Abstract

Wakefields with amplitude in the 10’s MV/m range can be routinely generated by passing electron beams through dielectric-loaded structures. The main obstacle in obtaining high field amplitude (in the GV/m range) is the ability to focus the high-peak-current electron beam in the transverse plane to micron level, and to maintain the focusing all the way along the dielectric structure. In this paper we explore the use of a flat, high-peak current, electron beams to be produced at the Fermilab’s NML facility to drive dielectric loaded structures. Based on beam dynamics simulation we anticipate that we can obtain flat beams with very small vertical size (under 100 microns) and peak current is in excess of 1 kA. We present simulations of the wakefield generation based on theoretical models and PIC simulations with VORPAL.

INTRODUCTION

Cherenkov radiation emitted during the passage of an electron beam through a dielectric-loaded waveguide (DLW) can be used to accelerate a closely following smaller charge witness beam. The wakefield phase velocity in such structures is the same with the drive bunch velocity [1], and the synchronism between the witness beam and the accelerating field can achieved when both electron bunches are ultra-relativistic and separated by a certain time delay.

In general, the intensity of the wakefield scales linearly with the drive bunch charge and it is inverse proportional with the transverse area of the waveguide. Therefore, GV/m gradient wakefields or higher can be obtained either by using high-charge drive beams (> 100 nC) and millimeter range transverse size DLW’s [2] or by focusing relatively low charge (1 nC) beams to the level of tens of microns [3]. By comparison with the standard iris-loaded copper structures, the DLW’s are inexpensive, do not need external RF power and the breakdown point is much higher [4].

Axis-symmetric DLW’s were intensively studied both theoretically and experimentally [5, 6]. Since the angular divergence of the beam also increases during focusing it is hard to maintain a low transverse size of the beam over a long propagation distance. A possible solution to this problem is to use electron flat beams ($\sigma_x \gg \sigma_y$) in rectangular shaped DLW’s [3, 7]. This possibility became more attractive since flat beams are routinely generated with normalized emittance ratios $\epsilon_{x,n}/\epsilon_{y,n} > 100$ and transverse beam size ratios $\sigma_x/\sigma_y > 20$ [8]. The main advantage of flat beams is that once they are produced, they remain flat over a long propagation distance due to their low vertical emittance. In addition, flat beams have reduced space charge effects.

In this paper we outline a two-dimensional theoretical model for the generation of wakefields in compact rectangular DLW’s by drive flat beams. Theoretical results are compared with VORPAL simulations [10] and the prospects of building DLW’s for flat beams with very high emittance ratios are examined.

WAKEFIELD GENERATION WITH FLAT BEAMS

Cherenkov wakefields generated by a drive charge passing through a slab-symmetric DLW (Fig. 1) with the horizontal size much bigger than the vertical gap can be completely described by a two-dimensional theoretical model [3].

Figure 1: Transverse view of the dielectric-loaded waveguide (DLW): dielectric slabs are colored in green and the metallic cladding is black. A rectangular-shaped drive beam is shown in blue.

It is assumed that the drive beam consists of a line of charge oriented along $x$-axis which moves ultra-relativistically in the $z$-direction. In this model the longitudinal electric field has no transverse coordinate dependence. The normal modes excited by the drive charge can

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be obtained from the transcendent equation 1
\[
\cot \left[ k_n \sqrt{\epsilon - 1} (b - a) \right] = k_n a \sqrt{\epsilon - 1} \frac{\epsilon}{\epsilon},
\]  
(1)
where the geometrical quantities \( a \) and \( b \) are defined in
Fig. 1 and \( k_n \equiv \omega_n / c \). The longitudinal wakefield corresponding to each normal frequency is just a plane wave with constant amplitude:
\[
W_{z,n} = \frac{4\pi \lambda}{a + \epsilon (b - a) A_n^2} \cos \left[ k_n (z - vt) \right]
\]  
(2)
where \( \lambda \) is charge per unit length and \( v \approx c \) is the drive
bunch velocity and \( A_n = \csc \left[ k_n \sqrt{\epsilon - 1} (b - a) \right] \).

For realistic drive beams the total wakefield can be expressed as a summation over the normal modes convoluted with the longitudinal charge distribution:
\[
W_z(z) = \sum_n \int_z^\infty f(z') W_{z,n}(z - z')dz'
\]  
(3)
where \( f(z) \) is the drive longitudinal charge distribution.

