FERMILAB-PUB-12-105-T

An explicit SU(12) family and flavor unification model with natural fermion masses and mixings

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(Dated: April 23, 2012)

We present an SU(12) unification model with three light chiral families, avoiding any external flavor symmetries. The hierarchy of quark and lepton masses and mixings is explained by higher dimensional Yukawa interactions involving Higgs bosons that contain SU(5) singlet fields with VEVs about 50 times smaller than the SU(12) unification scale. The presented model has been analyzed in detail and found to be in very good agreement with the observed quark and lepton masses and mixings.

PACS numbers: 12.10.Dm, 12.15.Ff, 14.60.Pq

I. INTRODUCTION

The elementary fermions in the Standard Model (SM) appear in three families, which are a triplication according to their gauge transformations in the unbroken electroweak Lagrangian. The masses vary by several orders of magnitude and are a mystery withhin the SM. A wide range of models introduce a spontaneously broken flavor symmetry, with the associated group being either continuous or discrete [1, 2]. Different charges assigned to the families account for the mass and mixing hierarchy by producing different mass terms with an appropriate Higgs sector. This is especially necessary in SO(10) Grand Unified Theories (GUTs), where the **16** spinor irreducible representation (irrep) is the only complex representation yielding chiral fermions but no exotic fermions [3]. Early on, unification groups based on higher rank orthogonal groups such as SO(18) were explored [4, 5], but the number of exotic fields introduced became prohibitive. Early studies of the case of SU(N) family symmetry include models based on SU(11) [6, 7] and SU(9) [8–10].

Grand Unified Theories based on the groups SU(N) with $N{>}5$ can give rise to a different approach: while all families transform in the same way under the SM gauge group it is possible to assign them to different antisymmetric multiplets of SU(N) to obtain a non-trivial flavor structure. Since in SU(5) a family can only be assigned to $\mathbf{10}+\overline{\mathbf{5}}$ (or to the conjugated pair) [11], the unification group must be larger, hence $N{>}5$. This idea has led to the supersymmetric SU(7) [12] and the non-supersymmetric SU(8) models [13] proposed by Barr. In a previous publication [14] two of the present authors (RF and TWK) and others have constructed a hybrid of the latter two approaches, with a partial assignment to different irreps of SU(9) and four discrete symmetries in a non-supersymmetric model. Since then we have developed a systematic scan of SU(N)'s that loops over all possible fermion assignments to find viable models with or without discrete symmetries, including the hybrid case mentioned above. We present here an SU(12) model found by this scan which is free of any imposed external flavor symmetries. We have now also included the assignment of right-handed neutrinos, which allows the analysis of the full lepton sector as well, which is more ambitious than [12–14].

In Sec. II we outline the construction of the model by effective higher dimensional operators, which produce the mass and mixing hierarchy. Sec. III gives a brief survey of the model scan procedure which enabled us to find the SU(12) model presented here. Sec. IV is the major section and presents the SU(12) model in detail: In Sec. IV A we demonstrate how three chiral families arise from SU(12) in our model. After listing the fermion assignments and the Higgs sector in Sec. IV B we construct the Yukawa interactions for the quark and lepton sector in Sec. IV C, compute the resulting mass matrices, which involves the seesaw mechanism for the neutrinos, and finally fit the mass matrices to the measured values of the known masses and mixings in Sec. IV D. The discussion of the results and implications are presented in Sec. V. We summarize and conclude in Sec. VI.

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Operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

II. FERMION MASS HIERARCHY FROM HIGHER DIMENSIONAL OPERATORS

The Yukawa couplings in the Standard Model correctly parametrize the observed masses and mixings of quarks and leptons, yet the SM fails to explain why the coupling strengths are spread over a range of five orders of magnitude. Assuming an underlying naturalness of the Yukawa couplings, one can understand their measured values in an effective field theory scenario as coefficients of effective operators encoding short distance physics above a scale Λ. In these effective theories only heavy fermions obtain their masses from renormalizable, four-dimensional Yukawa couplings, while the masses of the lighter fermions are due to higher dimensional operators. In non-supersymmetric models these operators may stem from loops involving Higgs fields, while in supersymmetric models these loops are suppressed by factors of $M_{\rm SUSY}/M_{\rm GUT}$. In the latter case the masses of the lighter fermions must come from tree-level diagrams at the $M_{\rm GUT}$ scale, which we have pursued in the construction of the model presented here. However, we do not consider the phenomenology of the supersymmetric partners of Standard Model particles by simply assuming that supersymmetry is broken at a scale high enough to be inaccessible to current collider experiments, but low enough not to upset the suppression of loops. Here we expect supersymmetry breaking in the 10^8–10^{10} GeV range, which would soften but not solve the hierarchy problem.

To this end we introduce vectorlike heavy fermions with masses M at the SU(12) unification scale and extend the Higgs sector by introducing SU(12) Higgs bosons containing SU(5) singlet vacuum expectation values (VEVs). These allow one to construct Froggatt-Nielsen-type diagrams [15], i. e. tree-level diagrams with heavy fermions as one or more mass insertions and Higgs bosons containing SU(5) singlet VEVs (see e. g. [16]), which are assumed to be about 50 times lighter than the SU(12) unification scale. In going to the electroweak scale or lower, these mass insertions can be integrated out leaving effective Yukawa couplings involving Higgs bosons with electroweak VEVs and SU(5) singlet VEVs, suppressed by the masses of the heavy fermions at the SU(12) unification scale, $M_{\rm SU(12)}$. After breaking the Higgs sector to SU(5) and subsequently to $G_{\rm SM}$, the SU(5) singlet VEVs $\langle 1\rangle_{\rm SU(5)}$ and SU(12) unification scale $M_{\rm SU(12)}$ appear in the ratio:

$$\varepsilon = \frac{\langle 1\rangle_{\rm SU(5)}}{M_{\rm SU(12)}} \sim \frac{1}{50}. \tag{1}$$

Yukawa interactions of dimension $4+n$ give rise to mass matrix elements of the form:

$$h_{ij}\varepsilon^n v\, u^T_{iL} u^c_{jL}, \tag{2}$$

where h_{ij} are the Yukawa couplings and $v = 174\,\text{GeV}$ is the electroweak VEV. The dimensionless quantity ε parametrizes the mass and mixing hierarchy in our model. The power n of ε in each Yukawa interaction is the number of mass insertions and SU(5) singlet VEVs and represents an order of the effective operator higher than four. The value is roughly the ratio of the bottom-quark mass to the top-quark mass. The assumption in our model is that all other mass and mixing ratios can be expressed in powers of ε, while the Yukawa couplings h_{ij} are of $\mathcal{O}(1)$ at the SU(12) unification scale, with the dimension of the corresponding effective Yukawa interaction chosen accordingly.

The top-quark Yukawa coupling in the Standard Model is of order unity, suggesting that the renormalizable, dimension four interaction is the correct description. Since all other quark masses are small compared to the top-quark mass they must arise from higher dimensional Yukawa couplings. The up-type quarks exhibit an especially strong mass hierarchy compared to the down-type quarks. The mixing angles of the CKM matrix are small, leading to similar up- and down-type mass matrices, but with somewhat stronger hierarchies for the former. The neutrinos on the other hand have comparable masses and large mixing angles, leading to a light neutrino mass matrix with either a mild or little hierarchy, while the charged leptons exhibit a strong mass hierarchy.

