EFFECTS OF THE RF FIELD ASYMMETRY IN SC CAVITIES OF THE PROJECT X*

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Abstract
Project-X is the proposed high intensity proton facility to be built at Fermilab in United States. First stage of the Project-X consists of H’ superconducting linac (SC) which will be operated in continuous wave (CW) mode to accelerate the beam from kinetic energy of 2.1 MeV to 3 GeV. The SC CW linac is divided into two parts: low energy part and high energy part. The low energy part is further segmented into three sections on the basis of families of cavities to be used for the acceleration of beam from 2.1 MeV to 160 MeV. It consists of one family of Half Wave Resonator (HWR) and two families of Single spoke resonator (SSR) i.e. SSR1 and SSR2 designed to operate at frequency of 325 MHz with βoptimal = 0.21 and βoptimal = 0.47 respectively. The high energy part of linac is also segmented into two sections for the rest of acceleration (160 MeV- 3 GeV). It consists of two families of elliptical cavities which are designed to operate at frequency of 650 MHz for geometrical beta (βgeo) 0.61 and 0.90 respectively. Asymmetry in cavity geometry results in multipole fields which lead to asymmetry in net transverse fields along the length of cavity. The preliminary studies of beam dynamics show that asymmetry in transverse fields is significant in low energy part of linac. In this paper we present the effects of asymmetry in transverse fields on beam dynamics and discuss the possible solutions to minimize these effects.

INTRODUCTION
Project-X is the proposed high intensity multi megawatt facility to be built at Fermilab [1]. The facility is based on 3GeV, 1mA SCRF CW linac. The linac is segmented into two sections: low energy section and high energy section. The low energy section (2.1 MeV- 160 MeV) uses one family of HWR and two families of spoke resonator i.e. SSR1 and SSR2. The HWR is designed to operate at 325 MHz with βoptimal = 0.11. SSR1 will be used for the acceleration of beam from 10 MeV to 35 MeV and SSR2 will be used for rest of acceleration in low energy part of linac up to 160 MeV. Further details about conceptual design of SRF linac is presented elsewhere [2].

The front-end of CW linac is most important part which influences the performance and reliability of rest of linac. To demonstrate the technical and beam dynamics feasibility of front-end, Fermilab decided to construct Project X Injector Experiment (PXIE) facility - a prototype of the front end of the Project X [3].

The SC part of PXIE (blue section on Fig.1) is composed of HWR section and SSR1 section. It consists of one cryomodule in each section. Cryomodule in HWR section consists of 8 HWR cavities and 8 solenoids and cryomodule in SSR1 section consists of 8 SSR1 cavities and 4 solenoids. The successful completion of PXIE will validate the concept and RF design for the Project X front end, thereby minimizing the primary technical risk element within Project X.

GENERAL
RF cavities provide not only longitudinal accelerating kick but radial transverse kick also. Magnitude of transverse kick is significant at low energy therefore; solenoids are most preferable choice of transverse focusing in SC low energy part of linac for effective compensation of radial defocusing of cavities. However, asymmetry in cavity geometry results in multipole fields which may lead to asymmetry in transverse kick of cavity. Asymmetry in transverse kick cannot be compensated with uniform radial focusing of solenoids. Thus, it disturbs the round beam profile in transverse plane and results in increase of beam size in respective plane.

HWR and SSR cavities will be used in low energy part of SC CW linac for Project-X. One of the advantages of using HWR and SSR in comparison of Quarter-Wave Resonators (QWR) is that there are no dipole fields on the beam axis due to the full symmetry of EM fields and as consequence any beam steering effects. However, the central electrode (HWR) and the spoke (SSR) break the azimuthal symmetry of cavities that can result in significant quadruple component (Q) of accelerating field:

$$Q = \frac{\Delta p_x c - \Delta p_y c}{(\Delta p_x c + \Delta p_y c)/2} \tag{1}$$

Where:

$$\Delta p_x(r, \alpha) c = \int_{z_1}^{z_f} \left( \frac{E_x(r, \alpha)}{\beta} - Z_0 i H_y(r, \alpha) \right) e^{ikz} dz \tag{2}$$

$$\Delta p_y(r, \alpha) c = \int_{z_1}^{z_f} \left( \frac{E_y(r, \alpha)}{\beta} + Z_0 i H_x(r, \alpha) \right) e^{ikz} dz \tag{3}$$

Figure 1: PXIE layout.
components of electric and magnetic fields. More detailed description of different approaches to evaluate the multipole effects in SC cavities are presented in elsewhere [].

FIELDS ASSYMETRY IN HWR
The initial design of HWR f=162.5 MHz, \( \beta = 0.11 \) for Project X was proposed in [2] and have racetrack electrode shape in circular \( (O=33\,\text{mm}) \) beam pipe area.

From Fig. 2, one can easily observed asymmetry in transverse electric fields in HWR. However, as shown in equations (1-3) that transverse kick is integral of fields along the cavity length. Thus, studies are performed to analyse the beam behaviour through HWR and SSR1 sections. Figure 3 shows \( 3\sigma \, x_{\text{rms}} \) (blue) and \( y_{\text{rms}} \) (red) beam size along the linac. It can be observed that beam trajectory split in two in transverse planes at the end of HWR section. Beam enters in SSR1 section and continues to split due to different focusing in respective planes. This splitting may results in mismatch with subsequent sections which can lead to emittance growth and beam losses.

