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Comment on "Beamstrahlung considerations in laser-plasma-accelerator-based linear colliders"

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Abstract

C. B. Schroeder, E. Esarey, and W. P. Leemans {Phys. Rev. ST Accel. Beams **15**, 051301 (2012)} have proposed a set of parameters for a TeV-scale collider based on plasma wake field accelerator principles. In particular, it is suggested that the luminosities greater than 10^{34} cm^{-2}s^{-1} are attainable for an electron-positron collider. In this comment we dispute this set of parameters on the basis of first principles. The interactions of accelerating beam with plasma impose fundamental limitations on beam properties and, thus, on attainable luminosity values.

In a recent paper [1], Schroeder, Esarey, and Leemans have proposed a set of parameters for a TeV-scale collider based on the plasma wake-field acceleration. To be comparable with conventional designs (such as the ILC and CLIC) the authors chose the design luminosity to be 2x10^{34} cm^{-2}s^{-1} (Table IV in Ref. [1]). Further, they assumed that the number of particles per bunch is equal to 4x10^9 and 5.2x10^9 for plasma densities, n_0, of 10^{17} cm^{-3} and 2x10^{15} cm^{-3}, correspondingly. To mitigate the beamstrahlung, they also employed quite short bunches with rms bunch lengths of 1 and 1.3 μm corresponding to the plasma densities mentioned above. The authors do not present beam emittances in Ref. [1]. Ref. [2] proposes the rms normalized emittance value of 0.1 μm for both beams. Such a choice is supported by the similarity of beam parameters considered in Refs. [1], [2], and [3].

The authors propose to use the so-called quasi-linear regime of acceleration with a low-density beam: "Reduction in the bunch length for fixed charge is limited by bunch density constraints, i.e., $n_b \le n_0$ to avoid the blow-out regime and the resulting strong beam self-focusing and emittance growth." Let us demonstrate that such a regime of low beam density is unachievable for the parameters of Ref. [1]. In order for the beam density to be lower or equal to the plasma density, the beta-function along the plasma acceleration channel must be equal to or greater than the following value (for a Gaussian distribution):

$$\beta(\gamma) = \frac{N_b \gamma}{(2\pi)^{3/2} \varepsilon_n \sigma_s n_0}, \tag{1}$$

where N_b is the number of particles per bunch, γ is the Lorentz factor, ε_n is the rms normalized beam emittance, and σ_s is the rms bunch length. With this beta-function value, the transverse rms beam size, σ_r, is kept constant along the acceleration channel such that $n_b = n_0$. For parameters of Ref. [1], this transverse beam size would be 50 μm for $n_0 = 10^{17}$ cm^{-3} and 350 μm for $n_0 = 2\text{x}10^{15}$ cm^{-3}. It is close to the transverse size of the plasma channel.

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The emittance growth in fully-ionized plasma due to the multiple Coulomb scattering is given by (in the ultra-relativistic case):

$$\frac{d\varepsilon_n}{d\gamma} = \frac{2\pi Z(Z+1) r_e^2 n_0 \beta(\gamma) \Lambda_c}{\gamma (d\gamma / ds)}, \tag{2}$$

where r_e is the classical electron radius, Z is the plasma ion charge, and $\Lambda_c (\approx 18)$ is the Coulomb log. Combining Eqs. (1) and (2) one obtains the following expression for the emittance growth:

$$\frac{d\varepsilon_n}{d\gamma} = \frac{Z(Z+1) r_e^2 N_b \Lambda_c}{\sqrt{2\pi}\varepsilon_n \sigma_s (d\gamma / ds)}. \tag{3}$$

Assuming that the acceleration rate, $d\gamma / ds$, is constant, and integrating the above equation one finally obtains:

$$\varepsilon_f = \sqrt{\varepsilon_i^2 + \frac{2Z(Z+1) r_e^2 N_b \Lambda_c}{\sqrt{2\pi}\sigma_s (d\gamma / ds)} (\gamma_f - \gamma_i)}, \tag{4}$$

where ε_i and ε_f are the initial and final normalized rms emittances, and γ_i and γ_f are corresponding γ's. Setting the initial emittance to zero one obtains the minimum emittance in the low-beam-density ($n_b \le n_0$) regime to be $\varepsilon_f \approx 10$ μm, where we assumed $n_0 = 10^{17}$ cm^{-3}, $Z = 1$ and other parameters from Ref. [1]. This is two orders of magnitude greater than the value suggested in Ref. [2]. As one can see from Eq. (4) the emittance growth increases with the decrease of an accelerating rate and, consequently, with the decrease of plasma density; so that it would be ~2 times larger for $n_0 = 2\text{x}10^{15}$ cm^{-3}. In addition, Eq. (4) shows that even if one increases the bunch length by a factor of 10, the number of particles per bunch still needs to be reduced by a factor of 10^3 in order to attain the desired emittance. Since the luminosity is proportional to fN_b^2, to compensate for this reduction would require a factor of 10^6 increase in the bunch repetition rate, f ! Thus, as far as we can judge, the proposal of Ref. [1] to use the low beam density acceleration regime is incompatible with low-emittance beams needed for collider schemes.

As one can see from the previous paragraph, the suppression of the emittance growth due to multiple scattering requires strong transverse focusing. Following Ref. [3] (Appendix A), one can demonstrate that for $\Delta\varepsilon_n$ to be less than 0.1 μm, the transverse rms beam size must be much smaller than k_p^{-1}, where $k_p = \omega_p/c$, and ω_p is the plasma frequency. In this case the beam energy loss (deceleration) in plasma has a weak dependence on the actual value of the beam radius and one can use the results of Ref. [4] (see Eq. (18)) to make an estimate of the decelerating field due to the beam interaction with plasma in the quasi-linear regime. For a short bunch, $\sigma_s << k_p^{-1}$, and the plasma density of 10^{17} cm^{-3} we obtain the average decelerating rate of ~30 GeV/m, which

significantly exceeds the accelerating rate of 10 GeV/m, suggested by the authors. Note also that the deceleration will not be equally distributed along the bunch. The bunch head "sees" the deceleration at a much smaller rate than the average, while the bunch tail experiences the deceleration at twice the average rate. The authors have chosen such short bunches in order to mitigate the beamstrahlung effect: the bunch length suggested in Ref. [1] (σ_s = 1 μm and 1.4 μm for $n_0 = 10^{17}$ and $2\text{x}10^{15}$ cm^{-3}, correspondingly) is much smaller than k_p^{-1}(≈17 μm and 120 μm for the plasma densities mentioned above). As a result of such short bunches, the total relative momentum spread after acceleration is approximately equal to the ratio of twice the energy loss rate to the average acceleration rate, and is well above 100% for the parameters suggested in Ref. [1]. Thus, we conclude again that the set of collider parameters proposed by the author is not self-consistent and not compatible with high-luminosity collider concepts.

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