III. MODEL SEARCH

The SU(12) model presented in this paper was found by a computer program developed by one of us (RPF) to scan models of the type described in Sec. II. The scan essentially seeks models by brute force, i. e., constructing all possible combinations of fermion assignments, Higgs irreps, and massive fermions and probing them for their phenomenological implications.

Due to the enormous number of combinations, the scan is constructed in five enclosing loops for a specific SU(N) group being searched: The first loop runs over anomaly-free sets of irreps that yield three chiral families at the SU(5) level. The fermions embedded in **10**'s of SU(5) are assigned to these sets of irreps first, which includes all up-type quark fields. This is sufficient to compute the up-type mass matrix, once the Higgs irreps and massive fermions are defined. All subsets of three of the anomaly-free, three-family sets of SU(N) irreps can be assigned to fermions of SU(5) **10**'s, which constitutes the second loop.

For each of these assignments, a third loop over all combinations of Higgs irreps and massive fermions taken from a basic set, is performed that computes the orders of the up-type mass matrix elements for each combination. Imposed requirements for the ordering can already filter out bad combinations of fermion assignments, Higgs and massive fermions. A crucial requirement is that only the top-quark mass term is of dimension 4, i. e., of zeroth order in ε.

For each of these filtered combinations, there is an analogous fourth loop over all assignments of fields embedded in the SU(5) $\overline{\mathbf{5}}$'s to irreps of the anomaly-free, three-family set the outer loop is currently investigating. Since assignments for the $\mathbf{10}$'s and $\overline{\mathbf{5}}$'s for all three families and definitions of Higgs irreps as well as massive fermions is sufficient to compute the orders of the up-type and down-type mass matrix elements and thus the CKM matrix, a fit of the prefactors of the up and down quark mass matrices and thus the complete quark sector is possible.

Having singled out quark models with reasonable phenomenology, a fifth loop adds assignments of right-handed neutrinos. For each of these assignments the orders of Dirac- and Majorana-neutrino mass matrix elements are computed, as well as the corresponding light-neutrino mass matrix via the type I seesaw mechanism. Together with the charged-lepton mass matrix, which is the transpose of the down-type mass matrix, the neutrino masses and mixings can be calculated.

For a fit to neutrino data analogous to the quark sector, only the mass differences squared are used, which allows for either a normal or an inverted hierarchy. The overall fit performed at this stage is a combined fit including the quark mass matrices as well. The fit yields a χ^2 value for each model, which allows one to select suitable models automatically. A second requirement imposed is that the prefactors be of order one. A more precise description of these steps and their results will be published in a follow-up paper by the authors.

IV. MODEL PROPERTIES

As a result of both the computer scan and by comparing many models by hand we have found a set of models of considerable interest. Here we will choose one specific SU(12) example to explore, which has many attractive features. Out of the thousands of models we have studied there is a large handful that fit the data quite well. Hence, our SU(12) model is neither generic nor unique.

A. Three Families in SU(12)

As a prime example of our procedure, we begin with the set of SU(12) irreps

$$6(\mathbf{495}) + 4(\overline{\mathbf{792}}) + 4(\overline{\mathbf{220}}) + (\overline{\mathbf{66}}) + 4(\overline{\mathbf{12}}) \tag{3}$$

which is anomaly free and consists of only totally antisymmetric irreps, to avoid the occurrence of exotic fermions. To see that this set contains precisely three chiral families we consider the breaking of the SU(12) gauge symmetry to SU(5), which can be accomplished by many different patterns of which we discuss two in the following paragraphs.

The totally antisymmetric irreps of SU(12) decompose to SU(5) as

$$\begin{aligned}
\mathbf{12} &\to (\mathbf{5}) + 7\,(\mathbf{1})\\
\mathbf{66} &\to 7\,(\mathbf{5}) + (\mathbf{10}) + 21\,(\mathbf{1})\\
\mathbf{220} &\to 21\,(\mathbf{5}) + 7\,(\mathbf{10}) + (\overline{\mathbf{10}}) + 35\,(\mathbf{1})\\
\mathbf{495} &\to 35\,(\mathbf{5}) + 21\,(\mathbf{10}) + 7\,(\overline{\mathbf{10}}) + (\overline{\mathbf{5}}) + 35\,(\mathbf{1})\\
\mathbf{792} &\to 35\,(\mathbf{5}) + 35\,(\mathbf{10}) + 21\,(\overline{\mathbf{10}}) + 7\,(\overline{\mathbf{5}}) + 22\,(\mathbf{1})\\
\mathbf{924} &\to 21\,(\mathbf{5}) + 35\,(\mathbf{10}) + 35\,(\overline{\mathbf{10}}) + 21\,(\overline{\mathbf{5}}) + 14\,(\mathbf{1})\\
\overline{\mathbf{792}} &\to 7\,(\mathbf{5}) + 21\,(\mathbf{10}) + 35\,(\overline{\mathbf{10}}) + 35\,(\overline{\mathbf{5}}) + 22\,(\mathbf{1})\\
\overline{\mathbf{495}} &\to (\mathbf{5}) + 7\,(\mathbf{10}) + 21\,(\overline{\mathbf{10}}) + 35\,(\overline{\mathbf{5}}) + 35\,(\mathbf{1})\\
\overline{\mathbf{220}} &\to (\mathbf{10}) + 7\,(\overline{\mathbf{10}}) + 21\,(\overline{\mathbf{5}}) + 35\,(\mathbf{1})\\
\overline{\mathbf{66}} &\to (\overline{\mathbf{10}}) + 7\,(\overline{\mathbf{5}}) + 21\,(\mathbf{1})\\
\overline{\mathbf{12}} &\to (\overline{\mathbf{5}}) + 7\,(\mathbf{1})
\end{aligned} \tag{4}$$

For the irreps in (3) including their multiplicities we have

$$3(\mathbf{10} + \overline{\mathbf{5}}) + 238(\mathbf{5} + \overline{\mathbf{5}}) + 211(\mathbf{10} + \overline{\mathbf{10}}) + 487(\mathbf{1}) \tag{5}$$

at the SU(5) level with three massless chiral families in $3(\mathbf{10} + \overline{\mathbf{5}})$. Vectorlike pairs of $(\mathbf{5} + \overline{\mathbf{5}})$ and $(\mathbf{10} + \overline{\mathbf{10}})$ as well as SU(5) singlet fermions $(\mathbf{1})$ acquire masses at the SU(5) unification scale. Of the sterile neutrinos in the form of SU(5) singlet fermions we assign three to the seesaw mechanism. The three massless chiral families will acquire mass via the Higgs mechanism at the electroweak scale.

