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Numerical Simulations of Transverse Beam Diffusion Enhancement by the use of Electron Lens in the Tevatron Collider

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Abstract

Transverse beam diffusion for the Tevatron machine has been calculated using the Lifetrac code. The following effects were included: random noise (representing residual gas scattering, voltage noise in the accelerating cavities) lattice nonlinearities and beam-beam interactions. The time evolution of particle distributions with different initial amplitudes in Hamiltonian action has been simulated for 6 million turns, corresponding to a machine time of about 2 minutes. For each particle distribution, several cases have been considered: a single beam in storage ring mode, the collider case, and the effects of a hollow electron beam collimator.

INTRODUCTION

The aim of this work is to evaluate the diffusion coefficient for the tevatron antiproton beam with the use of the tracking code Lifetrac [1], and compare it with the experimental results. The diffusion equation, introduced in the first section, is the foundation for the analysis of both the experimental data [2] and the simulation data. The limitations of this approach are investigated, and the diffusion coefficient results are presented and compared with the experimental results previously published [3]. In the last section the results of frequency map analysis for the beam beam case, with and without electron lens, are presented.

THE DIFFUSIVE MODEL

The time evolution of the particle distribution function ρ, both for beam core particles and beam tails, can be interpreted with the well-known diffusion equation:

$$\frac{\delta\rho}{\delta t} = \nabla \cdot (D\ \nabla(\rho)) \tag{1}$$

The diffusion equation is derived from the continuity equation, which requires that the change in particle population $\frac{\delta\rho}{\delta t}$ is equal to the flux ϕ of incoming particles

$$\frac{\delta\rho}{\delta t} = -\nabla \cdot \phi \tag{2}$$

and from the semi-empirical Flick's law

$$\phi = -D\ \nabla\rho \tag{3}$$

which states that the flux is proportional to the gradient of the population itself via a proportionality factor D. In order to understand the physical meaning of the diffusion coefficient D it is important to define the space in which the density function ρ is considered. In literature different approaches have been explored. The obvious choice to consider $\rho = \rho(x, y, z)$, i.e. the density function in the physical space, has been analyzed in detail in [4] In a single dimension, the diffusion equation reads as:

$$\frac{\delta\rho(W)}{\delta t} = \frac{\delta}{\delta W}\left(\frac{4D_{ph}(W)}{\beta} W \frac{\delta\rho}{\delta W}\right) \tag{4}$$

where the Courant Snyder invariant (single particle emittance) $W = (x^2 + p_x^2)/\beta = (x^2 + (\beta x' + \alpha x)^2)/\beta$ has been introduced. In the given formulation the general case of $D_{ph} = D_{ph}(W)$ has been taken into account, however it has been shown that, in case of a purely brownian motion, the diffusion coefficient in the physical space is independent on the particle emittance W.

It can also be convenient to consider the diffusion equation in the action space, where the Hamiltonian action J in the plane z for a single particle is defined as:

$$J = \frac{z_{max}^2}{4\beta_z} \tag{5}$$

for the generic z direction. This approach is particularly useful when analyzing experimental data, where only the particle maximum displacement z_{max} is known [2]. For linear machines it is straightforward to show the relation between the single particle emittance and the action, i.e. $W = 4J$. In this case Equation 4 becomes:

$$\frac{\delta\rho}{\delta t} = \frac{\delta}{\delta J}\left(\frac{D_{ph}}{\beta}\ J\frac{\delta\rho}{\delta J}\right) = \frac{\delta}{\delta J}\left(D_J(J)\frac{\delta\rho}{\delta J}\right) \tag{6}$$

where the diffusion function D_J in the action space is introduced. It follows that, in case of brownian motion in the physical space, $D_J(J)$ is expected to be linear in J, and inversey proportional to the local beta function.
For a thin particle distribution in the range $J_0 - \delta < J_0 < J_0 + \delta$, the function D_J can be considered constant, and the local diffusion equation becomes:

$$\frac{\delta\rho}{\delta t} = D_J\frac{\delta^2\rho}{\delta^2 J} \tag{7}$$

where the diffusion coefficient D_J can be calculated as [5]:

$$D_J = \frac{\Delta J^2}{2\Delta t} \tag{8}$$

The analysis of both the experimental results (see [Stancari]) and the simulation data are based on Equation 7.

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WORKING IN A COUPLED MACHINE

It is worth noticing that the Tevatron is a coupled machine, therefore it is not possible to treat the vertical, horizontal and longitudinal motion independently. However, for a linear machine, it is still possible to define three uncoupled planes, i.e. the eigenmodes of the one turn matrix, where the three normalized particle amplitudes A_1, A_2, A_3 [6] are invariants of the motion.

