

The $B^*B\pi$ coupling with relativistic heavy quarks

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We report on a calculation of the $B^*B\pi$ coupling in lattice QCD. The strong matrix element $\langle B\pi|B^*\rangle$ is directly related to the leading order low-energy constant in heavy meson chiral perturbation theory (HM χ PT) for B -mesons. We carry out our calculation directly at the b -quark mass using a non-perturbatively tuned clover action that controls discretisation effects of order $|\vec{p}a|$ and $(ma)^n$ for all n . Our analysis is performed on RBC/UKQCD gauge configurations using domain wall fermions and the Iwasaki gauge action at two lattice spacings of $a^{-1} = 1.73(3)$ GeV, $a^{-1} = 2.28(3)$ GeV, and unitary pion masses down to 290 MeV. We achieve good statistical precision and control all systematic uncertainties, giving a final result for the HM χ PT coupling $g_b = 0.569(48)_{stat}(59)_{sys}$ in the continuum and at the physical light-quark masses. This is the first calculation performed directly at the physical b -quark mass and lies in the region one would expect from carrying out an interpolation between previous results at the charm mass and at the static point.

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1. Introduction

The power of lattice QCD in probing of the Standard Model, and uncovering evidence for new physics, lies predominantly in the flavour sector. To constrain the CKM unitarity triangle there are many inputs required that can only be accessed non-perturbatively, particularly in the B -meson sector. For instance, lattice calculations of the decay constants f_B and f_{B_s} are necessary inputs for neutral B -meson mixing calculations and for the Standard Model predictions of $\text{BR}(B \rightarrow \tau\nu)$ and $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ respectively. Furthermore, lattice calculations of the $B \rightarrow \pi l\nu$ form factor allow a determination of the CKM matrix element $|V_{ub}|$. For both semileptonic form factors and mixing matrix elements, the lattice precision lags behind experiment. However the experimental measurements will continue to improve with the large data sets available at Belle II and the LHCb upgrade. Therefore it is essential to further reduce the theoretical uncertainties in the non-perturbative hadronic parameters in order to maximise the scientific impact of current and future B -physics experiments.

A major source of uncertainties in all previous lattice calculations is from practical difficulties simulating at physical light-quark masses. Theoretical insight from HM χ PT can guide extrapolations down to the physical point, but lack of knowledge of the low-energy constants (LECs) of the theory introduces unwanted uncertainties. For example, at next-to-leading order (NLO) in HM χ PT the dependence of f_{B_d} on the light-quark (or equivalently, pion) mass is given by

$$f_{B_d} = F \left(1 + \frac{3}{4}(1 + 3g_b^2) \frac{M_\pi^2}{(4\pi f_\pi)^2} \log(M_\pi^2/\mu^2) \right) + \dots, \quad (1.1)$$

where g_b is the leading order LEC of the theory, and is directly related to the strong coupling $g_{B^*B\pi}$. In this work we perform the first calculation of the coupling g_b directly at the b -quark mass.

2. Heavy Meson Chiral Perturbation Theory

In the infinite quark mass limit, symmetries predict that properties of heavy-light mesons will be independent of the heavy quark's spin and flavour quantum numbers. Combining this with the chiral symmetry present in the $m_q \rightarrow 0$ limit of QCD provides the basis for heavy meson chiral perturbation theory (HM χ PT). This effective theory of QCD is a joint expansion in powers of the inverse heavy-quark mass $1/m_h$ and the light-quark-mass m_q . At lowest order the interactions between the heavy and light mesons are determined by a Lagrangian with a single LEC [1]

$$\mathcal{L}_{\text{HM}\chi\text{PT}}^{\text{int}} = g \text{Tr} \left(\bar{H}_a H_b \mathcal{A}_\mu^{ba} \gamma^\mu \gamma_5 \right), \quad (2.1)$$

with

$$\mathcal{A}_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \quad (2.2)$$

and $\xi = \exp(i\mathcal{M}/f_\pi)$, where \mathcal{M} represents the usual octet of pseudo-goldstone bosons. The coupling g_b can be related to the coupling responsible for the strong decay $B^* \rightarrow B\pi$, defined as

$$\langle B(p)\pi(q) | B^*(p', \lambda) \rangle = -g_{B^*B\pi} q \cdot \varepsilon^\lambda(p'). \quad (2.3)$$

