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Geolocating the Higgs Boson Candidate at the LHC

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The latest results from the ATLAS and CMS experiments at the CERN Large Hadron Collider (LHC) unequivocally confirm the existence of a resonance, X, with mass near 125 GeV which could be the Higgs boson of the Standard Model. Measuring the properties (quantum numbers and couplings) of this resonance is of paramount importance. Initial analyses by the LHC collaborations disfavor specific alternative benchmark hypotheses, e.g. pure pseudoscalars or gravitons. However, this is just the first step in a long-term program of detailed measurements. We consider the most general set of operators in the decay channels $X \to ZZ, WW, Z\gamma, \gamma\gamma$ and derive the constraint implied by the measured rate. This allows us to provide a useful parametrization of the orthogonal independent Higgs coupling degrees of freedom as coordinates on a suitably defined sphere.

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Introduction. The determination of the spin J and parity P of the putative Higgs boson [1] is of paramount importance. The observed decay to $\gamma\gamma$ [2] already excludes the case of $J = 1$ by the Landau-Yang theorem [3]. Spin discrimination and coupling measurements are achieved by studying kinematic correlations of the final state objects, generally leptons. An optimal way of incorporating all available information is through the Matrix Element Method (MEM) [4–7]. Initial results from ATLAS [8] and CMS [9, 10] strongly disfavor $J^P = 0^-$, as well as $J^P = 2^+$ with graviton-like couplings. These results represent an important first foray into the general parameter space of generic boson couplings.

In this paper we provide a theoretical framework for spin and coupling measurements in full generality, without theoretical prejudice towards specific benchmarks. For concreteness and simplicity, we consider the example of a spin-zero "X" resonance, decaying to four leptons through two intermediate Z-bosons [11–14], though our approach can be readily extended to other spins and final states. The complete measurement of the X couplings in full generality will provide insights into the nature of electroweak symmetry breaking and may offer the first glimpses of physics beyond the Standard Model (SM).

Effective Theory. In what follows, we will consider a spin-zero state X. In general, X has no definite CP properties, and can be thought of as a linear combination of some CP-even state H and some CP-odd state A [15]:

$$X \equiv H \cos\alpha - A \sin\alpha. \tag{1}$$

In the special case of $\alpha = 0$, X is a pure $J^P = 0^+$ scalar, as predicted in the Standard Model, while for $\alpha = \pm\pi/2$, X is a pure pseudoscalar with $J^P = 0^-$.

The X couplings to two gauge bosons (e.g. ZZ) can be classified according to their symmetries in the CP-eigenstate basis (H, A). Three types of terms are possible:

$$\begin{aligned}\mathcal{L} \ \ni \ & \frac{M_Z^2}{v} H Z^\mu \hat{f}^{(H)}_{\mu\nu} Z^\nu + \frac{1}{2} H F^{\mu\nu} \hat{f}^{(H)}_{\mu\nu\rho\sigma} F^{\rho\sigma} \\ + \ & \frac{1}{2} A F^{\mu\nu} \hat{f}^{(A)}_{\mu\nu\rho\sigma} F^{\rho\sigma},\end{aligned} \tag{2}$$

where $F^{\mu\nu}$ is the Z-boson field strength tensor, $M_Z = 91.1876$ GeV is the Z-boson mass, $v = 246$ GeV is the electroweak scale, and $\hat{f}^{(H)}$ and $\hat{f}^{(A)}$ are (in general momentum-dependent) form-factors, which from the effective theory point of view should be thought of as infinite series expansions in terms of some new physics scale Λ. Let us discuss each one in turn.

The form-factor $\hat{f}^{(H)}_{\mu\nu}$ describes interactions of the CP even component H that necessarily violate $SU(2)$ gauge invariance. Expanding in powers of Λ^{-1}, one obtains

$$\begin{aligned}\hat{f}^{(H)}_{\mu\nu} \ \equiv \ & g_1 g_{\mu\nu} + \frac{g_5}{\Lambda^2}\left(\vec{\partial}_\mu \overleftarrow{\partial}_\nu + g_{\mu\nu} \vec{\partial}^\rho \overleftarrow{\partial}_\rho\right) \\ + \ & \frac{g_6}{\Lambda^2} g_{\mu\nu}\left(\overleftarrow{\Box} + \vec{\Box}\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right),\end{aligned} \tag{3}$$

where g_i are dimensionless coupling constants, and the derivative operator $\overleftarrow{\partial}$ ($\vec{\partial}$) acts on the Z field to its left (right). Eq. (3) does not include terms containing $\partial^\mu Z_\mu$, which vanish in the Lorenz gauge.

