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B and D meson decay constants from 2+1 flavor improved staggered simulations

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We give an update on simulation results for the decay constants f_B, f_{B_s}, f_D and f_{D_s}. These decay constants are important for precision tests of the standard model, in particular entering as inputs to the global CKM unitarity triangle fit. The results presented here make use of the MILC (2+1)-flavor asqtad ensembles, with heavy quarks incorporated using the clover action with the Fermilab method. Partially quenched, staggered chiral perturbation theory is used to extract the decay constants at the physical point. In addition, we give error projections for a new analysis in progress, based on an extended data set.

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1. Introduction

Within the standard model, the decay of mesons containing heavy quarks (in particular, B and D mesons) into purely leptonic final states provides an important testing ground for a number of theoretical ideas. Such decays involve both weak and strong interactions simultaneously, so that a complete understanding of the standard model is necessary to describe them. In particular, the decay width of a charged meson is proportional to both the meson decay constant (determined by strong interactions) and the CKM mixing angle,

$$\Gamma(H \to \ell \nu_\ell) \propto f_H^2 |V_{Qq}|^2. \tag{1.1}$$

Because this fully leptonic decay has no hadrons in the final state, the meson decay constant f_P can be readily and accurately determined by lattice simulations [1, 2, 3, 4]. Such a determination is in fact necessary in order to extract the CKM angles from experimental measurements of these decays, and precise computations of the decay constants could potentially reveal the presence of new physics through tension in the CKM unitarity triangle [5, 6, 7, 8]. In addition, certain leptonic decay channels (e.g. $B_s \to \mu^+\mu^-$) are both loop and CKM suppressed in the standard model, and so they may be particularly sensitive to flavor-changing interactions induced by new physics [9].

2. Simulation Details

We make use of the MILC asqtad-improved staggered gauge configurations, with $2+1$ dynamical quarks in the sea [10]. For the light valence quarks, we make use of the same staggered action, while charm and bottom valence quarks are incorporated using the clover action with the Fermilab interpretation [11]. The particular set of ensembles used to obtain the results presented here, along with the number of configurations and other relevant information, are detailed in Table 1. Data on the coarsest lattice spacing $a \approx 0.15$ fm are shown in the analysis but are used only for the purpose of estimating the discretization errors; these points are excluded from the final fit used for chiral and continuum extrapolation.

3. Analysis and Fitting

The heavy meson decay constant is determined through the overlap of the meson wavefunction $|H\rangle$ with the axial vector current:

$$\langle 0|\mathscr{A}^\mu|H(p)\rangle (M_H)^{-1/2} = i(p^\mu/M_H)(f_H\sqrt{M_H}) \equiv i(p^\mu/M_H)\phi_H. \tag{3.1}$$

The quantity $\phi_H \equiv f_H\sqrt{M_H}$ is thus proportional to the ground-state amplitude of the two-point function between the axial vector current and a heavy-light pseudoscalar operator $\mathscr{O}$. We therefore extract ϕ_H by fitting this two-point function simultaneously with the two-point pseudoscalar correlator,

$$\Phi_2^s(t) = \tfrac{1}{4}\textstyle\sum_{a=1}^4 \langle A_a^{4\dagger}(t,\mathbf{x})\mathscr{O}_a^{(s)}(0)\rangle, \tag{3.2}$$

$$C_2^{s,s'}(t) = \tfrac{1}{4}\textstyle\sum_{a=1}^4 \langle \mathscr{O}_a^{(s)\dagger}(t,\mathbf{x})\mathscr{O}_a^{(s')}(0)\rangle, \tag{3.3}$$

$\approx a$ [fm]	am_h	am_l	β	r_1/a	$N_{\rm conf}$
0.09	0.031	0.0031	7.08	3.75	435
		0.0062	7.09	3.79	557
		0.0124	7.11	3.86	518
0.12	0.050	0.005	6.76	2.74	678
		0.007	6.76	2.74	833
		0.010	6.76	2.74	592
		0.020	6.79	2.82	460
		0.030	6.81	2.88	549
0.15	*0.0484*	*0.0097*	*6.572*	*2.22*	*631*
		0.0194	*6.586*	*2.26*	*631*
		0.0290	*6.600*	*2.29*	*576*

Table 1: Table of gauge configurations used for the full analysis to be presented. The ensembles with $a \approx 0.15$ fm (italics) are excluded from the final chiral/continuum extrapolation, as described in the text.

where a is a staggered taste index. Precise definitions of the interpolating operators A^4 and $\mathscr{O}$ are given in Ref. [12].

