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Theoretical Perspective of Charm Physics

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A perspective on charm physics, emphasizing recent developments, future prospects, and the interplay with lattice QCD.

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1 Interplay of Charm and Lattice QCD

A talk opening a conference should give participants something to chat about while slurping coffee, touring a stately home, or dining at the Old Trafford. This talk is supposed to look at the theory of charm physics. I'd like to do so through the lens of a narrative, which was developed as port of the CLEO-c proposal [1], that experimental charm physics can and should be used to validate lattice QCD. This story is now twelve years old, and it is time to ask whether it is still as useful (scientifically) in 2013 as in 2001.

Ths first thing to keep in mind is that the least familiar aspects of numerical lattice gauge theory are very well tested. Monte Carlo integration codes are checked numerous ways, such as reproducing expansions around the strong- and weak-coupling limits, writing two separate programs, and, nowadays, carrying out unit tests of individual modules. The codes that compute quark propagators have always had a simple built-in test. They solve a big matrix problem of the form $MG = b$, and the solution G can always be muliplied with the Dirac matrix M to check whether b is reproduced. In fact, lattice QCD practitioners were in a position to pioneer scientific usage of GPUs [2, 3], because this check makes it possible to use gamers' cards that don't implement error-correcting arithmetic.

There are also several theoretical properties of gauge theories than can be tested without reference to experiment. For example, the eigenvalues of the Dirac matrix satisfy certain theorems [4]. The hep-lat arXiv contains numerous papers exploring this connection, far too many to cite here; for a review of one slice of this work, see Ref. [5]. A bit closer to experiment, observables such as the pion mass must depend on the quark mass in a way consistent with spontaneously broken chiral symmetry. Monitoring his behavior is a routine part of lattice QCD.

Finally, lattice gauge theory starts with a mathematically sound footing to define continuum gauge theory. To interpret numerical data at nonzero lattice spacing and at finite volume, we use the same kind of effective field theories used throughout theoretical physics. Once the numerical data have been generated, any well-trained theorist should be able to fit them to an EFT formula and learn something from the fit.

Most of these points could have been made twelve years ago, so what has changed? First, most lattice-QCD calculations of the twentieth century were marred by something called the quenched approximation, in which the dynamical effects of quark loops are omitted, and approximated very roughly with shifts in the bare parameters. The error associated with this approximation is difficult to estimate. In addition, the up and down quark masses in the computer took values around that of the strange quark, which stymied the chiral tests mentioned above. In 2001, algorithms and computers began to overcome these obstacles. The time was ripe to try to compute with lattice QCD some observables related to charm physics: the calculations were

feasible; the correct results were not known experimentally; and experiments were about to improve the measurements, especially CLEO-c, BaBar, Belle, and CDF. These calculations could test the foundations, the methodology, and—not least—the practitioners.

After several successful postdictions in 2003 [6], a collaboration of collaborations (of which I am a member) used the same methods (the fastest ones) to predict semileptonic form factor for $D \to Kl\nu$ [7], charmed-meson decay constants [8], and the mass of B_c meson [9]. These were all quickly confirmed by experiment. Meanwhile, all of these calculations have been updated and extended: the normalization [10] and shape [11] of the form factors, and the B_c mass with an untested prediction of the B_c^* mass, $M_{B_c^*} = 6.330(9)$ GeV [12, 13]. (References to updates of decay constants are given in Sec. 3.) Meanwhile, there are numerous other results of general interest (reviewed in Ref. [14]), including calculations of the baryon mass spectrum to 2–4% [14, 15]. Indeed, as discussed in other talks at this conference, lattice-QCD spectroscopy has advanced to excited [16] and exotic [17] states.

In light of this progress, we should ask ourselves several questions. Where do inexorably smaller lattice-QCD errors lead? Does it make sense to mount new experiments just to test the results of the (much less expensive) computers? What is the scope—and where are the challenges—of lattice QCD? Many interesting problems are precisely those that cannot be validated, because theory and experiment are in conversation. Tests of QCD, just like tests of QED, have their place, but it is more interesting to ponder the muon's anomalous magnetic moment as a probe of new physics rather than a test of QED. It is somewhat amusing to hear (from some experimenters) that simple matrix elements ("gold-plated" in the sense of Ref. [6]) still need validation, while very very messy things, like the structure of the XYZ states, will be understood only with lattice QCD [18].

The following sections survey recent developments in charm physics with this narrative in mind. I think that its utility has run its course, and that there are more interesting facets to discuss. I'll talk about the charm mass and its role in Higgs physics (Sec. 2), leptonic and semileptonic decays (Sec. 3), and D^0-$\bar{D}^0$ mixing and nonleptonic decays (Sec. 4). Section 5 provides an outlook.

