

# PRESSURE SENSITIVITY CHARACTERIZATION OF SUPERCONDUCTING SPOKE CAVITIES\*

D. Passarelli<sup>#,†,‡</sup>, M.H. Awida<sup>†</sup>, I.V. Gonin<sup>†</sup>, L. Ristori<sup>†</sup>, V.P. Yakovlev<sup>†</sup>

<sup>†</sup> Fermilab, Batavia, IL 60510, USA.

<sup>‡</sup> University of Pisa, Scuola di Dottorato “L. da Vinci”, Italy

## Abstract

The following proposal illustrates a method to characterize the pressure sensitivity behavior of superconducting spoke cavities. This methodology relies on evaluating the variation of resonant frequency of a cavity by observing only the displacements at designed regions of the cavity. The proposed method permits a reduced computational burden and a systematic approach to achieve a minimum value of pressure sensitivity in a complex system of dressed cavity. This method has been used to characterize the superconducting spoke cavities type-1 (SSR1), under development for Project X, and to design the helium containment vessel in such way to reduce the pressure sensitivity value to zero.

## INTRODUCTION

The cavity sensitivity to Helium pressure is an important parameter which must be taken in consideration during the design of a dressed cavity system. The pressure fluctuations in the Helium bath cause cavity detuning by elastic deformations and micro-oscillations of the cavity walls. Any small shift from the resonant frequency of the cavity requires significant increase in power to maintain the electromagnetic field constant and, at the same time, it produces phase errors that affect the beam. For a cavity on resonance, the electric and magnetic stored energies are equal. If a small perturbation is made on the cavity wall, this will generally produce an unbalance of the electric and magnetic energies, and the resonant frequency will shift to restore the balance. The *Slatter perturbation theorem* [1] describes the shift of the resonant frequency, when a small volume  $\Delta V$  is removed from the cavity of volume  $V$ . From the theorem is understandable that the frequency increases if the magnetic field is large where the walls are pushed in, and it decreases if the electric is large there. This result is easier to remember if one identifies a decrease in the effective inductance where the magnetic field is large and an increase in the effective capacitance where the electric field large.

The traditional evaluation of  $df/dp$  involves a series of electromagnetic and structural analyses that can be performed in parallel with multiphysics software, such as Comsol [2] or Ansys multiphysics [3]. The goal is to evaluate the resonant frequency of the cavity under two arbitrary pressure loads. In this way it is possible to calculate the  $df/dp$  as:

$$\frac{df}{dp} = \frac{f_1 - f_0}{p}$$

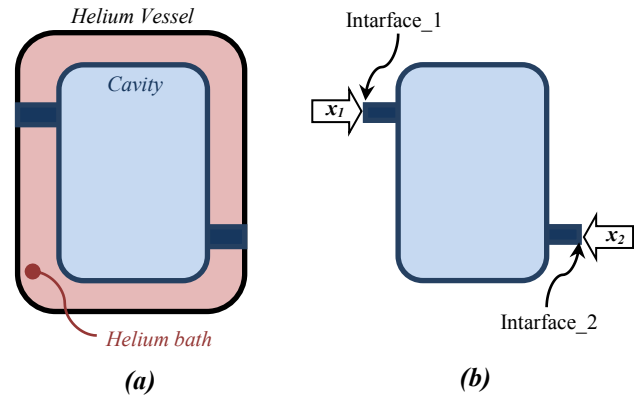
Where:

- $f_1, f_0$  are the resonant frequency, respectively, after and before the application of the pressure fluctuation that deform the volume of the cavity;
- $p$  is the pressure load applied to simulate the fluctuation of the helium bath.

It is of great importance to have a sense of how this sensitivity is affected by the different shapes of the helium vessel. During the phases of mechanical design of the vessel, its design is typically changed several times for engineering purposes and the iteration process is reduced considerably if there is a methodology to optimize and to estimate quickly the behavior of the system, in terms of sensitivity of resonant frequency to the pressure.

## METHODOLOGY

To describe the methodology let's consider a simple case at first.



**Figure 1.** Sketch of a cavity connected to a helium vessel (a) by two interfaces (b) where the directional displacements are probed to study the characterization.

An RF cavity enclosed in a helium vessel with only 2 areas of interface ( $DOF = 2$ ), Figure 1.

The definition and identification of such interfaces ( $DOF$ ) is of great importance for this methodology. The electromagnetic behavior of the RF cavity is probed in relation with the displacements at such interfaces.

The goal is to be able to evaluate the electromagnetic behavior of the RF cavity by observing only the displacements at such interfaces. The latter can be done by simple structural analyses of the complete assembly (cavity and vessel) under the same pressure loads ( $p$ )

\* Operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the U.S. Department of Energy.  
<sup>#</sup> email: donato@fnal.gov

used to extract the characteristic equation. The structural analyses require less time than electromagnetic-mechanical coupled analyses (traditional evaluation), also complex assemblies can be studied with these simpler analyses, therefore allowing a faster turnaround time.