For a correlation function with source type s and sink type s', we fit to the "factorized" functional form

$$C_{ss'}(t) = \sum_{n=0}^{N_X} \left[A_{s,n} A_{s',n} \left(e^{-E_n t} + e^{-E_n (N_t - t)} \right) - (-1)^t A'_{s,n} A'_{s',n} \left(e^{-E'_n t} + e^{-E'_n (N_t - t)} \right) \right] \tag{3.4}$$

where N_X denotes the number of excited states included in the fit. The pseudoscalar source and sink type s, s' can be either point-like or smeared, so that a total of six distinct correlators are available for analysis. In the results shown here, joint fits are carried out to various combinations of correlators, with Bayesian priors imposed as constraints on the fit parameters.

From the two-point fits, we extract a bare value for the ground-state amplitude between an axial-vector current and pseudoscalar operator, which must then be renormalized in order to obtain the decay constant:

$$\phi_H = \sqrt{2} Z_{A^4_{Qq}} A_{A^4_{Qq},0}. \tag{3.5}$$

To compute the heavy-light axial current renormalization constant $Z_{A^4_{Qq}}$, we divide it into flavor-diagonal contributions $Z_{V^4_{qq}}, Z_{V^4_{QQ}}$, which are determined non-perturbatively, and a flavor off-diagonal piece $\rho_{A^4_{Qq}}$ which is computed in lattice perturbation theory [13]. Our renormalized result for $\phi_H = f_H \sqrt{M_H}$ is thus given by

$$\phi_H = \sqrt{2} Z_{A^4_{Qq}} A_{A^4_{Qq},0} = \sqrt{2} (\rho_{A^4_{Qq}} \sqrt{Z_{V^4_{qq}} Z_{V^4_{QQ}}}) A_{A^4_{Qq},0}. \tag{3.6}$$

We use rooted staggered chiral perturbation theory (rSχPT) [14] to extrapolate our results simultaneously to the continuum limit and to the physical light-quark masses. (Heavy quark masses are tuned non-perturbatively to give physical heavy-light meson masses, so no extrapolation is necessary for them. Details of the tuning procedure are given in [15].) The chiral fit functions incorporate terms describing a number of different effects, including discretization errors, finite-volume corrections, and hyperfine splittings.

Table 2: Error budget for the decay constants, as obtained in Section 4. In addition, projected improvements to the decay constant error budget for the updated analysis in progress (discussed in Section 5) are shown italicized and in brackets.

Source	f_{D^+}(MeV)	f_{D_s}(MeV)	f_{B^+}(MeV)	f_{B_s}(MeV)
Statistics	2.3 *[1.1]*	2.3 *[1.1]*	3.6 *[1.8]*	3.4 *[1.7]*
Heavy-quark disc.	8.2 *[3.6]*	8.3 *[3.6]*	3.7 *[1.9]*	3.8 *[2.0]*
Light-quark disc.	2.9 *[0.7]*	1.5 *[0.3]*	2.5 *[0.6]*	2.1 *[0.5]*
Chiral extrapolation	3.2 *[1.6]*	2.2 *[1.1]*	2.9 *[1.5]*	2.8 *[1.4]*
Heavy-quark tuning	2.8 *[2.0]*	2.8 *[2.0]*	3.9 *[2.4]*	3.9 *[2.4]*
$Z_{V^4_{QQ}}$ and $Z_{V^4_{qq}}$	2.8 *[1.4]*	3.4 *[1.7]*	2.6 *[1.5]*	3.1 *[1.9]*
u_0 adjustment	1.8 *[0]*	2.0 *[0]*	2.5 *[0]*	2.8 *[0]*
Other sources	3.8 *[3.8]*	3.0 *[3.0]*	3.5 *[3.5]*	4.8 *[4.8]*
Total *[projected]* error	11.3 *[6.1]*	10.8 *[5.6]*	8.9 *[5.5]*	9.5 *[6.4]*

4. Results

Applying the procedure outlined above, we obtain the values for ϕ shown in Figures 1 and 2, renormalized and in units of the standard scale r_1. The chiral best-fit curve to the points is also shown, both explicitly at each lattice spacing and extrapolated to the continuum limit.