We comment further here on the spontaneous symmetry breaking from SU(12) to SU(5) and then on to the standard model gauge group by discussing two of the several possible patterns of symmetry breaking. Note that since our model is supersymmetric above $\sim 10^{11}$ GeV, one must investigate spontaneous symmetry breaking via the superpotential. For this purpose there already exists an analysis of the spontaneous symmetry breaking in SU(N) models due to VEVs for chiral superfields in the adjoint and totally antisymmetric tensor irreps [17–19]. It is straightforward to show that a single adjoint can break SU(N)$\to$SU($N-n$)$\otimes$SU(n)$\otimes$U(1) and preserve supersymmetry, except when $n = N/2$. Hence we can break SU(12)$\to$SU(5)$\otimes$SU(7)$\otimes$U(1) with a single $\mathbf{143}_{\mathrm{H}}$. Adding four more adjoints we can break to SU(5)$\otimes$U(1)^7 and keep supersymmetry unbroken. Finally another adjoint can break SU(5) to $\mathrm{SU}(3)_{\mathrm{C}}\otimes\mathrm{SU}(2)_{\mathrm{L}}\otimes\mathrm{U}(1)_{\mathrm{Y}}$. One can check that the addition of $\mathbf{143}_{\mathrm{H}}$ adjoint scalars does not upset the patterns of masses and mixings we have established in our SU(12) model. The safest way to proceed further is to keep all the U(1)'s unbroken until we reach the SUSY breaking scale where a set of singlet VEVs coming from the antisymmetric tensor irreps with charges under the various U(1)'s then breaks all the U(1)'s except $\mathrm{U}(1)_{\mathrm{Y}}$. These low scale VEVs for components of the antisymmetric tensor irreps will also not impact the masses and mixings. Hence we are left with $\mathrm{SU}(3)_{\mathrm{C}}\otimes\mathrm{SU}(2)_{\mathrm{L}}\otimes\mathrm{U}(1)_{\mathrm{Y}}$ at the SUSY breaking scale. This procedure is rather generic and should work for most if not all of the models in the scan.

A second somewhat more appealing and economical, but less generic, approach is to use a set of scalars coming directly from the antisymmetric chiral superfield irreps to break SU(12) directly to SU(5) and then use a single adjoint to break SU(5) to $\mathrm{SU}(3)_\mathrm{C}\otimes\mathrm{SU}(2)_\mathrm{L}\otimes\mathrm{U}(1)_\mathrm{Y}$. This can be accomplished if the set of antisymmetric chiral superfield VEVs has vanishing total Dynkin weight [17, 18]. At least some of these VEVs would be expected to be in the same SU(12) irreps as the quark and lepton families, but this would not necessarily be so at the SU(5) level. Sequestering the families from the VEVs at the SU(5) level would avoid some technical difficulties, but the new VEVs would still leave the model in danger of disrupted masses and mixings. This approach would necessarily require a full analysis for each model in the scan. For the rest of this work we follow the first more generic approach to avoid these complications.

B. Fermion Assignments, Higgs and Massive Fermions

A successful assignment of the three SU(5) chiral families and singlet right-handed massive neutrinos to the SU(12) irreps follows:

$$\begin{array}{lll}
\text{First Family} & (\mathbf{10})\mathbf{495}_1 & \to u_\mathrm{L}, u^c_\mathrm{L}, d_\mathrm{L}, e^c_\mathrm{L} \\
 & (\overline{\mathbf{5}})\overline{\mathbf{66}}_1 & \to d^c_\mathrm{L}, e_\mathrm{L}, \nu_{1,\mathrm{L}} \\
 & (\mathbf{1})\overline{\mathbf{792}}_1 & \to N^c_{1,\mathrm{L}} \\
\text{Second Family} & (\mathbf{10})\overline{\mathbf{792}}_2 & \to c_\mathrm{L}, c^c_\mathrm{L}, s_\mathrm{L}, \mu^c_\mathrm{L} \\
 & (\overline{\mathbf{5}})\overline{\mathbf{792}}_2 & \to s^c_\mathrm{L}, \mu_\mathrm{L}, \nu_{2,\mathrm{L}} \\
 & (\mathbf{1})\overline{\mathbf{220}}_2 & \to N^c_{2,\mathrm{L}} \\
\text{Third Family} & (\mathbf{10})\overline{\mathbf{220}}_3 & \to t_\mathrm{L}, t^c_\mathrm{L}, b_\mathrm{L}, \tau^c_\mathrm{L} \\
 & (\overline{\mathbf{5}})\overline{\mathbf{792}}_3 & \to b^c_\mathrm{L}, \tau_\mathrm{L}, \nu_{3,\mathrm{L}} \\
 & (\mathbf{1})\overline{\mathbf{12}}_3 & \to N^c_{3,\mathrm{L}}
\end{array} \tag{6}$$

with five unassigned $\mathbf{495}$'s, two unassigned $\overline{\mathbf{220}}$'s and three unassigned $\overline{\mathbf{12}}$ irreps, as required by anomaly cancellation, regarded as massive fields decoupled below the SU(5) GUT scale as in (5).

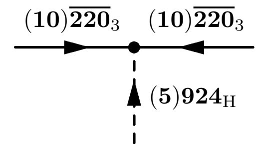
The model uses two conjugated Higgs representations containing the electroweak VEV, $\mathbf{5}$ and $\overline{\mathbf{5}}$ at the SU(5) level, which contain the SM Higgs doublet and its conjugate when SU(5) is broken via an adjoint Higgs. Two additional conjugate Higgs pairs containing SU(5) singlet VEVs and two massive fermion pairs are needed for the higher dimensional Yukawa couplings. As explained above and discussed in detail in Sec. IV A, a $\mathbf{143}_\mathrm{H}$ of SU(12) and a $\mathbf{24}_\mathrm{H}$ of SU(5) are needed for the symmetry breaking, where the latter may be embedded in the $\mathbf{143}_\mathrm{H}$. Four more adjoints for complete SU(7) breaking are not displayed here. To summarize, our scalar and massive fermion content is:

$$\begin{array}{ll}
\text{Higgs bosons} & \text{Massive fermions} \\
(\mathbf{5})\mathbf{924}_\mathrm{H},\ (\overline{\mathbf{5}})\mathbf{924}_\mathrm{H}, & \mathbf{220}\times\overline{\mathbf{220}}, \\
(\mathbf{1})\mathbf{66}_\mathrm{H},\ \ (\mathbf{1})\overline{\mathbf{66}}_\mathrm{H}, & \mathbf{792}\times\overline{\mathbf{792}} \\
(\mathbf{1})\mathbf{220}_\mathrm{H},\ (\mathbf{1})\overline{\mathbf{220}}_\mathrm{H}, & \\
(\mathbf{24})\mathbf{143}_\mathrm{H} &
\end{array} \tag{7}$$

C. Yukawa Interactions

By construction, the only renormalizable, dimension four Yukawa interaction is the top-quark mass term denoted as **U33**. At the SU(5) level the top-quark mass term is $\mathbf{10}_3\mathbf{10}_3\mathbf{5}_\mathrm{H}$,arising from $\overline{\mathbf{220}}_3\overline{\mathbf{220}}_3\mathbf{924}_\mathrm{H}$ at the SU(12) level, both containing singlets under their gauge groups, with the corresponding Feynman diagram

U33:

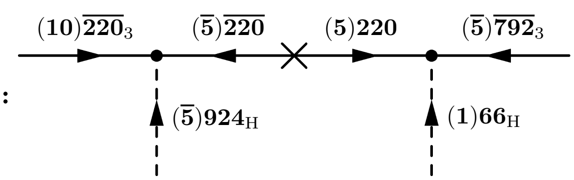


(8)

which displays both the SU(5), in parentheses, followed by the SU(12) multiplets. After spontaneous symmetry breaking including the electroweak symmetry, the top-quark mass term becomes: $h^\mathrm{u}_{33} v\, t^T_\mathrm{L} t^c_\mathrm{L}$.