When a strong non linearity (e.g. beam-beam effect) is included in the simulation, it generates a beating of the particle amplitudes. To compensate for the beating, the average amplitude over a large number of turn is considered. It has been verified that 50K turns are an appropriate value.

From sake of simplicity in the following we will focus only on the transverse modes, even though in the simulation the full 6D treatment is implemented.

SIMULATION PARAMETERS

The code Lifetrac[1] has been used to calculate the diffusion coefficient for anti protons in the Tevatron. Narrow bi-Gaussian distributions in the average amplitude space have been used as an input. The initial distribution width is about .02 σ in both planes and its center is $(n\sigma_1, n\sigma_2)$, for n between 1 and 8. The population is of 1000 particles per distribution, tracked (with fulll 6D treatment) for a total number of turns of 6 10^6 (equivalent to about 2 minutes). Different machine configurations has been considered:

- single beam: purely random noise (brownian motion);
- collider mode: random noise and beam beam, with and without electron lens.

For amplitudes larger than 8σ it has been observed that, in collision case, the particles gain large amplitudes (above 50σ) within few turns. This sudden particle loss is in good agreement with the experimental observation of the dynamic aperture [7]. It has been verified that the observed aperture limitation disappear when removing the parasitic IPs from the simulation, thus proving that the Tevatron dynamic aperture is defined by the presence of parasitic IPs.

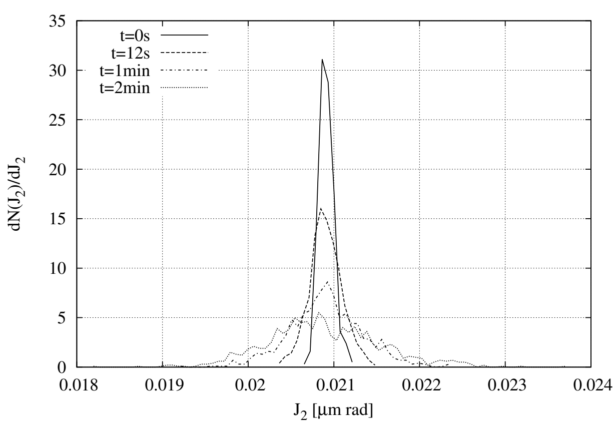

Figure 1: Evolution in time of a narrow gaussian distribution in the J_2 space, case with beam beam, no electron lens.

SIMULATION RESULTS

Diffusion coefficients

A typical evolution of the particle distribution distribution in Hamiltonian action space J_2 is shown in Figure 1. For each distribution the rms width of the distribution is calculated, and then the local diffusion coefficient D_2 is calculated trough a quadratic fit, accoridng to equation 8.

The summary results for the coefficient D_2 versus the action J_2 are presented in Figure 2. The results for the other direction are similar. For the first curve (single beam case) the only source of diffusion in the code is a random noise matrix: in this case the linear dependency of $D_2(J)$ (predicted by equation 6)) is verified. Including the beam beam effect (second curve) leads to diffusion coefficient values which are about a factor two to five larger. In the third curve, finally, the electron lens is activated, and it effect on the beam diffusion is clearly visible: for amplitudes lower than $3\sigma_2$ the core is untouched, while in the electron lens range (amplitudes larger than $4\sigma_2$) the diffusion coefficient is greatly enhanced. A moderate increase in diffusion coefficient for the $3\sigma_2$ case is justified by the fact that, as previously explained, the amplitude indicated on the x axis is intended to be the average amplitude, meaning that some of the particles can actually reach a physical aperture larger than the inner electron lens radius.

Comparison with experimental results

It is worth noticing that the overall diffusion coefficient D_y perceived by a vertical collimator (such as in the experiment described in [3]) is determined by the behavior of all the particle sitting in proximity to the collimator edge. In the normalized average amplitude space the collimator edge describes a curve y_{coll}. The total diffusion coefficient is a weighted average of the local diffusion coefficient along y_{coll}:

$$D_y = \int_{y_{coll}} D_y(A_1, A_2)\ \rho(A_1, A_2) dA_1 dA_2 \qquad (9)$$

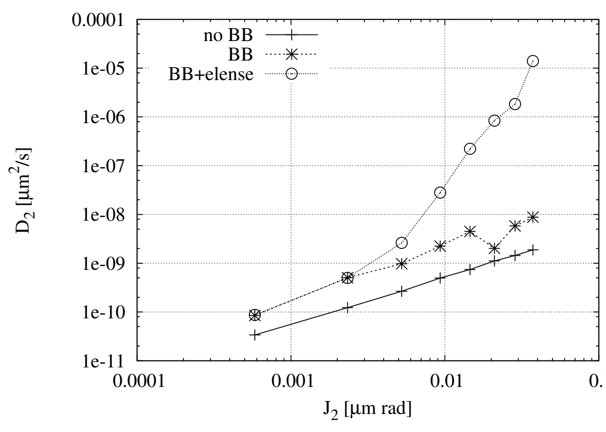

Figure 2: Diffusion coefficient D_2 versus the Hamiltonian action in the eigen mode 2.