Evaluating the continuum best-fit curves at the physical points, we obtain the following values for the decay constants and their ratios:

$$f_{B^+} = 196.9(8.9)\ \text{MeV}, \tag{4.1}$$

$$f_{B_s} = 242.0(9.5)\ \text{MeV}, \tag{4.2}$$

$$f_{B_s}/f_{B^+} = 1.229(0.026), \tag{4.3}$$

$$f_{D^+} = 218.9(11.3)\ \text{MeV}, \tag{4.4}$$

$$f_{D_s} = 260.1(10.8)\ \text{MeV}, \tag{4.5}$$

$$f_{D_s}/f_{D^+} = 1.188(0.025). \tag{4.6}$$

The error bars quoted here include both statistical and systematic sources of error, which are accounted for in a detailed error budget. A summary of the full error budget for the individual decay constants is given in Table 2. We discuss the error budget further in Section 5 below, but a thorough discussion is beyond the scope of this paper. Instead, we refer the reader to Ref. [12], which contains a complete discussion of the systematic error analysis, including the full error budget for the decay-constant ratios..

5. Outlook

A new analysis following the approach outlined above is currently in progress, based on an expanded set of gauge configurations as shown in Table 3. In addition to extending the available simulations to finer lattice spacing and smaller quark mass, the “new" data set includes large increases in statistics for several ensembles.

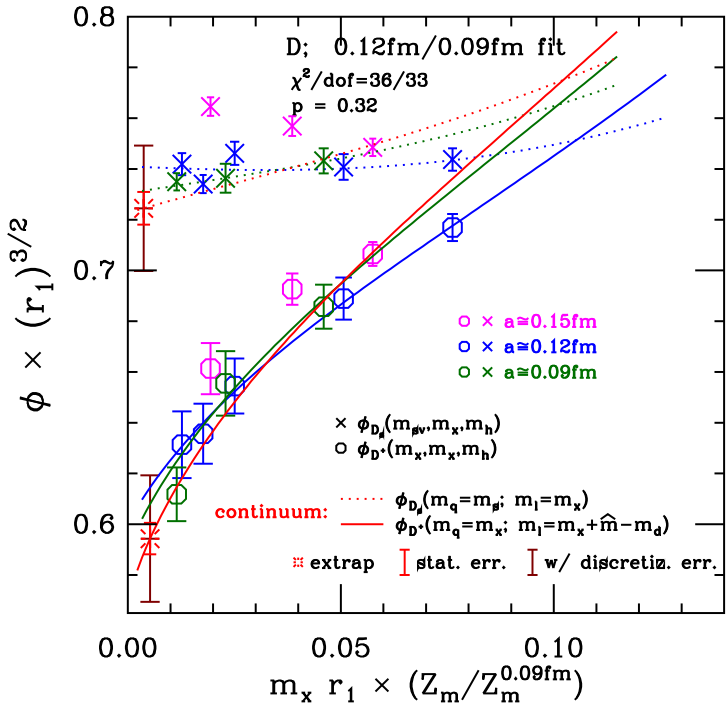

Figure 1: Extracted values of ϕ and chiral best-fit curves for the D system. Only points where valence and sea light-quark masses are equal are shown here. Data from the $a \approx 0.15$ fm ensemble are shown but not included in the fit. The red curve and symbols show the continuum extrapolation and the continuum/physical point extrapolation, respectively, for both ϕ_{D^+} and ϕ_{D_s}. For the fully extrapolated points, the inner error bars (bright red) represent statistical errors only, while the outer errors (dark red) include discretization errors.