2 Charm and the Higgs Boson

The biggest news in particle physics since Charm 2012 is the observation at the LHC of a new particle with mass 126 GeV [19, 20]. As measurements of its properties improve, it is beginning to look a lot like a standard Higgs boson [21]. The measured mass lies in the region for which the standard Higgs boson has many measurable branching ratios, including $H \to c\bar{c}$. This decay is interesting, because a comparison of the measured branching ratio with the standard-model expectation tests the hypothesis

that the standard Yukawa couplings generate the mass of up-like quarks. In the standard model, $\mathrm{BR}(H \to c\bar{c}) \approx 2 \times 10^{-2}$, with $M_H = 126$ GeV [22].

The essential point of contact with charm physics is the charmed quark mass: the very nature of fermion mass generation via Higgs bosons is that the branching ratio is proportional to m_c^2. For the standard Higgs boson, the branching ratio is also approximately proportional to m_b^{-2}, because $H \to b\bar{b}$ is the dominant branch. The most precise determinations of both masses comes from measuring the quarkonium correlator, in the timelike region, in e^+e^- collisions near $Q\bar{Q}$ threshold, or from computing the same correlator, in the spacelike region, with lattice QCD. The latest results are (in the $\overline{\mathrm{MS}}$ scheme) $m_c(m_c) = 1.279(13)$ GeV [23] and $m_c(m_c) = 1.273(6)$ GeV [24], respectively. For more details, see Ref. [25]. Note that the Particle Data Group [26] acknowledges the lattice-QCD result as the most accurate, as it does for α_s.

3 Leptonic and Semileptonic Decays

Semileptonic decays offer an example of the past decade's developments. The plot on the left of Fig. 1 shows the 2004 calculation of the $D \to Kl\nu$ semileptonic form factors [7, 27], together with measurements from Belle [28], Babar [29], and CLEO-c [30, 31]. To make the comparison, the measured $|V_{cs}| f_+(q^2)$ is divided by $|V_{cs}|$ as deduced from CKM unitarity. The plot on the left was widely hailed as a validation, and provided confidence that similar form factors could be used to determine $|V_{ub}|$ from $B \to \pi l\nu$ decay.

CLEO-c's final results [32] land on the lower edge of the error band from lattice QCD *anno* 2004. This development is shown on the right of Fig. 1: the data lie near the bottom of the straw-colored band. A 2013 calculation [11] (brown band), which is both more precise (higher statistics) and more accurate (smaller systematics),

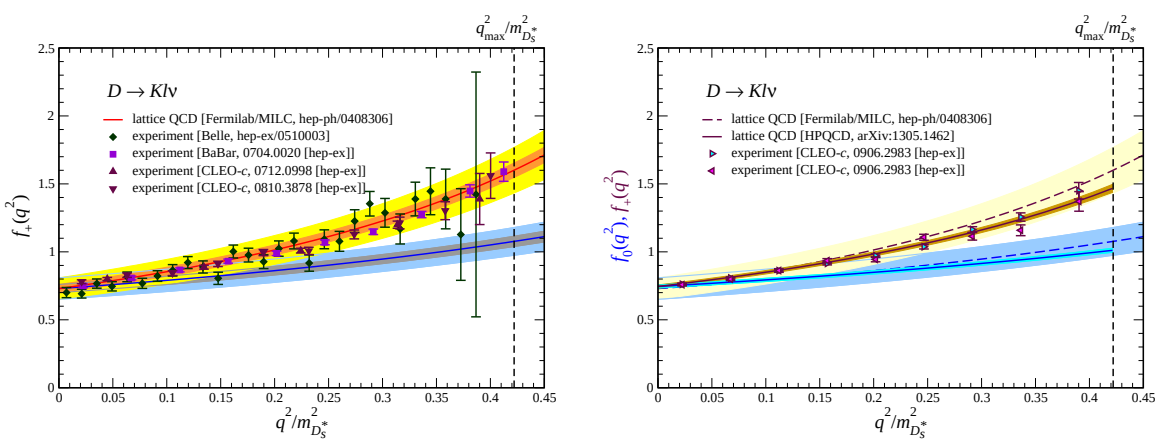

Figure 1: Semileptonic form factors, then and now.

goes straight through the experimental points. The systematic improvement stems from small-enough lattice spacing so that charm quarks are now "light" quarks, not "heavy" quarks. One now follows the strategy of B decays, determining $|V_{cs}|$ from a combined fit to lattice-QCD and experiment [11]. The agreement of the experimental data with the new lattice-QCD curve tells us that the unitarity tests of the second row and second column are well satisfied. For further discussion, see Ref. [34].

