FERMILAB-CONF-11-611-T

arXiv:1110.0016v3 [hep-ph] 22 Oct 2011

Workshop on Precision Measurements of α_S

MPI Munich, Germany
February 9 to 11, 2011

Editors

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Abstract

These are the proceedings of the Workshop on Precision Measurements of α_s held at the Max-Planck-Institute for Physics, Munich, February 9-11, 2011. The workshop explored in depth the determination of $\alpha_s(m_Z)$ in the $\overline{\text{MS}}$ scheme from the key categories where high precision measurements are currently being made, including DIS and global PDF fits, τ-decays, electro-weak precision observables and Z-decays, event-shapes, and lattice QCD. These proceedings contain a short summary contribution from the speakers, as well as the lists of authors, conveners, participants, and talks.

Supported by: MPI Munich, TUM/LMU Excellence Cluster, University of Vienna

Operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

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1 Introduction

The "Workshop on Precision Measurements of α_s" was held at the Max Planck Institute for Physics, February 9th through 11th of 2011. The meeting brought together experts from several different fields where high precision measurements of $\alpha_s(m_Z)$ are currently being made. Main goals of the workshop were to facilitate discussion between the groups, and in particular to give speakers the opportunity to explain details that one would normally not be able to present at a conference, but which have an important impact on the analyses. In each field the session was led off by a review speaker, followed by more specialized talks, and was closed with a dedicated time period for discussions.

There were 67 physicists who took part in the workshop, and 35 talks were presented. Slides as well as background reference materials are available on the conference website

http://www.mpp.mpg.de/alphas

The sessions and talks in the workshop program were

- Welcome
  - "World Summary of α_s (2009) and beyond", S. Bethke
- α_s from Deep Inelastic Scattering and Global Fits
  - "Review of α_s Determinations from Jets at HERA" by C. Glasman
  - "α_s from Deep-Inelastic Scattering: DESY Analysis" by J. Blümlein
  - "CTEQ-TEA Parton Distribution Functions and α_s" by C.P. Yuan
  - "α_s in MSTW Analyses", A. Martin
  - "Unbiased α_s from Global Fits: The NNPDF Approach" by S. Forte
  - "Hera PDF" by B. Reisert
  - Discussion Session on DIS and Global Fits, convened by V. Radescu
- Measurements of α_s from τ Decays
  - "α_s Determinations from Hadronic τ Decays" by A. Pich
  - "α_s from Contour Improved Perturbation Theory (CIPT)" by S. Descotes-Genon
  - "Fixed Order Perturbation Theory (FOPT) Analysis" by M. Beneke
  - "FOPT and CIPT in τ Decays" by S. Menke
  - "Duality Violations in Hadronic τ Decays" by M. Goltermaan
  - "Perturbative Input to τ Decays" by J. Kühn
  - "Running and Decoupling of α_s at Low Scales" by M. Steinhauser
  - Discussion Session on τ Decays, convened by A. Höcker
- α_s from Z Decays and Electroweak Observables
  - "α_s in Electroweak Physics" by J. Kühn
  - "α_s with Global Analysis of Particle Properties (GAPP)" by J. Erler
  - "α_s from the Hadronic Width of the Z" by K. Mönig
  - Discussion Session on Elecroweak Analyses, convened by W. Hollik
- α_s from Event Shape Measurements
  - "Review of event-shape measurements of α_s" by G. Salam
  - "α_s at NNLO and NNLA from (mainly) ALEPH data" by T. Gehrmann
  - "NNLO and Classic Power Corrections" by B. Webber
  - "α_s from Soft-Collinear Effective Theory analysis" by V. Mateu

- "Five jets at LEP at NLO and $\alpha_s(m_Z)$" by G. Zanderighi
- "Jet and Event Shape Observables at the LHC" by K. Rabbertz
- "Monte Carlo Tuning with Professor" by H. Schulz
- "Experimental Issues and Combination of Results" by S. Kluth
- Discussion Session on Event Shapes, convened by Y. Dokshitzer
- MPI Colloquium, "On the Other Side of Asymptotic Freedom" by Y. Dokshitzer

• Lattice Calculations and Quarkonia

- "Lattice QCD Calculations and α_s" by A. Kronfeld
- "The QCD Coupling from Lattice QCD (HPQCD)" by G.P. Lepage
- "α_s from Lattice QCD (PACS-CS)" by S. Aoki
- "α_s from Lattice QCD (JLQCD)" by S. Hashimoto
- "α_s from Lattice QCD (ALPHA)" by R. Sommer
- "α_s from Bottomonium" by N. Brambilla
- Discussion Session on Lattice QCD, convened by C. Davies

• Applications and Future of α_s Measurements

- "α_s for New Physics" by Y. Nomura
- "The Strong Coupling and LHC Cross Sections" by F. Petriello
- "α_s at the International Linear Collider (ILC)" by C. Pahl
- "The Principle of Maximum Conformality" by S. Brodsky
- Final Discussion Session, convened by S. Bethke

This web proceedings represent a collection of extended abstracts and references for the presentations, summarizing the most important results and issues. In these writeups, unless otherwise indicated, the strong coupling $\alpha_s(\mu)$ is defined in the $\overline{\text{MS}}$ scheme, and $\alpha_s(m_Z)$ is the coupling with five light quark flavors.

Acknowledgments

We would like to thank all participants for their effort to travel to Munich and for making the Workshop on Precision Measurements of α_s a very stimulating meeting. We cordially thank our secretary Mrs. Rosita Jurgeleit for her excellent assistance, and the staff of the Max-Planck-Institute for their invaluable support.

Munich, May 2011

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Workshop on Precision Measurements α_s 2011: Welcome and Introduction

Siegfried Bethke

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Heartiest welcome to all participants of the 2011 Workshop on Precision Measurements on α_s at the Max-Planck-Institute of Physics at Munich! The principal organisers, i.e. Andre Hoang (Vienna Univ.), Stefan Kluth (MPP Munich), Jochen Schieck (LMU Munich), Iain Stewart (MIT) and myself, are very glad to see so many representatives and world-experts from all subfields of the workshop topics present at this meeting.

Measurements of α_s, one of the basic constants of nature whose numerical value is not given by any theory, continue to be at the fore-front of theoretical and experimental endeavors in particle physics. The purpose of this workshop is

- to review the latest theoretical developments and experimental studies in this field,
- to discuss and possibly solve open questions and issues which currently limit the precision to which α_s is known,
- to provide input for calculating a new 2011 world average of $\alpha_s(m_Z)$

The latest comprehensive summary of α_s measurements and calculation of a world combined average value of $\alpha_s(m_Z)$ was given in [1] in 2009, which resulted in

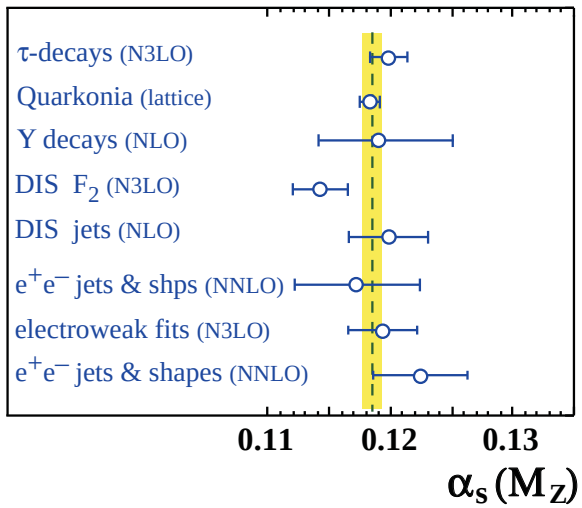
$$\alpha_s(m_Z) = 0.1184 \pm 0.0007 \ .$$

This result was also adopted by the latest review of the particle data group [2].

Table 1: Summary of recent measurements of $\alpha_s(m_Z)$. The rightmost two columns give the exclusive mean value of $\alpha_s(m_Z)$ calculated *without* that particular measurement, and the number of standard deviations between this measurement and the respective exclusive mean.

Process	Q [GeV]	$\alpha_s(m_Z)$	excl. mean $\alpha_s(m_Z)$	std. dev.
τ-decays	1.78	0.1197 ± 0.0016	0.11818 ± 0.00070	0.9
DIS [F_2]	2 - 170	0.1142 ± 0.0023	0.11876 ± 0.00123	1.7
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1198 ± 0.0032	0.11836 ± 0.00069	0.4
Lattice QCD	7.5	0.1183 ± 0.0008	0.11862 ± 0.00114	0.2
Υ decays	9.46	$0.119^{+0.006}_{-0.005}$	0.11841 ± 0.00070	0.1
e^+e^- [jets & shps]	14 - 44	0.1172 ± 0.0051	0.11844 ± 0.00076	0.2
e^+e^- [ew prec. data]	91.2	0.1193 ± 0.0028	0.11837 ± 0.00076	0.3
e^+e^- [jets & shps]	91 - 208	0.1224 ± 0.0039	0.11831 ± 0.00091	1.0

Figure 1: Summary of measurements of $\alpha_s(m_Z)$. The vertical line and shaded band mark the final world average value of $\alpha_s(m_Z) = 0.1184 \pm 0.0007$ determined from these measurements.



The individual results entering that determination of $\alpha_s(m_z)$ are given in Table 1, and a graphical representation in Fig. 1; see [1] and references therein. All results are compatible with the quoted overall world average; the largest deviation from an exclusive mean being less than two standard deviations (interpreting quoted uncertainties of α_s results as gaussian errors). In fact, the χ^2 of the overall average is close to unity per degree of freedom [1]. While the 2009 average came out in a rather consistent way, some questions and potential problems remained:

- the individual result with smallest quoted uncertainties is from lattice QCD; it largely dominates (not so much) the overall average and (mainly) its assigned error;
- there are large systematic differences between different studies of α_s from τ decays, depending on the type of perturbative prediction (CIPT and FOPT), leading to an increased uncertainty assigned to the value listed in Table 1;
- are there systematic unknowns in the DIS/F_2 result?
- theoretical uncertainties are defined using different definitions in most of the input studies; do they lead to a consistent guess-timate of the overall uncertainties?

As will be seen in the course of this workshop, the consistency of an overall new world summary may look more unfavorable in the light of some of the most recent results (see the summary contribution to this workshop [3]).

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Review of α_s determinations from jets at HERA

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The H1 and ZEUS Collaborations at HERA have performed precise determinations of $\alpha_s(M_Z)$ using observables such as jet cross sections, ratios of jet cross sections, and internal structure of jets in different regimes. Some of them are reviewed in this report. In addition, the energy-scale dependence of the coupling has been determined very precisely. The results are in good agreement with the predicted running of α_s with small experimental uncertainties in a wide range of the scale.

Values of $\alpha_s(M_Z)$ were extracted by the H1 Collaboration from the double-differential inclusive-jet, dijet and trijet cross sections at low Q^2 ($5 < Q^2 < 100$ GeV^2) using 43.5 pb^{-1} of integrated luminosity [1]. Jets were searched using the k_T cluster algorithm in the Breit frame with $P_T > 5$ GeV and $-1 < \eta^{\rm jet}_{\rm LAB} < 2.5$. The values of $\alpha_s(M_Z)$ extracted are shown in Table 1 (rows 1, 2 and 3; row 4 shows the combined value). These values have small experimental uncertainties; however, the theoretical uncertainty, dominated by terms beyond NLO, is large. A reduction of the theoretical uncertainties can be achieved by determining $\alpha_s(M_Z)$ from the trijet to dijet ratio; the value obtained is shown in row 5 and has a smaller theoretical uncertainty.

Values of $\alpha_s(M_Z)$ were extracted by the H1 Collaboration from the double-differential inclusive-jet, dijet and trijet normalised cross sections at medium Q^2 ($150 < Q^2 < 15000$ GeV^2) using 395 pb^{-1} of integrated luminosity [2]. Jets were searched using the k_T cluster algorithm in the Breit frame with $P_T > 5$ GeV and $-0.8 < \eta^{\rm jet}_{\rm LAB} < 2$. The values of $\alpha_s(M_Z)$ extracted are shown in rows 6, 7 and 8. Row 9 shows the combined $\alpha_s(M_Z)$ value, which has a very small experimental uncertainty; the theoretical uncertainties, though still dominated by the terms beyond NLO, are smaller than for the low Q^2 analysis. This reduction was accomplished by using normalised cross sections, for which correlated uncertainties cancel, and making the extraction at higher Q^2, where the contribution from the terms beyond NLO is reduced.

The ZEUS Collaboration extracted values of $\alpha_s(M_Z)$ from the inclusive-jet cross sections based on the k_T, anti-k_T and SIScone jet algorithms at high Q^2 ($Q^2 > 500$ GeV^2) using 82 pb^{-1} of integrated luminosity [3],[4]. Jets were searched in the Breit frame with $E^{\rm jet}_{T,\rm B} > 8$ GeV and $-2 < \eta^{\rm jet}_{\rm B} < 1.5$. The values of $\alpha_s(M_Z)$ extracted are shown in rows 10, 11 and 12. Row 13 shows an updated value using HERA II statistics ($\mathcal{L} = 300$ pb^{-1}) [5]. The experimental uncertainty is dominated by the jet energy scale and amounts to 2%. The theoretical uncertainties are dominated by the terms beyond NLO and amount to around 1.5% for the three jet algorithms. The reduction of the theoretical uncertainty was obtained by restricting to the high Q^2 region; even though the experimental uncertainty increases at high Q^2 (mainly due to statistics), the total uncertainty remains small.

Values of $\alpha_s(M_Z)$ were extracted by the ZEUS Collaboration from the inclusive-jet cross section in photoproduction at high $E^{\rm jet}_T$ ($21 < E^{\rm jet}_T < 71$ GeV) using 189 pb^{-1} of integrated luminosity [6],[7]. Jets were searched using the k_T, anti-k_T and SIScone jet algorithms in the laboratory frame. The values of $\alpha_s(M_Z)$ extracted are shown in rows 14, 15 and 16. These

have similar precision. In photoproduction, there is an additional uncertainty coming from the photon PDFs, which is as large as that coming from higher orders.

A completely different approach to the extraction of α_s at HERA is given by the analysis of the internal structure of jets. The internal structure of jets can be studied by means of the integrated jet shape and the subjet multiplicity. The ZEUS Collaboration extracted values of $\alpha_s(M_Z)$ from the measured mean integrated jet shape [8] for $E_T^{\rm jet} > 21$ GeV using 82.2 pb^{-1} of integrated luminosity and from the mean subjet multiplicity [9] for $E_T^{\rm jet} > 25$ GeV with 38.6 pb^{-1} of integrated luminosity. Jets were searched using the k_T cluster algorithm in the laboratory frame. The values of $\alpha_s(M_Z)$ extracted are shown in rows 17 and 18. In both cases, the experimental uncertainties are small; however, the theoretical uncertainty, dominated by the terms beyond NLO, is much bigger than for the extraction from jet cross sections.

From the results presented, it is concluded that the dominant uncertainty in the extraction of $\alpha_s(M_Z)$ at HERA comes from terms beyond NLO. This uncertainty decreases with increasing Q^2 or $E_T^{\rm jet}$, so an extraction at high Q^2 or $E_T^{\rm jet}$ is advantageous to minimise this contribution. In addition, such uncertainty cancels partially when ratios of jet cross sections are used for the extraction, and lower Q^2 or $E_T^{\rm jet}$ values can be used. The uncertainties coming from the PDFs are also smaller as Q^2 increases.

To summarise, jet physics at HERA provides precise values of $\alpha_s(M_Z)$ in different regimes and precise determinations of the running of the coupling over a wide range of the scale. There is still room for improvement. All HERA data have been analysed: the values of $\alpha_s(M_Z)$ obtained have very small experimental uncertainties; the theoretical uncertainties are dominant, in particular that coming from terms beyond NLO. Therefore, enormous benefit will be gained from NNLO calculations for jet cross sections at HERA to improve the precision in $\alpha_s(M_Z)$.

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Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	± 0.0011	± 0.0050	± 0.0051	± 4.3
HERA average 2007		0.1198	± 0.0019	± 0.0026	± 0.0032	± 2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

$\alpha_s(M_Z^2)$ in NNLO Analyses of Deep-Inelastic World Data

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The present world data of deep-inelastic scattering (DIS) reached a precision which allows the measurement of $\alpha_s(M_Z^2)$ from their scaling violations with an error of $\delta\alpha_s(M_Z^2) \simeq 1\%$. This requires at least NNLO analyses, since NLO fits exhibit scale uncertainties of $\Delta_{r,f}\alpha_s(M_Z^2) \sim 0.0050$. The NNLO values for α_s obtained are summarized in the following Table.

	$\alpha_s(M_Z^2)$	
BBG	$0.1134\ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO [1]
GRS	0.112	valence analysis, NNLO [2]
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$ [3]
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach [3]
JR	0.1124 ± 0.0020	dynamical approach [4]
JR	0.1158 ± 0.0035	standard fit [4]
MSTW	0.1171 ± 0.0014	[5]
ABM	0.1147 ± 0.0012	FFNS, incl. combined H1/ZEUS data [6]
Gehrmann et al.	$0.1153 \pm 0.0017 \pm 0.0023$	e^+e^- thrust [7]
Abbate et al.	$0.1135 \pm 0.0011 \pm 0.0006$	e^+e^- thrust [8]
BBG	$0.1141\ ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO [1]
world average	0.1184 ± 0.0007	[9]

NNLO non-singlet data analyses have been performed in [1] and [2]. The analysis of Ref. [1] is based on an experimental combination of flavor non-singlet data referring to $F_2^{p,d}(x, Q^2)$ for $x < 0.35$ and using the respective valence approximations for $x > 0.35$. The $\overline{d} - \overline{u}$ distributions and the $O(\alpha_s^2)$ heavy flavor corrections were accounted for. At low Q^2 and at large x also at low W^2 higher twist corrections have to be taken into account [10]. The corresponding region was cut out in [1] performing the fits for the leading twist terms only. The analysis could be extended to N^{3}LO effectively due to the dominance of the Wilson coefficient in this order [11] if compared to the anomalous dimension, cf. [1] and [12]. This analysis led to an increase of $\alpha_s(M_Z^2)$ by +0.0007 if compared to the NNLO value.

A combined singlet and non-singlet NNLO analysis based on the DIS world data, including the Drell-Yan and di-muon data, needed for a correct description of the sea-quark densities, was performed in [3]. In the fixed flavor number scheme (FFNS) the value of $\alpha_s(M_Z^2)$ is the same as in the non-singlet case [1]. The comparison between the FFNS and the BMSN scheme [13] for the description of the heavy flavor contributions induces a systematic uncertainty $\Delta\alpha_s(M_Z^2) = 0.0006$. The NNLO analyses of Ref. [4] are statistically compatible with the results of [1] and [3] while those of [5] yield a higher value.

In Ref. [6] the combined H1 and ZEUS data were accounted for in a NNLO analysis for the first time, which led to a shift of +0.0012. However, running quark mass effects [14] and the account of recent F_L data reduce this value again to the NNLO value given in [3]. We mention that other recent NNLO analyses of precision data, as the measurement of $\alpha_s(M_Z^2)$ using thrust in high energy e^+e^- annihilation data [7], [8] result in lower values than the 2009 world average [9] based on NLO, NNLO and N^{3}LO results. The sensitivity of the fits

to a precise description of the longitudinal structure function F_L has been demonstrated in [15] recently, in the case of the NMC data. Inconsistent descriptions of F_L induce a high value of α_s of ~ 0.1170 to be compared with that obtained in [5]. It is observed that the values of

$\alpha_s(M_Z^2)$	with $\sigma_{\rm NMC}$	with $F_2^{\rm NMC}$	difference
NNLO	0.1135(14)	0.1170(15)	$+0.0035 \simeq 2.3\sigma$
NNLO $+F_L$ $O(\alpha_s^3)$	0.1122(14)	0.1171(14)	$+0.0050 \simeq 3.6\sigma$

α_s found in NLO fits are systematically higher than those in NNLO analyses. α_s measurements based on jet data can be performed presently at NLO only. Here typical values obtained are $\alpha_s(M_Z^2) = 0.1156\ ^{+0.0041}_{-0.0034}$ [16], $\alpha_s(M_Z^2) = 0.1161\ ^{+0.0041}_{-0.0048}$ [17] in recent examples. The precise knowledge of $\alpha_s(M_Z^2)$ is of instrumental importance for the correct prediction of the Higgs boson cross section at Tevatron and the LHC [18].