All other mass terms in the full theory involve at least one mass insertion of a heavy-fermion pair and one Higgs with an SU(5) VEV. The corresponding tree-level diagrams are constructed by placing the fermion multiplets at both ends and assembling one Higgs containing the electroweak VEV and, depending on the dimension, one or more Higgs with an SU(5) singlet VEV and one or more massive fermions in (massive-)fermion-massive-fermion-Higgs vertices that individually form SU(12) as well as SU(5) singlets. The whole mass term thus contains an SU(12) and SU(5) singlet automatically. As an instructive example we give the bottom-quark mass term diagram (**D33**), which will be of dimension 5 after integrating out the massive fermions:

D33:



(9)

We list all leading order diagrams for the quark and charged lepton matrix elements in Table I using a shorthand notation for the Feynman diagrams, which abbreviates (9) to

$$(\mathbf{10})\overline{\mathbf{220}}_3.(\overline{\mathbf{5}})\mathbf{924}_\mathrm{H}.(\overline{\mathbf{5}})\overline{\mathbf{220}}\times(\mathbf{5})\mathbf{220}.(\mathbf{1})\mathbf{66}_\mathrm{H}.(\overline{\mathbf{5}})\overline{\mathbf{792}}_3. \tag{10}$$

After integrating out massive fermions the bottom-quark mass term becomes

$$(\mathbf{10})\overline{\mathbf{220}}_3(\overline{\mathbf{5}})\mathbf{924}_\mathrm{H}(\mathbf{1})\mathbf{66}_\mathrm{H}(\overline{\mathbf{5}})\overline{\mathbf{792}}_3, \tag{11}$$

and after spontaneous symmetry breaking including the electroweak one: $h^\mathrm{d}_{33}\varepsilon v\, b^T_\mathrm{L} b^c_\mathrm{L}$. Note that only one diagram for each matrix element appears at leading order, which is not self-evident in our model setup.

Up-Type Quark Mass-Term Diagrams		
Dim 4:	**U33**:	$(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
Dim 5:	**U23**:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
	U32:	$(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
Dim 6:	**U13**:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}_3$
	U31:	$(10)\overline{220}_3.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$
	U22:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
Dim 7:	**U12**:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}_2$
	U21:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$
Dim 8:	**U11**:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(5)924_{\mathrm{H}}.(10)\overline{220}\times(\overline{10})220$
		$.(1)66_{\mathrm{H}}.(10)\overline{792}\times(\overline{10})792.(1)220_{\mathrm{H}}.(10)495_1$
Down-Type Quark Mass-Term Diagrams		
Dim 5:	**D32**:	$(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
	D33:	$(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 6:	**D31**:	$(10)\overline{220}_3.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$
	D22:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
	D23:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 7:	**D12**:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_2$
	D21:	$(10)\overline{792}_2.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$
	D13:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}_3$
Dim 8:	**D11**:	$(10)495_1.(1)220_{\mathrm{H}}.(\overline{10})792\times(10)\overline{792}.(1)66_{\mathrm{H}}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\mathrm{H}}.(\overline{5})\overline{220}\times(5)220$
		$.(1)66_{\mathrm{H}}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\mathrm{H}}.(\overline{5})\overline{66}_1$

Table I. Leading order up- and down-type quark diagrams for each matrix element abbreviated as discussed in Sec. IV C.

We have defined the mass contributions in Table I with the left-handed fields to the left and the left-handed conjugate fields to the right. As can be seen from Eq. (6) the left- and right-handed components of the charged leptons are flipped assignments compared to the down-type quark components, due to the breaking of the underlying SU(5) to the SM gauge group. The corresponding diagrams for the charged leptons are then just the transpose of those listed for the down quarks, since no $\mathbf{143}_{\mathrm{H}}$ contributions appear in the diagrams.

1. Quark Masses and Mixings

Each mass term in Table I is accompanied by a coupling constant, which is assumed to be of order one at the SU(12) unification scale, as naturalness predicts. In Sec. IV D we will perform a fit to data for masses and mixings, where these coupling constants constitute the fit parameters. The coupling constants, also called "prefactors", are denoted by h^{u}_{ij} and h^{d}_{ij} for the up- and down-type quark mass terms, h^{ℓ}_{ij} for the charged-lepton mass terms and h^{mn}_{ij} and h^{dn}_{ij} for the Majorana- and Dirac-neutrino mass terms, with $i, j = 1, 2, 3$.

The number of Higgs bosons with SU(5) singlet VEVs for each mass term tells us the exponent of the parameter ε occurring after SU(5) symmetry breaking to the SM gauge group. We can thus derive the up-type, down-type and charged-lepton mass matrices with the coefficients of the effective mass operators involving the prefactors h^{u}_{ij}, h^{d}_{ij} and h^{ℓ}_{ij}, respectively.

As explained above, due to the SU(5) breaking to the SM gauge group, the charged-lepton mass matrix will be the transpose of the down-type quark mass matrix, which also holds true for its prefactors, $h^{\ell}_{ij} = h^{\mathrm{d}}_{ji}$. This is true to the extent that no adjoint Higgs bosons with VEVs pointing in the $B-L$ direction are present which would modify this transpose structure [20]. As such, the Yukawa coupling matrices are then given by

$$
\begin{aligned}
M_{\mathrm{U}} &= \begin{pmatrix} h^{\mathrm{u}}_{11}\varepsilon^4 & h^{\mathrm{u}}_{12}\varepsilon^3 & h^{\mathrm{u}}_{13}\varepsilon^2 \\ h^{\mathrm{u}}_{12}\varepsilon^3 & h^{\mathrm{u}}_{22}\varepsilon^2 & h^{\mathrm{u}}_{23}\varepsilon \\ h^{\mathrm{u}}_{13}\varepsilon^2 & h^{\mathrm{u}}_{23}\varepsilon & h^{\mathrm{u}}_{33} \end{pmatrix} v\,, \\
M_{\mathrm{D}} &= \begin{pmatrix} h^{\mathrm{d}}_{11}\varepsilon^4 & h^{\mathrm{d}}_{12}\varepsilon^3 & h^{\mathrm{d}}_{13}\varepsilon^3 \\ h^{\mathrm{d}}_{21}\varepsilon^3 & h^{\mathrm{d}}_{22}\varepsilon^2 & h^{\mathrm{d}}_{23}\varepsilon^2 \\ h^{\mathrm{d}}_{31}\varepsilon^2 & h^{\mathrm{d}}_{32}\varepsilon & h^{\mathrm{d}}_{33}\varepsilon \end{pmatrix} v\,, \\
M_{\mathrm{L}} &= \begin{pmatrix} h^{\ell}_{11}\varepsilon^4 & h^{\ell}_{12}\varepsilon^3 & h^{\ell}_{13}\varepsilon^2 \\ h^{\ell}_{21}\varepsilon^3 & h^{\ell}_{22}\varepsilon^2 & h^{\ell}_{23}\varepsilon \\ h^{\ell}_{31}\varepsilon^3 & h^{\ell}_{32}\varepsilon^2 & h^{\ell}_{33}\varepsilon \end{pmatrix} v = M_{\mathrm{D}}^T.
\end{aligned}
\tag{12}
$$

It is clear from the above that the up-quark matrix is symmetric, while the down-quark and charged-lepton mass matrices are doubly lopsided: the terms with h^{d}_{23} and h^{ℓ}_{32} are suppressed by one extra power of ε compared with the h^{d}_{32} and h^{ℓ}_{23} terms, respectively. For M_{D}, for example, this implies that a larger right-handed rotation than left-handed rotation is needed to bring the down quark matrix into diagonal form, while the opposite is true for M_{L} [3, 20, 21].

