# A 3 TEV MUON COLLIDER LATTICE DESIGN*

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*Abstract*

A new lattice for 3 TeV c.o.m. energy with $\beta^*$=5mm was developed which follows the basic concept of the earlier 1.5 TeV design [1] but uses quad triplets for the final focus in order to keep the maximum magnet strength and aperture close to those in 1.5 TeV case. Another difference is employment of combined-function magnets with the goal to lower heat deposition in magnet cold mass and to eliminate bending field free regions which produce "hot spots" of neutrino radiation that can be an issue at higher energy. The proposed lattice is shown to satisfy the requirements on luminosity, dynamic aperture and momentum acceptance.

## INTRODUCTION

Muon Collider (MC) is now considered as the most exciting option for the energy frontier machine in the post-LHC era. It has a number of important advantages over its competitor $e^+e^-$ collider: potentially higher energy, better energy resolution, larger cross-section for scalar particle production, smaller footprint, etc. However, in order to achieve a competitive level of luminosity a number of demanding requirements to the collider optics should be satisfied [1] arising from short muon lifetime and relatively large values of the transverse emittance and momentum spread in muon beams that can realistically be achieved with ionization cooling.

These requirements are aggravated by limitations on the magnet maximum operating fields as well as by the necessity to protect superconducting magnets and collider detectors from muon decay products [2].

In the case of a 1.5 TeV c.o.m. MC we found a particular solution for the interaction region optics whose distinctive feature was a three-sextupole local chromatic correction scheme [1]. Together with a new flexible momentum compaction arc cell design that scheme allowed to satisfy all the above-mentioned requirements and was relatively insensitive to the beam-beam effect.

However, that scheme could not be extended for significantly higher MC energies due to the final focus quadrupoles gradient and aperture limitations: the experiment requirement to keep luminosity rising as $\sim E^2$ dictates $\beta^* \sim 1/E$ so that the maximum $\beta$-function in quadrupoles increases at least as $E$ counteracting the effect of r.m.s. emittance reduction on beam sizes.

Another complication associated with higher energies are the "hot spots" of radiation which can be induced by neutrinos from muon decay in straight sections [3]. This radiation limits the admissible length of regions without bending field or large beam divergence to ~1 m for 1.5 TeV beam energy.

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In the present report we consider the lattice modifications that are necessary to overcome these complications for MC with 3 TeV c.o.m. energy.

## LATTICE DESIGN

We retain rather conservative limits on magnet strength adopted for the 1.5 TeV c.o.m. MC design: B=8 T for dipoles at high-beta locations and up to 10 T in the arcs, 250 T/m for quadrupoles with 80 mm aperture and proportionally lower gradient for larger aperture quadrupoles.

### *Interaction Region*

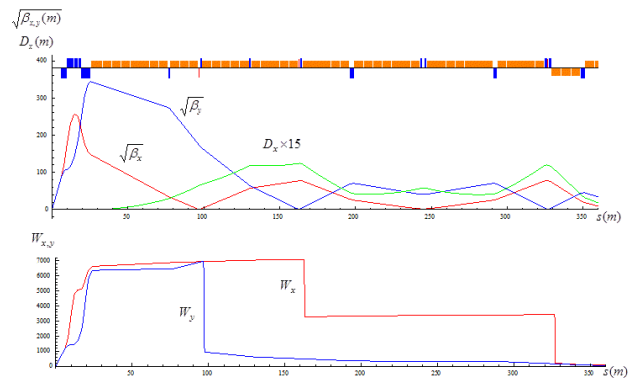


Figure 1 (color): IR and CCS layout and optics functions (top) and chromatic functions (bottom) for $\beta^* = 5$ mm. Orange and blue rectangles represent dipoles and quadrupoles respectively while sextupoles are shown in red. The lattice is symmetric w.r.t. the interaction point at $s = 0$.

The purpose of the Interaction Region (IR) lattice modification was to avoid a significant increase in the required quadrupole aperture. The design proposed in [1] used a doublet final focus with much larger vertical $\beta$-function in the quads than the horizontal one (53 km vs 4.4 km for $\beta^* = 1$ cm). The accordingly larger vertical chromatic function was corrected first with a single sextupole located at the first from IP minimum of horizontal $\beta$-function, while the horizontal chromatic function was corrected with a pair of sextupoles separated by a $-I$ section called Chromaticity Correction Section (CCS).

Attempts to use the same scheme for $\beta^* = 5$ mm 3 TeV c.o.m. MC IR lead to very large $\beta_y^{max}$ and quadrupole aperture due to a runaway effect: the increase of the aperture lowers the achievable gradient leading to an even larger $\beta$-function which in turn requires an even larger aperture.

The natural solution to this problem is to use a triplet final focus while keeping the chromaticity correction scheme the same as in [1]. By equalizing the maximum

values of horizontal and vertical $\beta$-functions it was possible to fit the beam within the same aperture as in the 1.5 TeV case. However, the horizontal dynamic aperture appeared rather small. Therefore in the presented here design (Fig. 1) we equalized the maximum values of chromatic functions rather than the $\beta$-functions.