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CTEQ-TEA Parton Distribution Functions and α_s

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Abstract

After summarizing a simple method to fully reproduce the correlated dependence of theoretical cross sections on the QCD coupling strength α_s and parameters of parton distribution functions (PDFs), we present a series of CTEQ6.6AS and CT10.AS PDFs, realizing this approach, for α_s values in the interval $0.113 \leq \alpha_s(M_Z) \leq 0.123$. If α_s is solely determined by the QCD global analysis data, and not by the world average value of $\alpha_s(M_Z)$ as done in the CT10 PDF analysis, we find that $\alpha(M_Z) = 0.1197 \pm 0.0061$ at the 90% CL.

In a recent paper [1], we examined the dependence of parton distribution functions (PDFs) on the value of the QCD coupling strength $\alpha_s(M_Z)$, in the QCD global analysis performed by the CTEQ-TEA (Tung et al.) group. We demonstrated a simple method that is rigorously valid in the quadratic approximation normally applied in PDF fitting, and fully reproduces the correlated dependence of theoretical cross sections on α_s and PDF parameters. This method is based on a statistical relation that allows one to add the uncertainty produced by α_s, computed with some special PDF sets, in quadrature with the PDF uncertainty obtained for a fixed α_s value (such as the CTEQ6.6 PDF set). A series of four CTEQ6.6AS PDFs realizing this approach, for α_s values between 0.116 and 0.120, was made available online [2]. Using these PDFs, the combined α_s and PDF uncertainty can be assessed for theoretical predictions at the Fermilab Tevatron and Large Hadron Collider. In a more recent paper [3], we provided a similar series, CT10.AS PDFs, but with a larger range of α_s values, from 0.113 to 0.123 [4].

Below, we summarize a few important features of, for example, the CTEQ6.6AS PDF analysis. Using the setup of the CTEQ6.6 PDF analysis and taking the world-average (w.a.) value [5] $(\alpha_s)_{w.a.} \pm (\delta\alpha_s)_{w.a.} = 0.118 \pm 0.002$ (at 90% CL [6].) as a separate data point in addition to the complete CTEQ6.6 set of hadronic scattering data, we allow $\alpha_s(M_Z)$ to vary within the global fits and find

$$\alpha_s(M_Z) = 0.1180 \pm 0.0019 \qquad \text{(at 90\% CL)}. \tag{1}$$

Thus, the constraint on $\alpha_s(M_Z)$ is dominated by the world-average uncertainty, $(\delta\alpha_s)_{w.a.}$. Using the α_s series of PDFs, and including the PDF uncertainty, we can estimate the uncertainties of cross section calculations. For any calculated quantity σ, we denote the central

prediction, corresponding to $\alpha_s(M_Z) = 0.118$, by σ_0. There are two contributions to the uncertainty: the PDF uncertainty ($\Delta\sigma_{\rm PDF}$) and the α_s uncertainty ($\Delta\sigma_{\alpha_s}$) of σ. The combined uncertainty $\Delta\sigma$ for CTEQ6.6+CTEQ6.6AS is $(\Delta\sigma)^2 = (\Delta\sigma_{\rm PDF})^2 + (\Delta\sigma_{\alpha_s})^2$.

The independence of the α_s uncertainty from the PDF uncertainty in the CTEQ6.6AS method does not preclude existence of some correlation between the α_s and PDF parameters. This correlation arises from the hadronic scattering experiments, which probe a variety of combinations of the PDFs and α_s, and can be closely examined by analyzing the correlation cosine as introduced in Refs. [7]. For that purpose, we performed the full fit CTEQ6.6FAS with the floating α_s and examined the correlation between $\alpha_{\rm s}(M_Z)$ and individual PDFs $f_a(x, Q)$. Fig. 8 of [1] shows the correlation cosine, $\cos\varphi$ versus x, for the PDFs that have the largest correlations with $\alpha_{\rm s}(M_Z)$, at $Q = 2$ and 85 GeV. The best-fit value of $\alpha_{\rm s}(M_Z)$ is thus determined by several types of the data, probing the gluon evolution in DIS at moderately small x, the singlet PDF evolution in DIS at large x, and HERA charm semi-inclusive DIS data. The correlation of each kind disappears if the relevant data set is removed.

After the completion of Refs. [1] and [3], we performed a similar analysis on the determination of $\alpha_{\rm s}(M_Z)$ value but with the CT10 setup. We find that, without including the world-average $\alpha_{\rm s}(M_Z)$ value as an input data, the constraints of the global fit on α_s are relatively weak, and

$$\alpha_s(M_Z) = 0.1197 \pm 0.0061 \qquad \text{(at 90\% CL)}, \tag{2}$$

where the error is an estimate of the uncertainty (at about 90 % C.L.) from all typical sources, including the statistical error, dependence on the parametrization form and free theoretical parameters. Within this range, $\chi 2$ exhibits nearly parabolic dependence on α_s. Eq. 2 shows clearly that constraints imposed on α_s by the hadronic scattering data only are significantly weaker than from the world average data point.

Finally, we note that replacing the NMC F_2 data by the corresponding differential cross section data in the CT10.AS PDF global analysis does not change our conclusion about the determination of the $\alpha_{\rm s}(M_Z)$ value.

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α_S in MSTW analyses

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In the MSTW2008 [1] global PDF analysis of DIS and related hard-scattering data the value of $\alpha_S(M_Z^2)$ was left as a free parameter and its optimum value determined. Subsequently, in [2], a detailed study was made of its 'experimental' uncertainty. The values found in the NLO and NNLO analyses are

$$\text{NLO:} \quad \alpha_S(M_Z^2) = 0.1202 \;\; {}^{+0.0012}_{-0.0015} \text{ (68\% C.L.)} \;\; {}^{+0.0032}_{-0.0039} \text{ (90\% C.L.)}, \tag{1}$$

$$\text{NNLO:} \quad \alpha_S(M_Z^2) = 0.1171 \;\; {}^{+0.0014}_{-0.0014} \text{ (68\% C.L.)} \;\; {}^{+0.0034}_{-0.0034} \text{ (90\% C.L.)}. \tag{2}$$

We note that the NNLO value is smaller than that at NLO since NNLO evolution is faster and since NNLO coefficient functions are generally positive. Fig. 1 shows the constraints on α_S coming from the individual data sets in the NNLO global analysis. Inclusive jet production is the only process proportional to α_S^2 at LO. Moreover, since the normalisation of the CDF jet production data is tied to the observed CDF Z rapidity distribution, we see that these jet data give the tightest constraint on α_S. In addition to the 'experimental' errors shown in (1) and (2), there is also a 'theory' uncertainty, which is estimated [2] to be ± 0.003 at NLO and at most ± 0.002 at NNLO.

The $\alpha_S(M_Z^2)$ values found by MSTW are, in general, greater than those of other PDF analyses, and are more consistent with the world average value [3]. There are two reasons for this. First, MSTW have a more flexible low-x parametrisation of the input gluon PDF. The flexibility is required by the data and gives a negative input behaviour at small x, which is confirmed by the NNPDF analyses [4]. The second reason is the inclusion of the Tevatron jet data in the analysis. It is informative to repeat the NNLO MSTW2008 analysis fitting *only* inclusive DIS data, which gives $\alpha_S(M_Z^2) = 0.1104$ compared to $\alpha_S(M_Z^2) = 0.1171$ for the global fit, but with a negative input gluon for $x > 0.4$ due to lack of a data constraint, implying a negative F_2^{charm} and an awful description of Tevatron jet data ($\chi^2/N_{\text{pts.}} \sim 10$). Fixing the high-$x$ gluon parameters to be the same as in the global fit gives $\alpha_S(M_Z^2) = 0.1172$ in the DIS-only fit. A DIS-only fit without BCDMS data gives $\alpha_S(M_Z^2) = 0.1193$, while a global fit without BCDMS data gives $\alpha_S(M_Z^2) = 0.1181$. We conclude that the Tevatron jet data are vital to pin down the high-x gluon, giving a smaller low-x gluon by the momentum sum rule, at the expense of some deterioration in the fit quality of BCDMS data. Thus, both reasons imply a larger α_S from the scaling violations ($\sim \alpha_S g$) of HERA data.

Concerning the NMC data, ref. [5] notes that it is better to use the NMC cross-section measurements rather than F_2, since for $x < 0.12$ NMC used a Q^2-independent $R = \sigma_L/\sigma_T$ value to extract their default F_2. Does this bias the MSTW value of α_S? As a check, we repeated the NNLO global fit with the NMC F_2 extracted using the SLAC $R_{1990}(x, Q^2)$ for all x, which is close to the MSTW NNLO $R(x, Q^2)$ in the most relevant x bins. This has a very small effect, with the NNLO $\alpha_S(M_Z^2)$ moving only from 0.1171 to 0.1167.

Finally, let us comment on the 'low' value $\alpha_S(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$ obtained in [6], since it is often taken as the 'DIS' value in world average compilations, where it is listed as DIS $F_2(\text{N}^3\text{LO})$ [3]. This determination assumes that F_2 is pure non-singlet for $x > 0.3$. We may

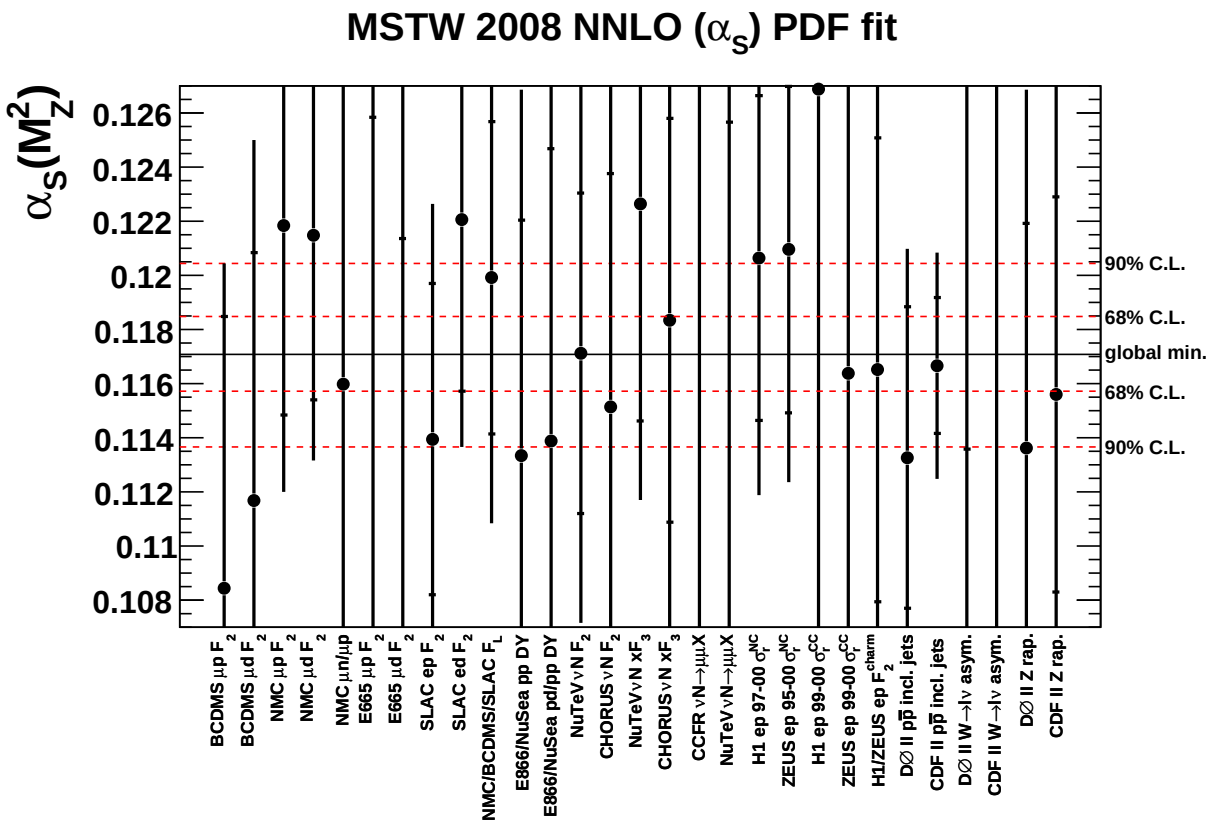

Figure 1: Ranges of $\alpha_S(M_Z^2)$ for which data sets are described within their 90% C.L. limit (outer error bars) or 68% C.L. limit (inner error bars) in the NNLO global fit. The points (•) indicate the values of $\alpha_S(M_Z^2)$ favoured by each individual data set n, that is, the values for which χ_n^2 is minimised. The figure is taken from [2].

examine this assumption using MSTW2008 PDFs. For the F_2^p and F_2^d measurements with $x > 0.3$ we have $\chi^2 = 329$ for 282 data points, of which 160 are from BCDMS. However, if we consider only non-singlet contributions then $\chi^2 = 1449$ for 282 data points. In fact, contributions other than valence quarks amount to about 10% at $x = 0.3$, and still 2% at $x = 0.5$. We conclude that the anomalously low value of α_S found in [6] is due both to the dominance of BCDMS data (cf. Fig. 1) and to the neglect of singlet contributions.

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$\alpha_s(M_Z)$ with NNPDF parton distributions

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The determination of $\alpha_s(M_Z)$ from the same wide of set of data which is used to determine PDFs is appealing because it simultaneously exploits the dependence on the coupling of scaling violations as well as that on individual hard matrix elements of the various processes under consideration. On the other hand, in such a determination the value of α_s is necessarily correlated to the best-fit form of the PDFs, and thus subject to potential sources of bias which may affect the PDFs, such as for example an insufficiently flexible parametrization.

Determining α_s with NNPDF partons has the advantage that in this approach parametrization bias is reduced to a minimum, thanks to the use of neural networks as unbiased interpolating functions coupled with a Monte Carlo approach. It has the disadvantage that, because NNPDF partons are delivered in the form of a Monte Carlo sample, the χ^2 of the comparison between data and theory is a random variable, which only tends to a constant value in the limit in which the size of the Monte Carlo sample tends to infinity. In order to determine α_S accurately one thus needs a number of Monte Carlo replicas of the same order of magnitude of the number of datapoints (more than 3500 for NNPDF and other global PDF sets).

We have determined α_s by constructing sets of at least 500 replicas for each of the eleven values of α_s for which NNPDF2.1 parton distributions have been provided [1]. The comparison to the data is done at NLO in QCD and includes heavy quark mass effects using the so-called FONLL method [2]. The uncertainty on each value of χ^2 due to the finite size of the replica sample is determined using the bootstrap method from the fluctuations of χ^2 computed from subsets of replicas in the given sample. The value of α_s is then determined by fitting a parabola to the χ^2 viewed as a function of α_s. The minimum of the parabola provides the best-fit value of $\alpha_s(M_Z)$ while the $\Delta\chi^2 = 1$ range gives the uncertainty on it. The further uncertainty due to the finite-size fluctuations is propagated from the uncertainty on each data point and is essentially negligible with the given sample size. The quality of the parabolic fit is assessed by evaluating the corresponding $\chi^2_{\rm par}/N_{\rm dof}$, with $N_{\rm dof} = N_{\alpha_s} - 3$.

We have performed [3] this determination both for the global NNPDF2.1 dataset, for all deep-inelastic data included in this dataset, and for HERA data only. The best-fit values and

	$\alpha_s(M_Z)$
NNPDF2.1	$0.1191 \pm 0.0006^{\rm stat} \pm 0.0001^{\rm proc}$
NNPDF2.1 DIS–only	$0.1177 \pm 0.0009^{\rm stat} \pm 0.0002^{\rm proc}$
NNPDF2.1 HERA–only	$0.1103 \pm 0.0033^{\rm stat} \pm 0.0003^{\rm proc}$

Table 1: Values of $\alpha_s(M_Z)$ and associated uncertainties. All uncertainties shown are 68% confidence levels.

uncertainties are collected in Table 1; the uncertainty from the $\Delta\chi^2 = 1$ range denoted with "stat" and the uncertainty from the finite size of the replica sample denoted with "proc".

Our best-fit value of α_s is in good agreement with the current PDG value, and it has a surprisingly small statistical uncertainty. The statistical uncertainty increases as the size of the dataset is reduced, as it ought to. We see no evidence that DIS data prefer a significantly lower value of α_s: the difference between the global and DIS-only determinations is compatible with a statistical fluctuation. HERA data, however, lead to a rather smaller value of α_s, though with much larger uncertainty. This may be related to deviations from predicted scaling violations, which have been observed [4] in HERA data at low x and Q^2. If so, the value of α_s could be brought in line with other determinations by an inclusion of small x resummation [5].

From the behaviour of the χ^2 as a function of α_s for individual datasets for the global and the DIS-only fit we find evidence [3] that BCDMS data prefer a lower value of α_s, but only in the DIS only fit. Also, the correlation between the χ^2 of the fit to various datasets and individual PDFs as a function of α_s provides evidence that this behaviour is due to the fact that at low α_s the χ^2 for BCDMS can be lowered by modifying the gluon distribution in a way which is incompatible with jet data. However, the best-fit α_s for the DIS fit is unaffected by all this, perhaps becuase of the great flexibility of the NNPDF parton parametrization.

Theoretical uncertainties on our results are likely to be dominant. The main ones are likely to be related to higher order QCD corrections and their resummation, and to the treatment of heavy quarks. These uncertainties are presumably of similar size here as in other determinations of α_s based on the same QCD processes. A detailed study of these uncertainties, which are likely to be dominant, would be of great interest once QCD corrections to the highest available order have been included.

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Tau Decay Determination of the QCD Coupling

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The inclusive character of the total τ hadronic width renders possible [1] an accurate calculation of the ratio $R_\tau \equiv \Gamma[\tau^- \to \nu_\tau \,\text{hadrons}]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]$. Its Cabibbo-allowed component can be written as [2]

$$R_{\tau,V+A} \;=\; N_C\, |V_{ud}|^2\, S_{\rm EW}\, \{1 + \delta_{\rm P} + \delta_{\rm NP}\}\,, \tag{1}$$

where $N_C = 3$ is the number of quark colours and $S_{\rm EW} = 1.0201 \pm 0.0003$ contains the electroweak radiative corrections. The non-perturbative contributions are suppressed by six powers of the τ mass [1] and can be extracted from the invariant-mass distribution of the final hadrons [3]. From the ALEPH data, one obtains $\delta_{\rm NP} = -0.0059 \pm 0.0014$ [4].

The dominant correction ($\sim 20\%$) is the perturbative QCD contribution [1] [3]

$$\delta_{\rm P} \;=\; \sum_{n=1} K_n\, A^{(n)}(\alpha_s) \;=\; \sum_{n=1} (K_n + g_n)\, a_\tau^n \;\equiv\; \sum_{n=1} r_n\, a_\tau^n\,, \tag{2}$$

which is determined by the coefficients of the perturbative expansion of the ($N_F = 3$) QCD Adler function, already known to $O(\alpha_s^4)$ [5]: $K_0 = K_1 = 1$; $K_2 = 1.63982$; $K_3(\overline{MS}) = 6.37101$ and $K_4(\overline{MS}) = 49.07570$. The functions [3]

$$A^{(n)}(\alpha_s) \;=\; \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi}\right)^n \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8}\right) \;=\; a_\tau^n + \mathcal{O}(a_\tau^{n+1}) \tag{3}$$

are contour integrals in the complex plane, which only depend on $a_\tau \equiv \alpha_s(m_\tau^2)/\pi$. Using the exact solution (up to unknown $\beta_{n>4}$ contributions) for $\alpha_s(-s)$ given by the renormalization-group β-function equation, they can be numerically computed with very high accuracy [3].

