# A high-resolution multi-slit phase space measurement technique for low-emittance beams

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**Abstract.** Precise measurement of transverse phase space of a high-brightness electron beam is of fundamental importance in modern accelerators and free-electron lasers. Often, the transverse phase space of a high brightness, space-charge dominated electron beam is measured using a multi-slit method. In this method, a transverse mask (slit/pepperpot) samples the beam into a set of beamlets, which are then analyzed on to a screen downstream. The resolution in this method is limited by the screen used which is around 20 um for a high-sensitivity Yttrium Aluminum Garnet screen. Accurate measurement of sub-micron transverse emittances using this method would require a long drift space. In this work, we propose a variation of the technique that incorporates quadrupole magnets between the multislit and the screen to improve the resolution of the transverse-phase-space measurement within a short footprint. This technique could be combined with a phase space exchanger to also measure longitudinal phase space in a single shot.

**Keywords:** multi-slit, emittance, high-resolution, emittance exchange, low-emittance
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## INTRODUCTION

Successful operation of accelerator-driven light sources such as free-electron lasers, energy-recovery linacs requires low emittance electron injectors [1]. Given the recent progress in low-charge operation of the photoinjector for a x-ray free-electron laser [2] and advanced beam manipulation such as flat-beam generation [3], the emittance of the beam can be as low as $\varepsilon_n = 0.1\ \mu m$. More recently there has also been increasing interest in building compact x-ray sources employing novel cathodes that also require ultra-low emittance beams [4]. In all of these machines, it is critical to measure the transverse phase space of the electron beam with high resolution and the beam emittance.

At lower energies, where the beam is space-charge dominated either a movable slit or a single multi-slit mask is used to measure the transverse-phase space of the beam. In this technique, the slit(s) mask samples the beam (space-charge dominated) into beamlets (emittance-dominated), which are drifted downstream and then imaged on to a screen. The weighted average of the r.m.s width of the beamlets together with the total beam size can reveal the transverse phase space of the electron beam [5]. The multi-slit technique has been used at many accelerators successfully for measuring the transverse phase space and the beam emittance [6][7]. In this work, we revisit the multi-slit technique and propose a modification that improves the resolution of this measurement significantly to measure ultra-low emittance beams.

### Resolution limit of the standard multi-slit technique

Traditional multi-slit methods are limited in their resolution when the beamlet size at the screen is comparable to the slit-width. To given an example, typical slit width is about 50 $\mu m$ (r.m.s width is 14.4 $\mu m$) and the beamlet size at the screen is given by $\sigma_x = L\sigma_{x'}$, where $\sigma_x$ is the beamlet size at the screen, $L$ is the drift space between the multi-slit and the screen, and $\sigma_{x'}$ is the particle beam divergence. If we assume the beam energy is 14.3 MeV, and the beam normalized emittance $\varepsilon = 0.1\mu m$, and the drift length as $\sim 3$ m, then the beamlet size at the screen is $18\mu m$ for a beam with a transverse spot size of 1 $mm$ at the slits. The beamlet width is comparable to the slit-width and hence the accuracy of this method is limited to $\sim 30\%$. In the next section, we describe our proposed technique which promises to increase the resolution of the multi-slit method using a quadrupole lattice inserted between the multi-slit mask and the screen.

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## ANALYSIS

In this section, we consolidate the analysis developed elsewhere[8]. The horizontal phase space at the multi-slit mask can be modelled as: $\phi(x,x') = \frac{1}{\sqrt{2\pi\varepsilon}} exp(-\frac{\gamma x^2 - 2\alpha x x' + \beta x'^2}{2\varepsilon})$, where $\alpha$, $\beta$, and $\gamma$ are the Twiss parameters and $\varepsilon$ is the rms emittance of the beam. Now for the calculating the effect of multi-slit mask, we assume the slit centers are located at $x_i$ and the slit width is $w$. Since the slit width is small , we assume that the mask can be modeled as a Dirac-comb function $\Pi(x) = \sum_{i=1}^{N} \delta(x - x_i)$ where $x_i = i\Delta x$, where $\Delta$ is the slit separation (typically 1 mm). The multi-slit samples the horizontal phase space and the distribution after the mask is given by: $\psi(x') = \sum_{i=1}^{N} \int \phi(x,x')\Pi(x)dx = \sum_{i=1}^{N} \phi(x_i,x')$. Therefore, the transverse horizontal phase space after the multi-slit screen can be written as

$$\psi(x') = \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi\varepsilon}} exp(-\frac{\gamma x_i^2 - 2\alpha x_i x' + \beta x'^2}{2\varepsilon}) \tag{1}$$

