

Top quark mass combination techniques and treatment of uncertainties at the Tevatron and LHC

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Abstract. We present an overview of the statistical method used to combine top quark mass (m_t) measurements at the Tevatron and the LHC. We discuss the breakdown of uncertainties and their correlations. Finally we present the results of the latest m_t combinations from the Tevatron and the LHC.

1. Introduction

The top quark plays a special role in the standard model (SM) because it is by far the most massive elementary particle. Top quark mass represents one of the fundamental parameters of the SM, and its precise knowledge provides an important consistency check of the SM. Through the SM radiative corrections to the mass of the W boson, top quark mass imposes the constraint on the mass of the Higgs boson, and together with other electroweak observables included in the global fit it constrains the contributions from physics beyond the SM.

Since the discovery of the top quark in 1995 [1] a lot of efforts went into the measurement of its mass with high precision. The measurements were performed in various decay channels of the $t\bar{t}$ pair and using different techniques. Depending on the channel and technique the measurements have different sensitivity to the effects of jet energy scale calibration, signal and background modelling. Given the limited statistics of the $t\bar{t}$ sample at the Tevatron majority of m_t measurements were statistically limited. Thus performing the combination of these measurements was important to improve the precision and to check the consistency of the different measurements.

2. BLUE method

The first combination of the direct m_t measurements at the Tevatron was performed in 2004 followed by the updates each year as new measurements become available [2]. The statistical method used for combination calculates a linear weighted sum of individual results with weights determined such that they minimize the total uncertainty on the combined result [3] and is known as Best Linear Unbiased Estimate (BLUE). It takes into account statistical and systematic uncertainties of each measurement and their correlations and allows combination of correlated measurements of one or more parameters. The output provides χ^2 value of the fit to evaluate the consistency of the input measurements.

Important features of the method can be understood by considering a combination of two measurements $x_1 \pm \sigma_1$ and $x_2 \pm \sigma_2$ with correlation $\rho \leq 1$, one of which, x_1 , is more precise than the other, i.e., $\sigma_2 > \sigma_1$. BLUE method will return a combined value of $x \pm \sigma_x$, where $x = \alpha x_1 + \beta x_2$ with $\alpha + \beta = 1$. The weight of the second measurement can be calculated as $\beta = (1 - \rho z)/(1 - 2\rho z + z^2)$, where z is the ratio of the uncertainties of the two measurements $z = \sigma_2/\sigma_1 > 1$. Then relative improvement of the uncertainty on the combined result with respect to the most precise one is $\sigma_x/\sigma_1 = \sqrt{z^2(1 - \rho^2)/(1 - 2\rho z + z^2)}$. Figure 1 shows the relative improvement as a function of correlation ρ between the measurements for different values of z . If two uncorrelated ($\rho = 0$) measurements of the same precision ($z = 1$, blue curve) are combined the combination will have 30% better uncertainty than each of the measurements. However, the combination of two 50% correlated measurements one of which is twice more precise than the other (turquoise curve in figure 1) will have the same uncertainty as the most precise of the two. In general, figure 1 demonstrates that the relative improvement of the combination and the weights of the input measurements depend only on their precisions and correlations and not on the actual measured values x_1 and x_2 . Furthermore, figure 2 shows that depending on the precision of the individual measurements and their correlation the less precise measurement can acquire negative weight in the combination. This happens if $\rho > 1/z$. In such case the combined value will be outside the interval between the two input values. The measurement with a negative weight still contributes to the improvement of the uncertainty of the combined result.

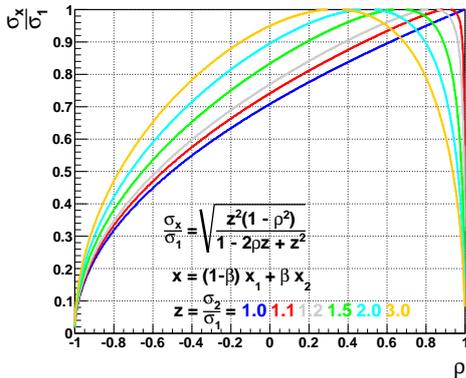


Figure 1. The relative improvement of the uncertainty on the combined measurement as a function of correlation for different values of z [4].

