FERMILAB-CONF-13-520-T

PROCEEDINGS OF SCIENCE

$K\pi$ scattering in moving frames

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We extend our study of the $K\pi$ system to moving frames and present an exploratory extraction of the masses and widths for the K^* resonances by simulating $K\pi$ scattering in p-wave with $I = 1/2$ on the lattice. Using $K\pi$ systems with non-vanishing total momenta allows the extraction of phase shifts at several values of $K\pi$ relative momenta. A Breit-Wigner fit of the phase renders a $K^*(892)$ resonance mass and $K^* \to K\pi$ coupling compatible with the experimental numbers. We also determine the $K^*(1410)$ mass assuming the experimental $K^*(1410)$ width. We contrast the resonant $I = 1/2$ channel with the repulsive non-resonant $I = 3/2$ channel, where the phase is found to be negative and small, in agreement with experiment.

31st International Symposium on Lattice Field Theory - LATTICE 2013
July 29 - August 3, 2013
Mainz, Germany

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Operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

1. Overview

In the lattice studies of meson excitations is has become clear that it is not sufficient to use quark-antiquark interpolators only. Although the dynamical quark vacuum allows in principle the coupling of $\bar{q}q$ operators to meson-meson intermediate states, those have usually not been observed in the correlation functions for $\bar{q}q \to \bar{q}q$ alone. If one enlarges the set of interpolators to include also 4-quark (meson-meson) operators the full system does exhibit energy levels related to these states.

In [1] we have studied the $K\pi$ system for both isospin values $\frac{1}{2}$ and $\frac{3}{2}$ in s-wave and p-wave in the rest frame. Due to the small lattice size $m_\pi L \simeq 2.7$ only one energy level was in the resonance region of the $K^*(892)$ vector meson. Changing to interpolators in moving frames allows us to obtain further energy levels on the same set of configurations. For that reason we extended our analysis for the p-wave to values of non-zero total momentum. The determination of $K^*(892)$ is challenging even in this case since the experimental width is only 50 MeV and even smaller for $m_\pi > m_\pi$phys.

Scattering of two mesons is described by correlation functions $C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j(0)\rangle$ where t denotes the Euclidean time and $\mathcal{O}$ operators with the channel's quantum numbers. The energy spectrum obtained from the eigenvalues of such propagators for finite spatial volumes is discrete. Under certain conditions (localised interaction region smaller than the size of the volume) one may relate these energy levels to values of the scattering phase shift in the continuum. The corresponding relations were first formulated for two identical particles in the rest frame by Lüscher [2, 3], for moving frames in [4, 5] and for mesons with unequal masses in [6 – 8]. Moving frames considerably complicate the analysis. Unfortunately, for moving frames there occurs mixing of partial waves and due to relativistic distortion the symmetry group changes from the simple cubic O_h to dihedral subgroups. One has to write the interpolators as representations of such groups.

Here we briefly discuss the main steps of our analysis and the principal results. More details can be found in [1, 9].

2. Analysis Details

Configurations: The analysis is based on one ensemble of gauge configurations on lattices of size $16^3 \times 32$, with clover Wilson dynamical, mass-degenerate u, d quarks and u, d, s valence quarks ($m_s > m_u = m_d$). This ensemble has been generated by the authors of [10, 11] while studying re-weighting techniques. The corresponding pion mass is $m_\pi = 266(4)$ MeV. The strange quark valence mass is fixed by m_ϕ, which corresponds to $\kappa_s = 0.12610$ and $c_{sw} = 1$, giving $m_K = 552(6)$ MeV. The parameters of the ensemble are given in [12, 1, 9]. Due to the limited data on just a single ensemble, our determination of the lattice spacing $a = 0.1239(13)$ fm results from taking a typical value of the Sommer parameter $r_0 = 0.48$ fm. We note that the uncertainty associated with this choice might lead to small shifts of all dimensionful quantities.

Variational analysis: We determine the energy levels of the coupled K^* and $K\pi$ system with help of the variational method [13 – 16]. For a given quantum channel one measures the Euclidean cross-correlation matrix $C(t)$ between several interpolators living on the corresponding Euclidean time

slices. The generalized eigenvalue problem disentangles the eigenstates $|n\rangle$. From the exponential decay of the eigenvalues one determines the energy values of the eigenstates by exponential fits to the asymptotic behavior. In order to reliably obtain the lowest energy eigenstates and energy levels one needs a sufficiently large set of interpolators with the chosen quantum numbers.

Wick contractions: In order to compute this correlation matrix one has to compute the Wick decomposition of the correlators in terms of the quark propagators. There are no completely disconnected terms, however there are contributions with backtracking quark lines. This necessitates an efficient algorithm to reliably determine those.