Dirac-Neutrino Mass-Term Diagrams		
Dim 4:	**DN23**:	$(\bar{5})\overline{792}_2.(5)924_{\rm H}.(1)\overline{12}_3$
	DN33:	$(\bar{5})\overline{792}_3.(5)924_{\rm H}.(1)\overline{12}_3$
Dim 5:	**DN13**:	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(5)924_{\rm H}.(1)\overline{12}_3$
	DN22:	$(\bar{5})\overline{792}_2.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$
	DN32:	$(\bar{5})\overline{792}_3.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$
Dim 6:	**DN12**:	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}_2$
	DN21:	$(\bar{5})\overline{792}_2.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$
	DN31:	$(\bar{5})\overline{792}_3.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$
Dim 7:	**DN11**:	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\rm H}.(5)792\times(\bar{5})\overline{792}.(1)66_{\rm H}.(5)220\times(\bar{5})\overline{220}.(5)924_{\rm H}.(1)\overline{220}\times(1)220.(1)66_{\rm H}.(1)\overline{792}_1$
Majorana-Neutrino Mass-Term Diagrams		
Dim 4:	**MN11**:	$(1)\overline{792}_1.(1)\overline{66}_{\rm H}.(1)\overline{792}_1$
	MN33:	$(1)\overline{12}_3.(1)66_{\rm H}.(1)\overline{12}_3$
Dim 5:	**MN12**:	$(1)\overline{792}_1.(1)\overline{66}_{\rm H}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\rm H}.(1)\overline{220}_2$
	MN21:	$(1)\overline{220}_2.(1)\overline{66}_{\rm H}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\rm H}.(1)\overline{792}_1$
Dim 6:	**MN13**:	$(1)\overline{792}_1.(1)\overline{66}_{\rm H}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\rm H}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\rm H}.(1)\overline{12}_3$
	MN31:	$(1)\overline{12}_3.(1)\overline{66}_{\rm H}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\rm H}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\rm H}.(1)\overline{792}_1$
	MN22:	$(1)\overline{220}_2.(1)\overline{66}_{\rm H}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\rm H}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\rm H}.(1)\overline{220}_2$
Dim 7:	**MN23**:	$(1)\overline{220}_2.(1)\overline{66}_{\rm H}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\rm H}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\rm H}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\rm H}.(1)\overline{12}_3$
	MN32:	$(1)\overline{12}_3.(1)\overline{66}_{\rm H}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\rm H}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\rm H}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\rm H}.(1)\overline{220}_2$

Table II. Leading order Dirac- and Majorana-neutrino diagrams for each matrix element abbreviated as discussed in Sec. IV C.

2. Neutrino Masses and Mixings

The assignment of heavy right-handed neutrinos to SU(12) multiplets containing an SU(5) singlet allows us to explore light-neutrino masses and mixings via the seesaw mechanism. To this end we have computed the resulting Dirac- and the Majorana-neutrino mass terms, which are of the form $(h^{\rm dn}_{ij}\varepsilon^n v)\bar{\nu}_{iL}N^c_{jL}$ and $(h^{\rm mn}_{ij}\varepsilon^n\Lambda_{\rm R})N^{cT}_{iL}N^c_{jL}$, respectively. The Majorana-neutrino mass terms are constructed from only SU(12) Higgs irreps containing SU(5) singlet VEVs. At the SU(5) level, a dimension four Majorana-neutrino mass term has the form $\mathbf{1}_{\rm i}\mathbf{1}_{\rm j}\mathbf{1}_{\rm H}$, while a higher dimensional mass term involves more SU(5) singlet Higgs. Thus the right-handed scale $\Lambda_{\rm R}$ coincides with the SU(5) singlet VEV $\langle 1\rangle_{\rm SU(5)}$. The Dirac-neutrino mass term couples the left-handed neutrino in the $\overline{\mathbf{5}}$ at the SU(5) level with the left-handed conjugate neutrino in the SU(5) singlet (see (6)). A four-dimensional Dirac-neutrino mass term thus has the form $\overline{\mathbf{5}}_{\rm i}\mathbf{1}_{\rm j}\mathbf{5}_{\rm H}$, while a higher dimensional Dirac mass term involves one or more SU(5) Higgs singlets. The Dirac- and Majorana-neutrino mass diagrams arising from the given fermion assignments and set of Higgs bosons and massive fermions are listed in Table II. As for the quark and charged lepton mass matrices, only one diagram for each matrix element appears at leading order.

The corresponding mass matrices are:

$$\begin{aligned} M_{\rm DN} &= \begin{pmatrix} h^{\rm dn}_{11}\varepsilon^3 & h^{\rm dn}_{12}\varepsilon^2 & h^{\rm dn}_{13}\varepsilon \\ h^{\rm dn}_{21}\varepsilon^2 & h^{\rm dn}_{22}\varepsilon & h^{\rm dn}_{23} \\ h^{\rm dn}_{31}\varepsilon^2 & h^{\rm dn}_{32}\varepsilon & h^{\rm dn}_{33} \end{pmatrix} v\,, \\ M_{\rm MN} &= \begin{pmatrix} h^{\rm mn}_{11} & h^{\rm mn}_{12}\varepsilon & h^{\rm mn}_{13}\varepsilon^2 \\ h^{\rm mn}_{12}\varepsilon & h^{\rm mn}_{22}\varepsilon^2 & h^{\rm mn}_{23}\varepsilon^3 \\ h^{\rm mn}_{13}\varepsilon^2 & h^{\rm mn}_{23}\varepsilon^3 & h^{\rm mn}_{33} \end{pmatrix} \Lambda_{\rm R}. \end{aligned} \tag{13}$$

Observe that not only are $M_{\rm D}$ and $M_{\rm L}$ doubly lopsided, but $M_{\rm DN}$ is as well. The symmetric light-neutrino mass matrix is obtained via the Type I Seesaw mechanism:

$$M_\nu = -M_{\rm DN}M^{-1}_{\rm MN}M^T_{\rm DN}. \tag{14}$$

In accordance with the construction of the up- and down-type quark mass matrices, we use only the leading term in ε for each matrix element of the light-neutrino mass matrix, yielding

$$M_\nu \approx \frac{v^2}{\Lambda_{\rm R}} \times \begin{pmatrix} \varepsilon^2\left(\frac{{h^{\rm dn}_{12}}^2 h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{{h^{\rm dn}_{13}}^2}{h^{\rm mn}_{33}}\right) & \varepsilon\left(\frac{h^{\rm dn}_{12}h^{\rm dn}_{22}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{13}h^{\rm dn}_{23}}{h^{\rm mn}_{33}}\right) & \varepsilon\left(\frac{h^{\rm dn}_{12}h^{\rm dn}_{32}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{13}h^{\rm dn}_{33}}{h^{\rm mn}_{33}}\right) \\ \varepsilon\left(\frac{h^{\rm dn}_{12}h^{\rm dn}_{22}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{13}h^{\rm dn}_{23}}{h^{\rm mn}_{33}}\right) & \frac{{h^{\rm dn}_{22}}^2 h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{{h^{\rm dn}_{23}}^2}{h^{\rm mn}_{33}} & \frac{h^{\rm dn}_{22}h^{\rm dn}_{32}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{23}h^{\rm dn}_{33}}{h^{\rm mn}_{33}} \\ \varepsilon\left(\frac{h^{\rm dn}_{12}h^{\rm dn}_{32}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{13}h^{\rm dn}_{33}}{h^{\rm mn}_{33}}\right) & \frac{h^{\rm dn}_{22}h^{\rm dn}_{32}h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{h^{\rm dn}_{23}h^{\rm dn}_{33}}{h^{\rm mn}_{33}} & \frac{{h^{\rm dn}_{32}}^2 h^{\rm mn}_{11}}{{h^{\rm mn}_{12}}^2 - h^{\rm mn}_{11}h^{\rm mn}_{22}} - \frac{{h^{\rm dn}_{33}}^2}{h^{\rm mn}_{33}} \end{pmatrix} \tag{15}$$