The second class of CP-even couplings, described by the form-factor

$$\hat{f}^{(H)}_{\mu\nu\rho\sigma} \equiv \frac{g_2}{\Lambda} g_{\mu\rho} g_{\nu\sigma} + \frac{g_3}{\Lambda^3} g_{\mu\rho} \partial_\nu \partial_\sigma + \mathcal{O}\left(\frac{1}{\Lambda^5}\right), \tag{4}$$

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may respect $SU(2)$ gauge invariance, if H is a SM singlet. Note that terms of this sort could be generated by a $\left(\vec{\partial}_\mu \overleftarrow{\partial}_\nu - g_{\mu\nu}\vec{\partial}^\rho\overleftarrow{\partial}_\rho\right)$ term in Eq. (3), which explains the absense of such a term there. Finally, the couplings of the CP-odd component A are given by

$$\hat{f}^{(A)}_{\mu\nu\rho\sigma} = \frac{g_4}{\Lambda}\varepsilon_{\mu\nu\rho\sigma} + \mathcal{O}\left(\frac{1}{\Lambda^5}\right) \tag{5}$$

and may be $SU(2)$ gauge invariant as well, if A is a SM singlet. However, here we do not make any assumptions about the $SU(2)$ quantum numbers of H and A; the terms (4) and (5) will also violate $SU(2)$ gauge invariance if H or A (respectively) transform under $SU(2)$.

We note that the g_5 and g_6 terms in (3) are typically omitted in the literature. The contributions to the $X \to ZZ$ amplitude resulting from the $\overleftarrow{\Box} + \vec{\Box}$ and $\vec{\partial}^\rho\overleftarrow{\partial}_\rho$ terms are proportional to $M^2_{Z_1} + M^2_{Z_2}$ and $M^2_X - M^2_{Z_1} - M^2_{Z_2}$, respectively, where M_X is the mass of the resonance and M_{Z_1} and M_{Z_2} are the invariant masses of the two Z-bosons (in the usual convention where $M_{Z_1} > M_{Z_2}$). Thus, if both Z-bosons were on-shell, the contribution from such terms would be essentially constant, and therefore could be absorbed into a redefinition of g_1. However, in the case of interest where $M_X \approx 125$ GeV, one or both of the Z's are off-shell and M_{Z_1} and M_{Z_2} vary from event to event. Thus, strictly speaking, the g_5 and g_6 terms cannot be absorbed into g_1, though their effects (relative to g_1) are expected to be rather small, due to the additional Λ^2 suppression.

It appears that a general analysis of the couplings of the Higgs boson candidate would have to include at the very minimum the terms identified in eqs. (3-5), and perhaps even the higher dimensional operators $\sim \Lambda^{-4}, \Lambda^{-5}, \ldots$ which were not explicitly listed. Such an analysis may indeed be desirable at some point in the future, when significantly more data will have been accumulated by the LHC experiments. However, there are strong theoretical and experimental motivations for making certain simplifying assumptions for analyses in the *immediate* future.

First, all experimental evidence so far suggests that the new physics scale Λ is high compared to v. Second, consistency of the effective theory description *requires* that Λ be sufficiently removed from the relevant experimental energy scale. Finally, the higher dimensional nature of the $1/\Lambda$ couplings suggests a radiative origin and hence suppressions by loop factors. It is therefore reasonable to expect that the higher order terms g_3, g_5, g_6, etc. in the expansions (3-5) are negligible relative to the corresponding leading order couplings g_1, g_2 and g_4.

At the same time, the relative size of the leading terms g_1, g_2 and g_4 is *a priori* unknown. For example, the g_2 and g_4 terms are equally suppressed by Λ, and may both preserve gauge invariance, thus it is difficult to argue that one should be preferred over the other. Similarly, we do not know the extent to which H is involved in breaking $SU(2)$ gauge invariance, and hence we cannot simply assume that the $SU(2)$ breaking term g_1 dominates over g_2, in spite of the additional Λ suppression in the latter [13]. This is why we keep all three terms and consider the effective Lagrangian for the mass eigenstate X to be