Distillation: We used the so-called distillation method [17]. On a given time slice one introduces separable quark smearing sources derived from the eigenvectors of the spatial lattice Laplacian. This allows high flexibility due to the disentanglement of the computation of the quark propagators ("perambulators") and the hadron operators.

Phase shifts: There are relations between the energy levels at finite volume and the phase shifts of the infinite volume, valid in the elastic region [2 – 5, 7, 8]. We apply these up to the region of the second resonance, as discussed below. More detailed studies would have to include more operators and deal with the coupled channel problem.

Moving frames: In order to facilitate the $K^* \to K\pi$ decay kinematically and to access further values of the center-of-momentum frame (CMF) energy, we implement interpolators with vanishing as well as non-zero total momenta $\vec{P}$ (we considered also all permutations of $\vec{P}$ and all possible directions of polarizations at given $\vec{P}$). The relativistic distortion of the lattices reduces the symmetries:

$$\begin{aligned}
&\vec{P}=\tfrac{2\pi}{L}\vec{e}_z: && C_{4v}\,,\ \text{irreps } E(\vec{e}_{x,y}),\ E(\vec{e}_x\pm\vec{e}_y), && l=1,2\,,\\
&\vec{P}=\tfrac{2\pi}{L}(\vec{e}_x+\vec{e}_y): && C_{2v}\,,\ \text{irreps } B_2,\ B_3, && l=1,2\,,\\
&\vec{P}=0: && O_h\,,\ \text{irrep } T_1^-, && l=1\,.
\end{aligned} \tag{2.1}$$

The zero-momentum case, studied in [1], is listed for completeness since we combine all these results. The analytic framework for p-wave scattering using the first two momenta is described in detail in [7], together with the symmetry considerations, appropriate interpolating fields and extraction of the phase shifts.

The symmetry group of the mesh viewed from the CMF of the $K\pi$ system is C_{4v} for $\vec{P}=\frac{2\pi}{L}\vec{e}_z$ and C_{2v} for $P=\frac{2\pi}{L}(\vec{e}_x+\vec{e}_y)$. These groups do not contain the inversion as an element, which in turn implies that even and odd partial waves can in principle mix within the same irreducible representation. An example would be the irrep A_1, where $\delta_{l=1}$ mixes with $\delta_{l=0}$, since $\delta_0(s)$ is known to be non-negligible in the whole energy region of interest. However, δ_1 does not mix with δ_0 in the irreducible representations E, B_2, B_3 (see equation (2.1)), so we can use these. In fact we employ two representations of the two-dimensional E: $E(\vec{e}_{x,y})$ with basis vectors along axis $(\vec{e}_x,\vec{e}_y)$ and $E(\vec{e}_x\pm\vec{e}_y)$ with basis vectors along the diagonal $(\vec{e}_x+\vec{e}_y,\vec{e}_x-\vec{e}_y)$.

Interpolators: The interpolators are constructed to transform according to irreducible representations B_2, B_3, $E(\vec{e}_{x,y})$, $E(\vec{e}_x\pm\vec{e}_y)$, and T_1^-. This will give energy levels E_n, values of $s=E_n^2-\vec{P}^2$ and scattering phases $\delta(s)$.

We use five operators (for $I = 1/2$) combining terms of the type

$$\mathscr{O}(\vec{p},t) = \sum_{\vec{x}} \mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}} \bar{s}(\vec{x},t)\,\Gamma\, u(\vec{x},t)\ , \tag{2.2}$$

where Γ includes Dirac structure and derivatives. The combinations depend on the used representations of the symmetry groups. For $I = 3/2$ there are no quark-antiquark interpolators.

In addition to the quark-antiquark operators we used two $K\pi$ interpolators; these are linear combinations of $K(\vec{p}_K)\pi(\vec{p}_\pi)$ where momenta for K and π are separately projected

$$K^+(\vec{p}_K,t) = \sum_{\tilde{\mathbf{x}}} \mathrm{e}^{\mathrm{i}\vec{p}_K\cdot\vec{x}} \bar{s}(\vec{x},t)\,\gamma_5\, u(\vec{x},t)\ , \quad \pi^+(\vec{p}_\pi,t) = \sum_{\vec{x}} \mathrm{e}^{\mathrm{i}\vec{p}_\pi\cdot\vec{x}} \bar{d}(\vec{x},t)\,\gamma_5\, u(\vec{x},t)\ , \tag{2.3}$$

and analogously for K^0 and π^0. We simulated three permutations of direction $P = \frac{2\pi}{L}\vec{e}_z$, and three of direction $P = \frac{2\pi}{L}(\vec{e}_x + \vec{e}_y)$. The complete list is given in [9].