In the result $\beta_y^{\max}$ and quadrupole aperture became larger but are still acceptable. A close up look at beam sizes (for emittance cited in Table 1) and magnet half-apertures, $a$, in the vicinity of the Interaction Point (IP) is given in Fig. 2.

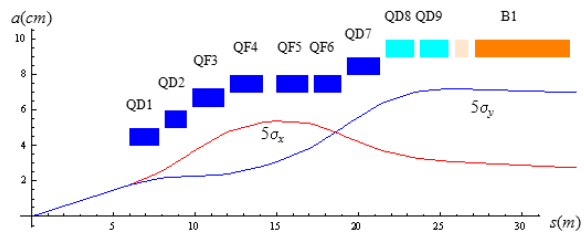


Figure 2 (color): Beam sizes and bore radii of the final focus magnets.

The bore radii of closest to IP magnets are determined by the requirement $a > 5\sigma_{\max}$+15 mm. A $5\sigma$ beam pipe inner radius may seem too small but it should be kept in mind that in MC - due to the short time muons spend in the collider - there will be practically no diffusion so that the beams can be collimated at less than $4\sigma$ amplitudes, the remainder $1\sigma$ providing room for possible closed orbit excursions. The additional 15 mm in the bore radius provide space for the beam pipe and annular helium channel.

The number of different apertures is increased from 3 in [1] to 6 here to follow the beam sizes more closely. The quadrupoles are split into short ($\leq$ 2 m) pieces to provide space for the tungsten masks in between to intercept decay electrons and the photons emitted by them.

Defocusing quadrupoles QD8-9 are shifted horizontally to create ~2 T dipole field to spread decay neutrinos. Large horizontal beam size in QD2-QD7 does not allow for such a shift, which however is not necessary there thanks to large angular spread in at least one of the planes.

The dipole field in IR quadrupoles can play two more roles: dispersion generation and sweeping away muon decay products. The quad contribution to dispersion is not very important, but the sweeping effect proved to be quite helpful in the 1.5 TeV case [2]. Unfortunately, with the present design the largest group of quads, QF3-6, is focusing so that the sweeping effect of horizontally bending field is very limited. We may contemplate creating vertically bending field in these quads by either shifting them vertically or adding special coils, but this will make the lattice significantly more complex. Detector background simulations should be performed first and show if such a complication is really necessary.

## *Arc Cell*

The interaction region produces large positive contribution to momentum compaction factor $\alpha_p$ which must be compensated by a negative contribution from the arcs.

In [1] we proposed a new version of the so-called Flexible Momentum Compaction (FMC) arc cell design which permitted to independently control all important parameters: tunes, chromaticities, momentum compaction factor and its derivative with momentum.

That design was based on separate-function magnets with rather long quadrupoles which are not good for neutrino radiation. Also, simulations of energy deposition by decay electrons in magnets [4] showed that the large vertical displacement these electrons can obtain in quadrupoles makes the choice of open-midplane dipole design ineffective.

Both above-mentioned problems can be alleviated by employing combined-function magnets.

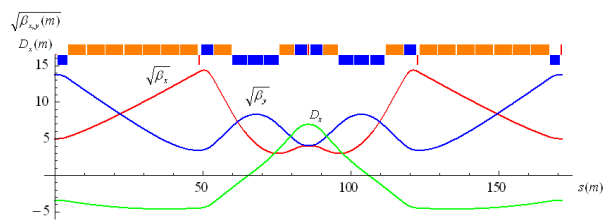


Figure 3 (color): Layout and optics of an arc cell with combined-function magnets.

Magnet parameters for the design presented in Fig. 3 are as follows: focusing magnets $B$ = 8T, $G$ = 85T/m, $L$ = 4m, defocusing magnets $B$ = 9T, $G$ = -35T/m, $L \leq$ 5m, pure dipoles $B$ = 10.4T, $L \leq$ 6m. Momentum compaction factor for a stand-alone cell is $\alpha_p$ = -0.004, betatron phase advance is 300° in both planes. Each arc consists of six such cells and two dispersion suppressors.

The horizontal beam size in the arcs is dominated by dispersion and reaches $\sigma_{\max}$=7mm in the cell centre. Since the magnitude of closed orbit excursions usually does not exceed $\sigma_\beta = (\beta\varepsilon_\perp)^{1/2}$, which is quite small compared to $\sigma_{\max}$, the requirement on the beam pipe radius can be relaxed as $a > 4\sigma_{\max}$.

## *Matching Section*

The design should be flexible enough to allow for a wide range of $\beta^*$ values for a number of reasons. First, it is easier to start machine running with large $\beta^*$. Second, there is an uncertainty in the muon beam emittance which can be obtained in the ionization cooling channel. With higher emittance $\beta$-functions in the IR magnets should become smaller in order to accommodate the beam inside the available aperture and in the result $\beta^*$ has to be larger. Conversely, with lower emittance smaller values of $\beta^*$ are allowed.

