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"SUPER"–DILATATION SYMMETRY OF THE TOP-HIGGS SYSTEM

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Abstract

The top-Higgs system, consisting of top quark (*LH* doublet, *RH* singlet) and Higgs boson kinetic terms, with gauge fields set to zero, has an *exact* (modulo total divergences) symmetry where both fermion and Higgs fields are shifted and mixed in a supersymmetric fashion. The full Higgs-Yukawa interaction and Higgs-potential, including additional $\sim 1/\Lambda^2$ NJL-like interactions, also has this symmetry to $\mathcal{O}(1/\Lambda^4)$, up to null-operators. Thus the interaction lagrangian can be viewed as a power series in $1/\Lambda^2$. The symmetry involves interplay of the Higgs quartic interaction with the Higgs-Yukawa interaction and implies the relationship, $\lambda = \frac{1}{2}g^2$ between the top–Yukawa coupling, g, and Higgs quartic coupling, λ, at a high energy scale $\Lambda \gtrsim$ few TeV. We interpret this to be a new physics scale. The top quark is massless in the symmetric phase, satisfying the Nambu-Goldstone theorem. The fermionic shift part of the current is $\propto (1 - H^\dagger H/v^2)$, owing to the interplay of λ and g, and vanishes in the broken phase. Hence the Nambu-Goldstone theorem is trivially evaded in the broken phase and the top quark becomes heavy (it is not a Goldstino). We have $m_t = m_h$, subject to radiative corrections that can in principle pull the Higgs into concordance with experiment.

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I. INTRODUCTION

The Higgs boson can be viewed as a "pseudo-dilaton" in a particular limit [1]. There is a fundamental distinction between a "scale invariant Higgs field" and a "dilatonic Higgs field." A scale invariant Higgs field has a vanishing mass term, but can have a nonvanishing gauge, quartic and Yukawa couplings. To qualify as a (pseudo) dilatonic Higgs boson, the Higgs potential must be (approximately) flat. Consider the pure Higgs lagrangian (no gauge fields):

$$\mathcal{L}_0 = \partial_\mu H^\dagger \partial^\mu H - \frac{\lambda}{2}(H^\dagger H - v^2)^2 \tag{1}$$

As usual, the groundstate has a minimum for:

$$\left\langle H^i \right\rangle = \theta^i \qquad \text{where} \qquad \theta^i = (v, 0) \tag{2}$$

where θ^i is an arbitrary orientation in gauge space and can be rotated under $SU(2)_L \times U(1)$.

In the limit of small λ, the Higgs potential plays the analogue role of an "applied external magnetic field" to a spin system, pulling $\langle H^i \rangle$ to the minimum VEV, v. If we then take $\lambda \to 0$ the lagrangian acquires a "shift symmetry,"

$$\delta H^i = \theta^i \epsilon \qquad \longrightarrow \qquad \delta\ \partial_\mu H^\dagger \partial^\mu H = 0 \tag{3}$$

The alignment, θ^i, is held fixed and the shift is parameterized by the variable ϵ. The Noether current is then:

$$J_\mu = \frac{\delta \mathcal{L}_0}{\delta \partial^\mu \epsilon} = \theta^\dagger \partial_\mu H + H^\dagger \partial_\mu \theta \tag{4}$$

We see that θ is a defining part of the current. If we view θ as co-rotating with H under the global $SU(2) \times U(1)$ transformations, the charge $\int d^3x\ J_0$ then commutes with the gauge group.

In the broken phase of the theory the current looks more dilatonic:

$$J_\mu \to \sqrt{2} v \partial_\mu h \tag{5}$$

The dilatonic nature of the Higgs implies that fields that acquire masses proportional to v are "scale invariant" in the sense of spontaneous scale breaking. That is, we can perform an ordinary scale transformation which would normally shift mass terms, but we can then undo this by a compensating shift in h. To see this, consider the top quark mass term:

$$g \overline{\psi_L} t_R H + h.c. \quad \longrightarrow \quad m_t \bar{t} t \left(1 + \frac{h}{\sqrt{2} v}\right) \tag{6}$$

Under an ordinary infinitesimal scale transformation we have $t(x) \to (1-\epsilon)^{3/2} t(x')$ and $h(x) \to (1-\epsilon)h(x')$ where $x_\mu = (1+\epsilon){x'}_\mu$, $d^4x = (1+\epsilon)^4 d^4x'$. Hence the action transforms as:

$$\int d^4x \; m_t \bar{t}t(x) \left(1 + \frac{h(x)}{\sqrt{2}v}\right) \to \; \to \int d^4x' \left((1+\epsilon) m_t \bar{t}t(x') + m_t \bar{t}t(x') \frac{h(x')}{\sqrt{2}v}\right) \tag{7}$$

The latter expression exhibits the fact that under ordinary scale transformations the $d = 4$ Higgs-Yukawa interaction is scale invariant, while the $d = 3$ mass term is not invariant.