If the integrals $A^{(n)}(\alpha_s)$ are expanded in powers of a_τ, one recovers the naive perturbative expansion of $\delta_{\rm P}$ shown in the rhs of Eq. (2). This approximation is known as *fixed-order perturbation theory* (FOPT), while the improved expression, keeping the non-expanded values of $A^{(n)}(\alpha_s)$, is usually called *contour-improved perturbation theory* (CIPT) [3]. Even at $\mathcal{O}(a_\tau^4)$, FOPT gives a rather bad approximation to the integrals $A^{(n)}(\alpha_s)$, overestimating $\delta_{\rm P}$ by 12% at $a_\tau = 0.11$. The long running of $\alpha_s(-s)$ along the circle $|s| = m_\tau^2$ generates very large g_n coefficients, which depend on $K_{m<n}$ and $\beta_{m<n}$ [3]: $g_1 = 0$, $g_2 = 3.56$, $g_3 = 19.99$, $g_4 = 78.00$, $g_5 = 307.78$. These corrections are much larger than the original K_n contributions, giving rise to a badly behaved perturbative series (at the four-loop level the expansion of $\alpha_s(-s)$ in powers of a_τ is only convergent for $a_\tau < 0.11$, which is very close to the physical value of a_τ). Thus, it seems compulsory to resum the large logarithms, $\log^n(-s/m_\tau^2)$, using the renormalization group. This is precisely what CIPT does.

It has been argued that in the asymptotic regime (large n) the renormalonic behaviour of the K_n coefficients could induce cancelations with the running g_n corrections, which would be missed by CIPT. In that case, FOPT could approach faster the 'true' result provided by the Borel summation of the full renormalon series. This happens actually in the large–β_1

limit, which however does not approximate well the known perturbative series (for $n \leq 4$ the true K_n coefficients add constructively with the g_n contributions). Models of higher-order corrections which assume a precocious asymptotic behaviour of the Adler function already at $n = 3, 4$ [6] [7] seem to favour the FOPT result. The CIPT procedure is much more reliable in all other scenarios.

The present experimental value $R_{\tau,V+A} = 3.4771 \pm 0.0084$ [8] implies $\delta_{\rm P} = 0.2030 \pm 0.0033$. The two different treatments of the perturbative series result in

$$\alpha_s(m_\tau^2)_{\rm CIPT} = 0.3412 \pm 0.0041_{\delta_{\rm P}}{}^{+0.0069}_{-0.0064}{}_{K_5}{}^{+0.0050}_{-0.0001}{}_{\mu}{}^{+0.0039}_{-0.0034}{}_{\beta_5} = 0.344 \pm 0.014\,, \quad (4)$$

$$\alpha_s(m_\tau^2)_{\rm FOPT} = 0.3194 \pm 0.0028_{\delta_{\rm P}}{}^{+0.0039}_{-0.0035}{}_{K_5}{}^{+0.0105}_{-0.0045}{}_{\mu}{}^{+0.0019}_{-0.0045}{}_{\beta_5} = 0.321 \pm 0.015\,. \quad (5)$$

Higher-order corrections have been estimated adding the fifth-order term $K_5\, A^{(5)}(\alpha_s)$ with $K_5 = 275 \pm 400$. We have also included the 5-loop variation with changes of the renormalization scale in the range $\mu^2/(-s) \in [0.4, 2.0]$. The error induced by the truncation of the β function at fourth order has been conservatively estimated through the variation of the results at five loops, assuming $\beta_5 = \pm\beta_4^2/\beta_3 = \mp 443$; in CIPT this slightly changes the values of $A^{(n)}(\alpha_s)$, while in FOPT it increases the scale sensitivity. The FOPT result shows as expected [3] [9] a much more sizeable μ dependence, but it gets smaller errors from $\delta_{\rm P}$ and K_5. The three theoretical uncertainties (K_5, μ, β_5) have been added linearly and their sum combined in quadrature with the 'experimental' error from δ_P.

Combining the two results with the PDG prescription (scale factor $S = 1.14$), one gets $\alpha_s(m_\tau^2) = 0.334 \pm 0.011$. We keep conservatively the smallest error, i.e.

$$\alpha_s(m_\tau^2) = 0.334 \pm 0.014 \qquad \longrightarrow \qquad \alpha_s(M_Z^2) = 0.1204 \pm 0.0016\,. \quad (6)$$

The resulting value is in excellent agreement with the direct measurement of $\alpha_s(M_Z)$ at the Z peak, providing a very significant experimental verification of *asymptotic freedom*.

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α_s from τ Decays using Contour-Improved Perturbation Theory

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Several recent results motivated the reanalysis of α_s from τ decays in ref. [1]. On the experimental side, new BABAR measurements of the e^+e^- annihilation cross section into $K\bar{K}\pi$ using the radiative return method increase the accuracy on the vector/axial-vector fractions in the corresponding τ decays, and better results are available on τ decays into strange final states from BABAR and Belle. On the theory side, the fourth-order term K_4 in the perturbative expansion of the Adler function was recently calculated [2]. Unitairty yields the vector/axial tau hadronic widths in terms of the experimentally accessible spectral functions (imaginary parts of two-current correlators Π). The integral can be reexpressed using analyticity and Operator Product Expansion (OPE) as the sum of contributions from non-perturbative condensates and $\delta^{(0)}$ the (dominant) perturbative contribution involving the Adler function $D(s) = -sd\Pi/ds$ along a circular contour $|s| = s_0 = M_\tau^2$:

$$1 + \delta^{(0)} = -2\pi i \int_{|s|=s_0} \frac{ds}{s} \left[1 - 2\frac{s}{s_0} + 2\frac{s^3}{s_0^3} - \frac{s^4}{s_0^4}\right] D(s)\,, \quad D(s) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} \tilde{K}_n(\xi) a_s^n(-\xi s_0)\,.$$

$a_s(s) = \alpha_s(s)/\pi$ on the contour is obtained by applying the Renormalisation Group Equation (RGE) known up to four loops. The phase space involved in the τ hadronic width leads to an integral for $\delta^{(0)}$ with a so-called pinched kernel, vanishing for $s = s_0$. We reexamined the convergence properties of the perturbative expansions for the τ hadronic widths, and the ambiguity between the fixed-order (FOPT) and contour-improved (CIPT) approaches for determining α_s along the circle of integration and summing up the series (as the difference between the two approaches exceeds the other sources of theoretical uncertainty). Starting from the value of α_s in the Euclidean region where OPE is known to be valid, CIPT corresponds to solving the RGE along the circle of integration by small steps, whereas FOPT corresponds to a Taylor expansion in powers of $\alpha_s(m_\tau^2)$ and $\log(s/s_0)$. We compared different implementations of FOPT, corresponding to keeping further terms derived from RGE along the circle of integration. The main difference between FOPT and CIPT was seen to arise from the truncation of higher orders perfomed in FOPT after the contour integral has been computed. We identified specific consistency issues for FOPT, which do not exist in CIPT: the convergence of the Adler function near the positive axis cut is problematic for FOPT (less visible once the Adler function convoluted with pinched weights vanishing at $s = m_\tau^2$), and the scale dependence of $\delta^{(0)}$ is much more pronounced than for CIPT.

Possible violations of quark-hadron duality at the τ mass scale have been considered using specific but crude models (equally-spaced resonances or instanton model). Their effect has been found to be well within our quoted overall theoretical uncertainty when we considered the $V + A$ spectral function. Due to the very simple models considered, we did not introduce additional theoretical uncertainties. We did not investigate the separate vector and axial channels, as discussed in refs. [3] and [4], since the more inclusive quantity $V + A$ provided a more stable determination of α_s than V or A separately. This was obtained by performing a combined fit of the τ hadronic width $R^{00}_{\tau,V/A} = R_{\tau,V/A}$ and hadronic spectral moments from the ALEPH collaboration of the form $R^{kl}_{\tau,V/A} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}$ with $(k, l) =$

$(1,0),(1,1),(1,2),(1,3)$, expressed through OPE in terms of 4 unknown quantities (α_s, $D=4$ gluon condensate, $D=6$ and $D=8$ effective condensates). The $D=4$ quark condensate was expressed in terms of quark masses through the Gell-Mann-Oakes-Renner relation, and $D \geq 10$ contributions neglected, even for higher moments (this approximation may feed back in the $D=6,8$ condensates). The fit for $V+A$ was indeed more satisfactory than the separate V and A fits, yielding $\alpha_s(\tau) = 0.344 \pm 0.005_{\rm exp} \pm 0.007_{\rm theo}$, consistent with the previous value obtained for three known orders, with a 20% reduced theoretical uncertainty. The evolved τ result at the M_Z scale was: $\alpha_s(M_Z) = 0.1212 \pm 0.0005_{\rm exp} \pm 0.0008_{\rm theo} \pm 0.0005_{\rm evol}$, which agrees remarkably well with the value directly extracted at this scale (the FOPT value evolved at M_Z would be lower by 0.0028).

After the completion of this work, it was pointed out in ref. [4] that the spectral functions from the ALEPH data seemed much more correlated than the currently available correlation matrix, which indeed does not take into account properly the correlations due to unfolding. A study is currently under way to assess these uncertainties precisely, but preliminary studies indicate that they have only a limited impact on the correlations between the pinched-weight moments used in the analysis of ref. [1]. Another issue was raised in ref. [5], where a model was introduced for the higher orders of the Adler function in perturbation theory, including the known properties derived from renormalon calculus. This model assumed that the perturbative series was saturated by three renormalons (two infrared ones and an ultraviolet one) and that the asymptotic behaviour for these renormalons could already be used with a good accuracy at order α_s^3. The Borel sum of the model (assumed to yield the "true" value of the Adler function) was used to compute $\delta^{(0)}$ and compared with CIPT and FOPT results at increasing orders of perturbation theory, leading to favouring FOPT. In ref. [6], we discussed some assumptions of this model, showing that different conclusions, favouring either FOPT, CIPT or neither could be drawn from rather simple variations of this model (asymptotic behaviour setting in than at a higher order than α_s^3, presence of several subleading infrared renormalons), as well as they depend on the particular weight chosen for the analysis. An alternative model [7] led to the conclusion that a different treatment of FOPT and CIPT could lead to the same result at high orders for some classes of models combining renormalon calculus and conformal mapping.

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Fixed-order analysis of the hadronic τ decay width

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The inclusive τ hadronic width provides one of the most precise determinations of the strong coupling, since despite the low scale, non-perturbative condensate contributions are below 5% of the perturbative correction [1]. Subtracting the condensate terms (PC) from the ALEPH data, the experimental value of the perturbative QCD correction relative to the parton model decay is $\delta^{(0)}_{\rm phen} = 0.2037 \pm 0.0040_{\rm exp} \pm 0.0037_{\rm PC}$. Theoretically, the perturbative expansion is obtained from the vector and axial-vector current correlation functions, whose expansion is known to $\mathcal{O}(\alpha_s^4)$ [2], by integrating the Adler function along a circle in the complex Q^2 plane. Depending on whether one performs a strict expansion of this integral in $\alpha_s(m_\tau)$ (fixed order, FO) or integrates the running coupling along the circle exactly (contour-improved, CI), one obtains

$$\delta^{(0)}_{\rm FO} = \sum_{n=1}^{\infty} a(m_\tau^2)^n \sum_{k=1}^{n} k\, c_{n,k}\, J_{k-1} \qquad \text{and} \qquad \delta^{(0)}_{\rm CI} = \sum_{n=1}^{\infty} c_{n,1}\, J_n^a(m_\tau^2), \tag{1}$$

respectively. In both cases, the dynamical input resides only in the expansion coefficients $c_{n,1}$ of the Adler function, but when one truncates the expansions at some value of n, the resulting value of $\delta^{(0)}$ is different, since the two expressions treat formally higher-order terms differently. The numerical effect is significant, as can be seen from

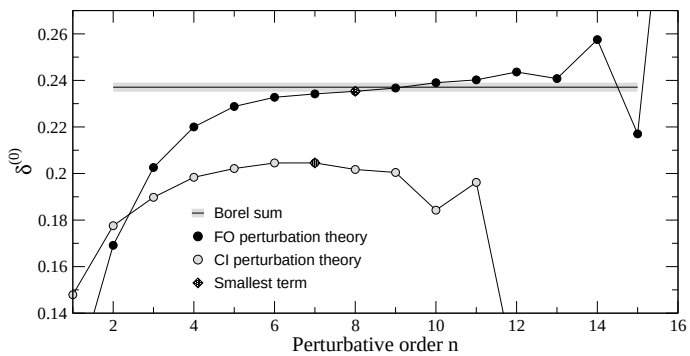
$$\begin{aligned} \delta^{(0)}_{\rm FO} &= 0.1082 + 0.0609 + 0.0334 + 0.0174 + 0.0088 = 0.2288 \\ \delta^{(0)}_{\rm CI} &= 0.1479 + 0.0297 + 0.0122 + 0.0086 + 0.0038 = 0.2021 \end{aligned} \tag{2}$$

(using the estimate $c_{5,1} = 283$ and $\alpha_s(m_\tau) = 0.34$), and *increases* as more terms are added, which points to a systematic problem in one of the two approaches. As will be explained below, the fixed-order approach is strongly preferred. Repeating the fixed-order analysis of [3] with $\delta^{(0)}_{\rm phen}$ as given above we obtain

$$\alpha_s(m_\tau) = 0.3199 \pm 0.0034_{\rm exp} \pm 0.0031_{\rm PC} \pm 0.0026_{c_{5,1}}{}^{+0.0105}_{-0.0052}\,(\text{scale}), \tag{3}$$

which implies $\alpha_s(M_Z) = 0.1185 \pm 0.0004_{\rm exp}{}^{+0.0013}_{-0.0008}\,(\text{th}) \pm 0.0002_{\rm evol}$.

The CI method is often argued to be the method of choice since it exhibits a faster apparent convergence and since the expansion of the running coupling on the circle is barely convergent near the Minkowskian axis $Q^2 < 0$. However, the expansion of the Adler function has zero radius of convergence (due to renormalon and other singularities in the Borel plane [4]), and is only asymptotic. The finite radius of convergence of the coupling expansion along the circle is therefore of subordinate significance – the assumption made in the CI approach is that the higher order terms in the Adler function series are negligible in comparison to running coupling effects despite their factorial divergence.



In order to clarify the discrepancy between the FO and CI approaches, in [3] we included in addition to the known $c_{n,1}$ all available information on the structure of the large-order behaviour of the Adler function. The analysis is based on the assumption that the Adler function series is sufficiently regular, as seen in the known terms, and that sufficiently many low-order terms are known, so that one can smoothly connect them with an ansatz for the Borel transform that contains the leading singularities. (This does *not* imply that the series is in the asymptotic regime for $n \approx 5$ – the known terms do not exhibit factorial behaviour. A pattern with several singularities being relevant at intermediate n is expected by comparison with the exact all-order series in the large-β_0 expansion [5].) With such an ansatz for the Borel transform we find that only the FO series approaches the "true" result represented by Borel sum within its ambiguity, shown as the grey band in the figure above (plotted for $\alpha_s(m_\tau) = 0.34$), while the CI approximation is systematically too low. In particular, this conclusion applies to $n = 4, 5$, up to which the series is already known (or estimated).

How general is this conclusion? An important point, specific to the hadronic τ width, is that the gluon condensate contribution is strongly suppressed. This is reflected in large cancellations in the large-order behaviour of the series coefficients $c_{n,1}+g_n$ of the expansion of the τ hadronic width associated with the first infrared renormalon: $(c_{n,1} + g_n)/c_{n,1} \sim 1/n^2$. These cancellations are destroyed in the CI expansion, which sums the running coupling terms g_n, but drops the Adler function coefficients $c_{n,1}$ in higher orders. The CI method therefore fails whenever the first infrared renormalon makes the dominant contribution to the series of the hadronic τ width in intermediate orders. We find that this is always the case unless we choose an ansatz in which we suppress this contribution by hand, or allow large and unnatural cancellations between the first and second infrared renormalon.

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FOPT and CIPT in τ Decays

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One of the largest theoretical uncertainties assigned to the strong coupling constant α_s as determined from hadronic tau decays [2,3,4,5] (for recent reviews see [6,7,8,9,10,11]) stems from the differences in the results for Fixed Order Perturbation Theory (FOPT), Contour Improved Perturbation Theory (CIPT) and Renormalon Chain Perturbation Theory (RCPT). The results of [1] are presented here where it is shown that FOPT and the FOPT part of RCPT have much larger errors than usually assumed and agree within their respective errors with CIPT. For the non-strange decay width of the τ the perturbative part is given by

$$1+\delta_\mathrm{pert} = \sum_{n=0}^{4} \frac{K_n}{2\pi i} \oint\limits_{|s|=m_\tau^2} \frac{\mathrm{d}s}{s} \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8}\right) \left(\frac{\alpha_\mathrm{s}(-s)}{\pi}\right)^n + O({\alpha_\mathrm{s}}^5), \tag{1}$$

with the known coefficients $K_{n=0..4}$ [12,13,14,15,16,17]. The fifth-order term has been estimated to $K_5 \simeq 275$ in [17], but the large deviation of the exact K_4 from it's prediction suggests that a 100% error on K_5 is realistic and $K_5 = 400 \pm 400$ is used in [1]. The methods FOPT and CIPT [5] differ in the way (1) is calculated. In the CIPT approach the β-function is used to numerically solve $\alpha_\mathrm{s}(-s)$ in the complex s-plane by starting with $\alpha_\mathrm{s}(m_\tau^2)$. The integrand is thus calculated in small steps on the circle $|s| = m_\tau^2$ and the sum of all pieces gives the total integral. For the FOPT method the β-function and its derivatives are Taylor expanded in s around $s_0 = m_\tau^2$ which leads to a power series representation of $\alpha_\mathrm{s}(-s)$ in powers of $\alpha_\mathrm{s}(m_\tau^2)$. The series is truncated at the desired order (here the 5th) in the strong coupling and inserted in the integral which becomes solveable now. For the purpose of showing the impact of the arbitrary choice to develop around $s_0 = m_\tau^2$ this approach can be generalized to first evolve $\alpha_\mathrm{s}(m_\tau^2)$ to $\alpha_\mathrm{s}(m_\tau^2 \exp(i\varphi_0))$ with the numerically solved β-function and derive the Taylor series of δ_pert around this new point. In order to preserve the symmetry

$$\alpha_\mathrm{s}(m_\tau^2 \exp(-i\varphi)) = \alpha_\mathrm{s}(m_\tau^2 \exp(i\varphi))^* \tag{2}$$

the upper semi-circle is developed around $s_0 = m_\tau^2 \exp(i\varphi_0)$ while the lower semi-circle is developed around $s_0^* = m_\tau^2 \exp(-i\varphi_0)$. Other schemes to preserve this symmetry and to avoid imaginary contributions to δ_pert lead to similar results and are not further discussed. Numerically the central result reads [1]:

$$\begin{aligned}
\delta_\mathrm{pert} = {} & a + 0.849\,b + (-4.5\,\varphi_0 + 0.808)\,a\,b + (1.910\,\varphi_0 + 5.202)\,(a^2 - b^2) + \\
& (-5.063\,\varphi_0^2 + 5.214\,\varphi_0 + 26.37)\,(a^3 - 3\,a\,b^2) + (4.297\,\varphi_0^2 + 27.41\,\varphi_0 + 12.36)\,(b^3 - 3\,a^2\,b) + \\
& (-9.669\,\varphi_0^3 - 101.5\,\varphi_0^2 - 71.63\,\varphi_0 + 127.1)\,(a^4 - 6\,a^2\,b^2 + b^4) + \\
& (45.56\,\varphi_0^3 - 100.9\,\varphi_0^2 - 918.6\,\varphi_0 - 521.1)\,(a^3\,b - a\,b^3) + \\
& (25.63\,\varphi_0^4 - 92.90\,\varphi_0^3 - 1221\,\varphi_0^2 - 1273\,\varphi_0 + K_5 + 307.8)\,(a^5 - 10\,a^3\,b^2 + 5\,a\,b^4) + \\
& (21.76\,\varphi_0^4 + 324.8\,\varphi_0^3 + 271.8\,\varphi_0^2 - 1612\,\varphi_0 + 0.849\,K_5 - 1414)\,(b^5 + 5\,a^4\,b - 10\,a^2\,b^3),
\end{aligned} \tag{3}$$

with $\varphi_0 \in [-\pi, 0]$, and $\alpha_s(m_\tau^2 \exp(i\varphi_0))/\pi = a + ib$, which resembles the usual FOPT result for $\varphi_0 = 0$ and $b = 0$.