Experience shows that in the elastic region one typically has only three situations:

- Two-meson levels close to the energy of the non-interacting two meson system; the energy shift is small and the corresponding phase shift close to multiples of π.
- Isolated resonance levels with little influence from the two-meson states; that is the case, when the resonance width is much smaller that the next nearby two-meson level.
- Energy levels in a region where two-meson states and the resonance state superimpose, the so-called region of avoid level crossing.

In fact, by including or removing the two-meson interpolators with different relative momenta one usually sees quite clearly an added or removed energy level and whether the other energy levels are shifted. The picture emerges that the quark-antiquark interpolators couple (and generate) the generic resonance states and the two-meson operators the generic two-meson states and only in the region, where their respective eigen-energies overlap, there are considerable effects on energy levels already observed in either the quark-antiquark or meson-meson basis. This is essentially the unitarization effect.

One thus observes, that the two-meson interpolator with higher relative momenta have little influence on the low lying eigenstates. Since (at least in the elastic region) there are – depending on the spatial size Lm_π – only a few (in our case one or two) such operators, we are confident, that the observed levels would not change when adding further interpolators.

3. Results

The phase shift for $K\pi$ scattering was extracted from experiment long time ago [20, 18, 19]. In phenomenological studies the K^* resonance-pole emerged for example within the Roy-equation approach and/or unitarized versions of the Chiral Perturbation Theory (for references see [9].

In [9] we present a table of the energy levels resulting from our analysis. The corresponding phase shift values are compared with the experimental values in Figs. 1 and 2.

The $I = 1/2$ channel: We obtain four phase shift points in the vicinity of $K^*(892)$ indicating a

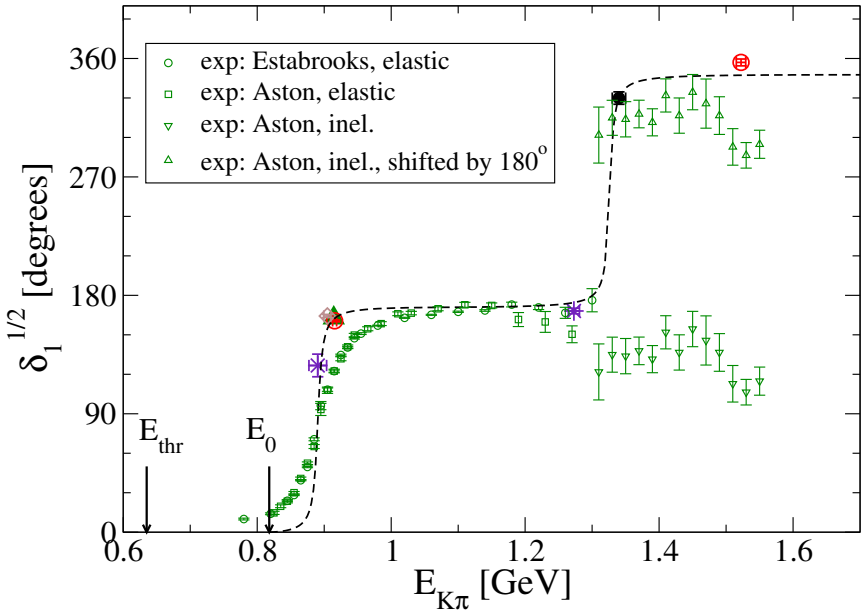

Figure 1: The p-wave scattering phase shift $\delta_{l=1}$, $I = 1/2$ as a function of $E_{K\pi} \equiv \sqrt{s}$. Different colors/symbols indicate results from different irreducible representations, while the point at $E_{K\pi} \simeq 1.34$ GeV, obtained by taking into account $\delta_{1,2}$ mixing, is indicated by black dot. Note that three points (circle, triangle and diamond) near $\sqrt{s} \simeq 0.91$ GeV are overlapping. The dashed line represents a fit over a pair of Breit-Wigner resonances as discussed in the text. The arrows indicate the physical threshold E_{thr} and the location $E_0 = m_K + m_\pi$ of the threshold for our unphysical (larger) masses. We also show experimental results (green symbols) by Estabrooks et al. [18] and Aston et al. [19]. Due to the definition of the phase shift modulo π we also plot the phase shift in the inelastic region [19] incremented by π.