However, with the dilatonic shift symmetry we can compensate the rescaled mass term by a shift in the Higgs-dilaton field:

$$h(x') \to h(x') - \sqrt{2}v\epsilon \tag{8}$$

and we see that:

$$\int d^4x' \left((1+\epsilon) m_t \bar{t}t(x') + m_t \bar{t}t(x') \frac{h(x')}{\sqrt{2}v}\right) \to \int d^4x' \left(m_t \bar{t}t(x') + m_t \bar{t}t(x') \frac{h(x')}{\sqrt{2}v}\right) \tag{9}$$

Hence, the simultaneous application of the scale transformation and Higgs shift symmetry allows us to maintain the symmetry of the top quark mass term. The scale symmetry can be viewed as spontaneously broken with the Higgs boson playing the role of the Nambu-Goldstone mode. The same invariance applies to the the gauge fields, W and Z. Higgs self-interactions that involve nonzero λ would not be invariant under scale transformations with dilatonic shifts in h, and the symmetry is broken by scale anomalies (running couplings).

II. GENERALIZE TO A "SUPER-DILATATION"

The Higgs boson is then a (pseudo) dilaton if the shift transformation is a (approximate) symmetry of the action. Fundamentally it stems from the exact shift or modular symmetry of the gaugeless Higgs kinetic term:

$$\delta H^i = \theta^i \epsilon \qquad \longrightarrow \qquad \delta \; \partial_\mu H^\dagger \partial^\mu H = 0 \tag{10}$$

A key point we wish to emphasize is that ϵ is the infinitesimal parameter of the transformation, while the orientation, θ^i, is held fixed. θ^i defines a "ray" and the shift moves the field along this direction in field space. We take θ^i to have the same mass dimension as the Higgs, *i.e.*, dimensions of mass and it is a normalized isospinor, $\theta^\dagger \theta = v^2$, where we conventionally

choose the alignment $\theta^i = (v, 0)$. Eq.(10) is a symmetry of the gaugeless Higgs boson kinetic terms, $\partial H^\dagger \partial H$. In such a theory the shift symmetry is exact.

We now propose a generalization of dilatation symmetry for the Higgs boson that involves a "super"-symmetric relationship between the top and Higgs fields [2]. The shift in the Higgs boson field is now promoted to an operator. This symmetry is exact in the top, with bottom-left, and Higgs, kinetic terms in the gaugeless limit (up to total divergences). Consider the top and Higgs kinetic terms of the standard model with *gauge fields set to zero*:

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/ \, \psi_L + \overline{t}_R i\partial\!\!\!/ \, t_R + \partial H^\dagger \partial H \tag{11}$$

We define the infinitesimal transformation:

$$\begin{aligned}
\delta\psi_L^{ia} &= \theta_L^{ia}\eta\epsilon - i\frac{\partial\!\!\!/ \, H^i \theta_R^a}{\Lambda^2}\epsilon; & \delta\overline{\psi}_{L\,ia} &= \overline{\theta}_{L\,ia}\eta\epsilon + i\frac{\overline{\theta}_{Ra}\partial\!\!\!/ \, H_i^\dagger}{\Lambda^2}\epsilon; \\
\delta t_R^a &= \theta_R^a\eta\epsilon - i\frac{\partial\!\!\!/ \, H_i^\dagger \theta_L^{ia}}{\Lambda^2}\epsilon; & \delta\overline{t}_{Ra} &= \overline{\theta}_{Ra}\eta\epsilon + i\frac{\overline{\theta}_{Lia}\partial\!\!\!/ \, H^i}{\Lambda^2}\epsilon; \\
\delta H^i &= \frac{\overline{\theta}_{Ra}\psi_L^{ia} + \overline{t}_{Ra}\theta_L^{ia}}{\Lambda^2}\epsilon; & \delta H_i^\dagger &= \frac{\overline{\psi}_{Lai}\theta_R^a + \overline{\theta}_{Lai}t_R^a}{\Lambda^2}\epsilon.
\end{aligned} \tag{12}$$

where i (a) is an isospin (color) index. η is a relative normalization factor that we determine subsequently.

It is readily seen that eq.(12) is an invariance of eq.(11) up to total derivatives:

$$\begin{aligned}
\delta(\overline{\psi}_L i\partial\!\!\!/ \, \psi_L) &= \frac{(\overline{\psi}_L\theta_R)\cdot\partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d. \\
\delta(\overline{t}_R i\partial\!\!\!/ \, t_R) &= \frac{(\overline{\theta}_L t_R)\cdot\partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d. \\
\delta(\partial H^\dagger\partial H) &= -\frac{(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)\cdot\partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d. \\
\text{hence,} \qquad & \delta\mathcal{L}_K = 0 + t.d.
\end{aligned} \tag{13}$$

The symmetry of the gauge free kinetic terms makes no use of equations of motion or on-shell conditions. At this stage, the shifts in ψ_L and t_R by $\theta_{L,R}$ proportional to η play no role, but will be essential with the Higgs-Yukawa interaction and Higgs mass term. Indeed, shifting $\overline{\psi}_L i\partial\!\!\!/ \, \psi_L \to \overline{\psi}_L i\partial\!\!\!/ \, \theta_L$ yields a total divergence provided we have switched off the local gauge fields.