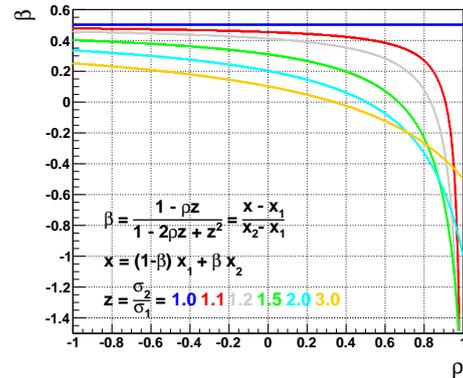


Figure 2. The weight of a less precise measurement in the combination as a function of correlation for different values of z [4].

3. Treatment of uncertainties

The most important part of the combination work is to understand correlations between the uncertainties of the individual measurements. The latter are combined in categories based on the physics origin of an uncertainty and the correlation pattern between analysis channels, data taking periods and experiments.

In the Tevatron m_t combination [2] the following correlation patterns are defined:

- **Uncertainties correlated between all measurements:** This class includes uncertainties related to the $t\bar{t}$ signal model and b -jets model based on Monte Carlo (MC) simulation.
- **Uncertainties correlated between all measurements of the same experiment:** This class includes uncertainties coming from calibration procedures used within the experiment.

- **Uncertainties correlated between measurements of the same experiment within the same data taking period:** This class includes uncertainties coming from calibration procedures used within the experiment which changed between data taking periods or use data from a specific period.
- **Uncertainties correlated between all measurements in the same channel:** These include uncertainties on the background determination based on simulation.
- **Uncorrelated uncertainties:** This class includes uncertainties of the statistical origin or the ones specific to a method.

The first LHC m_t combination [5] followed the breakdown of uncertainties introduced in the Tevatron combination as much as possible and reasonable. There are five categories of uncertainties which are common for both: statistical, method, background estimate by data driven techniques, background estimate from simulation and lepton model. Full breakdown consists of 14 categories for the Tevatron and 17 categories for LHC, 7 of which are related to jet energy calibration. The necessity of several categories for the jet energy calibration uncertainties is motivated by the fact that these were the dominating uncertainties on the m_t measurements and by the complex calibration procedures which differ between experiments and change between run periods. The calibration is based on both data and MC driven techniques which require different correlation patterns.

With the recently dramatically increased sample of top quarks the coherent approach to evaluation of various signal modelling uncertainties and understanding of their correlations became critical. At the Tevatron $t\bar{t}$ signal modelling uncertainties include hadronization model, higher order effects (since leading order MC is used as a default), uncertainty on the amount of initial and final state radiation, on modelling of color reconnection and the choice of PDF. All of them are combined in one class and taken as fully correlated between all measurements. Similar uncertainties are evaluated by the CMS and ATLAS collaborations. However, unlike at the Tevatron, the methods used to derive them differ significantly between ATLAS and CMS. For this reason LHC m_t combination keeps different modelling uncertainties as separate sources. Variation of the assumed correlation between two sources, the choice of the signal MC generator and the amount of radiation, turned out to have a non-negligible effect on the uncertainty on the combined m_t . A study showed that the uncertainty is the largest when correlation of these sources between the CMS and ATLAS measurements are assumed to be 50%. This value has been chosen to ensure a conservative uncertainty of the combined result.

4. Tevatron top quark mass combination results

The latest Tevatron top quark mass combination [2] is based on 12 measurements: 8 by the CDF and 4 by the D0 experiment, 5 performed in Run I of the Tevatron and 7 from Run II, 5 results from the l +jets, 4 from the dilepton and 2 from the all hadronic channel and one requiring \cancel{E}_T +jets. Table 1 summarizes the input measurements with the uncertainties and provides a breakdown of the uncertainty on the combined result which yields: $m_t = 173.18 \pm 0.56(\text{stat}) \pm 0.75(\text{syst})$ GeV with $\chi^2/NDF = 8.3/11$ corresponding to χ^2 probability of 69%. Figure 3 presents a summary of the input measurements and the combined m_t at the Tevatron. The combined measurement is 25% more precise than the most precise CDF measurement in the l +jets channel. Dominating uncertainties come from the limited statistics of the top quarks, expected to go down for the measurements using full Tevatron data set, and from the $t\bar{t}$ signal model.

