The D_s, D^+, B_s and B decay constants from $2+1$ flavor lattice QCD

A. Bazavov[a], C. Bernard[b], C. Bouchard[c], C. DeTar[d], D. Du[e], A.X. El-Khadra[e], J. Foley[d], E.D. Freeland[f], E. Gámiz[g], Steven Gottlieb[h], U.M. Heller[i], J. Kim[j], J. Komijani[b], A.S. Kronfeld[k], J. Laiho[l,m], L. Levkova[d], P.B. Mackenzie[k], D. Mohler[k], E.T. Neil[k,n,o,α], M.B. Oktay[d], S. Qiu[d], J.N. Simone[*,k,β], R.L. Sugar[p], D. Toussaint[j], R.S. Van de Water[k], and R. Zhou[h,k]

[a]*Physics Department, Brookhaven National Laboratory, Upton NY 11973, USA*
[b]*Department of Physics, Washington University, St. Louis, MO 63130, USA*
[c]*Department of Physics, The Ohio State University, Columbus, OH 43210, USA*
[d]*Physics Department, University of Utah, Salt Lake City, UT 84112, USA*
[e]*Physics Department, University of Illinois, Urbana, IL 61801, USA*
[f]*Liberal Arts Department, School of the Art Institute of Chicago, Chicago, IL 60603, USA*
[g]*CAFPE and Depto. de Física Teórica y del Cosmos, Universidad de Granada, Granada, España*
[h]*Department of Physics, Indiana University, Bloomington, IN 47405, USA*
[i]*American Physical Society, One Research Road, Ridge, NY 11961, USA*
[j]*Department of Physics, University of Arizona, Tucson, AZ 85721, USA*
[k]*Fermi National Accelerator Laboratory, Batavia, IL 60510, USA*
[l]*SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK*
[m]*Department of Physics, Syracuse University, Syracuse, NY 13244, USA*
[n]*Department of Physics, University of Colorado, Boulder, CO 80309, USA*
[o]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*
[p]*Department of Physics, University of California, Santa Barbara, CA 93106, USA*
Email: [α]ethan.neil@colorado.edu   [β]simone@fnal.gov

Fermilab Lattice and MILC Collaborations

We present a study of the D and B leptonic decay constants on the MILC $N_f = 2+1$ asqtad gauge ensembles using asqtad-improved staggered light quarks and clover heavy quarks in the Fermilab interpretation. Our previous analysis [1] computed the decay constants at lattice spacings $a \approx 0.14, 0.11$ and 0.083 fm. We have extended the simulations to finer $a \approx 0.058$ and 0.043 fm lattice spacings, and have also increased statistics; this allows us to address many important sources of uncertainty. Technical advances include a two-step two-point fit procedure, better tuning of the heavy quark masses and a better determination of the axial-vector current matching. The present analysis remains blinded, so here we focus on the improvements and their predicted impact on the error budget compared to the prior analysis.

---

[*]Speaker.

1. Introduction

   Decays of B and D mesons provide an important testing ground for the CKM paradigm for flavor-changing weak interactions in the Standard Model. The D and B decay constants, which encapsulate the role of QCD interactions in these decay processes, are a crucial theoretical input. Precise calculation of these strong-coupling quantities, enabled by lattice simulations, is very important in predicting experimental rates for rare decays, such as the process $B^0_{(s)} \to \mu^+ \mu^-$ observed recently by LHCb and CMS [2], where the B-meson decay constants enter into the Standard Model rates. In addition, precise knowledge of the B^+-decay constant in combination with the observed $B^+ \to \tau^+ \nu$ decay rate probes the V-A structure of the Wub vertex and helps in understanding the tension between inclusive and exclusive determinations of $|V_{ub}|$.