This transformation exploits the interplay of the quantum numbers of ψ_L, t_R and H. It resembles a scalar supermultiplet transformation of component fields [3], where the Higgs field is treated as a superpartner of ψ_L. We emphasize that this is not a representation of

the supersymmetry algebra (there is no "F" auxillary field, [3]; this is essentially a scalar supermultiplet transformation with fixed $F = 0$ and the superparameters replaced by $\theta\epsilon$). With the assignment of scales of the $\theta_{L,R}$ and the presence of Λ the commutators of subsequent transformations for different $\theta_{L,R}$ cannot close. Also, the $\theta_{L,R}$ carry flavor and color quantum numbers, and the failure of the algebra to close into a superalgebra is presumably a supersymmetric extension of the Coleman-Mandula no-go theorem. In fact, this is a $U(1)$ symmetry with the transformation parameter, ϵ, for fixed background values of $\theta_{L,R}$. As such, the commutator trivially vanishes on the fields: $[\delta_{\epsilon'}, \delta_\epsilon](\psi, H, t_R) = 0$

We presently turn to the full lagrangian of the top-Higgs system in the standard model with gauge fields turned off:

$$\begin{aligned}\mathcal{L}_H &= i\overline{\psi}_L \partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ &\quad + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \end{aligned} \tag{14}$$

From eq.(12) we compute the transformations:

$$\delta(-M_H^2 H^\dagger H) = -\frac{\epsilon}{\Lambda^2}\, M_H^2(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)\cdot H + h.c \tag{15}$$

$$\delta(-\frac{\lambda}{2}(H^\dagger H)^2) = -\frac{\epsilon}{\Lambda^2}\, \lambda(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)\cdot H H^\dagger H + h.c. \tag{16}$$

$$\begin{aligned}\delta(g\overline{\psi}_L t_R H + h.c.) &= g\eta\epsilon(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)H + g^2\frac{\epsilon}{2\Lambda^2}(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L)\cdot\left(H^\dagger H^\dagger H\right) \\ &\quad + g\frac{\epsilon}{\Lambda^2}\overline{\psi}_L t_R(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L) \\ &\quad + ig\frac{2\epsilon}{\Lambda^2}\overline{\psi}_L\gamma_\mu\frac{\tau^A}{2}\theta_L\left(H^\dagger\overleftrightarrow{\partial^\mu}\frac{\tau^A}{2}H\right) + ig\frac{\epsilon}{2\Lambda^2}\overline{\psi}_L\gamma_\mu\theta_L\left(H^\dagger\overleftrightarrow{\partial^\mu}H\right) \\ &\quad - ig\frac{\epsilon}{\Lambda^2}\overline{\theta}_R\gamma_\mu t_R\left(H^\dagger\overleftrightarrow{\partial^\mu}H\right) + h.c. + t.d.\end{aligned} \tag{17}$$

where we use the isospin Fierz identity, $[\tau^A]_{ij}[\tau^A]_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$, and: $\overleftrightarrow{\partial^\mu} = \frac{1}{2}(\overrightarrow{\partial^\mu} - \overleftarrow{\partial^\mu})$. and we have also applied the fermionic equations of motion:

$$i\partial\!\!\!/ t_R + g\psi_L\cdot H^\dagger = 0 \qquad\qquad i\partial\!\!\!/ \psi_L + g t_R H = 0 \tag{18}$$

and eq.(17) follows [2].

Notice in eq.(17) we have generated higher dimension operator terms of the form:

$$\begin{aligned}&\frac{g\epsilon}{\Lambda^2}\overline{\psi}_L t_R(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L) + \; i\frac{2g\epsilon}{\Lambda^2}\overline{\psi}_L\gamma_\mu\frac{\tau^A}{2}\theta_L(H^\dagger\overleftrightarrow{\partial^\mu}\frac{\tau^A}{2}H) \\ &+ i\frac{g\epsilon}{2\Lambda^2}\overline{\psi}_L\gamma_\mu\theta_L(H^\dagger\overleftrightarrow{\partial^\mu}H) - \; i\frac{g\epsilon}{\Lambda^2}\overline{\theta}_R\gamma_\mu t_R(H^\dagger\overleftrightarrow{\partial^\mu}H) + h.c. + t.d.\end{aligned} \tag{19}$$

In analogy to the "bottoms up" derivation of a nonlinear chiral lagrangian (see section (3)), these terms can be cancelled by adding higher dimension operators to the original lagrangian of the form:

$$\begin{aligned}\mathcal{L}_{d=6} &= \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H) \end{aligned} \tag{20}$$

where we will presently relate the coupling constant, κ, to M_H, Λ and g below.