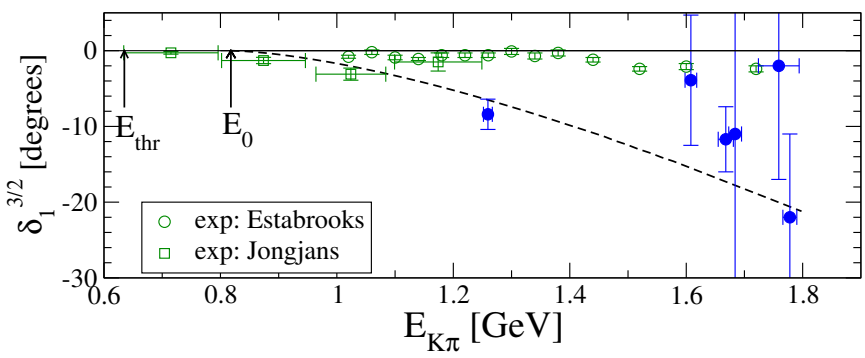

Figure 2: The p-wave scattering phase shift $\delta_{l=1}$, $I = 3/2$. as a function of $E_{K\pi} \equiv \sqrt{s}$. Our results are given by blue circles, the green symbols denote experimental values due to [18] and [20]. The broken line indicates a one-parameter fit (effective range fit with vanishing range parameter) to our data, setting $(p^3/\sqrt{s})\cot\delta(p) = \alpha$. The arrows indicate the physical threshold E_{thr} and the location $E_0 = m_K + m_\pi$ of the threshold for our unphysical (larger) masses.

fast rise of the phase in a narrow region around $E_{K\pi} \equiv \sqrt{s} \simeq 0.89$ G$eV \simeq m_{K^*}$. Note that phase shift points from B_3, E and T_1^-, that almost overlap in $E_{K\pi}$, overlap also in δ_1; this is non-trivial since Lüscher's relations for these three irreducible representations have a different form.

Due to our unphysical values of the pion and the kaon masses, the threshold is closer to the resonance position than in nature, leading to a smaller width. A Breit-Wigner fit gives a mass of $m_{K^*(892)} = 0.891(14)$ GeV and a coupling $g_{K*K\pi} = 5.7(1.6)$ (to be compared with the value 5.72(6) from experiments).

Above $E_{K\pi} \simeq 1.3$ GeV the process becomes inelastic; for this exploratory study we still assume elasticity and apply the Lüscher-tpye relations. Another complication arises from the fact that d-wave phase shift δ_2 cannot be neglected around $\sqrt{s} \simeq m_{K_2^*(1430)}$. We therefore derived Lüscher relations that contain δ_1 as well as δ_2 for irreps considered here: they are obtained from the so-called determinant condition. Combining results from different representation allowed us to disentangle p- and d-wave (for details see [9]).

We attempt an extraction of the $K^*(1410)$ resonance parameters by fitting the resulting δ_1 using a Breit-Wigner parametrization for two resonances in the elastic region,

$$\frac{p^{*3}}{\sqrt{s}} \cot \delta_1(s) = \left[\sum_{K_i^*} \frac{g_{K_i^*}^2}{6\pi} \frac{1}{m_{K_i^*}^2 - s} \right]^{-1}, \quad K_i^* = K^*(892),\, K^*(1410)\,. \tag{3.1}$$

We fix $m_{K^*(892)}$ and $g_{K^*(892)}$ in to the values obtained from the single resonance fit in the $K^*(892)$ region, fixing $g_{K^*(1410)}$ to the value $g^{exp}_{K^*(1410)} = 1.59(3)$ derived from $\Gamma^{exp}[K^*(1410) \to K\pi]$ and get a resonance position of $m_{K^*(1410)} = 1.33(2)$ GeV.

The $I = 3/2$ channel: We extract $\delta_1^{3/2}$ assuming elasticity and $\delta_2^{3/2} = 0$, employing the same phase shift relations as for $I = 1/2$. The resulting phase shift in Fig. 2 is small and negative (consistent with zero). We also show the result of a one-parameter fit to the leading term of an effective range approximation, $(p^3/\sqrt{s}) \cot \delta(p) = \alpha$.

At this conference the Hadron Spectrum Collaboration [21] presented results for $I = 3/2$ $K\pi$ phase shift [21] derived using techniques similar to ours; whereas our results from [1] agrees for the s-wave we disagree in the sign for the p-wave. Although we have different quark masses (in [21] the pion mass is larger) this discrepancy is considerable and should be clarified.

Acknowledgments: We thank Anna Hasenfratz for providing the gauge configurations used for this work. We would like to thank J. Bulava, S. Descotes-Genon, C. Morningstar and C. Thomas for valuable discussions. The calculations were performed at Jozef Stefan Institute. This work is supported by the Slovenian Research Agency. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

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