Now let us assume a 2 x 2 transfer matrix $\mathbf{R}$ inserted between the mask and the screen. Let $\Xi = \mathbf{RX}$, where $\mathbf{X}$ is the input $x, x'$ coordinate and $\Xi$ is the trace space co-ordinates at the screen. In order to obtain the beam divergence in terms of the R-matrix and the initial position at the screen, we can write $\xi = R_{11}x_i + R_{12}x'$, where $\xi$ is the $x$-position in the screen after passing through the R-matrix. Rearranging this equation we get for $x' = \frac{\xi - R_{11}x_i}{R_{12}}$. Substituting this expression in (1), we get

$$\psi(x') = \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi\varepsilon}} exp(-\frac{\gamma x_i^2 - 2\alpha x_i \frac{\xi - R_{11}x_i}{R_{12}} + \beta(\frac{\xi - R_{11}x_i}{R_{12}})^2}{2\varepsilon}) \tag{2}$$

Expanding and rearranging equation (2), we get

$$\phi(\xi) = \frac{1}{\sqrt{2\pi\varepsilon}} \sum_{i=1}^{N} exp(-\frac{x_i^2}{2\sigma_0^2}) exp\left\{ -\frac{1}{2R_{12}^2\sigma_0^4} \left\{ \xi - x_i(R_{11} + \frac{\alpha R_{12}}{\beta}) \right\}^2 \right\} \tag{3}$$

where $\sigma_0 = \sqrt{\varepsilon\beta}$ is the rms beam size at the slits mask location and $\sigma_0' = \sqrt{\frac{\varepsilon}{\beta}}$ is the rms uncorrelated divergence spread. From equation (3) we can see that the output at the screen consists of a series of Gaussian beamlets. Some important parameters are the rms size of these beamlets given by $R_{12}\sigma_0'$ and the separation between the beamlets given by $x_i(R_{11} + \frac{\alpha R_{12}}{\beta})$. By fitting the equation (3) in a multi-parameter fit, the Twiss parameters and the beam emittance at the mask location can be retrieved.

The beamsize at the screen is given by $\sigma_x = \sqrt{\frac{w^2}{12} + R_{12}^2\sigma_0'^2}$, where $w$ is the slit width. In this work, we note that in a standard multi-slit measurement $R_{12}$ is set to the drift length $L$ between the mask and the screen location and $R_{11}$ is set to unity. In order to improve the resolution in this technique, we have to increase the ratio $R_{12}/L$ and simultaneously decrease $R_{11}$. A quadrupole lattice inserted between the mask and the screen can achieve this by increasing $R_{12}$ (magnification) and decreasing $R_{11}$ (reduce sensitivity to the slit width), thereby improving the resolution of this method within a small footprint.

## SIMULATION

**TABLE 1.** Parameter list for the simulation

| | |
|---|---|
| Beam Energy | 14.3 MeV |
| Emittance (normalized) | 0.1 $\mu m$ |
| Slits width | 50 $\mu m$ |
| Slits spacing | 1 $mm$ |
| Distance between screen and slits | 3.3 m |