Leptonic decays were supposed to yield a similar narrative, but did not. Figure 2 shows the history of unquenched lattice-QCD calculations, together with recent measurements [35, 14]. (Earlier measurements had very large error bars.) The first two with $\sim 10\%$ errors [36, 37] agreed well with the first $n_f = 2+1$ calculation (i.e., up, down, and strange quarks in the sea) [8]. In 2007, however, the first calculation of f_{D_s} treating charm with light-quark methods [33] found a result that was significantly—nearly 4σ at one stage—lower than experiment. This discrepancy caused some consternation, including studies of how new physics could enter this process and not others [38, 39]; see also Ref. [40].

The history has been complicated by two notable shifts from normalization corrections. In one case, HFAG [41] proposed that the denominator of the relative branching ratio measured in Ref. [36] could be improved, using a more suitable mea-

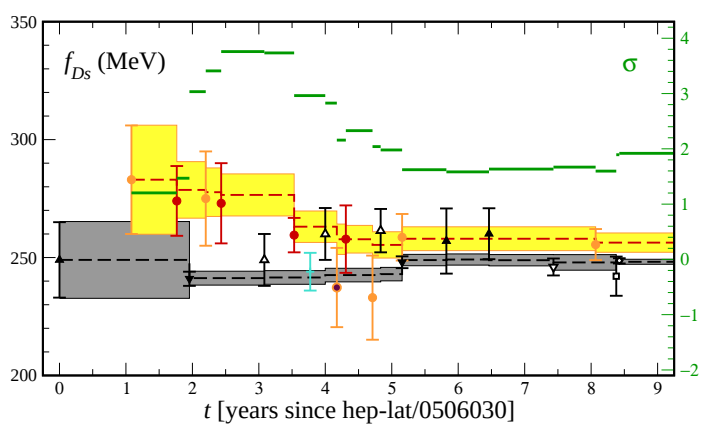

Figure 2: f_{D_s} puzzle over the years. Author's averages of f_{D_s} from lattice QCD (gray band) and $|V_{cs}|f_{D_s}$ from experiment, divided by $|V_{cs}|$ from CKM unitarity (yellow band). Measurements from charm threshold (red); and D_s in flight at the $\Upsilon(4S)$ (orange). For the calculations, the number of sides of the symbols corresponds to n_f, the number of sea quarks. Closed symbols are published; open are conference proceedings. Symbol rientation denotes charm formulation: Fermilab method denoted by up-pointing triangles; staggered (HISQ) by down-pointing triangles ($n_f = 2+1$) and diamonds ($n_f = 2+1$); twisted-mass Wilson fermions by flat bars ($n_f = 2$, cyan, not in the average) and squares ($n_f = 2+1$). The right axis and green curve denote the discrepancy in σ. See Refs. [35, 14] for a full set of references.

sure of $\phi \to KK$. As a consequence the first orange point (near $t = 1$) in Fig. 2 moved down, to the one with a maroon filling (near $t = 4$). Had this normalization been in place from the outset, no puzzle would have arisen. In another case, the conversion from lattice units to MeV changed with the analysis of a wider set of data (especially finer lattices), causing the step in the gray band near $t = 5$. The discrepancy, if it can be called that, now stands under 2σ (with σ principally from experimental statistics).

The total uncertainty for f_D and f_{D_s} from lattice QCD has now reached the level of 0.5% [42, 43, 44]. To push further—in kaon physics as well—we have to worry more about isospin and QED effects. Isospin is principally a computing problem. But how should one to connect a matrix element computed in finite volume to the physical photon cloud? Other effects are worth mentioning. Electroweak box diagrams lead to a correction factor $1 + (\alpha/\pi)\log(M_Z/\mu)$ for all leptonic and semileptonic decays of hadrons [45]. In between the electroweak scale of and that of bremsstrahlung [46] lie electromagnetic corrections that depend on hadron structure. The interfaces between these scales should be handled with effective field theory, as it is for kaons [47], with lattice QCD+QED, or with a judicious combination.

4 D^0-$\bar{D}^0$ Mixing and Nonleptonic Decays

At a superficial glance, D^0-$\bar{D}^0$ mixing is like mixing in other neutral meson systems, K^0-$\bar{K}^0$, B^0-$\bar{B}^0$, and B_s^0-$\bar{B}_s^0$. Indeed, short-distance contributions to ΔM_D and $\Delta\Gamma_D$—whether standard or BSM—follow the same pattern. In particular, any nonstandard model can be described by five matrix elements of $\Delta C = 2$ operators. Knowing the matrix elements can, thus, constrain extensions of the standard model [48]. Lattice-QCD calculations of all five matrix elements are underway [49].