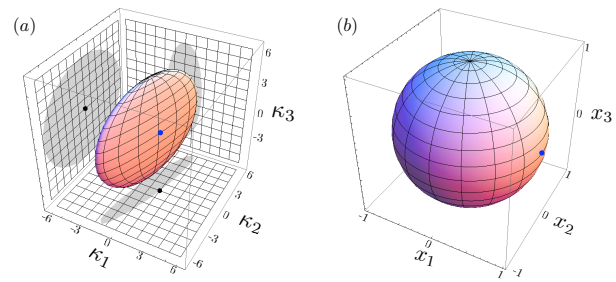

FIG. 1: The principle of geolocating: the change of variables (9) inflates (a) the hyper-surface defined by the measurement (8) into (b) a perfect sphere.

$$\mathcal{L} = X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu}\tilde{F}^{\mu\nu}\right], \tag{6}$$

where

$$\kappa_1 \equiv g_1 \cos\alpha,\ \kappa_2 \equiv g_2 \frac{v}{\Lambda}\cos\alpha,\ \kappa_3 \equiv g_4 \frac{v}{\Lambda}\sin\alpha \tag{7}$$

are the effective couplings which need to be measured by experiment. We note that this procedure means neglecting (for the reasons stated above) the g_3 operator from Refs. [5, 6] and the g_5 and g_6 terms listed in (3).

Counting degrees of freedom. From an effective theory point of view, the coefficients κ_i in the Lagrangian (6) are real numbers. Thus the problem of measuring the Higgs couplings is reduced to determining the three dimensionless degrees of freedom κ_i. However, one can go one step further and use the measured signal rate (X production cross-section times branching fraction for $X \to ZZ$) to eliminate one more degree of freedom, namely the overall κ scale. In order to do this, we will assume that the X can be treated as a narrow resonance. We then note that the partial width for $X \to ZZ$ is a quadratic function of the κ's:

$$\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij}\kappa_i\kappa_j, \tag{8}$$

where we have factored out the partial width Γ_{SM} predicted in the SM in order to define dimensionless coefficients γ_{ij} listed in Table I. The measured total rate then provides the overall normalization and constrains the κ_i couplings to lie on the closed hyper-surface shown in Fig. 1(a). The idea now is to change variables and parameterize the couplings κ_i in terms of the *two* coordinates on this hyper-surface. This re-parametrization is useful and meaningful, since the (normalized) angular and invariant mass distributions used to measure the

TABLE I: Numerical values for the coefficients defined in (8) for different diboson final states.

Process	γ_{11}	γ_{22}	γ_{33}	γ_{12}
$X \to ZZ$ (DF)	1	0.090	0.038	-0.250
$X \to ZZ$ (SF)	1	0.081	0.032	-0.243
$X \to \gamma\gamma$	0	1	1	0
$X \to WW$	1	0.202	0.084	-0.379
after cuts				
$X \to ZZ$ (DF)	1	0.101	0.037	-0.277

Higgs spin and CP properties are insensitive to the overall scale of the couplings κ_i. Operationally, we propose to do this by changing variables as

$$x_i = \sum_j O_{ij}\kappa_j, \tag{9}$$

where $O_{21} = O_{31} = O_{32} = 0$ and

$$\begin{aligned} O_{1i} &= \gamma_{1i}/\sqrt{\gamma_{11}}, \quad (i = 1, 2, 3), \\ O_{2i} &= \frac{\gamma_{11}\gamma_{2i} - \gamma_{12}\gamma_{1i}}{\sqrt{(\gamma_{11}\gamma_{22} - \gamma_{12}^2)\gamma_{11}}}, \quad (i = 2, 3), \\ O_{33} &= \sqrt{\det||\gamma_{ij}||/(\gamma_{11}\gamma_{22} - \gamma_{12}^2)}. \end{aligned}$$

In terms of the new variables x_i, the constraint implied by (8) is the surface of a sphere, as seen in Fig. 1(b).

Note that in the case of real couplings, γ_{13} and γ_{23} vanish identically (and hence are not listed in (8)). This can be understood in terms of parity; interference terms between amplitudes describing the decay of a parity-even scalar and a parity-odd scalar are odd under parity, hence vanish under integration provided the cuts respect parity (as is nearly always the case). The situation in the presence of cuts is less clear if the couplings of the three operators in (6) are complex valued; however, any γ_{13} or γ_{23} term generated in this case is expected to be very small.