which does not involve the prefactors h_{11}^{dn}, h_{21}^{dn}, h_{31}^{dn}, h_{13}^{mn} and h_{23}^{mn}. These prefactors remain undetermined by the fit described in Sec. IV D, reducing the number of fit parameters as opposed to using the full expression, and thereby improving the fit convergence somewhat.

The light-neutrino mass matrix exhibits a much milder hierarchy compared to the up-type and down-type mass matrices, as can be seen from the pattern of powers of ε. A mild or flat hierarchy of M_ν is conducive to obtaining large mixing angles and similar light neutrino masses. Furthermore, one observes that the light neutrino mass matrix obtained via the seesaw mechanism involves the doubly lopsided Dirac neutrino mass matrix twice. The lopsided feature of M_{DN} is such as to require a large left-handed rotation to bring M_ν into diagonal form.

D. Phenomenology

The phenomenological implications of the model presented here are encoded in the mass matrices. Normally the up-type, down-type, charged-lepton and light-neutrino masses are the eigenvalues of the corresponding mass matrices M_{U}, M_{D}, M_{L} and M_ν, but since not all of these matrices are hermitian we diagonalize $MM^\dagger$ instead. Thus, with left-handed rotations we obtain real and positive eigenvalues as squares of the corresponding masses, according to

$$\begin{aligned}
\mathrm{diag}(m_u^2, m_c^2, m_t^2) &= U_{\mathrm{U}}^\dagger M_{\mathrm{U}} M_{\mathrm{U}}^\dagger U_{\mathrm{U}}, \\
\mathrm{diag}(m_d^2, m_s^2, m_b^2) &= U_{\mathrm{D}}^\dagger M_{\mathrm{D}} M_{\mathrm{D}}^\dagger U_{\mathrm{D}}, \\
\mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2) &= U_{\mathrm{L}}^\dagger M_{\mathrm{L}} M_{\mathrm{L}}^\dagger U_{\mathrm{L}}, \\
\mathrm{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2) &= U_\nu^\dagger M_\nu M_\nu^\dagger U_\nu.
\end{aligned} \tag{16}$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} is calculated from the unitary transformations U_{U} and U_{D} that diagonalize the up-type and down-type mass matrices respectively:

$$V_{\mathrm{CKM}} = U_{\mathrm{U}}^\dagger U_{\mathrm{D}}, \tag{17}$$

encoding the mismatch of the flavor and mass eigenbases of the up-type and down-type quarks. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix V_{PMNS} is obtained analogously from U_{L} and U_ν that diagonalize the charged-lepton mass matrix M_{L} and the light-neutrino mass matrix M_ν:

$$V_{\mathrm{PMNS}} = U_{\mathrm{L}}^\dagger U_\nu. \tag{18}$$

From the doubly lopsided natures of the three matrices, M_{D}, M_{L}, and M_{DN}, discussed earlier, we anticipate that for the mixing of the left-handed fields described by V_{CKM} and V_{PMNS}, small mixing angles will appear in the former and large mixing angles in the latter.

1. Fit Setup

The results still depend on the prefactors h_{ij}^{u}, $h_{ij}^{\mathrm{d}} = h_{ji}^{\ell}$, h_{ij}^{dn} and h_{ij}^{mn}. All four independent sets of prefactors are of $\mathcal{O}(1)$ at the SU(12) unification scale in our Froggatt-Nielsen scenario. To test the model, the prefactors should be fit with data for the masses and mixing matrices. The best fit should give reasonable theoretical predictions, and a χ^2 value serves as goodness-of-fit measure. Obviously the data and prediction should be fit at a common scale, e.g., the top-quark mass scale. Hence, the running of the prefactors has to be calculated, and their values from the fit run to the SU(12) unification scale should turn out to be of $\mathcal{O}(1)$.

For the fit we consider only real prefactors of the CKM and PMNS matrix elements, to avoid too many fit parameters for a good convergence of the fit. We have adhered to the Particle Data Group (PDG) sign convention for the CKM matrix [22] but used the tri-bimaximal mixing sign convention for the PMNS lepton mixing matrix [23]. As common scale for the fit, we choose for the top-quark scale $m_t(m_t) \simeq 166\,\mathrm{GeV}$ and use extrapolated masses for the quarks and charged lepton masses from [24], where they have been calculated using three-loop QCD and one-loop QED beta functions.

We use the measured values of the CKM matrix elements, with PDG sign convention, without extrapolating to the top-quark mass scale. The renormalization group flow of the CKM matrix is governed by the Yukawa couplings, which are small except for the top-quark. Thus the effect is negligible, especially for the matrix elements of the first two families, and small for the third family in the Standard Model [25], which also holds true for the low scale of the SU(12) model presented here. As data for the fit of the neutrino sector, we use the mass squared differences of the light neutrinos and the neutrino mixing angles obtained by a global analysis of oscillation data [26]. The PMNS matrix entering the fit as data is computed from the neutrino mixing angles using the PDG parametrization of the PMNS matrix [22] but with the tri-bimaximal mixing sign convention [23]. Note that the 13 element of the PMNS matrix is non-zero, as opposed to that for tri-bimaximal mixing, but in accord with the evidence for a non-zero θ_{13} [26]. A negative sign for the 13 element gives us better fit convergence and theoretical predictions than a positive one, which coincides with the preference for the CP phase of $\cos\delta{=}{-1}$ in [26].

With respect to the SU(12) unification scale of $\mathcal{O}(10^{16})\,\mathrm{GeV}$, the scale of neutrino measurements is near the top-quark scale ($\sim$1 MeV for reactor and solar neutrinos and $\sim$1 GeV for accelerator and atmospheric neutrinos). We assume here that the running between the two neutrino scales is small compared to the uncertainties in neutrino measurements.

Up-type masses	**Down-Type masses**	**CKM Matrix**				
$m_{\rm u} = 2.2\,\text{MeV}$	$m_{\rm d} = 3.8\,\text{MeV}$	0.974	0.225	0.003		
$m_{\rm c} = 600\,\text{MeV}$	$m_{\rm s} = 75\,\text{MeV}$	−0.225	0.973	0.041		
$m_{\rm t} = 166\,\text{GeV}$	$m_{\rm b} = 2.78\,\text{GeV}$	0.009	−0.040	0.999		
Ch. Lepton masses	**Neutrino Mass Diff.**	**PMNS Matrix**			**Mixing Angles**	**Phase**
$m_{\rm e} = 0.501\,\text{MeV}$	$\lvert\Delta_{21}\rvert = 7.6\times10^{-5}\,\text{eV}^2$	0.824	0.547	−0.145	$\sin^2\theta_{12} = 0.306$	
$m_\mu = 104\,\text{MeV}$	$\lvert\Delta_{31}\rvert = 2.4\times10^{-3}\,\text{eV}^2$	−0.500	0.582	−0.641	$\sin^2\theta_{23} = 0.420$	$\delta = \pi$
$m_\tau = 1.75\,\text{GeV}$	$\lvert\Delta_{32}\rvert = 2.4\times10^{-3}\,\text{eV}^2$	−0.267	0.601	0.754	$\sin^2\theta_{13} = 0.021$	

Table III. Phenomenological data entering the fit with masses at the top-quark scale.