To begin, one must evaluate the influence on  $df/dp$  of arbitrary displacements at the interfaces. This characterization requires a number of coupled simulations (traditional evaluations) that will increase with the type of interpolating equation and number of interfaces.

For quasi-static considerations and small perturbations, the interpolating equation can be considered a line and one needs  $N$  simulations to define entirely the equation, where  $N = 1 + DOF$ .

This can be done in the following way:

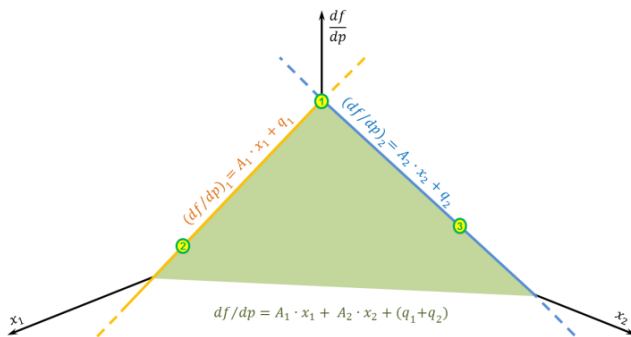
- Fix all interface locations ( $x_i = 0$ ), apply pressure ( $p$ ) (e.g.  $p = 1 \text{ atm}$ ) and evaluate the  $df/dp$  as described earlier (traditional evaluation). This is the reference  $df/dp$  for the next evaluations.
- Fix all interface locations but one at a time, apply the pressure and evaluate the directional displacement at the free interface and again the  $df/dp$ .
- Assuming a linear behavior, this time one extrapolates the linear function correlating the pressure sensitivity and the displacements of each DOF, calculating the slopes ( $A_i$ ) and the constant terms ( $q_i$ ), to obtain:

$$(df/dp)_i = A_i \cdot x_i + q_i$$

- In virtue of the superposition principle, true for the assumptions done, one can compile the function:

$$\frac{df}{dp} = \sum_{i=1}^{DOF} A_i \cdot x_i + q_i$$

The values  $A_i$ ,  $q_i$  are function of the structural design of the cavity. Each geometrical change of the cavity has influence on the characteristic function and a new function must be extracted.



**Figure 2.** Geometric function

For this simple case (Figure 1), one needs only  $N = 3$  coupled simulations to extract the three-points which are needed to define two linear equations, see Figure 2.

1. Line passing through points 1 and 2  
 $(df/dp)_1 = A_1 \cdot x_1 + q_1$
2. Line passing through points 1 and 3

$$(df/dp)_2 = A_2 \cdot x_2 + q_2$$

The final function is a plane in the following form:

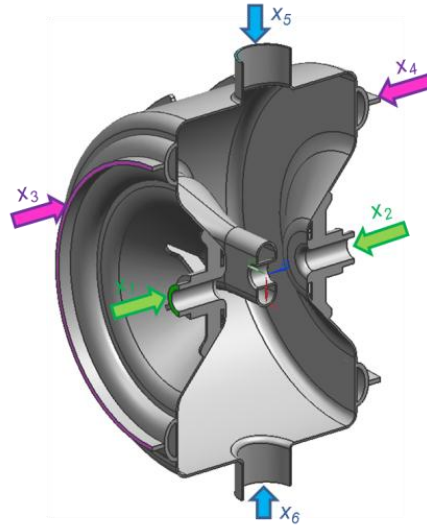
$$\begin{aligned} df/dp &= (df/dp)_1 + (df/dp)_2 = \\ &= A_1 \cdot x_1 + A_2 \cdot x_2 + (q_1 + q_2) \end{aligned}$$

## APPLICATION TO SSR1

The current plan for Project X [3] includes cavities with low bandwidths and the effects of microphonics have been taken into account from the early stages of cavity design.

The bare SSR1 cavities [4] previously designed and manufactured for HINS (a 4 K pulsed linac), will be utilized for Project X. These needed an improved design for the helium vessel to meet the new requirements for  $df/dp$  [5].

The SSR1 cavity has six interfaces with its helium vessel ( $DOF = 6$ ): two power-coupler (PC) ports, two beam-pipes (BP) ports and two transition rings (TR), shown in Figure 3.



**Figure 3.** Half model of SSR1 cavity showing the directional displacements applied at each DOF.

According to the above mentioned methodology, the characteristic expression of  $df/dp$  for such cavity will be comprised of six functions, each representing the influence of each DOF on  $df/dp$  of the cavity:

$$\frac{df}{dp} = \sum_{i=1}^6 A_i \cdot x_i + q_i$$

In this specific case we have found that:

$$A_1 = A_2; A_3 = A_4; A_5 = A_6.$$

This allowed to simplify the equation in the following way:

$$\frac{df}{dp} = A_1(x_1 + x_2) + A_3(x_3 + x_4) + A_5(x_5 + x_6) + \sum_{i=1}^3 q_i$$