5. LHC top quark mass combination results

The first LHC top quark mass combination [5] is based on 7 measurements: 3 by the ATLAS and 4 by the CMS collaboration, 3 performed using 2010 and 4 2011 data, 4 results from

Table 1. The uncertainty in GeV from each component for the twelve measurements of m_t and the resulting Tevatron combination [2]. The total uncertainties are obtained by adding the components in quadrature. The entries “n/a” stand for “not applicable” and “n/e” for “not evaluated.” The non-evaluated uncertainties were not considered as significant sources of uncertainty for Run I measurements.

Channel	Exp.-Run	Jet energy scale systematics							Other systematics							Total uncertainty			
		Light-jet response (1)	Light-jet response (2)	Out-of-cone correction	Offset	Model for b jets	Response to $b/q/g$ jets	$In-situ$ light-jet calibration	Jet modeling	Lepton modeling	Signal modeling	Multiple interactions model	Background from theory	Background based on data	Calibration method	Statistical uncertainty	Total JES uncertainty	Other systematic uncertainty	Total uncertainty
Lepton+jets	CDF-II	0.41	0.01	0.27	n/a	0.23	0.13	0.58	0.00	0.14	0.56	0.10	0.27	0.06	0.10	0.65	0.80	0.67	1.23
Lepton+jets	D0-II	n/a	0.63	n/a	n/a	0.07	0.26	0.46	0.36	0.18	0.77	0.05	0.19	0.23	0.16	0.83	0.83	0.94	1.50
Lepton+jets	CDF-I	3.4	0.7	2.7	n/a	0.6	n/e	n/a	n/e	n/e	2.7	n/e	1.3	n/e	0.0	5.1	4.4	2.8	7.3
Lepton+jets	D0-I	n/a	2.5	2.0	1.3	0.7	n/e	n/a	n/e	n/e	1.3	n/e	1.0	n/e	0.6	3.6	3.5	1.6	5.3
Alljets	CDF-II	0.38	0.04	0.24	n/a	0.15	0.03	0.95	0.00	n/a	0.64	0.08	0.00	0.56	0.38	1.43	1.06	0.91	2.00
Alljets	CDF-I	4.0	0.3	3.0	n/a	0.6	n/e	n/a	n/e	n/a	2.1	n/e	1.7	n/e	0.6	10.0	5.0	2.6	11.5
Dileptons	CDF-II	2.01	0.58	2.13	n/a	0.33	0.14	n/a	0.00	0.27	0.80	0.23	0.24	0.14	1.12	1.95	3.01	0.88	3.69
Dileptons	D0-II	n/a	0.56	n/a	n/a	0.20	0.40	0.55	0.50	0.35	0.86	0.00	0.00	0.20	0.51	2.36	0.90	1.11	2.76
Dileptons	CDF-I	2.7	0.6	2.6	n/a	0.8	n/e	n/a	n/e	n/e	3.0	n/e	0.3	n/e	0.7	10.3	3.9	3.0	11.4
Dileptons	D0-I	n/a	1.1	2.0	1.3	0.7	n/e	n/a	n/e	n/e	1.9	n/e	1.1	n/e	1.1	12.3	2.7	2.3	12.8
E_T +jets	CDF-II	0.45	0.05	0.20	n/a	0.00	0.12	1.54	0.00	n/a	0.78	0.16	0.00	0.12	0.14	1.80	1.64	0.78	2.56
Decay length	CDF-II	0.24	0.06	n/a	n/a	0.15	n/e	n/a	0.00	n/a	0.90	0.00	0.80	0.20	2.50	9.00	0.25	2.80	9.43
Tevatron Combination		0.12	0.19	0.04	0.00	0.15	0.12	0.39	0.11	0.10	0.51	0.00	0.14	0.11	0.09	0.56	0.49	0.57	0.94

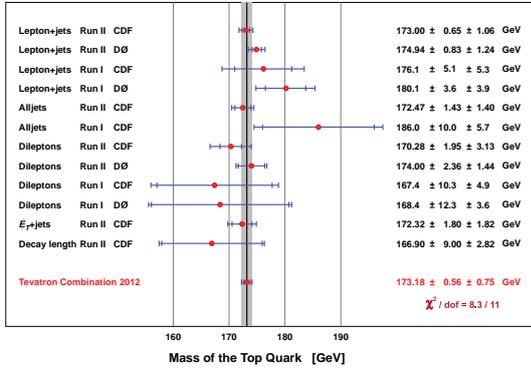


Figure 3. Summary of the Tevatron input m_t measurements and the resulting combined top quark mass [2].