Equivalently, the same matrix element can be evaluated at leading order in HM χ PT,

$$\langle B(p)\pi(q)|B^*(p',\lambda)\rangle = -\frac{2M_B}{f_\pi}g_bq\cdot\varepsilon^\lambda(p'), \quad (2.4)$$

giving the relationship

$$g_{B^*B\pi} = \frac{2M_B}{f_\pi}g_b. \quad (2.5)$$

Performing an LSZ reduction and using the partially-conserved axial current relation for a soft pion, Eq. (2.3) becomes

$$g_{B^*B\pi}q\cdot\varepsilon^\lambda(p') = iq_\mu \frac{M_\pi^2 - q^2}{f_\pi M_\pi^2} \int d^4x e^{iq\cdot x} \langle B(p)|A_\mu(x)|B^*(p',\lambda)\rangle, \quad (2.6)$$

where $A^\mu = \bar{q}\gamma^\mu\gamma_5q$ is the light-quark axial vector current. If we parameterise the axial-current matrix element in terms of form factors

$$\begin{aligned} \langle B(p)|A^\mu|B^*(p',\lambda)\rangle &= 2M_{B^*}A_0(q^2)\frac{\varepsilon\cdot q}{q^2}q^\mu + (M_{B^*} + M_B)A_1(q^2)\left[\varepsilon^\mu - \frac{\varepsilon\cdot q}{q^2}q^\mu\right] \\ &+ A_2(q^2)\frac{\varepsilon\cdot q}{M_{B^*} + M_B}\left[p^\mu + p'^\mu - \frac{M_{B^*}^2 - M_B^2}{q^2}q^\mu\right], \end{aligned} \quad (2.7)$$

we see that at $q^2 = 0$

$$g_{B^*B\pi} = \frac{2M_{B^*}A_0(0)}{f_\pi}. \quad (2.8)$$

On the lattice, we cannot simulate exactly at $q^2 = 0$ without using twisted boundary conditions. Furthermore, and from Eq. (2.6) we see the form factor A_0 contains a pole at the pion mass, so it will be difficult to extrapolate to $q^2 = 0$ in a controlled manner. However, the form factor decomposition in Eq. (2.7) must be free of nonphysical poles, which allows us to obtain the relation

$$g_{B^*B\pi} = \frac{1}{f_\pi}[(M_{B^*} + M_B)A_1(0) + (M_{B^*} - M_B)A_2(0)]. \quad (2.9)$$

3. Computational strategy

Our analysis is carried out using ensembles produced by the RBC and UKQCD collaborations [2] with the Iwasaki gauge action and 2+1 flavour dynamical domain-wall fermions. The configurations are at two lattice spacings, the finer 32^2 ensembles have an inverse lattice spacing of $a^{-1} = 2.28(3)$ GeV and the coarser 24^3 ensembles have $a^{-1} = 1.73(3)$ GeV, corresponding to approximately 0.08 fm and 0.11 fm respectively. All ensembles have a spatial extent of 2.6 fm. We simulate with unitary light-quarks corresponding to pion masses down to $M_\pi = 289$ MeV. On all ensembles, the sea strange-quark mass is tuned to within 10% of its physical value. The fifth dimensional extent of both lattices is $L_5 = 16$, corresponding to a residual quark mass of $(m_{\text{res}}a) = 0.003$ on the 24^3 lattice and $(m_{\text{res}}a) = 0.0007$ on the 32^3 lattice. Full details of the ensembles and propagators used are presented in Table 1.

In this work we use the Relativistic Heavy Quark (RHQ) action [3, 4, 5] to simulate fully rela-

L/a	$a(\text{fm})$	$m_l a$	$m_s a$	#Configs	#Sources	$M_\pi(\text{MeV})$
24	0.11	0.005	0.04	1636	1	329
24	0.11	0.010	0.04	1419	1	422
24	0.11	0.020	0.04	345	1	558
32	0.08	0.004	0.03	628	2	289
32	0.08	0.006	0.03	889	2	345
32	0.08	0.008	0.03	544	2	394

Table 1: Lattice simulation properties. All ensembles are generated using 2+1 flavours of domain-wall fermions and the Iwasaki gauge action. All valence pion masses are equal to the sea-pion mass.

tivistic bottom quarks whilst controlling discretisation effects. The RHQ action is an anisotropic Wilson action with a Sheikholeslami-Wohlert term:

$$S_{RHQ} = a^4 \sum_{x,y} \bar{\psi}(y) \left(m_0 + \gamma_0 D_0 + \xi \vec{\gamma} \cdot \vec{D} - \frac{a}{2} (D_0)^2 - \frac{a}{2} \xi (\vec{D})^2 + \sum_{\mu\nu} \frac{ia}{4} c_p \sigma_{\mu\nu} F_{\mu\nu} \right) \psi(x).$$

El Khadra, Kronfeld, and Mackenzie showed that for correctly tuned parameters the anisotropic Clover action can be used to describe heavy quarks with controlled cut-off effects to all orders in ma and of $\mathcal{O}(|\vec{p}a|)$ [3]. Christ, Li, and Lin [5] later showed that only three independent parameters need to be determined and, further, presented a method for performing this parameter tuning non-perturbatively [6]. This tuning has now been completed for b -quarks [7] on the RBC/UKQCD configurations and these results are exploited in this calculation.