Also the coefficients  $E$  and  $F$  appeared to be negligible compared to the others allowing further simplification:

$$\frac{df}{dp} = A_1(x_1 + x_2) + A_3(x_3 + x_4) + \sum_{i=1}^2 q_i$$

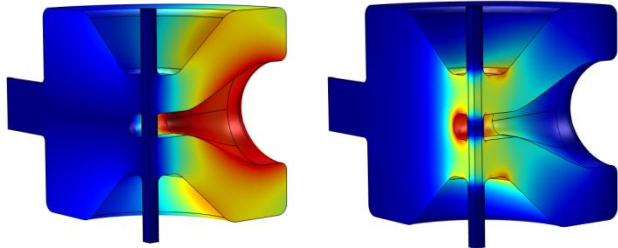
One could have come to the same conclusion, avoiding in this case four coupled simulations ( $N = 3$  and not 7) by observing the symmetrical distribution of the fields at the interfaces and the negligible fields at the PC port areas, see Figure 4.

The characteristic equation of the SSR1 cavities, shown in Figure 3, is:

$$\frac{df}{dp} [Hz/Torr] = -0.71(x_1 + x_2) - 0.07(x_3 + x_4) + 2.3$$

The pressure sensitivity is evaluated in  $[Hz/Torr]$  and the displacements are expressed in  $(\mu m)$ .

The fluctuation of pressure ( $p$ ) that one must apply to the structural analysis is  $p = 1atm$  because it has been the load used for the characterization.



**Figure 4.** Distribution of magnetic (left) and electric (right) distribution in SSR1 cavity volume.

#### Optimization and evaluations for final solution

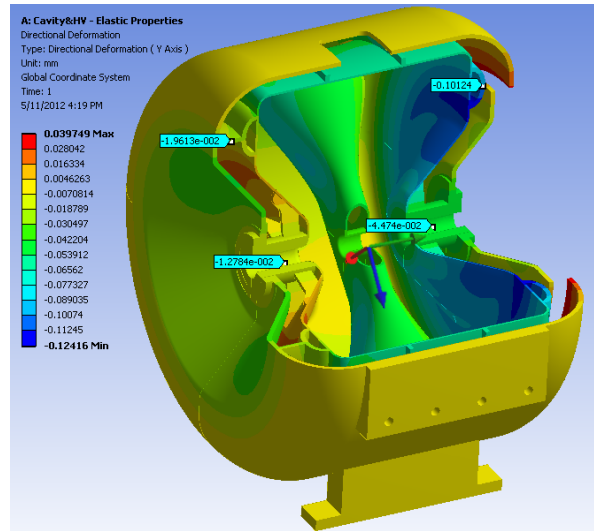
In most cases, the optimal design is a final system (cavity, helium vessel, tuner, etc.) which has  $df/dp \approx 0$  and the equation above gives a systematic approach to achieve that result.

Theoretically, for the SSR1 cavity, infinite (line) solutions allow meeting the optimal condition but limited range of deformations is acceptable to verify the required linear-elastic behavior of the material.

The equation has been useful to design the helium vessel around the cavity controlling only the deformations which cause a compensation of contributes.

Few steps of iteration of geometric changes were necessary to obtain the desired displacements at the interfaces which were evaluated by static structural analyses. This provided the advantage of shorter computational time than the traditional approach that uses coupled simulations. Obviously, the pressure load and the geometric model of the cavity must be the same used to define the characteristic function.

Coupled analyses have been performed with the final model (cavity and helium vessel) and they confirm the reliability of the pressure sensitivity ( $df/dp$ ) evaluated utilizing the described method. An example is shown in Figure 5.



**Figure 5.** Evaluation of  $df/dp = -14 Hz/Torr$  of jacketed SSR1\_G3 cavity (without Tuner effect) by the characteristic function with displacements probed by structural analysis:  $x_1 + x_2 = 32\mu m$  and  $x_3 + x_4 = 81.4\mu m$  under  $p = 1atm$ .

## CONCLUSIONS

The methodology described returns evaluations of  $df/dp$  with acceptable approximation compared to the results coming from coupled simulations (traditional method). Reduced computational burden has been achieved using this methodology.

The characterization of a bare cavity requires only few coupled analyses and it allows a systematic approach to the minimization of the pressure sensitivity, avoiding a trial-error approach.

Once the bare cavity is characterized, simpler structural analyses can evaluate  $df/dp$  of the complete assembly of cavity, vessel and tuner. This would be a difficult task for a coupled analysis.

More cavities already under study will be characterized following the proposed approach to confirm the methodology.

## REFERENCES

- [1] J.C. Slater, Microwave Electronics, Reviews of Modern Physics, Vol. 18, No. 4, October 1946;
- [2] Comsol Multiphysics v4.2 user guide.
- [3] Ansys Multiphysics v.13 user guide.
- [4] L. Ristori et al., "Design, Fabrication and Testing of Single Spoke Resonators at Fermilab", SRF2009, Berlin.
- [5] L. Ristori et al., "Functional Requirement Specification for SSR1 resonators", (project-docdb.fnal.gov)
- [6] L. Ristori, et al, "Design of Single Spoke Resonators for PXIE", presented at IPAC 2012, paper ID: 2689-WEPPC057