   This study uses simulations on the ensembles listed in Table 1. Since Ref. [1] we have added ensembles labeled A through E, at two finer lattice spacings, and ensemble F at a sea-quark mass $m_l = 0.05 m_h$, nearer the physical mass. Statistical accuracy is also better, with about $3.6\times$ more $N_{config} \cdot N_{tsrc}$ combinations than in Ref. [1]. Better statistics, finer lattice spacings and a nearly physical sea quark mass all help to control the leading systematic uncertainties observed in Ref. [1] that arise from heavy- and light-quark discretization effects and the (chiral) extrapolation. This analysis also benefits from reduced uncertainty from the input charm and bottom quark masses, due to a retuning of the masses with improved techniques and higher statistics, and a reanalysis of the nonperturbative matching of the flavor conserving heavy and light vector currents, again with higher statistics.

id	a [fm]	beta	m_l/m_h	am_h	m_h/m_s	r_1/a	N_{config}	N_{tsrc}
A	0.043	7.81	0.2	0.014	1.079	7.208	801	4
B	0.059	7.46	0.1	0.018	1.019	5.307	827	4
C	0.058	7.465	0.139	0.018	1.024	5.330	801	4
D	0.058	7.47	0.2	0.018	1.028	5.353	673	8
E	0.058	7.48	0.4	0.018	1.037	5.399	593	4
F	0.083	7.075	0.05	0.031	1.255	3.738	791	4
G	0.083	7.08	0.1	0.031	1.256	3.755	1015	4
H	0.083	7.085	0.15	0.031	1.262	3.772	984	4
I	0.082	7.09	0.2	0.031	1.267	3.789	1931	4
J	0.081	7.11	0.4	0.031	1.290	3.858	1996	4
K	0.11	6.76	0.1	0.05	1.489	2.739	2099	4
L	0.11	6.76	0.14	0.05	1.489	2.739	2110	4
M	0.11	6.76	0.2	0.05	1.489	2.739	2259	4
N	0.11	6.79	0.4	0.05	1.534	2.821	2052	4
O	0.14	6.572	0.2	0.0484	1.156	2.222	631	24

Table 1: MILC asqtad ensembles and parameters.

2. Two-point fits

   We use a two-stage procedure for performing the two-point fits. In the first stage, plots of the effective mass are inspected for a stable "plateau" at large values of the source-sink separation t. A range $[t_{p,min}, t_{p,max}]$ is then chosen based on the correlator signal-to-noise ratio (SNR), maintaining SNR ≥ 10 for all correlators and holding the SNR range approximately fixed over different ensembles; this translates to fits at approximately equal physical distances on the different lattice spacings. The first-stage fit is carried out to a standard two-state functional form (one oscillating), with no excited states included.

   For the second stage, the results of the first-stage fit are used to set empirical Bayesian priors; best-fit values give the prior means, and the width is set equal to the best-fit one-sigma error estimate times an inflation factor of 3, to ensure that the second-stage fit parameters are not over-constrained. We have tested that increasing the inflation factor beyond this point has negligible

effect on the resulting final classical error estimates; however, the use of this two-stage procedure serves to stabilize the fits performed over bootstrap resampled data, by reducing the occurrence of outliers. We fit a basis of four (or five) smeared and local source two-point correlators including the two correlators having an $\mathcal{O}(a)$-improved axial-current at the sink in stage two. We find good isolation of the ground state when including four to five states (plus an equal number of oscillating states) and fitting down to $t_{min} = 2$.

3. Chiral fits

The decay constant f_{H_q} for a meson H_q is related to $\phi_{H_q} = f_{H_q}\sqrt{M_{H_q}}$, where

$$r_1^{3/2}\phi_{H_q} = \left(\frac{r_1}{a}\right)^{3/2}\sqrt{Z_{V_{QQ}^4}Z_{V_{qq}^4}}\,\rho_{A_{Qq}^4}\,a^{3/2}\phi_{H_q}^{lat}\quad. \tag{3.1}$$

Values for r_1/a are shown in Table 1. The flavor-conserving vector-current renormalization factors $Z_{V_{QQ}^4}$ and $Z_{V_{qq}^4}$ are found nonperturbatively, while $\rho_{A_{Qq}^4} = 1 + \mathcal{O}(\alpha_V)$ is known to one-loop order and is near unity. The $a^{3/2}\phi_{H_q}^{lat}$ are determined by fitting two-point functions.