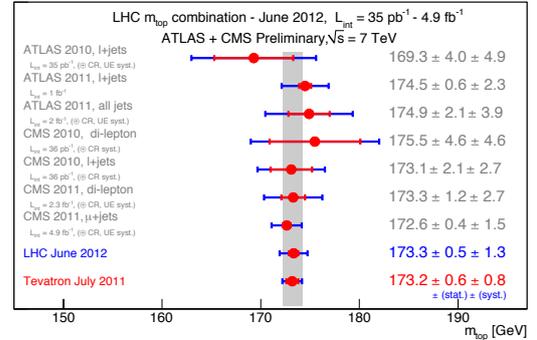


Figure 4. Summary of input measurements and result of LHC combination compared to the Tevatron combined m_t .

the l +jets, 2 from the dilepton and 1 from the all hadronic channel. Table 2 summarizes the input measurements with the uncertainties and provides a breakdown of the uncertainty on the combined result which yields: $m_t = 173.3 \pm 0.5(\text{stat}) \pm 1.3(\text{syst})$ GeV with $\chi^2/NDF = 2.5/6$ corresponding to χ^2 probability of 87%. The categories of uncertainties and their naming follow the breakdown used for the latest preliminary Tevatron m_t combination [6]. Figure 4 presents a summary of the input measurements and the combined m_t at the LHC compared to the Tevatron result. The combined measurement is 9% more precise than the most precise CMS measurement in the μ +jets channel. Dominating uncertainties come from the signal model, mainly from radiation and color reconnection models, from the b -jets and underlying event models.

Table 2. Inputs to the LHC combination and the results obtained using the categories grouping and correlation as defined in [5]. The entries “n/a” stand for “not applicable” and “n/e” for “not evaluated.”

	ATLAS			CMS				LHC
	2010	2011		2010		2011		
	l +jets	l +jets	all jets	di- l	l +jets	di- l	μ +jets	comb.
[GeV]								
Measured m_t	169.3	174.5	174.9	175.5	173.1	173.3	172.6	173.34
Stat	4.0	0.6	2.1	4.6	2.1	1.2	0.4	0.47
iJES	n/a	0.4	n/a	n/a	n/a	n/a	0.4	0.38
aJES	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
bJES	2.5	1.6	1.4	0.9	0.9	1.1	0.7	0.68
cJES	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
dJES	2.1	0.7	2.1	2.1	2.1	2.0	0.2	0.07
rJES	n/a	n/a	n/a	3.3	n/a	n/a	n/a	0.06
Lept	n/e	n/e	n/e	0.3	n/e	0.2	n/e	0.01
MC	1.0	0.4	0.5	0.4	n/e	0.1	n/e	0.04
Rad	2.5	1.0	1.7	0.9	1.2	0.8	0.8	0.69
CR	0.6	0.6	0.6	0.5	0.5	0.5	0.5	0.55
PDF	0.5	0.1	0.6	0.5	0.1	0.4	0.1	0.01
DTMO	1.2	0.3	0.5	0.6	0.4	0.7	0.3	0.19
UE	0.6	0.6	0.6	1.4	0.2	0.6	0.6	0.47
BGMC	1.8	0.1	n/a	0.1	0.2	n/a	0.1	0.01
BGDT	0.6	0.5	1.9	n/a	0.4	0.4	n/a	0.16
Meth	0.4	0.1	1.0	0.3	0.1	0.4	0.2	0.13
MHI	0.7	< 0.05	n/e	1.0	0.1	0.2	0.4	0.25
[GeV]								
Total Syst. Unc	4.9	2.3	3.9	4.6	2.7	2.7	1.5	1.33
Total Unc.	6.3	2.4	4.4	6.5	3.4	3.0	1.5	1.40

6. Conclusions and outlook

The technique to perform top quark mass combination is well established and was used at the Tevatron and LHC. New m_t measurements using full Tevatron data set and large LHC data sets are expected to have significantly reduced statistical uncertainties and uncertainties related to the detector model that depend on the amount of data available for the calibration of objects. The uncertainties on the various aspects of the $t\bar{t}$ signal model will be by far the dominating ones. Thus for the future LHC top quark mass combinations and for the first combination of m_t measurements at the Tevatron and LHC uniform approach to evaluating signal model systematic uncertainties becomes critical. Given a regime of high correlations in the future combinations a careful breakdown of all important uncertainties into model-related and detector-related is required to correctly take into account correlations between input measurements.

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