This FOPT result depends largely on the choice of φ_0. FOPT agrees with CIPT around $\varphi_0 \simeq -1$ but spans over a much larger range of $\delta_{\rm pert}$ values. Compared to the uncertainty from the neglected higher orders this intrinsic error is 4 times larger as none of the choices for φ_0 should be excluded. The default choice of $\varphi_0 = 0$ leads to the largest possible value of $\delta_{\rm pert}$ and therefore α_s from FOPT used to be smaller than from CIPT. The deviation can however not be attributed to higher order terms in the series of $\delta_{\rm pert}$.

For the Renormalon Chain Perturbation Theory (RCPT) $\delta_{\rm pert}$ can be written as

$$\delta_{\rm pert}^{\rm RCPT} = \delta_{\rm renormalon} - \delta_{{\rm large}-\beta_0}^{\rm FOPT} + \delta_{\rm pert}^{\rm FOPT}, \tag{4}$$

where the three terms in the sum refer to the renormalon chain result, the large-β_0 resummed result up to the order used in FOPT, and the FOPT result, respectively. For the FOPT correction and the fixed order large-β_0 correction the same arbitrariness of the choice of φ_0 as discussed above exists, as long as both the FOPT term and the fixed order large-β_0 term are expanded around the same φ_0. Therefore the variation of $\delta_{\rm pert}^{\rm FOPT} - \delta_{{\rm large}-\beta_0}^{\rm FOPT}$ with φ_0 is a source of uncertainty in the RCPT approach. The generalized $\delta_{{\rm large}-\beta_0}^{\rm FOPT}$ can be derived from the generalized FOPT solution by setting $\beta_n = 0$ for $n > 0$ and replacing the K_n with $\beta_0^{(n-1)}\kappa_n$, which are given up to $n = 4$ in [18] and up to $n = 12$ in [11]. The numerical form is given in [1]. Furthermore the α_s in the renormalon part and the fixed order part of eq. (4) refers to the same quantity. This is probably not the case. The renormalon part in [18] is derived from the one-loop coupling in the so-called V scheme, $\alpha_s^{\rm V}(\mu^2)$ which is matched on the 0-loop level to $\alpha_s^{\overline{\rm MS}}(\exp(-5/3)\mu^2)$. The problem therefore is that we have a coupling constant on the 3-loop level in the FOPT parts, but treat it as a one-loop coupling in the renormalon parts. A possible solution would be to use 2-loop matching to go from the $\overline{\rm MS}$-scheme to the V scheme [19,20]. The generalized RCPT result with 2-loop matching for $\alpha_s^{\rm V}$ shows a large overlap with the CIPT result and the uncertainties in the deduced strong couplings from both theories are much smaller than previously assumed [1].

Using the same numerical value for $\delta_{\rm pert} = 0.2042 \pm 0.0038_{\rm exp} \pm 0.0033_{\rm non-pert}$ as obtained in [10] and used in [11], where the first error is the experimental one, dominated by the non-strange hadronic decay ratio of the τ, $R_{\tau,V+A}$ and the second is due to the non-perturbative and quark-mass corrections, the results for CIPT and generalized FOPT and RCPT read [1]:

$$\begin{aligned}
\alpha_s^{\rm CIPT}(m_\tau^2) &= 0.3406 \pm 0.0047_{\rm exp} \pm 0.0041_{\rm non-pert} \pm 0.0066_{K_5}, \\
\alpha_s^{\rm FOPT}(m_\tau^2) &= 0.3535 \pm 0.0061_{\rm exp} \pm 0.0053_{\rm non-pert} \pm 0.0208_{\varphi_0}{}^{+0.0005}_{-0.0001}{}_{K_5}, \\
\alpha_s^{\rm RCPT}(m_\tau^2) &= 0.3440 \pm 0.0030_{\rm exp} \pm 0.0026_{\rm non-pert} \pm 0.0061_{\varphi_0} \pm 0.0019_{K_5}.,
\end{aligned} \tag{5}$$

All three results agree within the error due to φ_0 which is very large for FOPT but moderate in case of RCPT. The difference between CIPT and RCPT is of the same size as the error due to φ_0 for RCPT and the average between both values (and conservatively assigning the larger of the two results errors to the average) leads to [1]:

$$\alpha_s(m_\tau^2) = 0.3423 \pm 0.005_{\rm exp} \pm 0.007_{\Delta K_5} \pm 0.004_{\rm non-pert}{}^{+0.005}_{-0.001}{}_{\mu}, \tag{6}$$

where the fourth error is due to the variation of the renormalization scale. The total theoretical error (including the non-perturbative part) is with ± 0.008 only marginally larger

than the experimental error. Evolving α_s given by eq. (6) from $m_\tau = 1.7768\,\mathrm{GeV}$ to $m_{Z^0} = 91.1876\,\mathrm{GeV}$ gives [1]:

$$\alpha_s(m_{Z^0}^2) = 0.1213 \pm 0.0006_{\mathrm{exp}} \pm 0.0008_{\Delta K_5} \pm 0.0004_{\mathrm{non-pert}}{}^{+0.0005}_{-0.0001}{}_{\mu} \pm 0.0002_{\mathrm{ev}}, \qquad (7)$$

where the last error is the evolution uncertainty due to the variation of the thresholds $m_q < m_{\mathrm{thresh}} < 2m_q$ and the quark masses itself within their respective errors.

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Duality Violations in Hadronic τ Decays and the Value of α_s

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Current estimates of the size of nonperturbative effects on the precision determination of α_s from hadronic τ decays are incomplete for at least three reasons: (i) In none of the existing determinations has the effect of duality violations (DVs) been estimated quantitatively. We demonstrated in previous work [1], [2] and in this talk that effects from DVs may turn out to be numerically significant, and that the use of only doubly-pinched moments of the τ spectral functions in order to suppress DVs is not reliable. (ii) While the moments used by ALEPH[4] and OPAL[5] in principle probe the OPE up to dimension D=16, the analyses retain contributions only up to D=8. This was shown to be not self-consistent [3]. (iii) The most precise quoted values for α_s are based on ALEPH data. However, it is now known that correlations due to the unfolding of spectral functions were omitted in ALEPH's latest data analysis [2]. Our preliminary explorations indicate that the impact of this on the error in α_s is not necessarily negligible.

The physical reason for the appearance of DVs is the presence of resonances in the vector (V) and axial (A) τ spectral functions, which neither QCD perturbation theory nor the OPE describe adequately. As demonstrated in Ref. [1], it is possible that the pattern of resonances is rather different in vector and axial channels, and this could lead to a significant effect on the value of α_s even if the sum of V and A spectral functions ($V + A$) looks relatively flat in the region between $s \approx 1$ GeV^2 and $s = m_\tau^2$. In other words, the assumption that DVs can be safely neglected in $V + A$ may turn out not to be justified. Because of this and point (ii) above, the estimates of the nonperturbative part of R_τ (and other moments) used in the literature need to be revisited.

There is no quantitative theory of DVs in QCD, and in order to investigate this issue in more detail, one has to resort to a (physically reasonable) model [1]. This makes it possible to explore finite-energy sum rules with simple, nonpinched weights. Figure 1 shows a fit to the nonstrange integrated vector spectral function using the sum rule

$$\int_0^{s_0} ds\, \rho_{exp}(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \Pi_{OPE}(s) - \int_{s_0}^{\infty} ds\, \rho_{DV}(s)\ . \tag{1}$$

Here $\rho_{exp}(s)$ is the experimental spectral function, available up to $s_0 = m_\tau^2$, $\Pi_{OPE}(s)$ the OPE expression for the vacuum polarization, and $\rho_{DV}(s)$ the "duality-violating" part of the spectral function, *i.e.*, the part not present in Im $\Pi_{OPE}(s + i\epsilon)$. The contour integral over $\Pi_{OPE}(s)$ is parametrized by $\alpha_s(m_\tau^2)$ and the OPE condensates. The functional form of $\rho_{DV}(s)$ is not known from first principles. Following Ref. [1], we model it as

$$\rho_{DV}(s) = \kappa\, e^{-\gamma s}\ \sin(\alpha + \beta s)\ , \tag{2}$$

which introduces four new parameters into the fits to experimental data. We restrict s_0 to an interval $s_0 \in [s_{min}, m_\tau^2]$, and we vary s_{min} in order to check for stability. For the physical

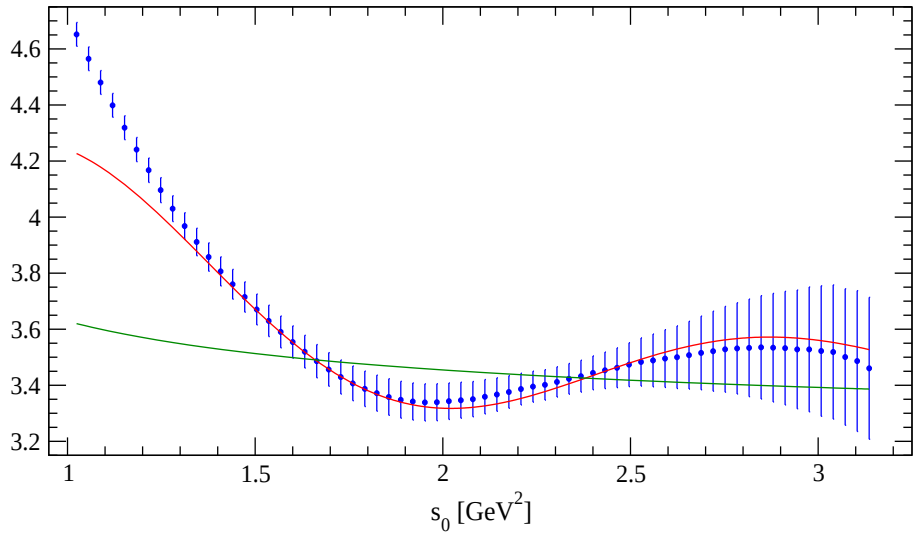

Figure 1: *Fit to Eq. (1), with the ansatz (2) (red curve) and without the ansatz (green curve).*

motivation of the model of Eq. (2), see Ref. [1]. The fit in Fig. 1 was done with OPAL data [5], and $s_{min} = 1.5$ GeV^2. The result for $\alpha_s(m_\tau^2)$ is

$$\alpha_s(m_\tau^2) = 0.307(18)(4)(5) \quad \text{(FOPT)} , \qquad \alpha_s(m_\tau^2) = 0.322(25)(7)(4) \quad \text{(CIPT)} , \tag{3}$$

where errors are the χ^2 error from the fit, from varying $s_{min} = 1.5 \pm 0.1$ GeV^2, and from varying the estimated coefficient of $(\alpha_s/\pi)^5$ in the Adler function in the range 0 to 566.

Fig. 1 demonstrates that DVs cannot, in general, be neglected in analyzing hadronic τ decay data, even integrated versions thereof. This is confirmed by explorations which also include the axial channel in the fits. Essentially all existing τ-based extractions of α_s neglect DVs, and thus assume model (2) with κ set to zero by hand. Such an assumption is necessarily accompanied by an uncertainty not accounted for in currently quoted errors for α_s. Fig. 1 shows that it is dangerous to assume, *a priori*, that this uncertainty can be neglected for analyses involving doubly-pinched weights. We are presently studying different combinations of V and A spectral-function moments with the goal of quantifying this uncertainty. This may help reducing the fitting errors on α_s, but at present, if one wishes to be conservative, and stick only to results not subject to additional, currently unquantified uncertainties, one should treat α_s from τ decays as being known to at best the accuracy given in Eq. (3).

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Perturbative Input to Tau Decays

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The aim of this brief contribution is to recollect the motivation for the evaluation of the α_s^4 corrections to the semileptonic tau decay rate, to describe the essential methods and, last not least the numerous cross checks which have been performed to establish the correctness of the result which, up to now, has not yet been verified by an independent calculation.

Removing the effect of the CKM matrix element V_{ud}, QED and as well as electroweak corrections and subtracting a small nonperturbative piece, the perturbative QCD corrections to the Cabbibbo allowed decay rate can be collected in the quantity δ_P which has been experimentally determined to be close to 0.2 with a relative error of about 2%. In view of the relatively large value of α_s at this low scale, corrections of higher orders play an extremely important role in the extraction of the strong coupling constant. In particular it seemed important to study the difference between the results based on contour improvement and fixed order perturbation theory, that had been observed and studied in [1], and to investigate the behavior of this difference upon inclusion of higher orders.

The key element is the evaluation of the α_s^4 correction to the vector current correlator $\Pi(q^2)$, more precisely, its absorptive part. Exploiting the renormalization group equation for the vacuum polarization function, it can be demonstrated that this requires the evaluation of the four-loop contributions to Π, including its constant (non-logarithmic) part and the five-loop anomalous dimension of Π, which can be obtained from a suitably chosen combination of four-loop propagator integrals [2]. The problem is thus reduced to the evaluation of the finite parts of four-loop massless propagators. This is performed in two steps: the algebraic reduction to a sum of master integrals $f^{(\alpha)}$, multiplied with coefficients $C^{(\alpha)}$ which depend on the amplitude to be evaluated and which are known to be rational functions of the space-time dimension d, $C = P^n/Q^m$, with polynomials P^n and Q^m and $m + n$ typically of order 60 or larger.(Dimensional regularization is understood throughout.) In [3],[4] a method has been proposed which allows to solve for the coefficients C in the limit of large d and to represent the coefficients of its $(1/d)$ expansion in terms of Gauss integrals. Given sufficiently many terms in the expansion, the rational function can be fully reconstructed, and the reduction to masters can be solved purely "mechanically", albeit at the expense of enormous algebraic effort. This has required the development of PARFORM [5] and TFORM [6], parallel versions of FORM [7] with speedups around ten for twenty processors. The analytic results for the 28 master integrals have been obtained [8] using the method of "glue and cut" and were obtained in 2004 already. Only recently these master integrals were also evaluated numerically [10], using the program FIESTA [11], and agreement to four up to five significant figures has been observed.

Using these ingredients, the Adler function and thus the perturbative corrections to the tau decay rate and to the R ratio as measured in electron-positron annihilation were evaluated in order α_s^4, first for QCD [9] and later for a general gauge theory [14]. A completely independent cross check of the reduction to master integrals has not yet been possible.

However, recently the four-loop corrections to the Bjorken sum rule have been obtained [14]. Using then a generalized version [13] of the Crewther relation [12] which connects in a nontrivial way the QCD corrections to the Adler function and to the Bjorken sum rule and the QCD beta function, one finds six nontrivial constraints which are indeed fulfilled [14] . This observation gives additional confidence in both results. A more detailed discussion of various technical aspects can be also found in Ref. [15].

Varying the renormalization scale μ^2/M_τ^2 in the range $0.4 - 2$ one finds [9] $\alpha_s^{FO}(M_\tau) = 0.322 \pm 0.004 \pm 0.02$ and $\alpha_s^{CI}(M_\tau) = 0.342 \pm 0.005 \pm 0.01$ for the two options discussed above. Here the first error results from the error in δ_P, the second one from the μ variation. The difference between FO and CI perturbation theory remains unchanged, as anticipated in [1]. Taking the mean value between the two results as central value and 0.015 as an estimate of the theory uncertainty we find

$$\alpha_s(M_\tau) = 0.332 \pm 0.005 \pm 0.015 \tag{1}$$

and, after evolving up to the Z mass

$$\alpha_s(M_Z) = 0.1202 \pm 0.0019 \tag{2}$$

as our final result. This is well consistent with the value of α_s as determined directly from the hadronic Z-boson decay rate, including the corresponding NNNLO corrections [1]

$$\alpha_s(M_Z) = 0.1190 \pm 0.0026 \tag{3}$$

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Running and Decoupling of α_s

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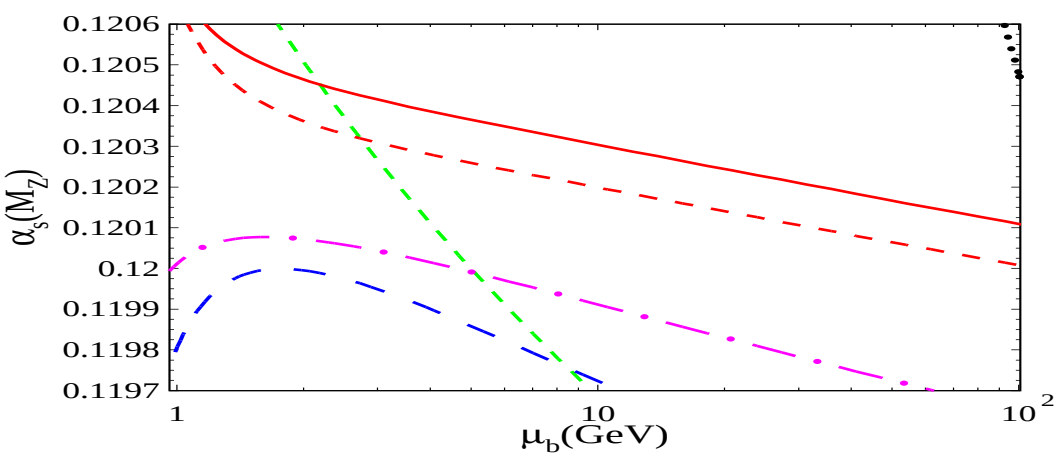
The β function governs the renormalization scale dependence of the strong coupling and is known to four-loop accuracy since almost 14 years [1][2]. There are different ways to solve the corresponding differential equation. The prefered method is the numerical solution with truncation of $\beta(\alpha_s)$ at the desired order. There are also several approximate (analytical) expressions, e.g., the one based on the iterative (perturbative) solution where the result for $\alpha_s(\mu)$ is given as an expansion in $1/L = 1/\ln(\mu^2/\Lambda^2)$ [3]. This formula should be used with care, in particular for small renormalization scales μ. If one considers, e.g., $\mu = M_\tau$ one observes a shift of $+0.004$ after including the four-loop corrections and negative shift of approximately the same order of magnitude at five-loop level. These numbers have to be compared with the current experimental precision which is cited as ± 0.005 in Ref. [4] (see also the other contributions on α_s from τ decays in these proceedings).

Next to the running itself also the decoupling of heavy quarks form the running of the strong coupling constant is a crucial ingredient of the precision determination of α_s. Every time a heavy quark threshold is crossed one has to apply the decoupling constants which relate α_s with n_f active quark flavours, usually denoted by $\alpha_s^{(n_f)}$, to the coupling with only $n_f - 1$ active quark flavours. The decoupling constants are obtained by matching n_f-flavour QCD to the effective theory with the number of quarks equal to $n_f - 1$. The theoretical framework for the calculation of the decoupling constants has been set up in Ref. [5] where formulae are given relating l-loop corrections to l-loop vacuum integrals.



As a consequence of the decoupling relations $\alpha_s(\mu)$ is not a continuous function of μ but has finite steps at the energy scale where the heavy quark is integrated out, $\mu_{\rm dec}$. This energy is not fixed by theory, should, however, be in the vicinity of the heavy quark mass. On general grounds the dependence on $\mu_{\rm dec}$ should become weaker if higher order perturbative corrections are included in the analysis. This is demonstrated in the figure above where $\alpha_s^{(5)}(M_Z)$ is computed using $\alpha_s^{(3)}(M_\tau)$ as a starting point. The decoupling of the charm

quark is performed at the fixed scale $\mu_c = 3$ GeV and the decoupling scale of the the bottom quark μ_b is varied in the broad range between 1 GeV and 100 GeV. N-loop running goes along with $(N-1)$-loop decoupling. Results are shown for $N = 1$ (upper right dotted line), $N = 2$ (steep dashed line), $N = 3$ (lower dashed line) and $N = 4$ (dash-dotted line). One observes a dramatic reduction of the μ_b dependence with increasing N resulting in a quite flat four-loop result (Note that the scale on the ordinate only varies by 0.0009.).