Guided by heavy meson staggered chiral perturbation theory [3], we fit to the function

$$\phi_{H_q}(m_q,m_h,m_l) = \Phi_H\left[1 + \Delta f_{H_q}(m_q,m_h,m_l) + P(m_q,m_h,m_l) + K(am_Q) + c_a a^2\right]\quad. \tag{3.2}$$

Term Δf_{H_q}, parameterizing the NLO chiral logarithms, includes hyperfine splitting effects. They are corrected for staggered taste effects at finite lattice spacing and for finite volume. The polynomial P includes analytic terms up to second order in the quark masses. The K terms, which parametrize the leading-order heavy quark discretization effects, are constrained in fits according to power counting estimates [1]. The residual heavy-quark discretization error for the physical ϕ_{H_q} is then incorporated into the overall statistical error.

Figure 1 shows a preliminary fit for the model in Equation (3.2) with all the D-meson ϕ_{H_q} simulation results at the five lattice spacings listed in Table 1. Figure 2 is the corresponding plot for the B-meson system. The figures indicate that the model adequately represents the simulation results. Points differing only by valence light-quark mass are shown in the individual plot subpanels. A reasonable fit is obtained even when the fit curve may visibly deviate from simulation results on a particular ensemble since points there are strongly correlated.

Figure 3 shows the combined continuum and chiral extrapolation curve (and error band) for the D- and B-meson systems. Shown overlaying each of the extrapolated B and D fit curves are the (finite a) "full QCD" points where $m_q = m_l$. No points correspond exactly to the tuned strange quark mass, consequently, we show a subset of points with roughly $m_q \approx m_s$ overlaying each of the B_s and D_s curves.