3.1 Ratios

To access the matrix element in Eq. (2.7) we calculate the lattice three-point function:

$$C_{\mu\nu}^{(3)}(t_x, t_y; \vec{p}, \vec{p}') = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{p}'\cdot\vec{y}} \langle B(y) A_\nu(0) B^*(x) \rangle_{t_x < 0 < t_y} \quad (3.1)$$

and the vector and pseudoscalar meson two point functions. If we set both the vector and pseudoscalar momenta to zero in Eq. (3.1) we can see from Eq. (2.7) that the only form factor accessible is A_1 . Therefore we form the ratio:

$$R_1 = \frac{C_{i,i}^{(3)}(t_x, t_y; \vec{p} = 0, \vec{p}' = 0) Z_B^{1/2} Z_{B^*}^{1/2}}{C_{BB}^{(2)}(t_y; \vec{p} = 0) C_{B_i^* B_i^*}^{(2)}(T - t_x; \vec{p} = 0)} = (M_{B^*} + M_B) A_1(q_0^2), \quad (3.2)$$

where Z_B and Z_{B^*} are the amplitudes extracted from the pseudoscalar and vector two-point functions.

To access the other form factors we need to inject a unit of momentum, such that $\vec{q} = \vec{p} = (1, 0, 0) \times 2\pi/L$ and $\vec{p}' = 0$. Following [8], we define further ratios R_2 , R_3 and R_4 which allows access to the form factor A_2 through

$$\frac{A_2}{A_1} = \frac{(M_{B^*} + M_B)^2}{2M_B^2 q_1^2} \left[-q_1^2 + E_{B^*}(E_{B^*} - M_B) - \frac{M_{B^*}^2 (E_{B^*} - M_B)}{E_{B^*}} \frac{R_3}{R_4} - i \frac{M_{B^*}^2 q_1}{E_{B^*}} \frac{R_2}{R_4} \right]. \quad (3.3)$$

The ratio in Eq. (3.3) is obtained at non-zero values of q^2 and needs to be extrapolated to $q^2 = 0$. However, its contribution is suppressed by the ratio $(M_{B^*} - M_B)/(M_{B^*} + M_B)$. The form factor A_1 is obtained at $q^2 = (M_{B^*} - M_B)^2$, but examination shows that the slight extrapolation to $q^2 = 0$ is not necessary at the resolution possible with the available statistics. If we define functions G_1 and G_2

$$\begin{aligned} G_1(q^2) &= (M_{B^*} + M_B)A_1(q^2), \\ G_2(q^2) &= (M_{B^*} - M_B)A_2(q^2), \end{aligned} \quad (3.4)$$

we can write the coupling as $G_1(0)$ plus a small correction from the ratio G_2/G_1 , giving

$$g_b = \frac{Z_A}{2M_B} G_1(0) \left(1 + \frac{G_2(0)}{G_1(0)} \right), \quad (3.5)$$

where Z_A is the light axial vector current renormalisation factor. We use the determination of Z_A from the RBC/UKQCD combined analysis of the light hadron spectrum, pseudoscalar meson decay constants and quark masses on the 24^3 and 32^3 ensembles [2].

4. Results

Figure 1 shows the ratios R_1 and R_2 on the 24^3 , $m_l a = 0.005$ ensemble fitted to a constant with statistical errors estimated using single elimination jack-knife. We perform a chiral extrapolation

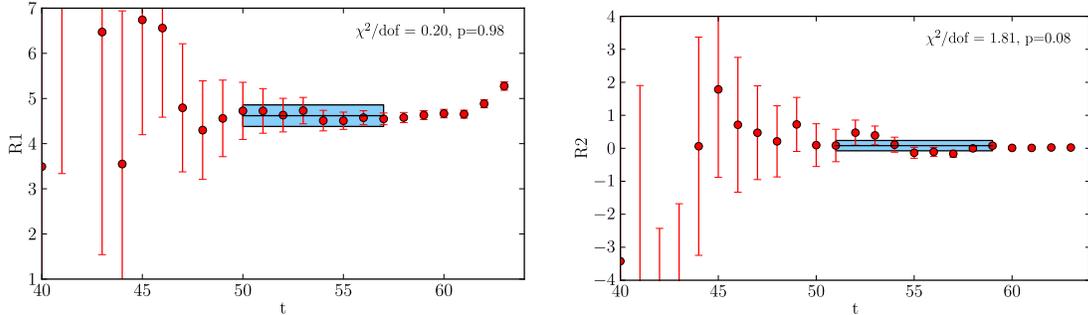


Figure 1: Ratios R_1 (left), R_2 (right) on the 24^3 , $m_l a = 0.005$ ensemble.