For comparison we show in the figure two more curves which correspond to $N = 5$. They incorporate the four-loop decoupling relations [6][7]. For the unknown five-loop coefficient of the β function we have chosen $\beta_4 = 0$ (solid line) and $\beta_4 = 150$ (dashed line parallel to the solid one; the normalization corresponding to $\{\beta_0, \beta_1, \beta_2, \beta_3\} \approx \{1.92, 2.42, 2.83, 18.85\}$ has been chosen).

From the figure above it is possible to estimate an uncertainty on $\alpha_s^{(5)}(M_Z)$ as obtained from $\alpha_s^{(3)}(M_\tau)$ due to missing higher order corrections. If we restrict ourselves to a range of μ_b between 2 GeV and 10 GeV and take the difference between the three- and four-loop curve as an estimate for the uncertainty we obtain $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$. The difference between the four- and five-loop (dashed) curve would even lead to $\delta\alpha_s^{(5)}(M_Z) \approx 0.0003$. The variation of $\alpha_s^{(5)}(M_Z)$ due to the variation of μ_b leads to an additional uncertainty of $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$. A similar uncertainty is obtained from the variation of μ_c between 2 GeV and 5 GeV. (This can easily be checked with the program `RunDec` [8].) Thus a total uncertainty of ± 0.0004 (obtained by adding the three uncertainties in quadrature) should be assigned to $\alpha_s^{(5)}(M_Z)$. The uncertainties induced by the errors in the quark masses are much smaller.

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α_s with GAPP

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The FORTRAN package GAPP [1] (Global Analysis of Particle Properties) computes so-called pseudo-observables and performs least-χ^2 fits in the $\overline{\rm MS}$ scheme. Fit parameters besides α_s and M_H include the heavy quark masses which are determined from QCD sum rule constraints thus affecting and being affected by α_s. When possible, analytical expressions (or expansions) are used to capture the full dependence on α_s and the other fit parameters.

Z-pole observables from LEP 1 and SLC include the Z-width, Γ_Z, hadronic-to-leptonic partial Z-width ratios, R_ℓ, and the hadronic peak cross section, $\sigma_{\rm had}$. These are most sensitive to α_s by far, but the weak angle enters and needs to be known independently. Thus, the extracted α_s depends on the set of other, purely electroweak (EW) measurements employed in the fits, such as various asymmetries and experiments exploiting parity violation. The statistical and systematic experimental correlations of Γ_Z, $\sigma_{\rm had}$ and the R_ℓ are known, small and included. The parametric uncertainties (such as from M_H) are non-Gaussian but treated exactly. The theoretical errors in Γ_Z, $\sigma_{\rm had}$, and the R_ℓ are identical, and induce a negligibly small uncertainty in $\Delta\alpha_s(M_Z) = \pm 0.00009$, dominated ($\pm 0.00007$) by the axial-vector singlet contribution [2] which is unknown at $\mathcal{O}(\alpha_s^4)$. As in the case of τ decays, one may opt for either fixed-order perturbation theory (FOPT) or contour-improved perturbation theory (CIPT) [3], and we take the difference[1] as the massless non-singlet uncertainty (± 0.00005). The W-width also features a strong α_s dependence, but it is currently not competitive and usually interpreted rather as a measurement of a combination of CKM matrix elements.

The global EW fit excluding τ decays (the Z-pole alone) yields $\alpha_s(M_Z) = 0.1203 \pm 0.0027$ (0.1198 ± 0.0028). These results are expected to be stronger affected by physics beyond the Standard Model than other α_s determinations which is the primary reason to include another α_s constraint in the fits as a control. If the new physics affects only the gauge boson propagators (oblique corrections) the resulting $\alpha_s(M_Z) = 0.1199^{+0.0027}_{-0.0030}$ hardly changes, while allowing new physics corrections to the $Zb\bar{b}$-vertex gives the lower $\alpha_s(M_Z) = 0.1167 \pm 0.0038$.

As the aforementioned α_s control we choose the τ lifetime, τ_τ, not least because of its transparent (even if controversial) theory uncertainty. Our master formula [4] reads,

$$\tau^{\rm expt} \equiv \tau[\mathcal{B}^{\rm expt}_{e,\mu}, \tau^{\rm expt}_{\rm direct}] = \hbar\,\frac{1-\mathcal{B}^{\rm expt}_s}{\Gamma^{\rm theo}_e + \Gamma^{\rm theo}_\mu + \Gamma^{\rm theo}_{ud}} = 291.09 \pm 0.48\ {\rm fs}\,, \tag{1}$$

where $\tau^{\rm expt}_{\rm direct} = 290.6\ (1.0)$ fs is the directly measured τ lifetime [5]. $\tau[\mathcal{B}^{\rm expt}_{e,\mu}] = 291.24\ (0.55)$ fs is the combination of indirect determinations, using $\tau[\mathcal{B}_{e,\mu}] = \hbar\,\mathcal{B}^{\rm expt}_{e,\mu}/\Gamma^{\rm theo}_{e,\mu}$ and the experimental branching ratios, $\mathcal{B}^{\rm expt}_e = 0.1785\ (5)$ and $\mathcal{B}^{\rm expt}_\mu = 0.1736\ (5)$, together with their 13% anti-correlation [5]. Decays into net strangeness, S, are plagued by the uncertainty in the $\overline{\rm MS}$ strange mass, $\hat{m}_s(m_\tau)$, and a poorly converging QCD series proportional to $\hat{m}_s^2$, so that in Eq. (1) we employ the measured $\Delta S = -1$ branching ratio, $\mathcal{B}^{\rm expt}_s = 0.0286\ (7)$ [5].

[1] This difference has the opposite sign from τ decays indicating that their theory errors are uncorrelated.

The partial τ-width into light quarks contains logarithmically enhanced EW corrections, $S(m_\tau, M_Z) = 1.01907 \pm 0.0003$ [6], and reads (employing FOPT as advocated in Ref. [7]),

$$\Gamma_{ud}^{\rm theo} = \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S(m_\tau, M_Z) \left(1 + \frac{3}{5}\frac{m_\tau^2}{M_W^2}\right) \times \left(1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.202\frac{\alpha_s^2}{\pi^2} + 26.37\frac{\alpha_s^3}{\pi^3} + 127.1\frac{\alpha_s^4}{\pi^4} - 1.393\frac{\alpha(m_\tau)}{\pi} + \delta_q\right), \quad (2)$$

where δ_q collects quark condensate, $\delta_{\rm NP}$ [8], as well as heavy and light quark mass effects. The dominant experimental and theoretical errors are given in the following tables, respectively:

source	uncertainty	$\Delta\alpha_s(M_Z)$
$\Delta\tau^{\rm expt}$	± 0.48 fs	∓ 0.00039
$\Delta\mathcal{B}_s^{\rm expt}$	± 0.0007	∓ 0.00017
ΔV_{ud}	± 0.00022	∓ 0.00007
Δm_τ	± 0.17 MeV	∓ 0.00002
total		0.00043

source	uncertainty	based on	$\Delta\alpha_s(M_Z)$
PQCD	∓ 0.0119	α_s^4-term	$^{+0.00167}_{-0.00137}$
RGE	$\beta_4 = \mp 579$	[1]	$^{+0.00038}_{-0.00034}$
$\delta_{\rm NP}$	± 0.0038	[8]	∓ 0.00048
~~OPE~~	± 0.0008	[9] & [10]	∓ 0.00012
total			$^{+0.00178}_{-0.00150}$

The perturbative QCD (PQCD) error dominates and is estimated as the α_s^4-term in Eq. (2). It is re-calculated in each call in the fits to access its α_s-dependence and features asymmetric. It basically covers the range from the higher values favored by CITP down to the lower ones one obtains from assuming that the roughly geometric form of FOPT continues. Note that if CIPT is used, the error from the renormalization group evolution (RGE) parametrized by the unknown 5-loop β-function coefficient, β_4, and part of the PQCD error are correlated. Effects breaking the operator product expansion, ~~OPE~~, are estimated by assuming the instanton motivated functional form [9], $A\,\alpha_s^{-6}\exp[-2\pi/\alpha_s(s_0)]$, and adjusting A to the difference between the OPE and data curves in Fig. 22 of Ref. [10]. Our result is $\alpha_s[\tau_\tau] = 0.1174^{+0.0018}_{-0.0015}$.

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Review of Event-Shape Measurements of α_s

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Event shapes have been widely used for extracting α_s values from e^+e^--collision data. They measure the extent to which the energy flow in an event departs from that of a simple 2-body $q\bar{q}$ configuration. As as example, the thrust $T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i.\vec{n}_T|}{\sum_i |\vec{p}_i|}$, is equal to 1 for a perfect back-to-back two-parton event and 2/3 for a symmetric 3-parton event. Since the distribution of thrust values is sensitive to gluon radiation, it can be used to extract a value for α_s. Various elements go into such an extraction: a perturbative fixed order calculation of the distribution (up to NNLO [1]); optionally, a resummation to all orders of logarithmically enhanced terms such as $\alpha_s^n \ln^{2n-1}(1-T)$ (up to N^{3}LL for thrust and heavy-jet mass [3], NLL for other observables [2]); a model for non-perturbative corrections (either Monte-Carlo based or analytical); a choice of the range of event-shape values to be fitted, generally taken to ensure that the relation between α_s and the prediction of the event-shape distribution can be considered reliable; and the choice of observables to use.

A summary of extractions of α_s is given fig. 1. Though they nearly all involve the same NNLO perturbative calculation [1], there is a substantial spread in central values. The associated uncertainties, nearly always dominated by theory and hadronisation uncertainties, are also quite different from one extraction to another.

For the upper two results of fig. 1, the perturbative input is just the NNLO prediction, supplemented with an estimate of hadronisation corrections derived from the difference between parton-level and hadron-level Monte Carlo distributions. The larger theory uncertainties for the JADE data reflect their lower centre of mass energies, where the larger value of α_s degrades the convergence of the perturbative series. The next two results, which still use MC hadronisation, supplement the NNLO calculation with matching to a NLL resummation, necessary in order to obtain a reliable perturbative prediction near the 2-jet limit. The results with resummations tend to be lower than those without (and JADE lower than ALEPH) but all remain consistent within total errors.

The use of hadronisation corrections from Monte Carlo generators brings with it the issue that a MC "parton level" is not easily related to the parton level of NNLO or resummed calculations. The systematic uncertainty associated with this difference of definitions is particularly subtle, insofar as the perturbative calculations implicitly integrate over the non-perturbative region. The 3 results that form the central block in fig. 1 attempt to work around this issue by using analytical hadronisation models [4,5], which explicitly attempt to remove the double counting between a parametrised hadronisation contribution and the non-perturbative component of the NNLO/resummed calculation. Most striking is the SCET result, both for its very small error and its apparent inconsistency with the world average. In favour of the SCET result, is the finding that as the perturbative order is increased, the result remains stable to within estimated uncertainties and the χ^2 per d.o.f. decreases. On the other hand, modest variations in the fit range lead to a change in α_s of ± 0.0015, 1.5 times the quoted theory uncertainty. This hints at missing contributions that are larger

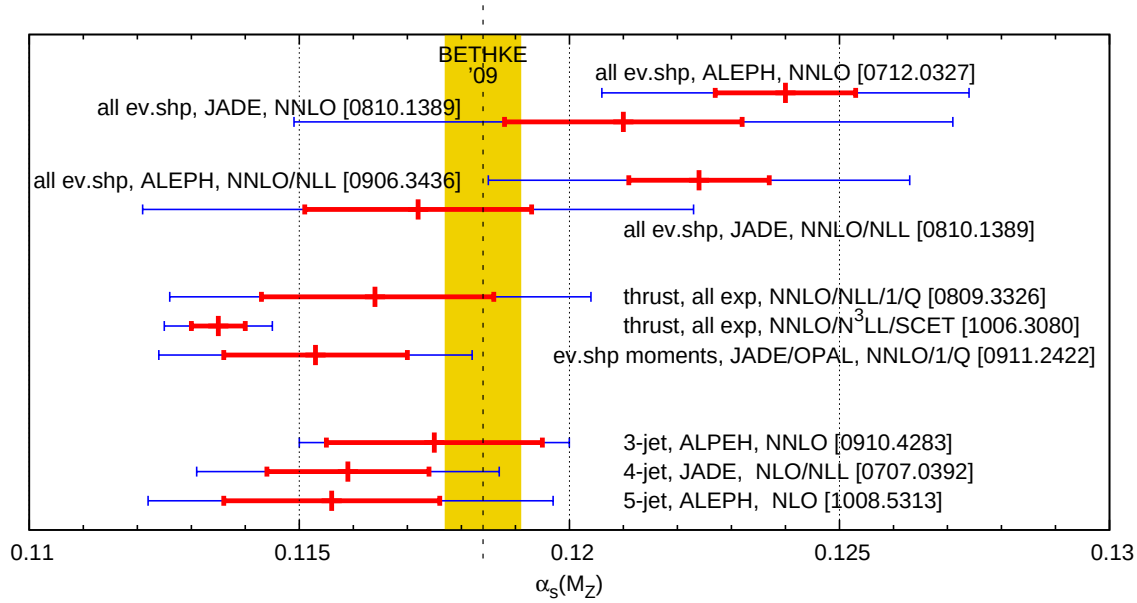

Figure 1: Summary of α_s extractions from e^+e^- event shape and jet-rate measurements. Thick (red) error bars are experimental only (statistical and systematic), while the thinner (blue) error bars additionally include the theoretical (and hadronisation) systematics.

than the estimated theory uncertainties. In this context, e.g., it is known that hadron-mass effects (and, experimentally, whether or not $\pi^\pm$, etc. are taken to have decayed) can have a significant impact on α_s extractions that use analytical hadronisation models [6]. Studies of observables beyond the thrust will also provide valuable insight.

One possible workaround for the difficulties in treating hadronisation is simply not to correct for it and, perhaps, take its whole estimated impact as a systematic error. This was essentially the approach for the 5-jet rate extraction, and is potentially an option for jet rates because hadronisation tends to be smaller than for event-shape type observables. It may be of interest to investigate it also for the 3-jet rate, which seems to have a more convergent perturbative series than other observables and has been studied so far with only a subset of the e^+e^- data.

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α_s at NNLO(+NLLA) from (mainly) ALEPH data

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Event shapes and jet cross sections in e^+e^- annihilation were measured to a high accuracy at LEP and at previous colliders at lower energies. By comparing the data with the QCD description of these observables, it is possible to extract α_s. During LEP times, only the next-to-leading order (NLO) QCD corrections (plus resummation of logarithmically enhanced terms) were available, and the theoretical uncertainty from missing higher order corrections was the dominant source of error on the α_s extraction. With the recent calculation of next-to-next-to-leading order (NNLO) corrections to three-jet production and related event shapes [1], these data can be revisited in view of an improved determination of α_s.

To compare with data, the perturbative NNLO prediction (or matched [2] to next-to-leading logarithmic resummation, NNLO+NLLA) is supplemented by quark mass effects (to NLO) and has to be corrected for hadronization effects. The standard procedure to estimate hadronization effects is to compare parton-level and hadron-level predictions obtained with multi-purpose (leading-order, leading-log) event generator programs for the observables. It has to be kept in mind that the hadronization models in these generators have been tuned extensively to precisely those LEP data sets that are analyzed now for the α_s extraction.

With this procedure, $\alpha_s(M_Z)$ has been determined from ALEPH [3], JADE [5] and OPAL [6] data, and the results are summarized in Table 1. Several important observations were made in these studies. First and foremost, inclusion of the NNLO corrections improves the mutual consistency of the α_s extractions from the different shape variables. Compared to previous NLO studies, the theory error is lowered by almost a factor two. It increases when NLLA resummation is included, since this order of resummation is insufficient to match onto NNLO. A systematic tension between the ALEPH and OPAL extractions can be observed, which can be explained in part by different fit ranges and by the different treatment of quark mass effects.

NNLO	ALEPH [3]	0.1240±0.0008(stat)±0.0010(exp)±0.0011(had)±0.0029(th)
	OPAL [6]	0.1201±0.0008(stat)±0.0013(exp)±0.0010(had)±0.0024(th)
NNLO +NLLA	ALEPH [4]	0.1224±0.0009(stat)±0.0009(exp)±0.0012(had)±0.0035(th)
	JADE [5]	0.1172±0.0006(stat)±0.0040(syst)±0.0030(th)
	OPAL [6]	0.1189±0.0008(stat)±0.0016(exp)±0.0010(had)±0.0036(th)

Table 1: Determinations of $\alpha_s(M_Z)$ from event shape data based on NNLO and NNLO+NLLA, combined with hadronization corrections from multi-purpose event generator programs.

Using hadronization corrections from more recent generators (HERWIG++, combined with POWHEG or MC@NLO), results are obtained that are substantially different from the PYTHIA values, and outside the band obtained from the different legacy generators. It may thus be fair to conclude that the the hadronization models in the generators may be

over-tuned in order to compensate the shortcomings of the perturbative description in these programs and that the hadronization uncertainty on the α_s extraction may be larger than assumed previously, and could actually be a dominant source of error. In order to circumvent this problem, one should either restrict to observables with small hadronization corrections (as for example jet rates) or turn to analytic hadronization models which allow a better order-by-order matching to the perturbative description of the event shape distributions. In two different studies, we pursued both approaches.

From analytic approaches to hadronization (e.g. dispersive model), it is known that the leading power corrections to most event shapes are of order $1/Q$. Among the standard set of shape variables, this power correction is absent only in Y_3 (hadronization corrections to this variable start at $1/Q^2$). The three-jet rate $R_3(y_{\rm cut})$, which derives from Y_3 is thus less sensitive to hadronization than the event shape distributions. We analyzed the LEP1 data on R_3 and extracted α_S from the measurement at $y_{\rm cut} = 0.02$ as [7]

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\text{exp}) \pm 0.0015\,(\text{theo}).$$

Extractions at different jet resolution are highly correlated, and yield consistent results.

Hadronization corrections to the moments of event shape distributions are additive. The moments are therefore very well suited to to disentangle perturbative contributions from hadronization effects. After extending the dispersive model for hadronization corrections to match onto the perturbative NNLO expression, we used data on event shape moments from JADE and OPAL to determine $\alpha_s(M_Z)$ and the hadronization parameter in a combined fit [8], yielding:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017\,(\text{exp}) \pm 0.0023\,(\text{theo}).$$

The hadronization corrections obtained in this approach are considerably larger than what is predicted by the legacy generators. The proper matching of the hadronization corrections onto the perturbative expression results in a considerable reduction of the theory uncertainty.

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Next-to-Next-to-Leading Order and "Classic" Power Corrections

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Here I report on an analysis of the thrust distribution in e^+e^- annihilation, performed in collaboration with Richard Davison [1], in which the next-to-leading-logarithmic (NLL) resummed prediction of [2] was matched to the next-to-next-to-leading (NNLO) fixed-order results of [3], with non-perturbative effects included using the dispersive approach of [4].

To match the resummed and NNLO fixed-order predictions we use the log-R matching scheme of [2]. The NLL resummed result is expressed as

$$R(t) \equiv \int_{1-t}^{1} dT \frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dT} = \exp[L g_1(\alpha_{\rm s} L) + g_2(\alpha_{\rm s} L)] = \exp\left[\sum_{n=1}^{\infty}\sum_{m=n}^{n+1} G_{nm}\, \alpha_{\rm s}^n L^m\right] \tag{1}$$

where the coefficients G_{nm} are known and $L = \ln(1/t - 1/t_{\rm max} + 1)$, $t_{\rm max} = 0.42$ being the kinematic limit for 5 partons, the maximum at NNLO. The terms with $n \leq 3$ are subtracted and replaced by the full NNLO result for $\ln R(t)$.