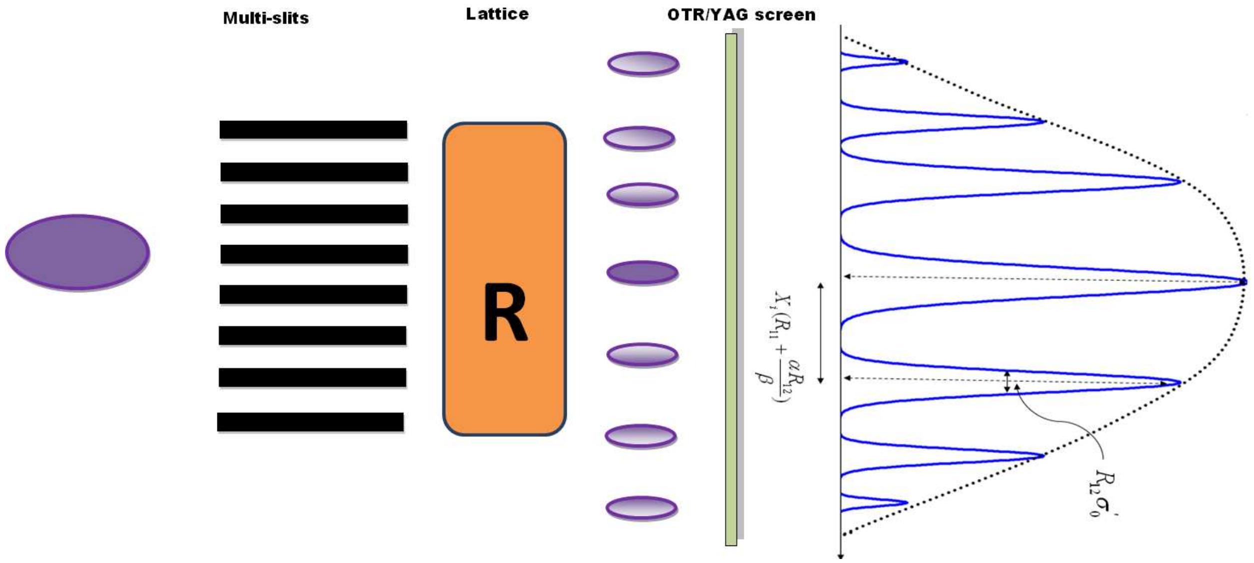


**FIGURE 1.** Schematic of the multi-slit method. The multi-slit mask converts the beam into beamlets, which are then propagated through a lattice and imaged on to a screen. In standard technique, the lattice is just a drift.

In order to test this method, we used the tracking code *elegant* [9] to both optimize and simulate a beam lattice for a predetermined $R_{11}$ and $R_{12}$. The parameter table (Table. 1) list all the details of the simulation. A schematic of the setup is shown in Fig. 1. In our case, we used a drift distance of 3.3 m and hence our ratio of $\frac{R_{12}}{R_{11}} = 3.3$. The goal of the simulation was to make the $\frac{R_{12}}{R_{11}}$ around 20 times higher than this nominal value. Three quadrupoles were used in the simulation to prevent overdriving a single (or double) quadrupole and thus minimize second order chromatic aberration. Lesser number of quadrupoles may be used depending on the ratio needed. After optimization, we obtain a quadrupole lattice that has $R_{11} = 0.78$ and $R_{12} = 16$. The beam profile on the screen after passing through the multi-slit mask in a drift is compared with the new method and is shown in the Fig. 2. The average x-size of the beamlets is $\sim$ 57 $\mu m$ compared to $\sim$ 18 $\mu m$ in case of the drift. This increase in the spot size and the reduction in the value of $R_{11}$ (thereby reducing the sensitivity of the measurement to the slit width), boosts the accuracy of this method to $\leq 0.5\%$ . We note that an added benefit of this method is the better signal-to-noise ratio (SNR) because we are defocusing in $x$ and simultaneously focusing in $y$, whereas in the pure drift the beam is defocused on both planes leading to a poorer SNR.

## Effect of fractional energy spread

In the simulation, we assume that the electron beam has no energy-position correlation. Typically laboratory beams, say after an accelerating cavity, have an RF-chirp. The use of quadrupole in our method can lead to chromatic aberrations that might influence the measurement of the beamsize and hence limit the resolution of this technique. The dependency of the fractional energy spread on the measured beam profile is shown in the Fig. 3. The aberration causes the lower energy particles to defocus more leading to increase in the spread of the particle position. This increase in beamlet width affects the calculated emittance as shown in Fig. 3. One possible way to overcome this effect is to use lesser magnification ratio and thus reducing the required strength on the quadrupoles.