using the SU(2) HM χ PT formula for the axial coupling matrix element derived in [9]:

$$g_b = g_0 \left(1 - \frac{2(1 + 2g_0^2)}{(4\pi f_\pi)^2} M_\pi^2 \log \frac{M_\pi^2}{\mu^2} + \alpha M_\pi^2 + \beta a^2 \right), \quad (4.1)$$

which is next-to-leading order in the chiral expansion, but only leading order in the heavy-quark expansion. We parameterize the light-quark and gluon discretisation effects with an a^2 term, as expected for the domain-wall light-quark and Iwasaki gauge actions. The lattice-spacing dependence from the RHQ action is more complicated. However, we estimate the heavy-quark discretisation effects using power counting arguments [10] and find them to be negligible, such that extrapolating in a^2 captures the leading scaling behaviour. Figure 2 shows the chiral-continuum extrapolation to the physical light-quark mass and continuum using Eq. (4.1). We consider systematic errors arising

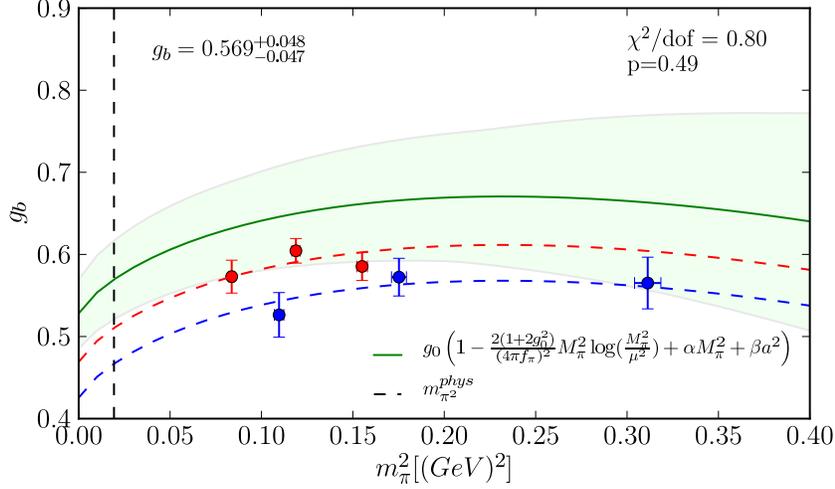


Figure 2: Chiral and continuum extrapolation. The bottom (blue) dashed line is the fit through the 24^3 ensemble points. The dashed line above (red) is the fit through the 32^3 ensemble points and the green solid line is the continuum extrapolation with a shaded error band. The intersect with the vertical dashed line corresponds to the physical pion mass.

from uncertainty in the lattice scale, the difference of our sea strange quark mass from its physical value, and uncertainties propagated through from the tuning of the RHQ parameters. However, we find our dominant source of uncertainties to be the combined chiral and continuum extrapolation. To estimate this error we tried a number of variations to our fitting procedure which included dropping the heaviest masses from each ensemble, considering a linear fit, and a fit function with no lattice scale dependence. We also varied the value of f_π that appears in Eq. (4.1) to simulate changing the relative sizes of the NLO and NNLO terms that appear in the chiral expansion. Our overall estimate of the uncertainty arising from the chiral and continuum extrapolations is 10%, and adding this in quadrature to the systematic errors from all other sources we arrive at a total error of 10.4%. Our final value of the g_b coupling including statistical and systematic errors is:

$$g_b = 0.567(52)_{stat}(59)_{sys}. \quad (4.2)$$

A publication with full details of our analysis and error estimates is in progress.

5. Conclusions

The determination of physical quantities from lattice-QCD simulations with unphysically heavy up- and down-quark masses requires a chiral extrapolation to the physical point. For heavy-light mesons, theoretical guidance is provided by heavy-meson chiral perturbation theory. At leading order the HM χ PT Lagrangian has one low-energy constant g , which we have calculated for the theory with heavy b -quarks. Our calculation is the first directly at the physical b -quark mass, and has a complete systematic error budget. Comparing our result with other determinations [11, 12, 13] shows it lies in the region that would be expected from interpolating between the charm- and infinite-mass. Our result will be used by the RBC/UKQCD collaboration in the

chiral extrapolations of numerical lattice data for the B -meson leptonic decay constants [14] and $B \rightarrow \pi \ell \nu$ semileptonic form factor [15], and can also be used by other lattice collaborations working on B -physics. This will help to reduce the important, and in many cases dominant, systematic uncertainty from the chiral extrapolation.

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