To include non-perturbative effects we note that the NLL resummed distribution has the form

$$\frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dT} = \frac{Q^2}{2\pi i} \int_C d\nu\, {\rm e}^{(1-T)\nu Q^2} \left[\tilde{J}^q_\nu(Q^2)\right]^2 \tag{2}$$

where $\tilde{J}^q_\nu(Q^2)$ is the Laplace transform of the jet mass distribution. The idea of [4] is to subtract out the infrared contribution to this expression and substitute a non-perturbative contribution based on the assumption of a universal low-scale effective strong coupling $\alpha_{\rm eff}$. The NNLO perturbative contribution from the infrared region $q < \mu_{\rm I}$ is

$$\begin{aligned}\delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\rm pert} = -\frac{2C_F}{\pi}\frac{\mu_{\rm I}}{Q}\Bigg\{ &\alpha_{\rm s}(\mu_R) + \alpha_s^2(\mu_R)\frac{\beta_0}{\pi}\left(\ln\frac{\mu_R}{\mu_I} + \frac{K_2}{2\beta_0} + 1\right) \\ &+\alpha_s^3(\mu_R)\left(\frac{\beta_0}{\pi}\right)^2\left[\ln^2\frac{\mu_R}{\mu_I} + \left(\ln\frac{\mu_R}{\mu_I}+1\right)\left(2 + \frac{\beta_1}{2\beta_0^2} + \frac{K_2}{\beta_0}\right) + \frac{K_3}{4\beta_0^2}\right]\Bigg\}\nu Q^2\end{aligned} \tag{3}$$

where μ_R is the renormalization scale, $K_2 = C_A(67/18 - \pi^2/6) - 5n_f/9$ is the two-loop and K_3 the three-loop cusp anomalous dimension [5]. At the time of [1] K_3 was not known and we assumed $K_3 = K_2^2$, which is just equivalent to a change of renormalization scheme. This turns out to be a good guess ($K_3 = 11.0$ versus $K_2^2 = 11.9$ for 5 flavours), although this term anyway has a negligible effect.

Assuming a universal low-scale coupling $\alpha_{\rm eff}(q)$, the non-perturbative contribution from the region $q < \mu_{\rm I}$, where $\mu_{\rm I}\nu Q \ll 1$, i.e. $1 - T \gg \mu_{\rm I}/Q$, is

$$\delta \ln \tilde{J}^q_\nu(Q^2)\Big|_{\rm n.p.} \simeq -\frac{2C_F}{\pi}\int_0^{\mu_{\rm I}} dq\, \alpha_{\rm eff}(q)\, \nu Q \equiv -\frac{2C_F}{\pi}\frac{\mu_{\rm I}}{Q}\,\alpha_0(\mu_{\rm I})\,\nu Q^2\,. \tag{4}$$

The change in the thrust distribution is given by the difference between (4) and (3), which we see is equivalent to a shift by an amount δT that is a power correction proportional to $1/Q$, modulo some logarithmic dependence if we take $\mu_R = Q$. Notice that the perturbative part (3) is the start of a divergent series in $\alpha_{\rm s}(\mu_R)$, representing the presence of an infrared renormalon. Thus universality of $\alpha_{\rm eff}$ implies that the power correction should decrease as we include more terms in the perturbation series. In the fit of [1], the NNLO (order-$\alpha_{\rm s}^3$) term does represent a significant decrease in the power correction, of 25% relative to NLO.

The resulting predictions have two free parameters, $\alpha_{\rm s}(\mu_R)$ and $\alpha_0(\mu_{\rm I})$. They were fitted to all the available e^+e^- thrust data in the c.m. energy range $14 \le Q \le 207$ GeV. The fitting range was $\max\{\mu_{\rm I}/Q, 0.05\} \le t < 0.33$, giving a total of 430 data points. The best fit was at $\alpha_{\rm s}(91.2\,{\rm GeV}) = 0.1164$, $\alpha_0(2\,{\rm GeV}) = 0.59$, with $\chi^2 = 466$. In view of the large χ^2 contributions from certain data sets, the combined experimental statistical and systematic error was quoted in [1] for a $\Delta\chi^2$ of 14.6. For $\Delta\chi^2 = 1$ the experimental error is reduced by a factor of 3.4. On the other hand, the theoretical uncertainty was assessed by varying the renormalization scale μ_R by a factor of $\sqrt{2}$ instead of the more usual factor of 2, which would double the theoretical error. With these revised error estimates, and taking into account loop corrections which give the 'Milan factor' [6], $\alpha_0 \to 2\mathcal{M}\,\alpha_0/\pi = 0.95\,\alpha_0$, the resulting parameter determinations were

$$\alpha_{\rm s}(91.2\,{\rm GeV}) = 0.1164^{+0.0034}_{-0.0032}\,, \quad \alpha_0(2\,{\rm GeV}) = 0.62 \pm 0.02\,. \tag{5}$$

It is a pleasure to thank the organizers for a very stimulating meeting and to acknowledge the support of a Leverhulme Emeritus Fellowship.

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α_s from Thrust at N^{3}LL with Power Corrections

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We perform a global fit of e^+e^- data for the thrust event shape at energies Q between 35 and 207 GeV [1]. We will consider the results with six stages of theoretical improvements:

1. Matrix elements and fixed order nonsingular terms at order α_s^3 using results from [2]. This includes the non-logarithmic term in the hard function at three loops. (A fixed order analysis at $\mathcal{O}(\alpha_s^3)$ was carried out in [3] using ALEPH data.)
2. Resummation of the most singular logs with N^{3}LL accuracy using Soft-Collinear Effective Theory (SCET) [4]. (A purely perturbative N^{3}LL$+\mathcal{O}(\alpha_s^3)$ fit to ALEPH and OPAL data can be found in [4].)
3. Profile functions (τ-dependent scales μ_J, μ_S, R, μ_{ns}) that account for the multijet boundary condition, which ensures fixed-order results are used in the far tail region.
4. Description of nonperturbative effects with field theory and a fit to a single nonperturbative matrix element of Wilson lines Ω_1. Ω_1's uncertainty dominates other OPE terms in the tail region, and its Wilson coefficient is included with N^{3}LL$+O(\alpha_s^3)$ accuracy.
5. Switching from $\overline{\text{MS}}$ to an R-scheme to define Ω_1. This ensures Ω_1 and the perturbative cross-section are free of $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon ambiguities. An RGE is used to sum large logarithms in the perturbative renormalon subtractions. The fit gives Ω_1 to 16%.
6. QED final state corrections at $O(\alpha)$ and NNLL (counting $\alpha \sim \alpha_s^2$); bottom mass corrections are included using a factorization theorem with log resummation; $O(\alpha_s^2)$ axial-singlet terms arising from the large top-bottom mass splitting.

We fit simultaneously for $\alpha_s(m_Z)$ and Ω_1 in the tail region of the thrust distributions (our default 487 bin dataset has $\tau = 1 - T = 6\,\text{GeV}/Q$ to 0.33). For fixed Q there is a strong degeneracy between $\alpha_s(m_Z)$ and Ω_1. It is lifted by the global fit with different Q values. To estimate perturbative uncertainties we performed a 500 point random scan over 12 parameters, refitting for each point. This accounts for theory uncertainty from higher orders via varying scales, statistical theory errors, and three unknown perturbative coefficients. We observe very good convergence when increasing the orders in perturbation theory [1], and achieve $\chi^2/\text{dof} = 0.91$ at stage 6. The results are shown in Fig. 1, including a comparison with [3] and [4] at the appropriate stages. For the global fit at stage 2 our random scan uses the μ_i variations of [4]. Stage 3 yields a result midway between 1 and 2.

The most dramatic effect is the -7.5% decrease to $\alpha_s(m_Z)$ from adding the Ω_1 nonperturbative fit parameter at stage 4. The size of this shift is consistent with a back of the envelope estimate based on data [1]. The field theory treatment is in sharp contrast with the traditional method of estimating nonperturbative effects employing Monte Carlo generators (MC) (used also in [3] and [4]). MC has less perturbative accuracy than level 1, and its tune parameters unavoidable include both nonperturbative and higher order perturbative effects, leading to a double counting in the MC estimate. At the Z-scale the Pythia

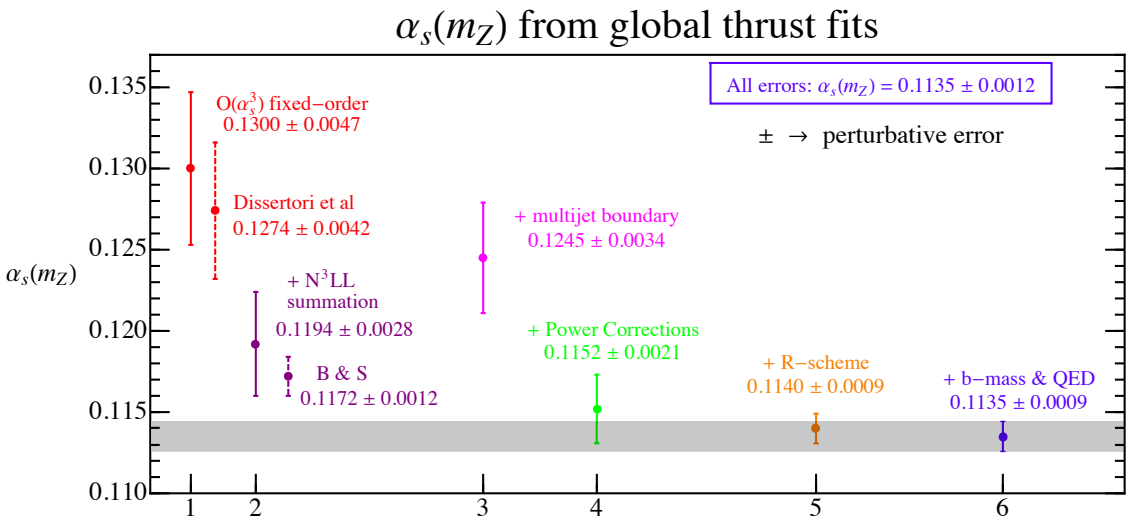

Figure 1: Global fit results (solid error bars) at various theoretical stages from [1]. Ω_1 is fit at stages ≥ 4. Dashed error bars are ALEPH $Q = m_Z$ fits from [2] and ALEPH+OPAL fits from [4] at stage 1, 2 respectively (hadronization corrections from Monte Carlo are quite small and their inclusion therefore does not change the identification with these stages).

tune has a very small power correction, inconsistent with our determination, and hence cannot be used for high precision α_s determinations. In [5] a global fit was performed at NLL respectively $+\mathcal{O}(\alpha_s^3)$ with the effective coupling model to treat nonperturbative effects. They find $\alpha_s(m_Z) = 0.1164 \pm 0.0022_{\rm exp} \pm 0.0017_{\rm pert}$ consistent with our stage 2, 4, 5, 6 fits. This model agrees with the parametrics of the OPE in the tail region and has renormalon subtractions. Advantage of our approach are that the power correction is defined as a field theory matrix element, one can include subleading OPE terms, and sum large logs in the subtractions. Our final result with all sources of uncertainty is shown in the box in Fig. 1.

Given the desired precision, an interesting issue is to assess the most appropriate way of computing the binned theory distribution. Two approaches being used are:

$$\text{(a)}: \int_{\tau_1}^{\tau_2} \mathrm{d}\tau' \, \frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau', \mu_i(\tau')) \ , \qquad \text{(b)}: \quad \Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) \ , \tag{1}$$

where $\Sigma(\tau, \mu) = (1/\sigma) \int_0^\tau \mathrm{d}\tau' \, \mathrm{d}\sigma/\mathrm{d}\tau(\tau', \mu_i(\tau))$ and μ_i are jet and soft scales depending on τ. They are equivalent at a given order in resummed perturbation theory, but differ by noticeable amounts from included higher order terms [1]. In particular, Σ gives an uncertainty estimate that is designed for comparison to cumulant data, and (b) is sensitive to contributions from τ below the bin $[\tau_1, \tau_2]$. The uncertainties in $\mathrm{d}\sigma/\mathrm{d}\tau$ are designed for distributions and hence (a) is best suited for the bins. (a) was used in [1], while (b) was used in classical resummation analyses and [4]. At stage 2 this issue is small for thrust, and only becomes noticeable at stage 3. For the Heavy Jet Mass (HJM) event shape it is already noticeable at stage 2. The stage 2 thrust and HJM results in Refs. [4] use (b) and yield: 0.1172 vs 0.1220. Using (a) yields consistent $\alpha_s(m_Z)$ values for thrust and HJM: 0.1169 vs 0.1175, resolving the apparent discrepancy in central values.

Work supported in part by the European Community's Marie-Curie Research Training Network PITN-GA-2010-264564 (LHCphenOnet), and by the Offices of Nuclear and Particle Physics of the U.S. Department of Energy under grants FG02-94ER40818, DE-FG02-06ER41449 and DE-SC003916. VM has been partially supported by a Marie Curie Fellowship under contract PIOF-GA-2009-251174. IS was also supported in part by a Friedrich Wilhelm Bessel award from the Alexander. von Humboldt foundation. Preprint number: MIT-CTP 4301

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α_s from five-jet observables at LEP

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Jet observables represent powerful tools to measure the strength of the strong interactions, encoded in α_s. The more jets are present in the final state, the stronger is the dependence of the relevant observables on α_s. At LEP the highest exclusive multiplicity measured is five. However, since five-jet events could be described only at leading order (LO) in QCD, the theoretical uncertainty was too large to extract a competitive value for α_s. We recently computed the next-to-leading (NLO) QCD corrections to five-jet events [1] using D-dimensional unitarity for the one-loop calculation [2] and Madgraph and MadFKS for the real radiation, subtraction, and phase-space integration [3]. We consider two observables: the five-jet resolution parameter, y_{45}, and the five-jet rate, R_5. Their perturbative distributions start at $\mathcal{O}(\alpha_s^3)$, and are therefore very sensitive to the value of α_s. We find that, once NLO corrections are included, ALEPH data is described well by perturbative QCD if the world-average of α_s is used as input of the theoretical prediction. Furthermore, theoretical uncertainties are reduced from $[-30\%, +45\%]$ at LO to $[-20\%, +25\%]$ at NLO. One can then turn this around and use the NLO calculation for a novel extraction of α_s using ALEPH data. Before this can be done, one needs to address how non-perturbative hadronization corrections can be accounted for. Traditionally, they are extracted using parton showers such as Herwig of Pythia. We find however that, if this is done, the hadronization corrections are very large ($\sim 50\%$) and generator dependent. We therefore use the Sherpa event generator [4] with its default cluster model [5] to extract hadronization effects. Sherpa implements the five-jet LO matrix element exactly, therefore hadronization corrections are less contaminated by missing perturbative effects. When this is done, we find smaller hadronization corrections ($\lesssim 20\%$). We also observe that while the hadronization corrections *per se* are not negligible, they have a very small effect on the extraction of α_s at LEP I, therefore we neglect hadronization corrections at LEP II.

In order to extract α_s we consider each bin of y_{45} and R_5 at a given energy as an observable $\mathcal{O}_i = [X_i, \sigma_i^{\rm stat}, \sigma_i^{\rm syst}]$ that can be used to measure α_s by solving the equation $T_i H_i = E_i$, where T_i is the perturbative theoretical prediction, H_i is the hadronization correction and E_i is the experimental measurement. From each bin i we obtain a central value of $\bar{\alpha}_s$ with corresponding errors

$$\alpha_s^i = \overline{\alpha}_s^i \pm \delta\alpha_s^{i,\rm stat} \pm \delta\alpha_s^{i,\rm syst} \pm \delta\alpha_s^{i,\rm scale} \pm \delta\alpha_s^{i,\rm hadr}. \tag{1}$$

The systematic and statistical errors are obtained by solving the same equation with $E_i = X_i \pm \sigma_i^{\rm stat/syst}$, the perturbative error is obtained by varying the default scale ($\mu_R = 0.3 M_Z$) by a factor two up and down, and the hadronization error is obtained by solving the same equation using the Lund hadronization model [6]. We take the fit range as large as possible, where our computation is reliable and data are good enough. We then vary the range to estimate a fit range error. Full details about the fit ranges and the fitting procedure are given in [1]. The result of this procedure it a set of values of α_s that need to be combined. To do this we define a covariance matrix as the sum of individual covariance

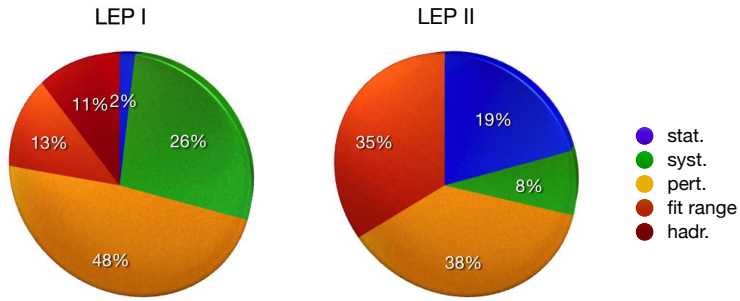

Figure 1: Breakdown of the errors on α_s at LEP I and LEP II.

matrices $V = V^{\rm stat} + V^{\rm syst} + V^{\rm scale} + V^{\rm hadr}$ with

$$V_{ij} = \delta_{ij}\left(\delta\alpha_s^i\right)^2 + (1-\delta_{ij})\,\min\left\{(\delta\alpha_s^i)^2, (\delta\alpha_s^j)^2\right\}, \tag{2}$$

where C_{ij} describes the correlation. We assumed that statistical errors are uncorrelated at different energies. At a given energy we take y_{45} bins as uncorrelated, R_5 bins as fully correlated and y_{45} and $R_5(y_{\rm cut})$ to be correlated for $y_{\rm cut} < y_{45}$. Systematic errors are taken as fully correlated at fixed energy, but uncorrelated at different energies. Perturbative errors are taken to be fully correlated for all observables and energies, except for the LEP I/LEP II correlation that we neglect. Finally, hadronization errors are assumed to be fully correlated. We then compute the weights w_i and use them to obtain the estimate of the average of the strong coupling constant and of its error

$$w_i = \sum_{j=1}^{N}(V^{-1})_{ij}\Big/\sum_{k,l=1}^{N}(V^{-1})_{kl}, \qquad \alpha_s = \sum_{i=1}^{N} w_i\bar{\alpha}_s^i\,, \qquad \sigma^2(\alpha_s) = \sum_{i,j=1}^{N} w_i V_{ij} w_j\,. \tag{3}$$

We obtain the value of α_s from five-jets (a breakdown of the errors is displayed in Fig. 1)

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}. \tag{4}$$

This value is compatible with the current world average, but on its lower side.

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Jet and Event Shape Observables at LHC

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During the first, very successful year 2010 of LHC operation, which saw an increase in instantaneous luminosity by a factor of ≈ 10000, an excellent performance of the LHC experiments ATLAS and CMS has been observed. The achieved understanding in detector operation and behaviour has lead to uncertainties of the order of 11% on the luminosity, 3–5% on the jet energy scale and about 5–20% on the jet energy resolution depending on experiment and phase space. With respect to their impact on QCD measurements three general analysis strategies can be differentiated: Absolute measurements like the inclusive jet p_T, shape measurements like jet angular variables and event shapes, or ratios of cross sections. The latter two exhibit much smaller experimental uncertainties due to cancellation effects and are, at least in the beginning, better suited for precise QCD studies in order to constrain parton distribution functions (PDFs) and/or determine the strong coupling constant α_s. The total experimental uncertainties on the jet cross sections currently are of the order of 20 to 40% for transverse momenta larger than $\approx 50\,\mathrm{GeV}$ while theoretical uncertainties from the PDFs, the renormalization and factorization scales, and non-perturbative corrections amount to 8 to 15%. With further improvements and much more data to come the jet cross sections are very promising for the future. In the following, some aspects of a shape measurement, the azimuthal decorrelation $\Delta\Phi_{jj}$, and the 3-jet ratio R_{32} will be discussed.

Azimuthal decorrelations $\Delta\Phi_{jj}$, recently published for LHC data by CMS and ATLAS [1], are normalized to the total dijet cross section eliminating luminosity uncertainties. They are a direct measure of additional activity like a radiated third jet in comparison to a normal dijet event perfectly balanced in transverse momentum. Restricting the observation to differences in azimuthal angle between the leading two jets reduces significantly uncertainties due to the jet energy scale and resolutions. In case of balanced dijet events the azimuthal angles between the leading jets are completely correlated to give distances of $\Delta\Phi_{jj} = \pi$. Additional radiation of any of the two jets decrease this distance where the extent to which this is possible depends on the jet multiplicity as demonstrated e.g. in Fig. 1 of the ATLAS analysis in [1]. Comparisons to pQCD at NLO are possible in the range $2\pi/3 < \Delta\Phi_{jj} < \pi$ with NLOJet++ [4] providing the required 3-parton and 4-parton final states. Below $2\pi/3$ mostly final states with four or more partons contribute such that the result of NLOJet++ effectively becomes LO only. This can also be seen by an associated increase of the scale uncertainties. At $\Delta\Phi_{jj} = \pi$ the 3- and 4-parton final states need to be complemented by the 2-parton ones requiring 2-loop corrections for a complete NNLO result which is not available as of today. This demonstrates that azimuthal decorrelations are a precisely measurable QCD observable providing a lot of information on multijet production. Redefining the azimuthal distance to $\Delta\Phi'_{jj} = \Delta\Phi_{jj} - \pi$ such that balanced dijets have $\Delta\Phi'_{jj} = 0$ this quantity can also be considered an event shape for which resummed predictions are conceivable [2]. Azimuthal decorrelations can also be investigated for leading particles.