We thus obtain the effective lagrangian,

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ &+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H) \end{aligned} \tag{21}$$

Performing the super-dilatation transformation of eq.(12) we now demand that:

$$\delta \mathcal{L}_H = \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \tag{22}$$

The generated $\mathcal{O}(1/\Lambda^4)$ terms can be compensated by adding additional $1/\Lambda^4$ terms to the lagrangian. By continued application of eq.(12) we would generate a power series of contact interactions that are scaled by $\sim 1/\Lambda^{2n}$.

First we see that the transformation of the Higgs mass term of eq.(21), from eqs.(15–17), cancels against the first term of the transformed Higgs-Yukawa interaction, provided:

$$g\eta = \frac{M_H^2}{\Lambda^2} \tag{23}$$

This establishes the normalization factor, η. It also establishes the relative sign (we assume Λ^2 positive). If we're in the symmetric (broken) phase, M_H^2 positive (negative), then we have $g\eta > 0$ ($g\eta < 0$). We have the freedom of choosing arbitrary η since the defining kinetic term invariance involves only ϵ.

One might think we can now take Λ^2 to be arbitrarily large compared to M_H^2 by adjusting $|\eta| << 1$. However, the second term of eq.(17) must also cancel against the transformation of the first κ term appearing in eq.(21). This requires that:

$$\eta\kappa = -g, \qquad \text{or, using eq.(23):} \qquad \frac{\kappa}{\Lambda^2} = -\frac{g^2}{M_H^2} \tag{24}$$

This is a striking result: a seesaw relation between the weak scale and Λ-scale terms. In the $d = 6$ operators we have a Nambu-Jona-Lasionio component. The Nambu-Jona-Lasinio attractive interaction corresponds to $\kappa > 0$, and we see in eq.(24) that the super-dilatation is then consistent only if $M_H^2 < 0$. Moreover, to make η small requires taking κ large.

Finally, the most interesting relationship, which is the analogue of the Goldberger-Treiman relationship in a chiral lagrangian (see section 3), arises from the cancellation of the $\sim \epsilon(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R) \cdot HH^\dagger H$ terms of eqs.(16) and (17) under the super-dilatation symmetry:

$$0 = (\lambda - \frac{1}{2}g^2)\frac{\epsilon}{\Lambda^2}(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R) \cdot HH^\dagger H + h.c \tag{25}$$

or,

$$\lambda = \frac{1}{2}g^2 \tag{26}$$

Note that his transformation does not involve η.

The $\lambda = g^2/2$ relationship refers to the coefficient of the $D = 6$ operator, $(\overline{\theta}_L t_R) \cdot HH^\dagger H + h.c$. We therefore assume that it applies at the scale Λ. The low energy relationship between the couplings g^2 and λ then depends upon the renormalization group running from Λ to $v_{weak} \approx 175$ GeV. If we ignore the RG running then eq.(26) would hold at the weak scale, and implies the supersymmetric relationship $m_h^2 = 2\lambda v_{weak}^2 = m_t^2$ in the broken phase. This is an improvement over the usual NJL result, $m_h^2 = 4m_t^2$.

Some comments on the structure of these higher dimension operators are in order. Note that we can Fierz rearrange the first term of eq.(20):

$$(\overline{\psi}_L^a t_{Ra})_i(\overline{t}_{Rb}\psi^b)^i \rightarrow -(\overline{\psi}_{iL}\gamma_\mu\frac{\lambda^A}{2}\psi_L^i)(\overline{t}_R\gamma^\mu\frac{\lambda^A}{2}t_R) + \mathcal{O}(\infty/\mathcal{N}) \tag{27}$$

where $N = 3$ is the number of colors. This term is a pure Nambu-Jona-Lasinio interaction as arises in topcolor [4, 9] in the form of a (color current)$\times$(color current). Indeed, massive Yang–Mills boson exchange for a boson of mass M^2 and momentum exchange $q^2 < M^2$ produces the negative sign for (current)$\times$(current) interactions. A positive sign for the first term of eq.(20) is the attractive sign for the Nambu-Jona-Lasinio model, and we thus see that the attractive sign corresponds to the correct (negative) sign for topgluon exchange. However, we see that the isospin (current)$\times$(current) interaction (second term of eq.(20)) then has the wrong sign (positive) for a gauge boson exchange. We will discuss this "frustration of signs" further in Section IV.A.

Since all of the higher dimension $d = 6$ operators are of the form (current)×(current), they preserve the chirality of the fermions. That is, the terms of eq.(20) contain no cross terms of the form $\overline{\psi}_L H t_R (H^\dagger H)^p$. They thus admit the discrete symmetry, $\psi_L \to (-1)^N \psi_L$ and $t_R \to (-1)^{N+1} t_R$. Operators of mixed chirality can therefore be excluded, or suppressed, on symmetry grounds.