4. Predicted errors and outlook

Our results remain blinded, hence, we do not quote values for the decay constants here. We will continue to use the blinded analysis to understand systematic effects. In Figure 4 we compare predicted errors in this study to Ref. [1]. This analysis has higher statistics and includes results
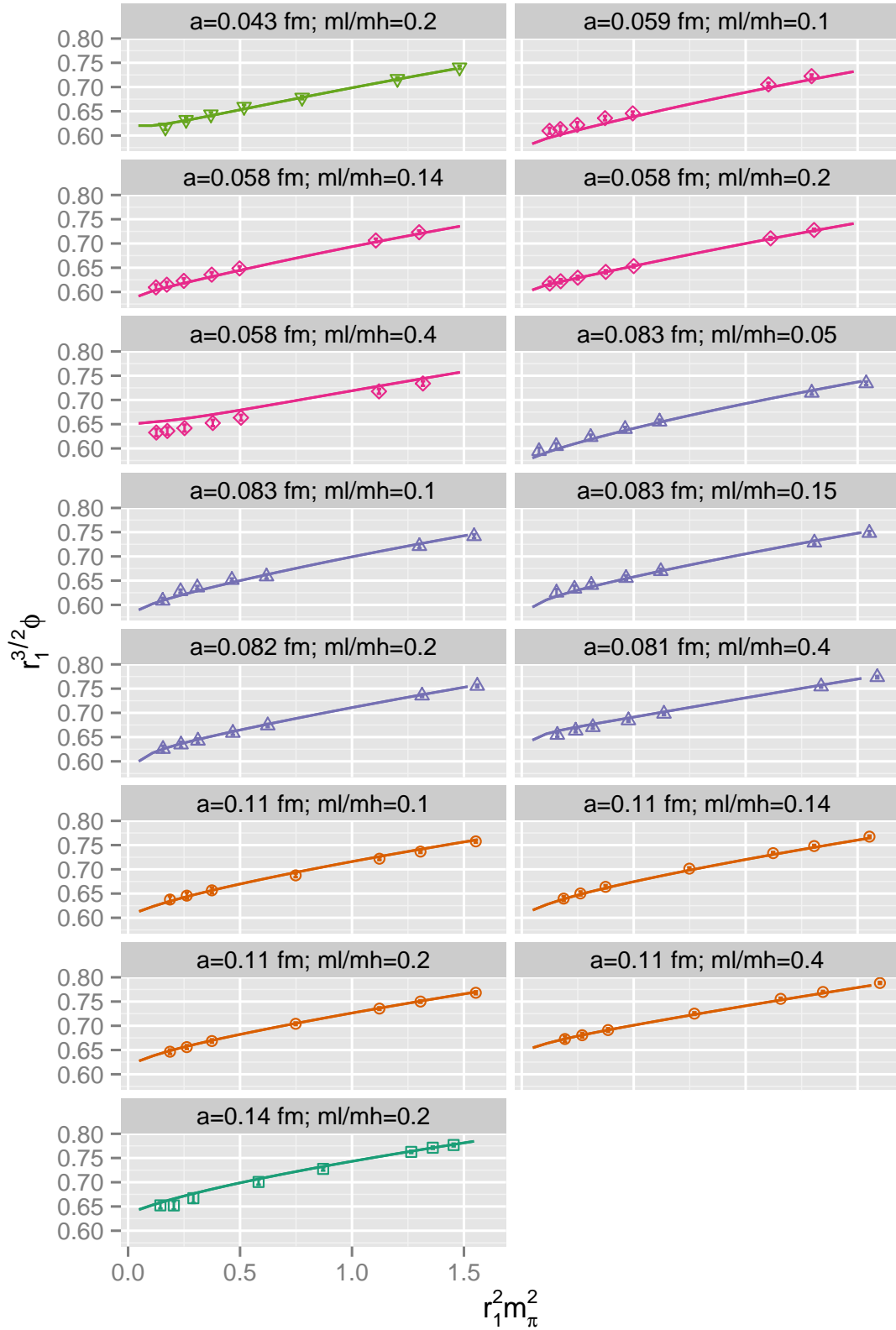
Figure 1: Fit of all *D* system partially quenched data to the *SXPT* model function.
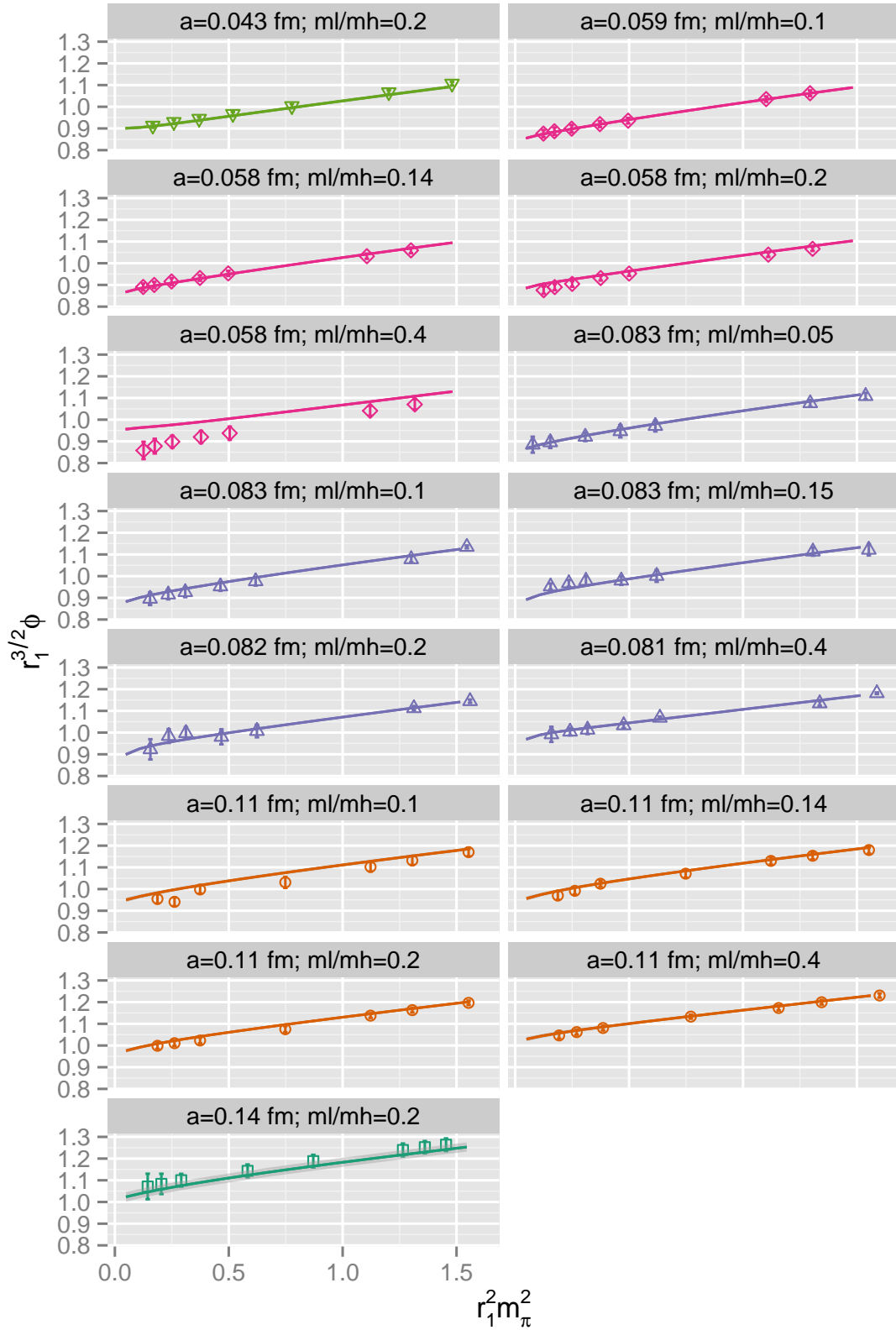
Figure 2: Fit of all *B* system partially quenched data to the *SXPT* model function.
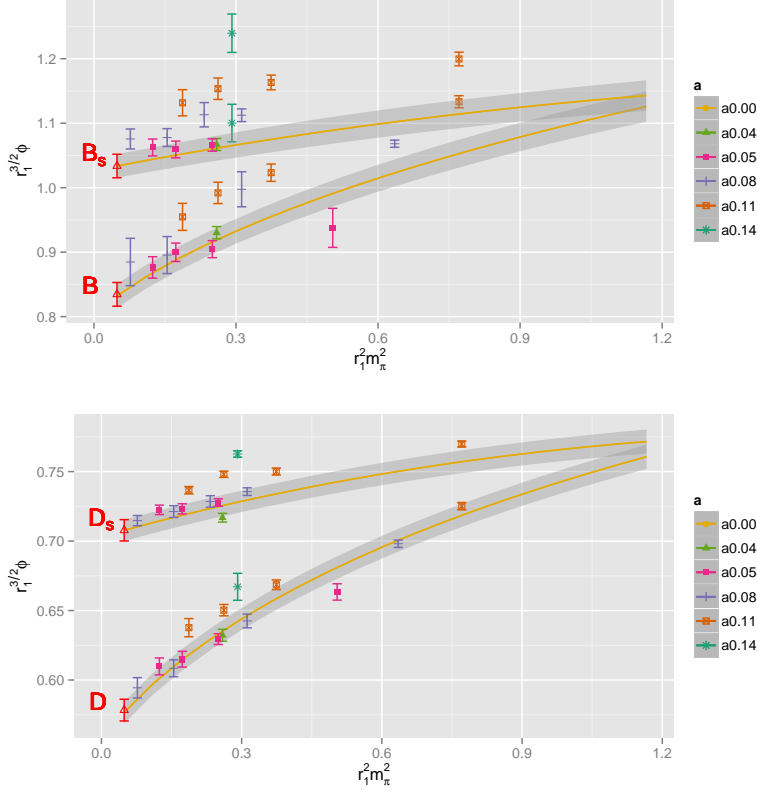
Figure 3: Combined chiral continuum extrapolation for the B (top) and D (bottom) systems. The curves and statistical error bands in the $a \rightarrow 0$ limit at physical quark masses are labeled "a0.00". The extrapolated values (and errors) are the points labeled by triangles at the physical pion mass. Note: these results are blinded by normalization factors known only to a few collaboration members, who are outside the analysis group. Note: the subset of points shown for each of the B_s and D_s extrapolations serve merely as a guide since their valence masses only approximate the physical strange quark mass: $r_1|m_q - m_s| < 0.07$.