Geolocating the Higgs. A sphere may be parameterized using latitude ϕ and longitude λ. Making an obvious analogy, we can think of the tree-level SM Higgs couplings $(\kappa_1, \kappa_2, \kappa_3) = (1, 0, 0)$ as being represented by the point $(\phi, \lambda) = (0, 0)$ in the Gulf of Guinea (see Fig. 2). The case of a pure pseudoscalar with $(\kappa_1, \kappa_2, \kappa_3) = (0, 0, 1)$ corresponds to the North pole with $(\phi, \lambda) = (90, 0)$. Those are the two main benchmark scenarios considered so far by the LHC experiments. The Higgs sphere from Fig. 1(b) now opens up the full range of possibilities. For example, one may consider a certain amount of mixing as in (1) between the SM Higgs and a pseudoscalar, placing us on the Greenwich meridian. Alternatively, one may allow non-trivial values for the two CP-even couplings κ_1 and κ_2, thus spanning the equator. Finally, one could also envision the most general case with non-trivial values for all three couplings,

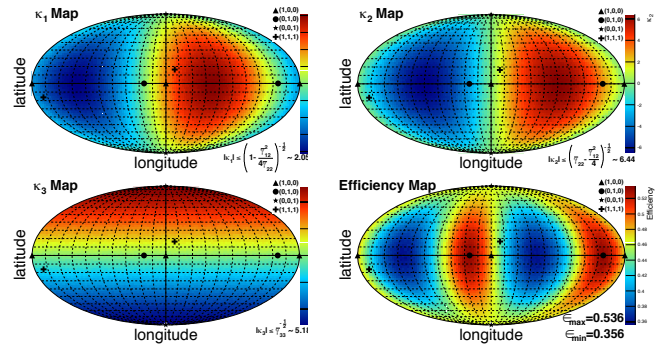

FIG. 2: The effective couplings κ_i and the signal efficiency after cuts as a function of latitude (ϕ) and longitude (λ) for the different flavor 4 lepton channel $X \to ZZ \to 2e2\mu$.

e.g. $(\kappa_1, \kappa_2, \kappa_3) = (-0.945804, -3.88525, 2.44522)$, corresponding to $(\phi, \lambda) = (29.64945, -82.3486)$, which happens to be a location in the south end zone of the Swamp.

We note that CMS has recently undertaken [10] an exploration of the sphere along the Greenwich meridian ($\lambda = 0$) by measuring the parameter $f_{a3} \equiv |A_3|^2/(|A_3|^2 + |A_1|^2)$, parametrizing the relative contributions $|A_1|^2$ and $|A_3|^2$ to the total cross-section of the two benchmark models $(\kappa_1, \kappa_2, \kappa_3) = (1, 0, 0)$ and $(\kappa_1, \kappa_2, \kappa_3) = (0, 0, 1)$, respectively.

Selection cut bias. By definition, the measurement of the Higgs signal rate (and, by association, of the partial width (8)) is done using only events which pass selection cuts. Therefore, it needs to be corrected for the efficiency. Unfortunately, as demonstrated in the bottom right panel in Fig. 2, the signal efficiency is noticeably model dependent, i.e. it is quite sensitive to the actual values of the individual κ_i parameters, and may vary from as low as 35% to as high as 54%. One source of this variation is the cut on M_{Z_2} (see e.g. [11]). The effect is quantified in Fig. 3, which shows unit-normalized M_{Z_2} distributions for several choices of κ_i, including an example with a high efficiency ($(\kappa_1, \kappa_2, \kappa_3) = (0, 0, 1)$, black circles) and a suitably chosen example with a low efficiency ($(\kappa_1, \kappa_2, \kappa_3) = (1.8, 5, 0.08)$, red crosses). The M_{Z_2} distribution in the latter case is softer, and many events fall below the minimum accepted M_{Z_2} value of 12 GeV. We conclude that it would be virtually impossible to correct for the efficiency without knowledge of the couplings κ_i, which we are trying to measure in the first place, thus falling into a vicious circle.

Here we advocate an alternative approach. It is sufficient to realize that the coefficients γ_{ij}, while also affected by the efficiency, do *not* depend on κ_i. Therefore, in defining the Higgs sphere (9) one could simply use the corresponding values of γ_{ij} *after cuts*, which need to be calculated once and for all. As shown in Table I, the changes are subtle, yet noticeable. Note that even without any cuts, the same flavor (SF) and different flavor (DF) coefficients are different, due to the interference ef-

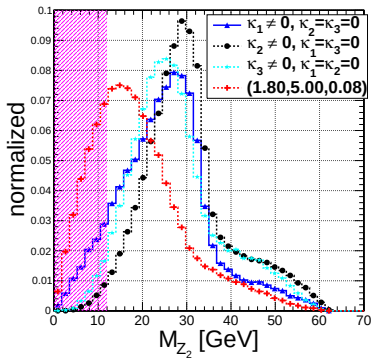

FIG. 3: Unit-normalized M_{Z_2} distributions for four different theory benchmarks $(\kappa_1, \kappa_2, \kappa_3)$.