III. ANALOGY TO A CHIRAL LAGRANGIAN

For comparison, we quickly review a familiar derivation of a pion chiral lagrangian from the "bottoms-up." Consider the kinetic terms:

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/ \, \psi_L + \overline{\psi}_R i\partial\!\!\!/ \, \psi_R + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi \tag{28}$$

We'll consider the RH-chiral symmetry:

$$\begin{aligned} \delta\psi_L &= 0 \\ \delta\psi_R &= i\theta\psi_R \qquad \delta\overline{\psi}_R = -i\theta\overline{\psi}_R \\ \delta\pi &= f_\pi \theta \end{aligned} \tag{29}$$

and we demand the lagrangian is invariant under this global transformation:

$$\delta\mathcal{L}_K = 0 \tag{30}$$

The RH-chiral current is:

$$-\frac{\delta\mathcal{L}_K}{\delta\ \partial^\mu\theta} = \overline{\psi}_R \gamma_\mu \psi_R - f_\pi \partial_\mu \pi \tag{31}$$

and we assume f_π is "determined from experiment," *e.g.*, $\pi \to \mu\nu$ (of course, in the real world this is the left-handed current).

Consider the interactions consisting of a massive "nucleon" coupled to pion:

$$\mathcal{L}_V = M\overline{\psi}\psi - ig\pi\overline{\psi}\gamma^5\psi = M\overline{\psi}_L\psi_R - ig\pi\overline{\psi}_L\psi_R + h.c. \tag{32}$$

We perform the RH-chiral transformation transformation:

$$\delta\mathcal{L}_V = (i\theta M - igf_\pi\theta + g\pi f_\pi\theta)\,\overline{\psi}_L\psi_R + h.c. \tag{33}$$

so:

$$\delta\mathcal{L}_V = 0 \qquad \longrightarrow \qquad g = \frac{M}{f_\pi} \tag{34}$$

which is the Goldberger-Treiman relation.

However, we must cancel the "higher order term" $\propto \pi\theta\overline{\psi}_L\psi_R$. We thus include an $\mathcal{O}(\pi^2)$ term:

$$\mathcal{L}_V \to M\left(1 - \frac{i\pi}{f_\pi} + c\frac{\pi^2}{f_\pi^2}\right)\overline{\psi}_L\psi_R + h.c. \tag{35}$$

Now:

$$\delta\mathcal{L}_V \to M\left(i\theta - i\frac{f_\pi\theta}{f_\pi} + \frac{\pi\theta}{f_\pi} + 2c\frac{\pi}{f_\pi^2}f_\pi\theta + ic\frac{\pi^2}{f_\pi^2}f_\pi\theta\right)\overline{\psi}_L\psi_R h.c. \tag{36}$$

so:

$$\delta\mathcal{L}_V = 0 \qquad \longrightarrow \qquad g = \frac{M}{f_\pi}, \qquad c = -\frac{1}{2} \tag{37}$$

But, now we must cancel higher order term $\propto \pi^2\theta\overline{\psi}_L\psi_R$ which implies an $\mathcal{O}(\pi^3)$ interaction, and so-forth.

We can sum the resulting power series and we find, iteratively, the solution:

$$\mathcal{L}_V = M\overline{\psi}_L U\psi_R + h.c. \qquad\qquad U = \exp(i\pi/f_\pi) \tag{38}$$

whence,

$$\mathcal{L} = \overline{\psi}_L i\partial\!\!\!/\,\psi_L + \overline{\psi}_R i\partial\!\!\!/\,\psi_R + \frac{f_\pi^2}{2}\partial_\mu U^\dagger\partial^\mu U + M\overline{\psi}_L U\psi_R + h.c. \tag{39}$$

and we have obtained the"nonlinear σ-model lagrangian."

Our present strategy is similar. We begin with the super-dilatational invariance of the top-Higgs kinetic terms. We then analyze the transformation of the Higgs-Yukawa, Higgs mass and quartic interactions. We demand overall invariance of the lagrangian. We thus find the "Goldberger-Treiman" relationship, $\lambda = g^2/2$, which implies $m_t = m_h$ in the broken phase. This induces higher dimension operators. Ultimately, we expect to sum the tower of operators, though in the present case we expect that these arise via new dynamics, such as heavy recurrences of composite Higgs bosons and vector–like top quarks.

IV. CURRENT STRUCTURE AND THE NAMBU-GOLDSTONE THEOREM

The critical aspect of our construction is that the operator shift of δH in the quartic Higgs interaction is cancelling against the super-rotation (*i.e.*,the "twist") of $\delta\psi$ in the Higgs-Yukawa interaction. Moreover, the pure fermionic shift in $\delta\psi \sim \eta\epsilon\theta$, in the Higgs-Yukawa interaction, *i.e.*, proportional to η, cancels against the δH shift in the Higgs mass term. This ties the transformations together into a single structure.