COMPENSATION OF FIELDS ASSYMETRY IN HWR
One of the possible solutions to decrease the field asymmetry in the initial design of HWR cavity is the modification of the beam pipe cross-section from circle to ellipse. In order to minimize the Q, different shapes are designed with different elliptical aspect ratio \( b/a \), where \( b \) and \( a \) are horizontal and vertical half-axis respectively. Fig. 4(a) shows the HWR with modified geometry of the beam axis cross-section.

Table 1 summarizes the values of formulas (1)-(3) for different \( b/a \) ratio for operating conditions. One can see that Q is almost zero for \( b/a=1.14 \) at \( \beta_{\text{optimal}} =0.11 \). Beam studies are performed for this particular case. Figure 5 shows beam profile through HWR and SSR1 sections. It can be observed that beam starts to split even earlier than previous case. To understand this beam behaviour, Q is plotted for complete range of \( \beta \) in HWR section in Fig. 6. It can be seen that Q (red) of this shape is higher than Q (blue) of previous shape with circular aperture at the beginning of HWR section which results in initial mismatch that propagates in following sections. It can be concluded that shape with elliptical beam pipe aperture helps in setting the zero point of the Q vs. beta curve but it is not effective to compensate asymmetry for wide range of beta. Argonne National Laboratory (ANL) proposed a new “ring-shaped” design [4] of HWR for PXIE project as shown in Fig. 4(b). The ring-shaped geometry has not only lower peak surface magnetic field and higher shunt impedance, but also negligible quadruple field asymmetry in comparison of the race-track geometry.
Figure 5: Beam profile: \( x_{\text{rms}} \) (blue) and \( y_{\text{rms}} \) (red) through the HWR with elliptical beam pipe and SSR1 sections.

Figure 6: Comparison of Q for the various geometries of HWR

Figure 6 shows the Q value (green) for new shape for given range of \( \beta \). It can be seen that Q is minimum for complete range of beta in comparison with previous two shapes discussed earlier. Table 2 summarizes Q values for different values of \( \beta \) where \( A_0 \) is the monopole amplitude (constant kick on the x-y plane)

Table 2: Q parameter for ring shaped HWR.

<table>
<thead>
<tr>
<th>Particle ( \beta )</th>
<th>Q</th>
<th>( \frac{2 \times A_2}{A_0} )</th>
</tr>
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<tbody>
<tr>
<td>( \beta = 0.07 )</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>( \beta = 0.11 )</td>
<td>0.0109</td>
<td>0.0109</td>
</tr>
<tr>
<td>( \beta = 0.15 )</td>
<td>-0.0284</td>
<td>-0.0286</td>
</tr>
</tbody>
</table>

Beam studies have been performed to analyze influence of this shape on beam profile in transverse plane. It can be seen from Fig. 7 that beam splitting is minimum in this case in comparison with other previous two shapes. The performance of ring shaped HWR is to be tested at PXIE facility. Figure 8 shows transverse fields in ring shaped HWR along its length at radial offset of 10 mm. It can be observed that fields are very symmetric in comparison with previous cases which results in symmetric kick. Table 3 compares multipole fields of different shapes of HWR.

Table 3: Multipole fields for different shapes of HWR at \( \beta_{\text{optimal}} = 0.11 \)

<table>
<thead>
<tr>
<th></th>
<th>HWR circ</th>
<th>HWR ell</th>
<th>HWR ring</th>
</tr>
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<tbody>
<tr>
<td>( A_n/r^{n-1} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 1^{\text{st}} ) [keV]</td>
<td>2.19E-13</td>
<td>1.05E-13</td>
<td>8.22E-04</td>
</tr>
<tr>
<td>( 2^{\text{nd}} ) [keV/mm]</td>
<td>18.793</td>
<td>5.652</td>
<td>1.429</td>
</tr>
<tr>
<td>( 3^{\text{rd}} ) [keV/mm²]</td>
<td>1.55E-15</td>
<td>1.83E-15</td>
<td>8.22E-06</td>
</tr>
<tr>
<td>( 4^{\text{th}} ) [keV/mm³]</td>
<td>5.611E-03</td>
<td>5.460E-03</td>
<td>6.49E-04</td>
</tr>
<tr>
<td>( 5^{\text{th}} ) [keV/mm⁴]</td>
<td>2.24E-17</td>
<td>2.47E-17</td>
<td>8.22E-08</td>
</tr>
<tr>
<td>( 6^{\text{th}} ) [keV/mm⁵]</td>
<td>2.25E-06</td>
<td>2.18E-06</td>
<td>1.75E-06</td>
</tr>
<tr>
<td>( 7^{\text{th}} ) [keV/mm⁶]</td>
<td>2.06E-19</td>
<td>1.83E-19</td>
<td>8.22E-10</td>
</tr>
<tr>
<td>( 8^{\text{th}} ) [keV/mm⁷]</td>
<td>3.29E-09</td>
<td>3.43E-09</td>
<td>1.36E-08</td>
</tr>
</tbody>
</table>

FIELD ASSYMETRY IN SSR1 & SSR2

Magnitude of Q for SSR1 is smaller than 0.1 [5] for the complete accelerating range of SSR1 section. However, SSR2 exhibits significant asymmetry. It is shown elsewhere [6] that rotation of cavities by 90° with respect to its neighbouring cavities reduces the effects of asymmetry in transverse kick.

CONCLUSIONS

A complete study of the RF field asymmetry for Project X HWR cavity has been carried out. The early design was affected by a strong quadrupole component that has been overcome in the latest geometry thanks to a ring-shaped central conductor. This inner conductor symmetrizes the electric field transverse components [5] making the quadrupole perturbation of HWR field smaller than the same effect in SSR1 cavity, which does not show any beam dynamic issue.

REFERENCES