Following a publication of D0, preliminary results by ATLAS and CMS [3] are reported for the inclusive 3- to 2-jet ratio R_{32} versus the scalar sum of all transverse jet momenta H_T. The suggested study, however, is far from optimal with respect to comparisons to pQCD. First

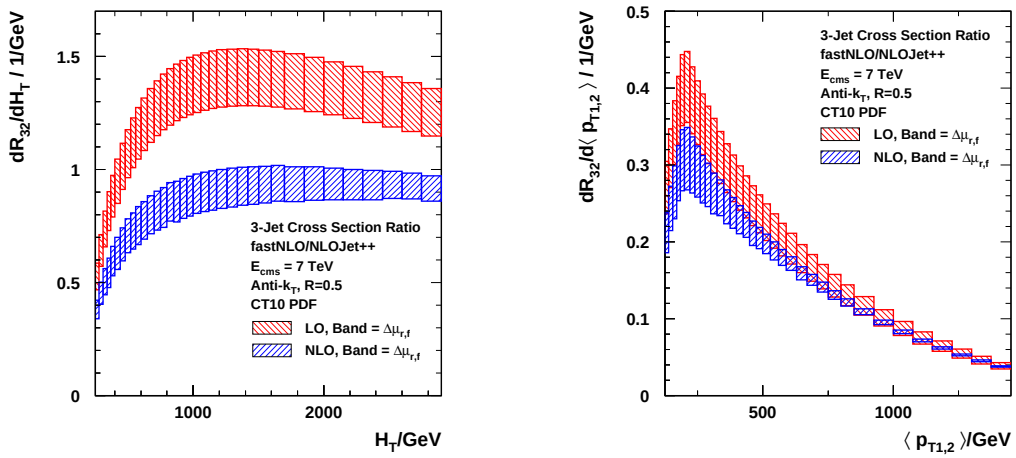

Figure 1: LO (red) and NLO (blue) predictions including scale uncertainties for an analysis similar as in [3] (left) in comparison to the one suggested here (right) requiring a minimal p_T of 25% of the leading dijet average $< p_{T1,2} >$.

of all, the LO prediction including the scale uncertainty as shown in Fig. 1 left as upper red hatched band is unreliable and gives unphysical values larger than one. The NLO presented in blue looks more reasonable and is not far off the CMS data, but the associated scale uncertainties are not much smaller with respect to LO. This points to a bad convergence of the perturbative series. In fact, ratios R_{32} of ≈ 0.8 where jets with $p_T > 50\,\mathrm{GeV}$ are counted even for leading jet momenta of $p_{T,max} \approx 1\,\mathrm{TeV}$ are not representative for hard processes with probabilities related to α_s at high scales. For that purpose it would be better to require a minimal hardness of third jets of e.g. a certain percentage like 25% of the average transverse momentum of the two leading jets. An example calculation performed with NLOJet++ in the fastNLO framework [5] for such an event selection is presented in Fig. 1 right leading to reasonable predictions already at LO and larger reductions of the scale uncertainties at NLO at least at high p_T. The abundant production of jets at highest transverse momenta at the LHC will enable many new precision tests of perturbative QCD in the near future.

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MC tuning with Professor

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Monte Carlo simulations of high energy physics (and particularly QCD) processes rely on phenomenological assumptions for regimes where perturbation theory does not apply. Of particular interest for α_S extractions from event-shape data are hadronisation-effects for which two kind of models are available. On the one hand cluster based models, e.g. in the codes of Herwig and Sherpa, and the Lund-string model as it is implemented in the Pythia event generator.

Both types have in common that a large number of tunable phenomenological parameters are necessary to achieve agreement with data and therefore predictive power. The task of tuning such generator parameters can be cumbersome if done by hand due to the high dimensionality of the parameter-space and the large number of observables that need to be studied for changes due to parameter-shifts.

The Professor[1] tuning-tool which tries to address the aforementioned problems was presented. It is based on the idea of calculating bin-wise parameterisations of the generator response to shifts in a certain parameter space and a subsequent numerical minimisation of a simple goodness-of-fit measure defined between the parameterisation and corresponding experimental data.

Furthermore, special applications of Professor have been introduced that exploit the statistical nature of the tool such as the quantification of tuning uncertainties as well as the calculation of so-called "Eigentunes", a set of parameter points on the $k\sigma$-contour of the χ^2 valley that are therefore representative for allowed tuning variations within data-uncertainty (Fig. 1).

Further uses of the parameterisation like calculating the sensitivity of observables to shifts in parameter space and an interactive Monte Carlo simulator ("prof-I") have been presented as well.

On request of the organisers, a quick study on a possible $\sqrt{s}$ dependence of a hadronisation tune, namely the Pythia6 tune pro-$p_\perp$[1], has been performed. This most recent hadronisation tuning was obtained from tuning the generator parameters to event shape data on the Z-pole. This is in large parts due to the fact that the development of Professor and tunings obtained with it are closely linked to the development of Rivet[2], an application designed for generator independent validation by means of built-in analyses that mimic published studies of collider experiments. By the time of the workshop only event shape data from the LEP I and II experiments was available in Rivet. An outcome of the workshop was the implementation of event shape studies of the PETRA experiments JADE and TASSO at energies between 14 and 44 GeV[3,4] which allowed to study the low-energy behaviour of the tune after the workshop.

Equipped with event shape data and corresponding Rivet analyses at energies between 14 and 209 GeV[3,4,5], the per-bin goodness-of-fit of the tune pro-$p_\perp$ and data could be

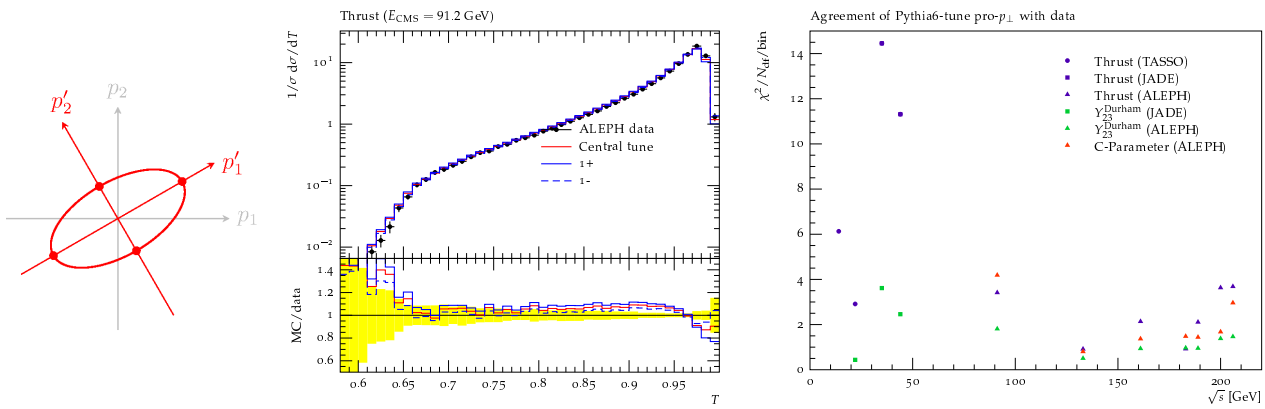

Figure 1: Left: Idea of "EigenTunes" — calculating principal directions of diagonalised covariance matrix of a tuning and determination of intersection with $k\sigma$ contour. Middle: The 1+ and 1- lines correspond to 1σ Eigentune variations in the positive and negative direction of one out of six principal axes. The corresponding parameter points are on the 1σ contour of the χ^2 valley in the vicinity of the central tune point. Right: Goodness-of-fit of the Professor hadronisation tune of Pythia6, pro-$p_\perp$, with data as function of the center-of-momentum energy, $\sqrt{s}$, indicating a non-universality of the hadronisation tune.

studied for the Thrust, C-Parameter and 2-jet-resolution parameter, y_{23}^{Durham}, as a function of $\sqrt{s}$. The result is given in Fig. 1.

The data seems to suggest that Pythia6's hadronisation model or at least the selected tuning is not capable of describing the data at all, especially low energies. Further studies should include more data, preferably at low energies. It may be fruitful to attempt a retuning to all available event shape data simultaneously to check if the available hadronisation models are in fact not universal.

We would like to thank Stefan Kluth for the discussions that lead to the implementation of the JADE analysis in Rivet.

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α_S from event shapes in e^+e^-: Experimental issues and combination of results

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The experimental issues of event shape measurements are summarised in table 1 [1][2][3][4][5].

	PETRA/JADE	LEP1	LEP2
Rad. return	cuts on 4-momentum balance	n.a.	cuts on 4-momentum balance, cuts on s'
τ pairs	$N_{ch} > 3$, reject 1- vs. 3-prong events	$N_{ch} > 6$	
2γ events	cuts on 4-momentum sum		
4-fermion events	n.a.		"4-jet event shape" cuts

Table 1: Summary of experimental effects in e^+e^- event shape analyses. N_{ch} is the multiplicity of charged particles, 4-fermion events are mainly W^+W^- pairs, s' refers to the invariant mass of the virtual gauge boson producing the hadronic final state.

Detector corrections take account of limited acceptance and resolution of the experiments, and of QED initial state radiation (ISR) effects, and are performed using MC based unfolding of the measured distributions. Table 2 gives an overview over the treatment of various experimental effects by the LEP experiments, the selection efficients and final background fractions, and the resulting experimental systematic uncertainties. The uncertainties are average values and can change significantly within a given distribution. The LEP1 experimental systematics are still to be matched by theory while the LEP2 experimental systematics have been reached by recent theory uncertainties. The JADE data have experimental systematics varying between 0.7 and 4%.

Combinations of NLO+NLLA α_S results from the analysis of event shape observables were performed by the collaborations using procedures assuming normal distributions for all uncertainties. Values of α_S are always evolved to a common scale before combining them. The observables are statistically correlated and have correlated systematic uncertainties. ALEPH, DELPHI and OPAL use a common method while L3 calculates unweighted averages. In the common method experimental uncertainties are taken as partially correlated (see below) within an experiment and as uncorrelated between experiments. Hadronisation and theory systematics are taken as uncorrelated, and the combination procedures are repeated for different hadronisation models or theory parameters such as the renormalisation scale. Statistical correlations within or between observable distributions are measured or obtained from MC simulations. For correlations of systematic uncertainties models are employed: i) no correlation ($\rho = 0$), ii) partial correlation ($\rho = \min(\sigma_i, \sigma_j)^2/(\sigma_i\sigma_j)$ or iii) full correlation ($\rho = 1$). In [6] a combination of all NLO+NLLA results from LEP and JADE using the common method is shown. However, the recent analyses of LEP or JADE data using NNLO(+NLLA) calculations have not yet been combined.

	ALEPH	DELPHI	L3	OPAL
Rad. return	find EM jets, force remaining event in 2 jets	kin. fit for jet and possible missing γ energies along beam	cuts on E_{vis}, N_{cls}, E_γ, $E_{\|\|}/E_{vis}$	find isolated γ, force remaining event into 4 jets, kin. fit for for jet and possible missing γ energies along beam
4-fermion bkg	force 4 jets, cut on α_{34} and $\|m_{jj} - m_W\|$	2d cut on N_{ch} vs. B_N	force 4 jets, cuts on y_{34}, N_{ch}, N_{cls}, E_{jet}	cut on likelihood from partial NLO QCD and EW ME, y_{34}, Sphericity
LEP1 exp. syst.	$< 1\%$ (exp. corr.)	1.6%	1.5% (exp. corr.)	$< 1\%$ (exp. corr.)
LEP1 ϵ	–	84.5%	$98.5\% \cdot A$	88.5%
LEP2 exp. syst.	$\sim 1\%$ (non-rad.)	$\sim 1.9\%$	1.0-2.5% (non-rad or 4-f)	0.8-4% (4-f cuts)
LEP2 ϵ	–	$85 - 90\% \cdot A$	$85{-}90\% \cdot A$	$\sim 80\%$
LEP2 f_{bkg}	–	5-14%	20%	2-6%
b-Quarks	NLO massive ME, no corr. syst.	udsc/udscb had. corr., 20% syst.	NLO massive ME	neglect, udsc had. corr. syst.

Table 2: Overview of treatment of experimental effects. E_{vis} is the total measured energy, N_{cls} is the number calorimeter clusters, E_γ is the energy of a cluster, $E_{||}$ is the energy component parallel to the beam direction, α_{34} is the angle between jets 3 and 4 after energy ordering, B_N is the narrow jet broadening, y_{34} is the jet resolution parameter where the event changes from 3 to 4 jets, ϵ is the selection efficiency, f_{bkg} is the background fraction and A is the geometric acceptance.

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Lattice QCD Calculations and α_s

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Lattice gauge theory provides a mathematically rigorous definition of quantum field theories, such as QCD, encompassing the perturbative and nonperturbative regimes [1]. In principle, it is ideally suited for the determination of α_s and the quark masses, which are the irreducible parameters of QCD. In practice, the most productive approach to lattice QCD has been computational, evaluating the functional integrals of quantum field theory with Monte Carlo methods and importance sampling.

The first such calculations were marred by the omission of sea quarks, often called the "quenched approximation." The sea quarks are the most computationally demanding part of the calculations and, in the early days, a lack of computer power made the quenched approximation a necessary compromise. Quenched QCD calculations are on a footing similar to models of nonperturbative QCD. Consequently, the first lattice-QCD determinations of α_s required a model-dependent corrections for the omitted sea. In particular, potential models could be used to patch up the mass splittings in charmonium and bottomonium [2].

Since the start of the new millenium, however, lattice QCD simulations with 2+1 flavors of sea quarks have become routine [3]. The notation "2+1" implies one sea quark tuned to the strange quark and two taken with masses as small as possible, for (degenerate) up and down. The physical limit is reached by extrapolating numerical data, guided by chiral perturbation theory. More recently, the first simulations obviating this extrapolation have been carried out [4,5,6] (though not used for α_s—yet). Furthermore, at least two collaborations have begun wide-ranging programs of lattice-QCD calculations of 2+1+1 sea quarks—adding the charmed sea to the rest [5,7]. Thanks to these developments, lattice QCD now plays an increasingly important role in many areas, for example, flavor physics [8].

Dating back to Ref. [2], many summaries of perturbative QCD still identify α_s from lattice QCD with the quarkonium splittings. This is outdated. In fact, the possibilities are much broader and share many features with determinations of α_s from high-energy scattering or the decays of heavy particles. To explain this perspective, let us list the ingredients needed to determine α_s. One needs

1. A dimensionless quantity, R, sensitive to QCD at a (range of) short distance(s), Q^{-1}; if R is not dimensionless, one can use Q to make it so; then asymptotic freedom implies $R = \mathcal{R}\left(\alpha_s(Q_s)\right) + \mathrm{o}\left((\Lambda_{\mathrm{QCD}}/Q)^r\right)$, where Q_s is a specific scale choice for scheme s with $Q_s \propto Q$, and the power r depends on the observable R.

2. A theoretical framework—or at least a notion—to separate short-distance scales from Λ_{QCD} and other long-distance scales.

3. A perturbative series for the short-distance contribution $\mathcal{R}\left(\alpha_s(Q_s)\right)$, certainly to NLO and preferably to higher order, and summing logarithms of scale ratios.

4. Measurements of R over a range of Q large enough to control the power-law effects, denoted $(\Lambda_{\mathrm{QCD}}/Q)^r$ in item 1.

5. Control of non-QCD physics at scales probed by Q (*e.g.*, electroweak or new physics).

6. A measurement of Q: usually a calibration, in contrast to event counting for R.

It is worth emphasizing that the whole procedure can be thought of as dimensionless until the last step, where the energy calibration of the calorimeters (high-energy scattering) or the τ-lepton mass (τ decays) is input.

In lattice QCD, an evaluation of a functional integral replaces the measurement of R. One is assured that non-QCD physics does not enter, but, in practice, issues like unphysical quark masses and nonzero lattice spacing play an analogous role. We know how to control these effects, *i.e.*, how to distill them into an error bar. Many choices of (R, Q) are possible, each with strengths and weaknesses. The dimensionless suite of lattice calculations becomes dimensionful when converting from lattice units to GeV—this precisely means deducing the scale to which the α_s corresponds (here s labels the scheme chosen in the lattice calculation). Since lattice QCD with 2+1 sea quarks reproduces a wide variety of masses, mass splittings, and decay constants, the quantity used to convert units is no longer crucial [3], and the error on the conversion is subdominant when propagated to α_s.

The table compiles several recent determinations of α_s from lattice QCD with 2, 2+1, or 2+1+1 flavors. Near-term global averages should consider using those set in **bold**, after some dialogue with the authors to ensure a consensus between reviewers and authors of the completeness and meaning of the error budgets. The others either are superseded [9,12], are a re-analysis of a bold entry [11], omit the possibly large uncertainty from quenching the strange sea [16], or are not yet complete [17]. Longer-term global averages of α_s will have more results from lattice QCD to choose from.

$\alpha_{\overline{\mathrm{MS}}}^{(n_f=5)}(M_Z)$	R	Q	range	$\mathcal{R}$	sea	Ref.
0.1170(12)			3			[9]
0.1183(8)	Wilson loops	a^{-1}	7	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[10]
0.1192(11)			7			[11]
0.1174(12)	quarkonium	$2m_c$	1–2	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[12]
0.1183(7)	correlators	$2m_Q$	3–6	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[13]
$\mathbf{0.1181(3)^{+14}_{-12}}$	Adler function	Q	5	NNLO*	2+1 overlap	[14]
$\mathbf{0.1205(8)(5)^{+0}_{-17}}$	"QCD in a can"		80		2+1 Wilson	[15]
0.1000(16)†	aka Schrödinger	L^{-1}	270	asymptotic	2 Wilson	[16]
0.1___(__)	functional		1000		2+1+1 Wilson	[17]

* The Adler function's $\mathcal{R}$ is known to N^{3}LO, but Ref. [14]'s analysis is NNLO.

† This entry gives $\alpha_{\overline{\mathrm{MS}}}^{(n_f=\mathbf{2})}$, obtained from Ref. [16]'s $r_0\Lambda_{\overline{\mathrm{MS}}}^{(n_f=2)} = 0.62(6)$ with $r_0 = 0.46(1)$ fm and Eq. (9.5) of the 2006 PDG.