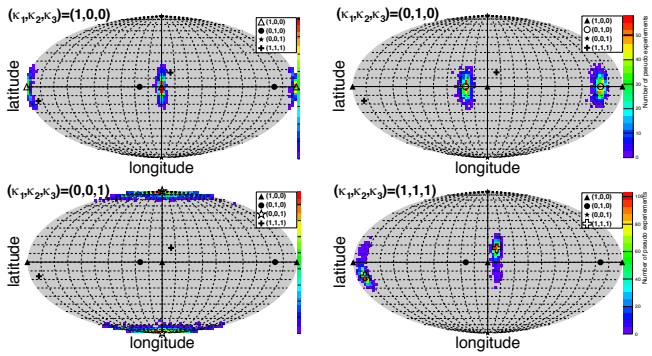

FIG. 4: Distribution of the best fit values for the Higgs geolocation in 1000 pseudoexperiments with 300 $X \to ZZ \to 2\mu 2e$ signal events each, for four different benchmark points.

fects present in the SF case (see, e.g. [6]).

Interpretation. We emphasize that by studying the angular and invariant mass distributions of the final state leptons in the $X \to ZZ \to 4\ell$ channel *it is experimentally possible* to determine the exact geolocation of the Higgs boson candidate, without any simplifying theoretical assumptions. A proof of principle is shown in Fig. 4. We consider 4 benchmark scenarios for $(\kappa_1, \kappa_2, \kappa_3)$: ($\triangle$) $(1,0,0)$, ($\circ$) $(0,1,0)$, ($\star$) $(0,0,1)$, and ($+$) $(1,1,1)$. We show results from 1000 pseudoexperiments with 300 signal events each (after cuts). Fig. 4 shows that in each case, the maximum likelihood fit indeed selects the correct geolocation (marked with an open symbol) of the Higgs candidate.

(In)determination of the mixing angle α. We note that geolocating the Higgs candidate by itself is not sufficient to determine the value of the mixing angle α, and we will still not know the relative composition of X in terms of H and A. As seen in eq. (7), the measurement of all three effective couplings κ_1, κ_2 and κ_3 only places 3 constraints on the 4 input parameters α, g_1, g_2/Λ and g_4/Λ, so one degree of freedom (which can be chosen to be the mixing angle α) will always be left undetermined.

Music of the Spheres. Until now, we have been treating the coefficients κ_i of the operators in the effective Lagrangian (6) to be real. We term this "Scenario 0". Of course, this must be the case at tree level, and even beyond, as long as those operators are generated by loop diagrams involving heavy particles, such as the top, which may be consistently integrated out of the full theory to form an effective theory. However, in accordance with the optical theorem, loops with light particles (such as b-quarks, which cannot be integrated out of the theory) will contribute imaginary parts to those couplings. In this case, each of the κ_i is complex and there are five physical degrees of freedom (an overall phase in the κ_i may be removed). Nevertheless, the width constraint $\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij} \kappa_i^* \kappa_j$ can still be enforced. As before, the diagonalization and rescaling (9) renders this a sphere in $\mathbb{C}^3$ (rather than an ellipsoid, as is the case in general).

Even in this general complexified case, there are three interesting and simple scenarios where the number of degrees of freedom is reduced to three. Namely, if one of κ_i is too small to have observable effects, then the remaining two κ_i, taken to be arbitrary complex numbers, also comprise two observable degrees of freedom (as an overall phase may be removed). There are, of course three such scenarios, "Scenario 1" where κ_1 is negligible, "Scenario 2" where κ_2 is negligible, and "Scenario 3" where κ_3 is negligible.

All four scenarios have physical motivation. Scenario 0 holds when the important couplings occur at tree-level or come largely from top-loops. Scenario 1 describes the most general couplings to a massless particle such as the photon or the gluon, though it could be obtained in the ZZ or WW cases if the X has nothing to do with EWSB. Scenario 2 describes, e.g., a state where X is primarily A, but the H with which it mixes is an SM-like Higgs boson. Scenario 3 describes X as a general CP-even scalar. The scenarios are not totally exclusive; for instance, the ZZ case in the Standard Model is well approximated by points in Scenarios 0, 2, or 3.