The wakefields in rectangular DLW’s with \( L_x \gg L_y \)
can be well described by the two-dimensional model outlined in this section. Since the \( x \)-component of the wakefield wavenumber \( k_x = \frac{2 \pi n}{L_x} \to 0 \), the drive charge distribution which would maximize the wakefields must also be dominated by the \( k_x \approx 0 \) component. So, flat beams with \( \sigma_x \gg \sigma_y \) are perfectly suited as drive beams. Flat beams obtained at Fermilab A0 facility have a large transverse emittance ratio \( \frac{\sigma_x}{\sigma_y} > 100 \) and consequently a large aspect ratio \( \frac{a}{\sigma_y} > 20 \) [8].

\( L_x = 20 \text{ mm} \) and \( \epsilon = 4.0 \) has the mode frequency spectrum shown in Fig. 2. In this case, the modes are excited by a drive flat beam with gaussian longitudinal charge distribution (\( \sigma_z = 50 \mu \text{m} \)) and a uniform transverse rectangular shape with horizontal and vertical lengths of 3.0 mm
and 150 \( \mu \text{m} \) respectively. Such a drive flat beam with total bunch charge of about 3 nC and kinetic energy of about 850 MeV can be easily obtained at STF@NML.

The transverse wakefields are \( \sin \)-like functions
and therefore the forces on the drive beam vanish in the \( \sigma_z \to 0 \)
limit. When the longitudinal beam size is not negligible compared with the wakefield wavelength, there is a net defocusing vertical force acting on the drive beam. When the ratio \( L_x / L_y \) is large but finite there is also a small horizontal focusing force.

**VORPAL SIMULATIONS**

The software package VORPAL [10] is used to evaluate the wakefields and to study the eventual beam-breakup effects for electron bunches that pass through compact rectangular DLW’s. VORPAL is a three-dimensional electromagnetic and electrostatic PIC code. VORPAL uses the finite difference-time domain (FDTD) method to solve Maxwell’s equations and includes an advanced technique known as cut-cell boundaries to allow accurate representation of curved geometries within a rectangular grid.

![Figure 2: Normal frequencies and associated amplitudes for wakefields induced in a structure with \( L_x = 10.0 \text{ mm} \), \( a = 0.1 \text{ mm} \), \( b = 0.3 \text{ mm} \) and \( \epsilon_r = 4.0 \).](image)

![Figure 3: Top: VORPAL simulation for the y-z view of the longitudinal electric field in the slab-symmetric structure described in the text. Bottom: comparison between simulations and theoretical model. The wakefields are generated by a 3.0 nC flat drive beam.](image)
DLW, previously described in this paper, is shown in Fig. 3. The simulation results obtained with VORPAL agree very well with the theoretical evaluations. The amplitude of the accelerating wakefield is about 0.3 GV/m when the electron bunch charge is 3 nC.

![Figure 4: Top plot: comparison between initial and final vertical drive beam particle distributions. The propagation distance through the dielectric structure is 8.6 cm. Bottom plot: final kinetic energy distribution. At the entrance in the dielectric structure all particles had the same kinetic energy (1.0 GeV).](image)

Since the drive bunch length ($\sigma_z = 50 \mu m$) is much shorter than the wavelength of the fundamental mode (1.5 mm) the defocussing vertical forces are expected to be small. This assertion can be verified by comparing the initial and final distributions of the particle position projections on the vertical axis. For the case shown in Fig. 4 the initial flat beam has a uniform particle distribution in the vertical direction which extends from $-75.0 \mu m$ to $+75.0 \mu m$ and all macroparticles have the same kinetic energy 1.0 GeV. To obtain a good resolution for wakefield evaluation, the number of macroparticles in the simulation is relatively large 1 million. The final state of the flat beam is determined by the positions and momenta of the macroparticles after 8.6 cm propagation through the dielectric structure. The histogram of the vertical positions is wider by about 5 % and the drive beam kinetic energy decreases by 0.6 %. The energy gain for a test beam following this drive beam is expected to be as high as 25 MeV.

**CONCLUSIONS**

The use of flat beams to generate wakefields in compact rectangular dielectric loaded structures have certain advantages in comparison with the more common axisymmetric DLW’s. First, the normal modes in the rectangular structures can be efficiently excited by drive beams strongly focused just in one direction. Second, flat beams stay flat over a long propagation distance. This property allows the use of long structures and hence a large energy gain for the witness beam.

The theoretical estimates based on the two-dimensional model outlined in this paper agree well with the VORPAL simulations. When the vertical size of the drive flat beam is about $50 \mu m$ and the longitudinal bunch length is also $50 \mu m$ the accelerating wakefield amplitude is about 0.3 GV/m for each nC/mm linear charge density. The beam stability depends on bunch length and the geometry of the structure. For the case studied in the paper, the vertical beam size increases by about 5 % over a propagating distance corresponding to about 60 times the fundamental wavelength. High peak-current (1 kA) flat beams suitable for these experiments can be obtained at Fermilab NML facility.

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**REFERENCES**