Projected improvements in the error budget when the new data set is included are shown alongside the previous error estimates in Table 2. Statistical errors are projected to improve as $\sqrt{N_{\rm cfg}}$, with $N_{\rm cfg}$ the number of gauge configurations available for a given ensemble. For the various discretization errors, the projected improvements are a result of reducing the smallest lattice spacing available from $a = 0.09$ fm to $a = 0.045$ fm. Light-quark discretization errors are estimated to scale as $\mathcal{O}(\alpha_s a^2)$; the heavy-quark discretization errors are estimated using the known functional dependence, which has several terms. The decrease in the chiral extrapolation error is projected based on the lightest available value of the quark mass in r_1 units, $m_x r_1$. The heavy-quark tuning error is based on a combination of statistical and discretization errors, and is treated as such. The "u_0 adjustment" error is the result of using different tadpole improvement factors for the valence and sea quarks. This is rectified in the new data analysis, eliminating the associated error. Finally, the error estimates for the heavy-quark renormalization factors $Z_{V^4_{QQ}}, Z_{V^4_{qq}}$ are based on preliminary non-perturbative results for those quantities.

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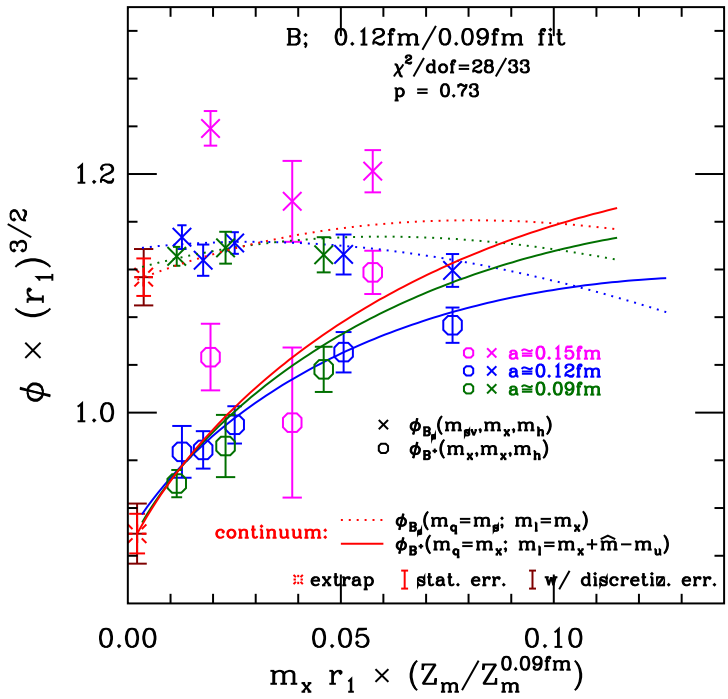

Figure 2: ϕ values and chiral best-fit curves as in Figure 1, but for the B system.

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$\approx a$ [fm]	am_h	am_l	β	r_1/a	$N_{\rm conf}$ (old run)	$N_{\rm conf}$ (new run)
0.045	0.014	0.0028	7.81	7.21	—	800
0.06	0.018	0.0018	7.46	5.31	—	825
		0.0025	7.465	5.33	—	800
		0.0036	7.47	5.35	—	631
		0.0072	7.48	5.40	—	591
0.09	0.031	0.00155	7.075	3.74	—	790
		0.0031	7.08	3.75	435	577
		0.00465	7.085	3.77	—	983
		0.0062	7.09	3.79	557	1377
		0.0124	7.11	3.86	518	1476
0.12	0.050	0.005	6.76	2.74	678	1419
		0.007	6.76	2.74	833	1274
		0.010	6.76	2.74	592	1664
		0.020	6.79	2.82	460	1637
		0.030	6.81	2.88	549	—
0.15	*0.0484*	*0.0097*	*6.572*	*2.22*	*631*	—
		0.0194	*6.586*	*2.26*	*631*	—
		0.0290	*6.600*	*2.29*	*576*	—

Table 3: Table of gauge configurations for the updated analysis in progress. Only configurations labelled "old run" were used to obtain the results presented in Section 4. The analysis in progress will make use of all configurations from both runs combined.

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