The Nambu-Goldstone theorem for a fermionic shift $\delta\psi \sim \eta\epsilon\theta$, which would naively imply a massless top quark (a "Goldstino"), is evaded in the broken phase. How does our theory evade the existence of a zero-mode associated with the fermionic shift? Naively, this would seem to prohibit a massless top quark. In fact, this happens in a subtle way. One must carefully construct the currents given our use of equations of motion in $\delta(g\overline{\psi}_L t_R H + h.c.)$. We therefore wish to clarify the the relationship to the Nambu-Goldstone theorem in the present set up.

We consider, for technical simplicity, the simpler "minimal" transformation defined by $\theta_L = 0$:

$$\delta\psi_L^{ia} = -i\frac{\partial\!\!\!/ H^i \theta_R^a}{\Lambda^2}\epsilon; \qquad \delta\overline{\psi}_{L\,ia} = i\frac{\overline{\theta}_{Ra}\partial\!\!\!/ H_i^\dagger}{\Lambda^2}\epsilon; \tag{40}$$

$$\delta t_R^a = \theta_R^a \eta\epsilon; \qquad \delta\overline{t}_{Ra} = \overline{\theta}_{Ra}\eta\epsilon; \tag{41}$$

$$\delta H^i = \frac{\overline{\theta}_{Ra}\psi_L^{ia}}{\Lambda^2}\epsilon; \qquad \delta H_i^\dagger = \frac{\overline{\psi}_{Lai}\theta_R^a}{\Lambda^2}\epsilon. \tag{42}$$

The parameter η is still fixed by the symmetry interplay of the Higgs mass term and Yukawa interaction as in eq.(23),

$$g\eta = \frac{M_H^2}{\Lambda^2} = \frac{\lambda v^2}{\Lambda^2} \tag{43}$$

Consider the top and Higgs system of the standard model with gauge fields set to zero:

$$\mathcal{L}_H \;=\; \mathcal{L}_K + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H) \tag{44}$$

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/ \psi_L + \overline{t}_R i\partial\!\!\!/ t_R + \partial H^\dagger \partial H \tag{45}$$

It is readily seen that eqs.(40–42) is a global invariance of eqs.(45) up to total derivatives. We presently allow ϵ to be a local function of spacetime $\epsilon(x)$ (note that the derivatives in eq.(40) act only upon H and not upon $\epsilon(x)$). We have:

$$\begin{aligned}
\delta(\overline{\psi}_L i\partial\!\!\!/ \psi_L) \;&=\; \frac{(\overline{\psi}_L\theta_R)\cdot\partial^2 H}{\Lambda^2}\epsilon + \frac{(\overline{\psi}_L\gamma_\mu\partial\!\!\!/ H\theta_R)}{\Lambda^2}\partial^\mu\epsilon + h.c. + t.d. \\
\delta(\overline{t}_R i\partial\!\!\!/ t_R) \;&=\; i(\overline{t}_R\partial\!\!\!/ \theta_R)\eta\epsilon + i(\overline{t}_R\gamma_\mu\theta_R)\eta\partial^\mu\epsilon + h.c. + t.d. \\
\delta(\partial H^\dagger\partial H) \;&=\; -\frac{(\overline{\psi}_L\theta_R)\cdot\partial^2 H}{\Lambda^2}\epsilon + \frac{(\overline{\psi}_L\theta_R)\cdot\partial_\mu H}{\Lambda^2}\partial^\mu\epsilon + h.c. + t.d.
\end{aligned} \tag{46}$$

The kinetic terms thus lead to a current:

$$J_\mu^K = \frac{\delta\mathcal{L}_K}{\delta\partial_\mu\epsilon} = i(\overline{t}_R\gamma_\mu\theta_R)\eta + \frac{(\overline{\psi}_L\gamma_\mu\partial\!\!\!/ H\theta_R)}{\Lambda^2} + \frac{(\overline{\psi}_L\theta_R)}{\Lambda^2}\partial_\mu H \tag{47}$$