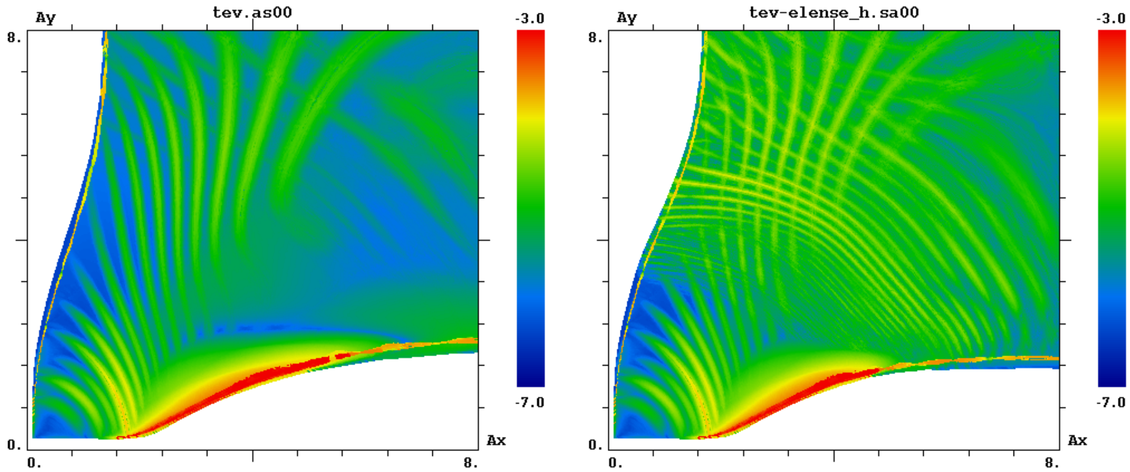

Figure 3: Frequency map analysis for the tevatron, collision case, without electron lens (left hand side) and with electron lens (right hand side).

Where $\rho(A_1, A_2)$ is the particle density function, and $D_y(A_1, A_2)$ is an appropriate combination of the local diffusion coefficient $D_1(A_1, A_2)$ and $D_2(A_1, A_2)$, e.g.

$$D_y(A_1, A_2) = D_1(A_1, A_2)\cos\theta_y + D_2(A_1, A_2)\sin\theta_y \quad (10)$$

with θ_y being the angle between the y direction and the mode 2. In general $\theta_y = \theta_y(A_1, A_2)$. The proper calculation would require a full sampling of the amplitude space, meaning massive computational resources. In this article we only compare some representative points of the amplitude space with the experimental results presented in [3, Figure 5]. The experimental data are obtained for the collision case, with no electron lens, and they are here compered to the second curve in Figure 2. While the comparison for the particles in the beam core is encouraging (same order of magnitude [7]), for amplitude between 4 and $8\sigma_y$ (range measured with the collimator scan method), the experimental values are up to a factor 10^5 larger than the simulated data. This large difference is not yet understood.

Frequency map analysis

In order to overcome the complexity of performing a full sampling of the amplitude space, an alternative approach has been explored, i.e. the frequency map analysis. FMA is a convenient way to identify the machine resonances either in the tune or in the amplitude space. The quality factor of an fma is the diffusion index i_d[8], which is equal to the jitter of the main betatron tune in logaritmic scale. Even if there is no explicit relation between the diffusion index and the diffusion coefficient, the fma is still a useful method for a qualitative evaluation of the diffusive behavior. The comparison between the fma plots with and without electron lens (Figure 3) shows clearly the effect of the device, which generates a dense region of additional resonances in the beam halo area, leaving the beam core unaffected.

SUMMARY

The diffusion coefficient for some representative points in the amplitude space has been calculated by fitting the time evolution of delta-like particle distributions using the diffusion equation, for different average amplitudes and different machine conditions. The result successfully reproduce the diffusion coefficient for the beam core, as measured in past experiments, but presents a large discrepancy for halo particles. This difference has not been understood yet, and it is still under investigation. Frequency map analysis has also been generated to show the global diffusive behavior in the amplitude space, showing clearly the effect of the electron lens on the large amplitude particles.

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