Up-type masses	**Down-Type masses**	**CKM Matrix**				
$m_{\rm u} = 2.1\,\text{MeV}$	$m_{\rm d} = 2.7\,\text{MeV}$	0.974	0.227	0.003		
$m_{\rm c} = 600\,\text{MeV}$	$m_{\rm s} = 90.7\,\text{MeV}$	−0.227	0.973	0.042		
$m_{\rm t} = 166\,\text{GeV}$	$m_{\rm b} = 2.32\,\text{GeV}$	0.007	−0.042	0.999		
Ch. Lepton masses	**Neutrino Mass Diff.**	**PMNS Matrix**			**Mixing Angles**	**Phase**
$m_{\rm e} = 2.7\,\text{MeV}$	$\lvert\Delta_{21}\rvert = 7.5\times10^{-5}\,\text{eV}^2$	0.824	0.548	−0.145	$\sin^2\theta_{12} = 0.306$	
$m_\mu = 90.7\,\text{MeV}$	$\lvert\Delta_{31}\rvert = 2.5\times10^{-3}\,\text{eV}^2$	−0.500	0.582	−0.641	$\sin^2\theta_{23} = 0.420$	$\delta = \pi$
$m_\tau = 2.32\,\text{GeV}$	$\lvert\Delta_{32}\rvert = 2.4\times10^{-3}\,\text{eV}^2$	−0.267	0.601	0.754	$\sin^2\theta_{13} = 0.021$	
Heavy Neutrinos	**Light Neutrinos**					
$M_1 = 1.67\times10^{12}\,\text{GeV}$	$m_1 = 0.0\,\text{meV}$					
$M_2 = 6.85\times10^{13}\,\text{GeV}$	$m_2 = 8.65\,\text{meV}$					
$M_3 = 5.30\times10^{14}\,\text{GeV}$	$m_3 = 49.7\,\text{meV}$					

Table IV. Theoretical mass and mixing results obtained from the fitting procedure.

The quark and charged-lepton masses and light-neutrino mass differences, as well as the CKM and PMNS matrix elements we use as data in the fit are listed in Table III. The fit uses 6 quark masses, 3 charged-lepton masses, 3 light-neutrino mass squared differences, and 9 CKM and 9 PMNS matrix elements as observations, for a total of $n_{\rm data} = 30$.

The fit parameters are the prefactors of the four mass matrices and the right-handed scale $\Lambda_{\rm R}$, i.e. $n_{\rm params} = n_{\rm prefactors} + 1$. Since the up-type mass matrix as well as the Majorana-neutrino mass matrix are symmetric, they involve only 6 independent fit parameters each, while the down-type mass matrix and the Dirac-neutrino mass matrix each contribute 9 parameters. As explained in Sec. IV C 2, only the leading order in ε of the light-neutrino mass matrix is used in the fit, which does not involve 3 prefactors of the Dirac-neutrino and 2 of the Majorana-neutrino mass matrix; thus 5 neutrino related prefactors remain undetermined, yielding a total of $n_{\rm prefactors} = 25$ prefactors used in the fit.

It is clear that the ratio of the SU(5) singlet VEV to the SU(12) unification scale used as the basic parameter, $\varepsilon = \langle 1\rangle_{\rm SU(5)}/M_{\rm SU(12)} \sim 1/50$, in our model should be determined by the fit as well. However, we observe a bad convergence of the fit, when we allow it to vary. Thus, we were forced to fix its value and found $\varepsilon = 1/6.5^2 = 0.0237$ to be an appropriate value in accord with [24]. The resulting number of degrees of freedom is then $n_{\rm dof} = n_{\rm data} - n_{\rm prefactors} - 1 = 4$.

2. Fit Results

The mass matrices with the results for the prefactors inserted are listed below:

$$
\begin{aligned}
M_{\rm U} &= \begin{pmatrix} -1.1\varepsilon^4 & 7.1\varepsilon^3 & 5.6\varepsilon^2 \\ 7.1\varepsilon^3 & -6.2\varepsilon^2 & -0.10\varepsilon \\ 5.6\varepsilon^2 & -0.10\varepsilon & -0.95 \end{pmatrix} v, \\
M_{\rm D} &= \begin{pmatrix} -6.3\varepsilon^4 & 8.0\varepsilon^3 & -1.9\varepsilon^3 \\ -4.5\varepsilon^3 & 0.38\varepsilon^2 & -1.3\varepsilon^2 \\ 0.88\varepsilon^2 & -0.23\varepsilon & -0.51\varepsilon \end{pmatrix} v, \\
M_{\rm DN} &= \begin{pmatrix} h^{\rm dn}_{11}\varepsilon^3 & 0.21\varepsilon^2 & -2.7\varepsilon \\ h^{\rm dn}_{21}\varepsilon^2 & -0.28\varepsilon & -0.15 \\ h^{\rm dn}_{31}\varepsilon^2 & 2.1\varepsilon & 0.086 \end{pmatrix} v, \\
M_{\rm MN} &= \begin{pmatrix} -0.72 & -1.5\varepsilon & h^{\rm mn}_{13}\varepsilon^2 \\ -1.5\varepsilon & 0.95\varepsilon^2 & h^{\rm mn}_{23}\varepsilon^3 \\ h^{\rm mn}_{13}\varepsilon^2 & h^{\rm mn}_{23}\varepsilon^3 & 0.093 \end{pmatrix} \Lambda_{\rm R}, \\
M_\nu &= \begin{pmatrix} -81.\varepsilon^2 & -4.3\varepsilon & 2.4\varepsilon \\ -4.3\varepsilon & -0.25 & 0.28 \\ 2.4\varepsilon & 0.28 & -1.1 \end{pmatrix} \frac{v^2}{\Lambda_{\rm R}},
\end{aligned}
\tag{19}
$$

with the right-handed scale determined to be $\Lambda_{\rm R} = 7.4\times10^{14}\,\text{GeV}$ and Δ_{32} fit with $m_3 \sim 50$ meV. As explained in Sec. IV C 2, $\Lambda_{\rm R}$ coincides with the SU(5) singlet VEV, $\langle 1\rangle_{\rm SU(5)}$, which allows us to determine the SU(12) unification scale from the fit to be $M_{\rm SU(12)} = \Lambda_{\rm R}/\varepsilon = 3.1\times10^{16}\,\text{GeV}$.

The corresponding theoretical predictions for the masses and mixings are listed in Table IV. The predictions are nearly in perfect agreement with the phenomenological data entering the fit, which is due to the fact that almost as many fit parameters as data points are used. This is reflected in the abnormally small $\chi^2/n_{\rm dof}$ and large P-value: With χ^2=0.239 and $n_{\rm dof}$=4 we obtain $\chi^2/n_{\rm dof}$=0.060 and a P-value of prob(0.239, 4)=0.993. Only the lepton and down-quark masses deviate significantly from their measured value, since our SU(12) model forces them to be equal which keeps the χ^2 from dropping even further.