The symmetry of the gauge free kinetic terms makes no use of equations of motion or on-shell conditions. However, the symmetry of the full action, as we have emphasized, involves a cancellation of the shift of eqs.(42) in the Higgs quartic interaction against the "twist" of eq.(40) in the Higgs-Yukawa interaction. In calculating the transformation of the Higgs-Yukawa interaction, however, we make use of an integration by parts and discard total divergences (and subsequently use the fermion equations of motion). This integration by parts in the "twist" of eq.(40) causes the derivative to act upon the parameter $\epsilon(x)$:

$$\delta(-M_H^2 H^\dagger H) = -\frac{\epsilon}{\Lambda^2}\, M_H^2(\overline{\psi}_L\theta_R)\cdot H + h.c \tag{48}$$

$$\delta(-\frac{\lambda}{2}(H^\dagger H)^2) = -\frac{\epsilon}{\Lambda^2}\, \lambda(\overline{\psi}_L\theta_R)\cdot HH^\dagger H + h.c. \tag{49}$$

$$\begin{aligned}\delta(g\overline{\psi}_L t_R H + h.c.) &= g\eta\epsilon(\overline{\psi}_L\theta_R)H + g^2\frac{\epsilon}{2\Lambda^2}(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L)\cdot\left(H^\dagger H^\dagger H\right)\\ &\quad + g\frac{\epsilon}{\Lambda^2}(\overline{\psi}_L t_R)(\overline{\theta}_R\psi_L) - ig\frac{\epsilon}{\Lambda^2}\overline{\theta}_R\gamma_\mu t_R\left(H^\dagger\overleftrightarrow{\partial^\mu}H\right) + h.c. + t.d.\\ &\quad -i\frac{g}{2\Lambda^2}\overline{\theta}_R\gamma_\mu t_R(H^\dagger H)\partial^\mu\epsilon\end{aligned} \tag{50}$$

The last term in eq.(50) shows explicitly that the result of the integration by parts leads to an additional term $\propto \partial^\mu\epsilon$. This, in turn, modifies the current, which now becomes:

$$J_\mu = \frac{\delta\mathcal{L}_H}{\delta\partial_\mu\epsilon} = i(\overline{t}_R\gamma_\mu\theta_R)\left(\eta - \frac{gH^\dagger H}{2\Lambda^2}\right) + \frac{(\overline{\psi}_L\gamma_\mu\partial\!\!\!/ H\theta_R)}{\Lambda^2} + \frac{(\overline{\psi}_L\theta_R)}{\Lambda^2}\partial_\mu H \tag{51}$$

Using the relationship of eq.(43) and $\lambda = g^2/2$, the current can be written:

$$J_\mu = \frac{\delta\mathcal{L}_H}{\delta\partial_\mu\epsilon} = i\eta(\overline{t}_R\gamma_\mu\theta_R)\left(1 - \frac{H^\dagger H}{v^2}\right) + \frac{(\overline{\psi}_L\gamma_\mu\partial\!\!\!/ H\theta_R)}{\Lambda^2} + \frac{(\overline{\psi}_L\theta_R)}{\Lambda^2}\partial_\mu H \tag{52}$$

The modifcation of the current occurs in the first term which is associated with the fermionic "shift"-symmetry of t_R. Again, this arises from the crucial linkage of the δH shift in the quartic Higgs interaction to the $\delta\psi_L$ shift in the Higgs-Yukawa interaction.

Note that, in the broken phase where $\langle H\rangle = v$ the modifcation of the current has the effect of "turning off" the fermionic shift. Indeed, we will now see that this has a remarkable effect in evading the Nambu-Goldstone theorem in the broken phase, and permitting the top quark to be massive.

Consider the two-point function of our current of eq.(52) with t_R:

$$S(y) = \int d^4x e^{iq\cdot x}\, \partial^\mu \langle 0|\, T^* J_\mu(x)\, t_R(y)\, |0\rangle \tag{53}$$

(T^* implies anti-commutation in the ordering of fermion fields). Formally, with $\partial^\mu J_\mu = 0$, we have, from the ∂_0 acting upon the T^* ordering a $\delta(x^0 - y^0)$, and:

$$\begin{aligned} S(y) &= \int d^3x\, e^{iq\cdot x} \langle 0| \{J_0(x),\ t_R(y)\} |0\rangle \\ &= \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \langle 0| \{J_0(\vec{x}),\ t_R(\vec{y})\}_{e.t.} |0\rangle \\ &= \langle 0| \{Q,\ t_R(\vec{y})\} |0\rangle \end{aligned} \tag{54}$$

where the charge operator Q is:

$$Q = \int d^3x\, J_0(\vec{x}). \tag{55}$$

In the symmetric phase of the standard model we have the Higgs VEV, $\langle H\rangle = 0$, and we can neglect all terms in the current that involve H. The charge operator then involves only the first term in $J^K_\mu = i\eta\overline{t}_R\gamma_\mu\theta_R$, whence it generates a shift in the fermion field:

$$\langle 0| \{Q,\ t_R(\vec{y})\} |0\rangle = \eta\theta_R \tag{56}$$

On the other hand we have:

$$\begin{aligned} S(y) &= \int d^4x\, e^{iq\cdot x}\, \partial^\mu \langle 0| T^* i\overline{t}_R(x)\gamma_\mu\theta_R\eta \quad t_R(y) |0\rangle \\ &= -\int d^4x\, e^{iq\cdot x}\, i\partial^\mu\gamma_\mu S_F(x-y)\theta_R\eta \\ &= \left.\frac{q^2 + \not{q}\, m}{q^2 - m^2}\theta_R\eta\right|_{q\to 0} \end{aligned} \tag{57}$$

In the $q^2 \to 0$ limit the consistency of eq.(57) with eqs.(54, 56) requires that the fermion mass satisfy $m = 0$. This is the fermionic Nambu-Goldstone theorem and it informs us that any fermionic action which has a pure fermionic shift symmetry, must correspond to a massless fermion. This is, indeed, the case in the symmetric phase in which the top quark is massless and $\langle H\rangle = 0$. Naively we might conclude that the top quark is forced to be a Goldstino and remain massless, even in the broken phase.