Not to destroy chromaticity correction the IR and CCS parameters should not change so that the $\beta^*$ variation

should be produced by quadrupoles in the adjacent IR-to-Arc matching section.

There will be four such sections in the ring which will also serve as utility sections. They should satisfy a number of requirements:

- allow for $\beta^*$ variation in wide range (e.g. 3 mm-3 cm),
- have no long straights without bending field,
- provide space with low $\beta$-functions and dispersion for RF cavities,
- provide space with high $\beta$-functions but low dispersion for halo extraction.

The first two requirements are difficult to reconcile: $\beta_x$ variation at a bend will change dispersion; trying to adjust the bending field will change the orbit. A possible solution to this problem is the use of a chicane with adjustable bending field which does not perturb the orbit outside and changes the total orbit length only slightly.

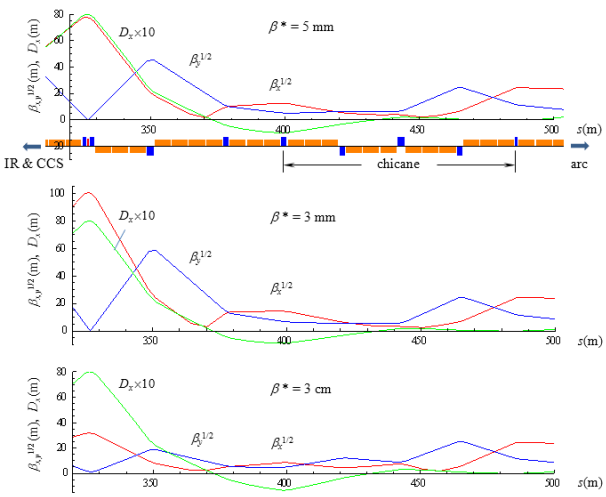


Figure 4 (color): Layout and optics functions in the matching section tuned for indicated values of $\beta^*$.

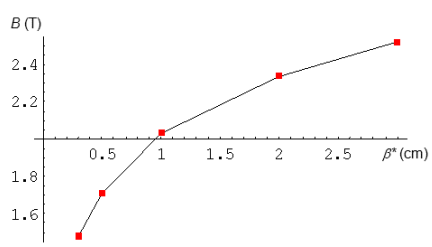


Figure 5: Bending field in chicane vs. $\beta^*$.

A generic solution for the matching section satisfying the requirements listed above is presented in Fig. 4. The chicane includes four groups of three 6m long dipoles. The bending field required for dispersion matching at different values of $\beta^*$ is shown in Fig. 5.

The relatively low magnetic field required in the chicane permit to significantly reduce the dipole length and place additional equipment like RF cavities, kickers and halo deflectors.

### *Performance*

High-order chromaticity correction with this design is not completed yet, the achieved momentum acceptance for $\beta^* = 3$ mm being ±0.4% and higher for larger $\beta^*$.

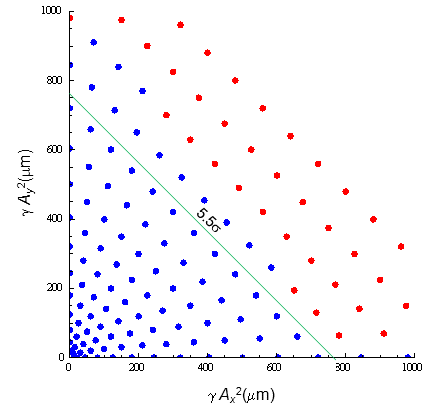


Figure 6 (color): 1024 turns dynamic aperture.

The on-momentum dynamic aperture (DA) was computed with MAD8 LIE4 option by tracking particles for 1024 turns. Figure 6 shows initial positions of stable (blue) and lost (red) particles in the plane of Courant-Snyder amplitudes for $\beta^*$ =0.5 cm. Conventional DA can be calculated as $A_{\min}\cdot(\gamma/\varepsilon_{\perp N})^{1/2}$ and amounts to 5.5σ for nominal emittance cited in Table 1.

Table 1: Muon Collider Parameters

| | | |
|---|---|---|
| Beam energy | TeV | 1.5 |
| Number of IPs | - | 2 |
| Circumference, $C$ | km | 4.45 |
| $\beta^*$ | cm | 0.5 (0.3-3) |
| Momentum compaction, $\alpha_p$ | $10^{-5}$ | -1.0 |
| Normalized emittance, $\varepsilon_{\perp N}$ | π·mm·mrad | 25 |
| Momentum spread | % | 0.1 |
| Bunch length, $\sigma_s$ | cm | 1 |
| Number of muons / bunch | $10^{12}$ | 2 |
| Repetition rate | Hz | 12 |
| Average luminosity / IP | $10^{34}$/cm$^2$/s | 4.4 |

## REFERENCES


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