at finer $a \approx 0.058$ and 0.043 fm lattice spacings leading to improved estimates for discretization effects modeled in the chiral fit function. We anticipate this will lead to a reduction in the residual discretization errors for the physical decay constants. The addition of results with $m_l = 0.05 m_h$, nearer to the physical quark (ensemble F in Table 1), narrows the extent of the chiral extrapolation, which is expected to reduce the residual chiral extrapolation uncertainty.

The current study includes several technical improvements compared to Ref. [1]. We have introduced a two-step procedure for two-point fitting which allows us to stably model more excited states and, hence, better utilize two-point data at small times where the signal-to-noise ratio is larger. This fit procedure also better preserves expected (significant) correlations among points by reducing the likelihood of finding outliers in output bootstrap distributions that tend to wash out correlations. The new procedure and better statistics help to directly reduce statistical errors as well as the "2-pt fit" uncertainty previously estimated from plausible variations in fitting procedures. We have reduced the error in the decay constants due to the input heavy quark mass through improvements to the tuning process used to determine the charm and bottom quark masses: a) We
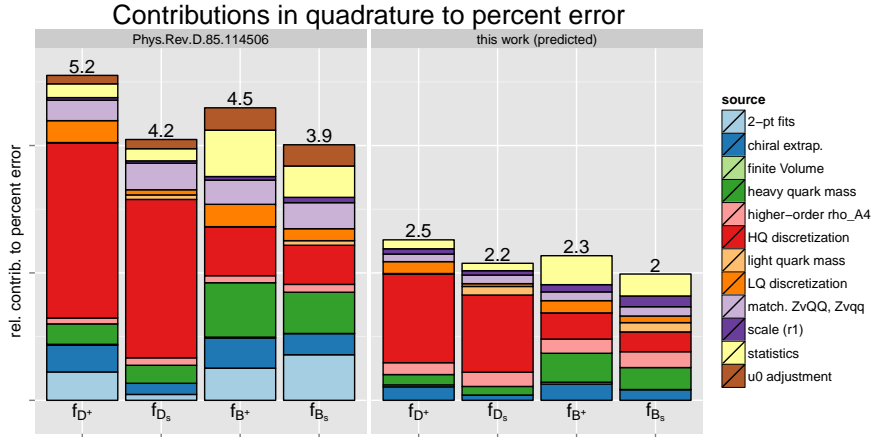
Figure 4: The relative contributions to the error in quadrature from sources of uncertainty. On the left a summary of our previous analysis [1]. On the right are our predictions for this analysis.

have four times the statistics than in prior tuning runs. b) We employ priors in energy-momentum dispersion relation fits which help stabilize the kinetic masses that are matched to the physical D_s and B_s masses. c) We compensate for mistuning of the strange sea-quark masses in simulations. d) We smoothly extend the charm and bottom tunings from the subset of ensembles used for tuning to all other ensembles. Uncertainties in the decay constants due to the flavor-conserving matching factors Z_{V^4} for both clover and staggered currents have been reduced by new determinations using better stochastic color wall meson sources and higher statistics.

From Figure 4, we anticipate that the total error in quadrature for the decay constants in this study will be around half of the error found in Ref. [1]. We are continuing to refine this analysis before unblinding the values for the decay constants.

Acknowledgments

References

[1] **Fermilab Lattice and MILC Collaborations**, A. Bazavov, *et al.*, *B- and D-meson decay constants from three-flavor lattice QCD*, *Phys. Rev.* **D85** (2012) 114506, [1112.3051].

[2] CMS and LHCb Collaborations, *Combination of results on the rare decays $B^0_{(s)} \to \mu^+\mu^-$ from the CMS and LHCb experiments*, Tech. Rep. CMS-PAS-BPH-13-007. CERN-LHCb-CONF-2013-012.

[3] C. Aubin and C. Bernard, *Staggered chiral perturbation theory for heavy-light mesons*, *Phys. Rev.* **D73** (2006) 014515, [hep-lat/0510088].