs potential. One simple possibility is to extend the

MSSM Higgs sector by adding more scalars, such as an electroweak singlet (the NMSSM) or a triplet. When one takes the limit that the singlet/triplet scalars are heavy and integrated out, new contributions to the scalar quartic couplings are generated. More specifically, new contributions to $\tilde{\lambda}_3$ are generated, thereby allowing for alignment without decoupling even if λ_6 and λ_7 continue to be zero.

Last but not least, we consider the effectiveness of using precision measurements of Higgs couplings to fermions to probe the wedge region in the $m_A - \tan\beta$ plane. If in the future no relevant deviations of the Higgs-fermion couplings from their SM values are observed, it could be naively inferred that large values of m_A are required, thereby disfavoring the wedge region. The possibility of alignment without decoupling opens up this region of parameter space, implying that new physics beyond the SM would have to be probed by other means. These would include, for example, measurements of loop-induced Higgs couplings, flavor physics, relic density and direct detection of dark matter, as well as direct searches for new physics beyond the 2HDMs. Moreover, alignment without decoupling also highlights the necessity of devising new search strategies to look for light non-standard Higgs bosons. We will return to these important topics in the near future.

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