## Measuring longitudinal phase-space

High-resolution longitudinal phase-space measurement is critical for tuning the electron beam for fourth-generation light sources. For e.g. seeded FELs need an accurate measurement of slice energy spread. This technique can be used

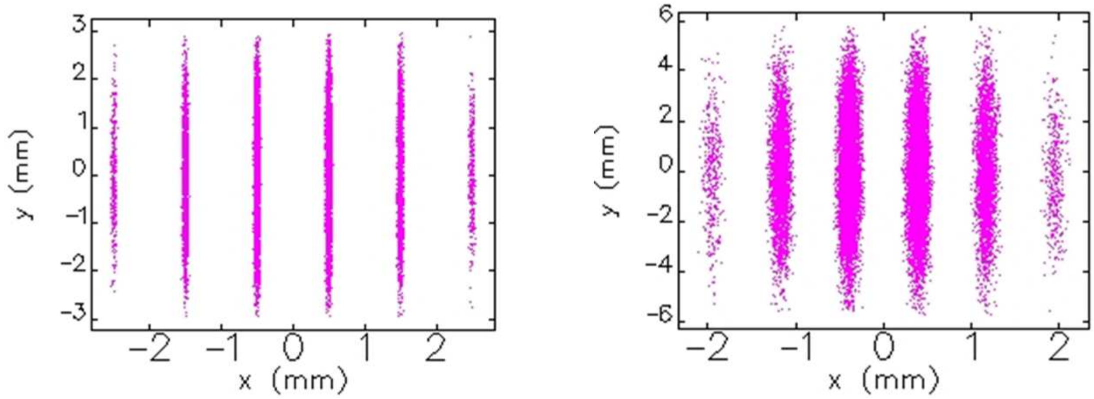


**FIGURE 2.** The transverse beam profile on the screen in a pure drift lattice (left) and a tuned quadrupole lattice (right). The average x-size of the beamlets is $\sim 57\ \mu m$ compared to $\sim 18\ \mu m$ in case of the drift.

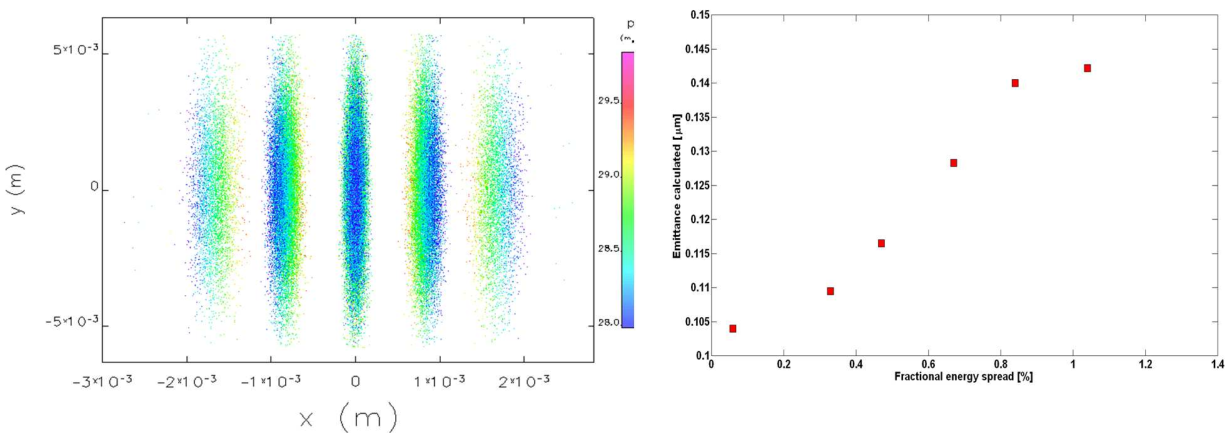


**FIGURE 3.** The effect of fractional energy spread on the measured beamlet width. The energy spread increases the width of the beamlet thereby overestimating the emittance.

to measure longitudinal phase-space with high resolution when used in conjunction with an phase-space exchanger (PEX)[10]. The beam after exchange is sent through a multi-slit mask and then magnified appropriately to construct the longitudinal phase space. Note, in this case, by adjusting the quadrupoles upstream of the PEX, the outgoing energy-position (axial) of the beam could be changed thereby reducing chromatic aberrations. Other limitations can arise from coherent synchrotron radiation emittance growth in the dipoles and geometric aberrations due to the large beam sizes through the dipoles.

## CONCLUSION

A method for improving the multi-slit mask technique to measure ultra low-emittance beams has been proposed. The advantages of this method are high-resolution measurement within a short footprint using traditional optical systems. No modification in imaging optics is required. The method has the added advantage of better signal-noise ratio and single-shot operation. Also, when combined with a phase-space exchanger, a high-resolution longitudinal phase-space of the beam can be measured as well. Practical limitations of the method include chromatic effects from quadrupoles for large energy spread beams and the need to accurately measure the transfer matrix during the experiment.

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