Other final state channels. While we have so far only focused on the $ZZ \to 4\ell$ final state, the preceding discussion can be readily applied to the other di-boson final states, W^+W^-, $Z\gamma$, and $\gamma\gamma$. The interactions relevant for those channels involve corresponding new couplings κ_i^{WW}, $\kappa_i^{Z\gamma}$, and $\kappa_i^{\gamma\gamma}$. Assuming $SU(2)$ gauge symmetry, these new couplings are related, e.g.

$$\kappa_1^{\gamma\gamma} = \kappa_1^{Z\gamma} = 0, \quad \kappa_1^{WW} = \kappa_1^{ZZ}, \tag{10}$$

$$\kappa_i^{Z\gamma} = \frac{1}{2}(\kappa_i^{ZZ} - \kappa_i^{\gamma\gamma}) \tan 2\theta_W, (i = 2, 3), \tag{11}$$

$$\kappa_i^{WW} = \frac{\kappa_i^{ZZ} \cos^2\theta_W - \kappa_i^{\gamma\gamma} \sin^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W}, (i = 2, 3), \tag{12}$$

with obvious definitions of κ's. As before, the measured rate in each of those channels defines a sphere similar to eq. (8), where the corresponding values for the coefficients γ_{ij} are given in Table I. It is interesting to note

that these spheres are not overlapping (even for the DF and SF case in the same $X \to ZZ$ channel), and in principle couplings can be measured from intersecting spheres, although we expect that this method is of only academic interest, since the corresponding precision would be very poor.

Experimental implementation. While the example analyses performed here are simplified, the additional steps needed to perform an actual analysis are relatively straightforward. One needs to

1. Calculate the parameters γ_{ij} from Table I using the actual detector geometry and effciencies and the specific cuts to be used in the analysis.
2. Use the γ_{ij} to define a sphere in coupling space, either in the scenario where all couplings are real (Eq. (8)) or in one of the other three scenarios.
3. Determine the point on the sphere in coupling space that maximizes the likelihood associated with the observed events. The likelihood employed should include background information [5–7, 14].

One potential experimental concern stems from the difficulty of generating events to perform pseudo-experiments for sufficiently many points in the continuous parameter space. In experiments, the significance of a measurement is often determined by comparing the observed value of some test statistic to a distribution of that test statistic obtained from pseudo-experiments. In principle, such an approach is not necessary when the test statistic used is the likelihood, since the likelihood has a well-defined statistical meaning. However, one may still wish to perform pseudo-experiments in this case, for example to take into account reducible backgrounds not included in the likelihood expression used.

We point out that this concern is unjustified; one can perform pseudo-experiments without generating events for every point in parameter space. Our recommended procedure is to generate events for one particular point in parameter space, which we label by (ϕ_0, λ_0). Then one can perform pseudo-experiments for a different point in parameter space, $(\phi_{test}, \lambda_{test})$, using the same events, but calculating the likelihood not from the usual formula

$$\ln \mathcal{L}(\phi, \lambda) = \sum_i \ln P(\mathbf{p}_i, \phi, \lambda), \tag{13}$$

but from

$$\ln \mathcal{L}(\phi, \lambda) = \sum_i \frac{P(\mathbf{p}_i, \phi_{test}, \lambda_{test})}{P(\mathbf{p}_i, \phi_0, \lambda_0)} \ln P(\mathbf{p}_i, \phi, \lambda), \tag{14}$$

where $\mathbf{p}_i$ is the kinematic information in the i-th event. In other words, we suggest recycling the events in the pseudo-experiments by reweighting, which is relatively trivial by using `JHUGen` [5] or `MEKD` [6], rather than generating a new event sample for each parameter space point.

Summary. We have proposed several related parameterizations of the couplings of the 125 GeV boson discovered at the LHC. These parameterizations allow the LHC experiments to go beyond comparing benchmark points, etc. in a relatively simple, yet very general, theoretically motivated fashion. The low dimensionality of the parameter space is helpful experimentally as well as in the visualization and interpretation of results. We look forward to the implementation of this method by the LHC collaborations and to the continued exploration of the couplings of the putative Higgs boson.

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