Because, however, the neutral D system mixes via diagrams with down-type quarks in the loop, long distance effects, in which two $\Delta C = 1$ interactions transpire at distances of order $\Lambda_{\rm QCD}^{-1}$ apart, can be just as large as the short distance contributions. These effects are notoriously difficult to control: m_c is too small for heavy-quark methods, while M_D is too large for hadronic methods [50]. Note that lattice QCD has recently tamed the long-distance contribution to ΔM_K [51] with methods like those needed to tackle nonleptonic decays.

So far, lattice QCD has not played a significant role in (strong or weak) nonleptonic decays. Nonexperts should read the next sentence carefully: The problem is not the lattice, i.e., the discrete spacetime; instead it arises from a conflict between Euclidean space and finite volume [52], two other choices necessary to place the problem on a computer. It would be wonderful to surmount this obstacle, because some of the most exciting physics is nonleptonic—for example the saga of $\mathcal{A}_{\rm CP}(D \to \pi\pi) - \mathcal{A}_{\rm CP}(D \to KK)$ [53]. Until recently, progress in nonleptonic D decays seemed hopeless (to me anyway), but now a recent spurt of activity provides some hope.

The foundation for the new work was developed over 25 years ago with Lüscher's formalism for (nearly) elastic problems such as $\pi\pi \to \pi\pi$ scattering [54] and $\rho \to \pi\pi$ decay [55]. Two notable extensions of this work have been to moving frames [56] and weak decays such as $K \to \pi\pi$ [57]. These have only recently become tractable in numerical lattice QCD [58].

The two key insights are that the energy spectrum of a quantum field theory in a finite volume is discrete and, moreover, that the energy levels are intimately related to the S matrix. Because the Euclidean-space evolution operator $\exp(-\hat{H}x_4)$ is just as suitable as $\exp(i\hat{H}t)$ for computing eigenvalues of the Hamiltonian $\hat{H}$, these insights allow us to use the favorite tool of lattice gauge theory—the exponential fall-off of correlations functions—to access information about the S matrix.

Considerable mathematical physics (scattering theory on a torus) leads to a master formula. If one partial wave dominates, the formula simplifies, and one directly obtains the 2×2 scattering amplitude $\mathcal{M}_{2\to 2}$ from

$$F(E_n, \boldsymbol{P}, L)\, \mathcal{M}_{2\to 2}(\mathcal{E}_n) = -1, \tag{1}$$

where $\mathcal{E}_n^2 = E_n^2 - \boldsymbol{P}^2$, and E_n is the nth energy level of momentum $\boldsymbol{P}$ in a periodic box of size L. Mathematics, not dynamics, provides the function F. The algorithm, then, is to choose box size L at the outset, pick several two-body $\boldsymbol{P} = 2\pi(\boldsymbol{n}_1 + \boldsymbol{n}_2)/L$, and compute the levels E_n for these $\boldsymbol{P}$. If a few partial waves matter, then one must resolve a few$\times$few determinant, but the basic structure of choosing $\boldsymbol{P}$, computing E_n, and plugging into a formula like Eq. (1) still holds.

As mentioned above, the old formalism tackled only elastic (or nearly elastic) kinematics. For nonleptonic D decays, or for the long-distance part of D^0-$\bar{D}^0$ mixing, rescattering effects play a role, however. In the past year or so, many authors have been generalizing these methods, taking steps to understand hadronic systems with more than two hadrons [59, 60, 61]. In particular, it is now known (conceptually) how to compute the 3×3 scattering amplitude [62, 63, 64, 65]. Weak amplitudes, needed for D decay, can then be obtained from formalism for strong interactions via perturbation in weak Hamiltonian [57, 59]. All this ideas, and more (probably including effective field theories, as in Ref. [66]) will be needed before tackling processes such as $\pi\pi \to \pi\pi\pi\pi$ and $\pi\pi \to \pi\pi\pi\pi\pi\pi$, which are a prerequisite to nonleptonic D decays and long-distance D mixing. It is too early to see the light at the end of the tunnel, but at least the tunnel has been breached.

5 Outlook

The LHC experiments have observed a particle that looks like the standard Higgs boson, but not the bevy of other particles anticipated by TeV-scale model builders. Without such new states, the way forward in particle physics is through precision

physics, such as rare and sensitive processes. Precise experiments require commensurately precise theoretical calculations. In my view, the past decade has seen lattice QCD move into this arena, down to the 1–2% level. Experiments studying charmed quarks played an important role in this enterprise. That said, it seems to me that the time has come to use lattice QCD and experiment together, to understand physics better.

Indeed, lattice QCD has entered an era where the challenges lie in aspects that will be harder for experiments and other theoretical approaches to verify. In some cases, small effects such as QED and isospin violation must be incorporated. In other areas, precision is not yet paramount. Rigorous treatment of many problems in charm physics will require a rigorous treatment of multi-hadron states. This includes excited-state and exotic spectroscopy [16, 17], as well as nonleptonic D decays.

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