However, we have seen that the current is modified in a significant way in the present case:

$$J_\mu = i\eta(\overline{t}_R\gamma_\mu\theta_R)\left(1 - \frac{H^\dagger H}{v^2}\right) + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) \tag{58}$$

In the broken phase, when $\langle H\rangle = v \neq 0$, this implies that the pure fermionic shift operator in the current "turns off:"

$$J_\mu = 0 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \tag{59}$$

This is a consequence of the interplay between the quartic interaction and the Higgs-Yukawa interaction in our construction. It implies that there can exist dynamical situations in which a Goldstino is massless in a symmetric phase, but acquires mass in a broken phase of a theory. The underlying fermionic shift, $\delta\psi = \eta\theta$ is intact, but the current is modified dynamically to evade the naive Nambu-Goldstone theorem.

Indeed, the symmetry yields $\lambda = g^2/2$ in both phases of the Standard Model. However, the fermionic shift part of the current is nontrivially modified by the quartic-Yukawa interplay and is $\propto (1 - H^\dagger H/v^2)$. It thus vanishes in the broken phase with $\langle H \rangle = v$. Therefore, the top quark becomes massive in the broken phase in the usual way, with the relationship $m_h = m_t$. This is consistent with the Nambu-Goldstone theorem that would otherwise naively force the top quark to be a massless Goldstino. This relationship $\lambda = g^2/2$ is the analogue of a Goldberger-Treiman" relationship. It holds at a high scale, Λ, and is subject to renormalization group and higher dimension operator effects that can bring the physical masses into concordance with $m_h^2 \approx m_t^2/2$.

V. UV-COMPLETION

The effective lagrangian we obtain from the minimal transformation of eq.(**??**) is simpler:

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ &\quad + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &\quad + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned} \tag{60}$$

The frustrated (current)×(current) signs are now absent given that we have banished the isospin interaction. We retain the relationships of eqs.(23,24,26).

Note the structure of the higher dimension operators. One of these is a Nambu-Jona-Lasinio four-fermion interaction, as expected in topcolor citetopc. The other is a current-current interaction of the Higgs with the top quark. With Λ sufficiently large, κ must become large in accord with the seesaw relationship of eq.(24).

This is suggestive of a dynamics in which a boundstate recurrence of the Higgs boson, composed of $\overline{t}t$ is generated via the NJL interaction [8**?**]. The Higgs-top interaction can likewise generate a composite Dirac fermion that is composed of $\sim H^\dagger t_R$ with quantum

numbers of of the left-handed top quark [**?**]. The Dirac fermion has a RH component that is an s-wave, $\sim H^\dagger t_R$, and a LH component that is a p-wave, $\sim H^\dagger i\partial\!\!\!/ t_R/\Lambda$.

We believe the tower of operators $\sim 1/\Lambda^{2n}$ may be determined, and the dynamical model admitting our super-dilatation may be understood as a full solution to this dynamics. This may be facilitated by introducing the composite fields explicitly as auxilliary fields.

VI. CONCLUSIONS

While the usual superalgebra of SUSY does not permit the Higgs to be the superpartner of the (t, b) quarks, the symmetry we present here does accomplish this. We emphasize that the "super"-dilatation symmetry is not a conventional superalgebra, *i.e.*, it is not a grading of the Lorentz Group, and is not associated with a nontrivial nonabelian closed superalgebra (at least not in our present exploratory formulation). The symmetry is a bosonic sibngle parameter, $\sim U(1)$, invariance, and closes trivially.

Our symmetry is remniscent of a "reparameterization invariance," *e.g.*, as occurs in heavy quark effective field theory (HQET) [5–7]. In the latter case one considers an $M \to \infty$ limit for a heavy quark and constructs a field theoretic lagrangian for a given four-velocity "supersector," v_μ. The lagrangian takes the form of a series expansion in higher dimension operators weighted by powers of $1/M$. The leading terms in the theory display heavy-spin symmetry (*e.g.*, degenerate 0^- and 1^- mesons). The reparameterization invariance is a residual symmetry that constrains the full operator structure and relates the coefficients of the terms in the lagrangian to higher orders of $1/M$. The reparameterization invariance is essentially the vestige of the underlying hidden Lorentz invariance [7].

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