It is evident conclusions that can be drawn from the fit results for our SU(12) model are somewhat limited, for only a slightly different phenomenology could be accommodated by an according shift of the prefactors. The prefactors obtained from the fit can be considered to be of $\mathcal{O}(1)$ as required by naturalness, aside from the 11 element of M_ν. However, according to the phenomenological input discussed above, their values apply at the top-quark scale. For a complete analysis one has to run them to the SU(12) unification scale, where their compliance with the naturalness paradigm is supposed to be probed. This calculation as well as a fully fledged fit, including uncertainties, correlations, complex prefactors and the running of the CKM matrix and of the neutrino data to the top-quark scale goes beyond the scope of this paper.

Given the above caveats, we note that a normal mass hierarchy for the light neutrinos is obtained with one massless neutrino. Allowing for the sizable reactor neutrino angle confirmed by the fit and the fully allowed ranges of the Dirac and Majorana phases not present in our analysis, the effective mass prediction for neutrino-less double beta decay lies in the range 1.5 - 3.7 meV.

V. DISCUSSION

Most flavor symmetry models studied to date involve discrete flavor groups. A typical model in this class based on the standard model gauge group or on a more general SU(N) family gauge group has an additional discrete flavor symmetry G with the matter spectrum living in irreps of G. However, such models have several disadvantages compared to models that have no additional discrete symmetry.

First disadvantage to having a discrete symmetry is that if it is a global symmetry, it will be broken by gravity [27–29], and the breaking will not in general be in the pattern one wishes to arrange for the family symmetry.

Second, it is difficult to explain the origin of a discrete symmetry in a more fundamental theory. It could arise from breaking a gauge symmetry and avoid the problems with gravity, but this is difficult to arrange. In that case it would be necessary for G to be anomaly-free [30]. Another disadvantage of including a discrete symmetry is that when it breaks, cosmic domain walls are produced. The walls need to be removed, and they can be inflated away in some models, but not all. In particular, if there is a discrete symmetry breaking after inflation, then the cosmology of the model will be untenable.

If we go to larger N to avoid G as in the present work, then there is no domain wall problem. There is usually still a magnetic monopole problem that needs to be solved by inflation. However, this can be done at the GUT scale, and it does not re-emerge at a lower scale. (The $SM{\otimes}G$ and SU(N)$\otimes G$ models also have similar magnetic monopole problems.) So we conclude that the cosmology of the discrete symmetry free models is typically more attractive. Their one disadvantage is that the initial gauge group is usually larger, but not below the GUT scale.

We see a balance between the two types of models. Including a discrete symmetry to arrange a desired behavior for masses and mixings in $SM{\otimes}G$ and SU(N)$\otimes G$ models can be offset by increasing N in pure gauge SU(N) models to avoid the inclusion of G. Since no domain wall problem or problem with gravity arises if G can be avoided, we conclude that pure gauge family symmetric models like the SU(12) model presented here, have several advantages over flavor-symmetric models that contain discrete symmetries.

In our studies of models of different SU(N)'s we find that with increasing N it is possible to obtain models with more and more desired features implemented. Those features show not only compliance with phenomenology but additional esthetic properties such as simplicity. Nevertheless, selecting a specific assignment of fermions and Higgs scalars out of millions of possible assignments, because of its ability to reproduce phenomenology, is yet another application of the anthropic principle and is reminiscent of the string theory landscape.

VI. SUMMARY AND CONCLUSION

We have developed a systematic computer scan for SU(N) family and flavor unification models that reproduce the observed fermion mass and mixing hierarchy with higher-dimensional effective Yukawa couplings involving an extended Higgs sector. These models are of the supersymmetric type since the higher-dimensional Yukawa couplings stem from Froggatt-Nielson-type diagrams involving massive fermion insertions. The three families of fermions, the massive fermions and the Higgs scalars are assigned to various SU(N) representations and may also involve the assignment of discrete symmetry charges. A basic parameter in this setup is the ratio of the scale of imposed SU(5) singlet VEVs to the SU(N) unification scale, denoted as ε, with a value of roughly the ratio of the bottom-quark to the top-quark mass, i.e. $\sim$1/50.

In this paper we have presented an example of an SU(12) model obtained by our computer scan, which does not involve any discrete flavor symmetry. This particular model belongs to a subset of economic SU(12) models having only two pairs of Higgs bosons with SU(5) singlet VEVs besides the conjugate Higgs field with the pair of electroweak VEVs, and two massive fermion pairs at the SU(12) level.

However, we need several additional SU(12) irreps for anomaly cancellation, and owing to the large SU(12) gauge group we predict a host of fermions which become massive after symmetry breakdown to SU(5). Only three SU(5) sets of $\mathbf{10} + \overline{\mathbf{5}}$ fermions remain massless down to the electroweak scale, while three SU(5) left-handed neutrino conjugate singlets take part in the seesaw mechanism.

The model presented has only one diagram at leading order in ε for each matrix element of the five up, down, charged lepton, Dirac- and Majorana-neutrino mass matrices. The down-type, lepton and Dirac-neutrino mass matrices are found to be doubly lopsided. The mass matrices involve undetermined Yukawa couplings, called "prefactors," which are supposed to be of $\mathcal{O}(1)$ at the SU(12) unification scale. Being able to compute all quark and lepton masses and mixings from their dependence on these prefactors, we performed a simple fit to experimental data to test their naturalness and the compliance of the model with phenomenology. We have presented here a fit result with prefactors that can be considered of $\mathcal{O}(1)$ and a near to perfect agreement of theoretical prediction with phenomenological data. In addition our analysis of the neutrino sector involving the type-I seesaw mechanism allows us to determine the light-neutrino masses and thus their hierarchy, as well as the heavy-neutrino masses and the full PMNS matrix. We find a normal hierarchy with one light-neutrino mass being zero.

Still the predictive power of our simple analysis is limited: We used the top-quark scale as common scale for the fit. Thus the determined values of the prefactors apply at this scale and should be run to the SU(12) unification scale to test their naturalness in a rigorous analysis. We also have not included any CP phases in the mass matrices. Furthermore, the nearly perfect agreement of theoretical prediction with phenomenological data is due to a large number of fit parameters, which are mostly prefactors. Besides being of $\mathcal{O}(1)$, there is no a-priory estimate of their value as initial value. Since differences in numerators and denominators of mixing-matrix elements are involved, the uniqueness of the χ^2 minimum must be doubted. In a fully fledged analysis pull distributions generated by toy Monte Carlos clarify this aspect of the fit quality. A rigorous analysis would also include uncertainties of experimental data as well as estimations of the theoretical uncertainties. Nevertheless, we believe the alternative approach to unification of families and flavors explored here warrants further study despite the limitations of our analysis cited above.

ACKNOWLEDGMENTS

We thank Qaisar Shafi and William Bardeen for useful discussions. The work of RPF was supported by a fellowship within the Postdoc-Programme of the German Academic Exchange Service (DAAD). The work of RPF and TWK was supported by US DOE grant E-FG05-85ER40226. One of us (CHA) thanks the Fermilab Theoretical Physics Department for its kind hospitality, where part of this work was carried out. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the U.S. Department of Energy.

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