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HPQCD: α_s from Lattice QCD

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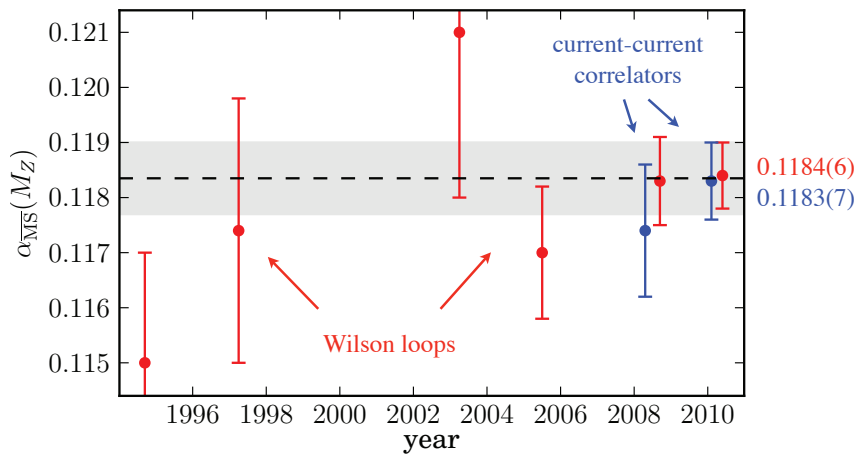
The HPQCD collaboration has been using lattice QCD simulations for more than a decade to determine the QCD coupling from experimental data for hadron masses. That history, which has produced consistent results over time, is summarized in the following:

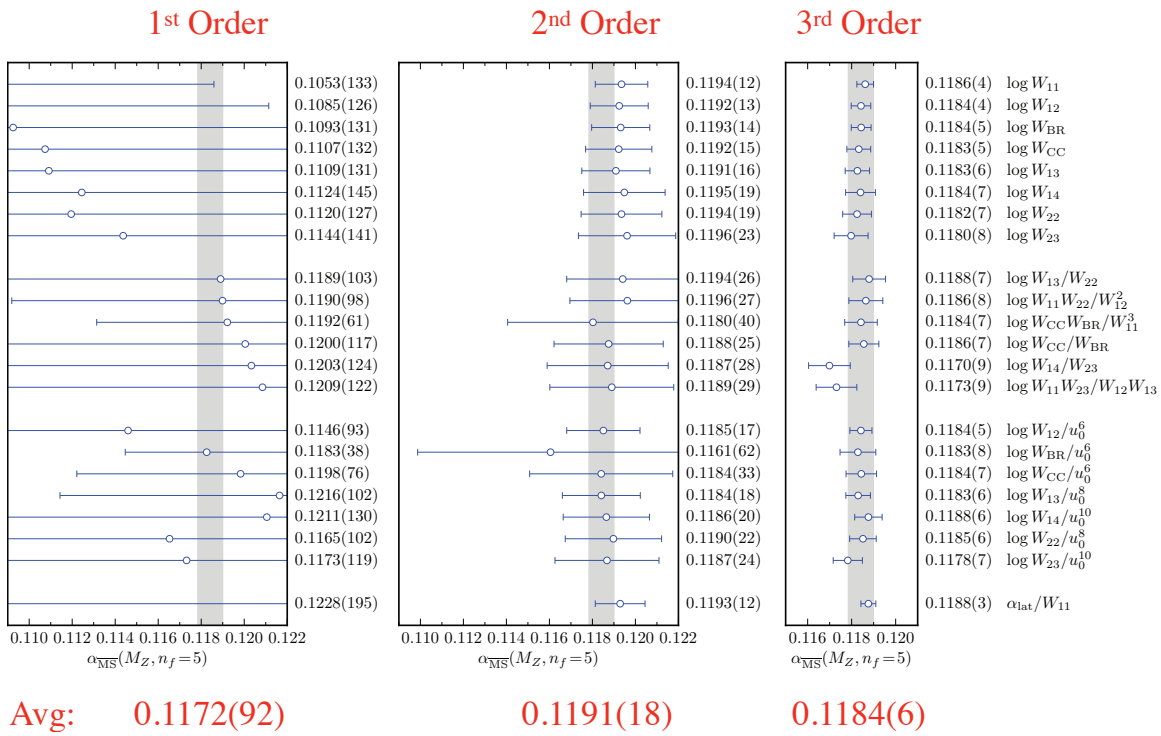


HPQCD now uses two very different approaches (red and blue in the plot), with very different systematic errors, for extracting the coupling [1]. These give almost identical results for the coupling: $\alpha_{\overline{MS}}(M_Z, n_f = 5) = 0.1184(6)$ and $0.1183(7)$.

The only experimental inputs to a QCD simulation are the small number of well-measured hadronic quantities (m_π, $m_K \ldots$) used to tune the bare quark masses and bare QCD charge in the action. Typically simulations are done for a variety of light-quark masses and lattice spacings a (and volumes) in order to extrapolate away lattice artifacts. Extrapolated simulation results should agree with continuum QCD to within simulation and extrapolation errors (of order 1% in many, many tests); there are no free parameters after tuning. We determine α_s by "measuring" various short-distance quantities Y (nonperturbatively) in the simulation, and comparing with the corresponding perturbative expansions: $Y = \sum_{n=1}^{\infty} c_n \alpha_s^n(q_Y^*)$, where the c_n are computed using Feynman perturbation theory.

Small Wilson loops are the easiest short-distance quantities to measure in a simulation. Five-digit precision is routine and the loops are very ultraviolet, and therefore also highly perturbative provided (lattice UV cutoff) π/a is large [2]. HPQCD used 22 different combinations of 8 small loops and the bare coupling, for each of 12 different combinations of u-d quark mass and lattice spacing ($\pi/a = 3.5 \to 14\,\mathrm{GeV}$), to extract 22 different values for the coupling. Perturbation theory was calculated, through third order, using lattice QCD perturbation theory, which includes lattice-spacing artifacts to all orders in the lattice spacing. Fourth and higher orders were estimated by comparing results from different lattice spacings (since $q^* \propto 1/a$). Nonperturbative effects, such as condensate contributions, were included, and can vary by factors of 100 or more between different quantities. Nevertheless all 22 quantities agree to within errors with $\alpha_{\overline{MS}}(M_Z, n_f = 5) = 0.1184(6)$ [1].

To test HPQCD's error estimates, we pretend that perturbation theory is known only to first or second order but otherwise use the same analysis, including the estimation of higher-order perturbation theory by comparing different lattice spacings. Values for $\alpha_{\overline{MS}}(M_Z)$ from each of the 22 quantities are shown in the following figure:



The gray bars show the final result, 0.1184(6), from the full analysis. The first, second and third order analyses agree to within their errors of order 0.0092, 0.0018 and 0.0006, respectively. Convergence is excellent and the error estimates robust.

The second approach for determining α_s used by HPQCD is to measure moments of current-current correlators of the form $\sum_x (am_{0h})^2 \langle 0|j_5(x,t)j_5(0)|0\rangle$ where $j_5 = \overline{\psi}_h \gamma_5 \psi_h$ and h is a heavy quark (c or b). Low-order moments are short-distance and therefore perturbative. They also are renormalized quantities, unlike Wilson loops, and so should agree with continuum QCD results when extrapolated to zero lattice spacing. HPQCD analyzed four moments at approximately 8 different values of the heavy-quark mass between m_c and m_b and 5 different lattice spacings. The analysis produced very accurate values for the coupling, $\alpha_{\overline{MS}}(M_Z, n_f=5) = 0.1183(7)$, and also for the c and b masses [1]. The masses agree with results from pure continuum analyses of moments to within about 0.5% — an important check on the lattice analysis.

The agreement between HPQCD's Wilson-loop and current-current correlator determinations of the coupling is perhaps the most compelling check on each method since the systematic errors are very different in the two cases. Many additional tests are discussed in [1].

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α_s from PACS-CS

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We evaluate the running coupling constant for 2+1 flavors in lattice QCD simulations[1]. The Schrödinger Functional(SF) scheme is employed as the intermediate scheme to carry out non-perturbative running from the low energy region, where physical scale is introduced as $1/L_{\max}$, to deep in the higher energy perturbative region, where a conversion to the $\overline{\rm MS}$ scheme is safely performed: For 3 flavors we have

$$\Lambda^{(3)}_{\overline{\rm MS}} = 2.61192\Lambda^{(3)}_{\rm SF}, \tag{1}$$

$$\Lambda^{(N_f)}_{\rm SF} = \frac{1}{L}(b_0\overline{g}(L))^{-\frac{b_1}{2b_0^2}}\exp\left(-\frac{1}{2b_0\overline{g}(L)}\right)\exp\left(-\int_0^{\overline{g}(L)} d\,g\,\left(\frac{1}{\beta(g)}+\frac{1}{b_0g^3}-\frac{b_1}{b_0^2g}\right)\right), \tag{2}$$

where $\overline{g}(L)$ is the running coupling constant in the SF scheme at the box size L, and the 3-loop beta function $\beta(g) = -g^3(b_0 + b_1g^2 + b_2g^4)$ in the SF scheme with

$$b_2 = \frac{1}{(4\pi)^3}\left(0.483(7) - 0.275(5)N_f + 0.0361(5)N_f^2 - 0.00175(1)N_f^3\right). \tag{3}$$

and the universal b_0, b_1. The SF scheme is the non-perturbative scheme whose scale is determined solely by the box size without any other scales. The continuum($a \to 0$) limit can be taken in this scheme where a is the lattice spacing.

To convert $\Lambda^{(3)}_{\overline{\rm MS}}$ to $\alpha^{(5)}_{\overline{MS}}(M_Z)$, we first run down $\alpha^{(3)}_{\overline{MS}}(\mu)$ at $\mu = m_c(m_c)$ using the 4-loop beta function and then convert it to $\alpha^{(4)}_{\overline{MS}}(\mu)$ at $\mu = m_c$ using the 3-loop matching. Next we run up $\alpha^{(4)}_{\overline{MS}}(\mu)$ and convert it to $\alpha^{(5)}_{\overline{MS}}(\mu)$ at $\mu = m_b(m_b)$. We finally run up $\alpha^{(5)}_{\overline{MS}}(\mu)$ at $\mu = M_Z(M_Z)$.

The low energy scale $L_{\max}$ should be determined by other simulations. We use the scale determination in $N_f = 2+1$ QCD simulations at three lattice spacings by CP-PACS/JLQCD collaborations[2] whose up-down quark mass covers a rather heavy region corresponding to $m_\pi/m_\rho = 0.63 - 0.78$.

Our final results in the continuum limit are

$$\alpha_s(M_Z) = 0.12047(81)(48)(^{+0}_{-173}), \tag{4}$$

$$\Lambda^{(5)}_{\overline{MS}} = 239(10)(6)(^{+0}_{-22}), \tag{5}$$

where the first error is statistical, the second one is systematic due to the perturbative matching at $\mu = m_c(m_c)$, while the last is due to the continuum extrapolation.

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Alpha_s from JLQCD

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Lattice determination of α_s is performed using some perturbative (or short distance) quantity calculated non-perturbatively on the lattice. Typical examples are the heavy quark potential at short distances and small Wilson loops. One could also consider the vector current correlator in the space-like region. Perturbative expansion for this quantity is available to α_s^4 (!) in the continuum theory, and the long-distance contribution is theoretically well understood using the Operator Product Expansion (OPE). We propose to use this quantity for the determination of α_s [1]. The correlator itself is ultraviolet divergent, but its derivative in terms of Q^2 is finite and unambiguously calculable also in lattice QCD.

There are potential sources of systematic errors due to (i) discretization effect that could be significant at short distances, (ii) truncation error of the perturbative expansion, (iii) uncertainty of lattice spacing. These are studied and found to be under good control. Our result obtained using our lattices generated with 2+1-flavors of dynamical overlap fermion is $\alpha_s^{(5)}(M_Z) = 0.1181(3)(^{+14}_{-12})$ [2]. The error is dominated by the uncertainty of lattice spacing, that is significant in our simulation on a relatively small and coarse lattice.

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$\alpha_{\rm s}$ from the ALPHA collaboration

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The conflict in determining $\alpha_{\overline{\rm MS}}(\mu)$: We recall the obvious fact that $\alpha_{\overline{\rm MS}}(\mu)$ is only perturbatively defined and not directly measurable. The resulting inherent and practical uncertainty vanishes as the renormalization scale becomes very large. In fact the Λ-parameter is well defined from the asymptotic behaviour of $\alpha_{\overline{\rm MS}}(\mu\to\infty)$ for QCD (in isolation). At low μ, where typically the experimental error is small, there is a large theory error in the extraction of $\alpha_{\overline{\rm MS}}(\mu)$ from observables due to the truncation of the divergent perturbative series with its intrinsically limited precision.

Alternatively one would like to connect low and high energies non-perturbatively by lattice methods as first suggested for a two-dimensional model [1]. Since on a finite lattice that can be simulated, large scale ratios can not be accomodated, low and high energy scales are bridged in several steps each changing scale by a factor of 2. This is possible by identifying $\mu = 1/L$, where L is the size of the box in which QCD is simulated and by using an intermediate (Schrödinger functional) coupling, $\alpha_{\rm SF}$, which is a non-perturbatively defined quantity – a finite size effect.
The master formula of the ALPHA collaboration is

$$\frac{\Lambda^{(5)}_{\overline{\rm MS}}}{F_{\rm K}} = \frac{1}{F_{\rm K}L_{\rm max}} \times \frac{L_{\rm max}}{L_k} \times \Lambda^{(4)}_{\rm SF} L_k \times \frac{\Lambda^{(4)}_{\overline{\rm MS}}}{\Lambda^{(4)}_{\rm SF}} \times \frac{\Lambda^{(5)}_{\overline{\rm MS}}}{\Lambda^{(4)}_{\overline{\rm MS}}}\,, \tag{1}$$

where $\Lambda^{(4)}_{\overline{\rm MS}}/\Lambda^{(4)}_{\rm SF} = 2.9065$ and typically $L_{\rm max} \approx 0.5\,{\rm fm}$, $L_k = 2^{-k}L_{\rm max}$. The first factor is computed non-perturbatively and the last three perturbatively. The non-perturbative connection of low and high energies yields the relation between $\alpha_{\rm SF}(1/L_{\rm max})$ and $\alpha_{\rm SF}(1/L_k) = \alpha_{\rm SF}(2^k/L_{\rm max})$ shown in the figure and we then use the perturbative formula giving $\Lambda^{(4)}_{\rm SF}L_k$ as a function of $\alpha_{\rm SF}(1/L_k)$ in terms of the three-loop β-function in the Schrödinger functional scheme. Since perturbation theory is applied at $\alpha_{\rm SF}(1/L_k) \approx 0.1$ and this renormalization scheme has been demonstrated to be well behaved, the perturbative error is negligible. The last factor $\Lambda^{(5)}_{\overline{\rm MS}}/\Lambda^{(4)}_{\overline{\rm MS}}$ will finally be taken from PT, where we can use the full 4-loop precision of [7]. From the numerical results shown by M. Steinhauser at the meeting we estimate an uncertainty of 2% on $\Lambda^{(5)}_{\overline{\rm MS}}/\Lambda^{(4)}_{\overline{\rm MS}}$ from this step, which is equivalent to decoupling across the b-threshold. In contrast, estimating the ratio $\Lambda^{(4)}_{\overline{\rm MS}}/\Lambda^{(3)}_{\overline{\rm MS}}$ would require matching at the charm threshold, which seems very difficult to control perturbatively. We therefore insist on the whole strategy being followed with 4 flavours. The approximate decoupling of the charm quark at low energy is then implemented non-perturbatively.

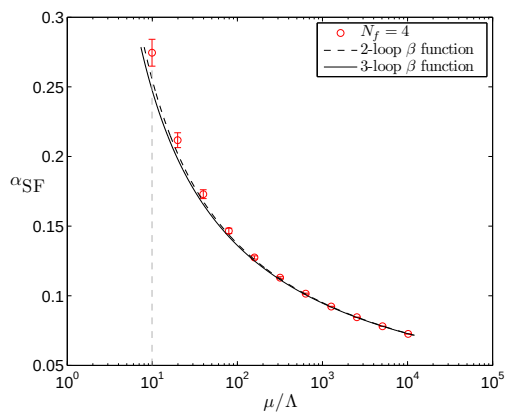
The strategy is transparent and based on minimal assumptions: asymptotic freedom, the existence of the continuum limit of the lattice theory and the fact that non-perturbative "corrections" vanish quickly as $\mu\to\infty$. *Practical issues* mainly concern discretization effects. The Schrödinger functional is very efficient in rendering simulations with very light quarks possible *and* to achieve the perturbative control (3-loop β-function). It does, however, introduce unpleasant linear lattice spacing effects. For the desired precision, it is important

that the improvement coefficient $c_{\rm t}$, which is dominant for removing these linear effects, is known to 2-loops in our discretization and furthermore lattice spacing effects have been seen to be small in PT and beyond[2].

Status and future: The strategy has been studied for $N_{\rm f} = 0$ (pure gauge theory), and for $N_{\rm f} = 2, 4$ but for the interesting 4-flavour theory, the first factor in eq. (1) is not yet know.

Present results are $\Lambda^{(0)}_{\overline{\rm MS}} r_0 = 0.60(5)$ [4], $\Lambda^{(2)}_{\overline{\rm MS}} r_0 = 0.73(3)(5)$ [5][6] compared to $\Lambda^{(4)}_{\overline{\rm MS}} r_0 = 0.71(3)$, $\Lambda^{(5)}_{\overline{\rm MS}} r_0 = 0.52(2)$ from Bethke's world average of 2009, where we use $r_0 = 0.475\,$fm. For the world average numbers we have not propagated an error in $r_0 = 0.475\,$fm, since the overall error is likely to be revised after this workshop.

In the near future we will repeat the computation of the scale dependence of $\alpha_{\rm SF}(\mu)$ on faster computers. Smaller lattice spacings and better statistical precision will mean an even more precise continuum extrapolation. The future total error of Λ as we see it now, is given in the table and appears to be achievable in a few years. It translates into an absolute error of 0.0008 in $\alpha_{\rm s}(M_{\rm Z})$.



Non-perturbative running of $\alpha_{\rm SF}(\mu)$ from $\mu \approx 1$GeV to $\mu \approx 1$TeV compared to perturbation theory[3].

	source	$N_{\rm f}$	$\delta\Lambda/\Lambda$	Ref.
now	running	4	0.07	[3]
future	running	4	0.04	
now	$L_{\rm max} F_{\rm K}$	$2+1+1$	unknown	
	$L_{\rm max} F_{\rm K}$	0	0.02	[4]
	$L_{\rm max} F_{\rm K}$	2	0.04	[5][6]
future	$L_{\rm max} F_{\rm K}$	$2+1+1$	0.03	
	PT	$4 \to 5$	0.02	[7]
	$O(\alpha_{em})$		0.01	
future	total	4	0.05	

Error budget.

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α_s Determination at the $Y(1S)$ Mass

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Heavy quarkonium leptonic and non-leptonic inclusive decay rates have historically provided ways to extract α_s. Ratio of these quantities are very sensitive to α_s, if the data are sufficiently precise. Determinations of α_s at the quarkonium masses are important because they are among the few ones at a relatively low energy scale. In PDG2006 the reported determination of α_s from Υ decays was lower than all the other determinations and about two standard deviations away the $\alpha_s(M_Z)$ average at the time.

In [1] we have obtained a new extraction of $\alpha_s(M_{\Upsilon(1S)})$ from the decay ratio

$$R_\gamma \equiv \Gamma(\Upsilon(1S) \to \gamma\, X)/\Gamma(\Upsilon(1S) \to X) \tag{1}$$

X being hadrons. R_γ is calculated in theory using the nonrelativistic QCD (NRQCD) factorization for each of the two quarkonium decay amplitudes:

$$\Gamma = \sum_n \frac{c_n(\alpha_s(M_{\Upsilon(1S)}), m_b)}{m_b^{d_n-4}} \langle \Upsilon(1S)|O_n^{\rm 4fermions}|\Upsilon(1S)\rangle \tag{2}$$

where c_n are process-dependent NRQCD Wilson coefficients, m_b is the bottom mass and $\langle \Upsilon(1S)|O_n^{\rm 4fermions}|\Upsilon(1S)\rangle$ are NRQCD matrix elements of four-fermions operators. They contain color singlet contributions that can be calculated in terms of the $\Upsilon(1S)$ wave function and color octet contributions. Adopting the NRQCD counting rules in v (v being the bottom quark velocity in the bound state) and $\alpha_s(M_{\Upsilon(1S)})/\pi \sim v^2 \sim \alpha_s^2(m_b v)$, we have included all the radiative, relativistic and octet contributions in the expansion of eq.(1) up to order v^2. Higher-order corrections that were not considered are of order v^3. We have assumed $v^2 = 0.08$ and $m_b v \gg \Lambda_{\rm QCD}$.

We have used the experimental value $R_\gamma^{\rm exp} = 0.0245 \pm 0.0001 \pm 0.0013$, (where the first error is statistical, the second one is systematical) corresponding to the Garcia-Soto (GS) [3] parameterization of the $\Gamma(\Upsilon(1S) \to \gamma\, X)$ data [2], which follows from a QCD calculation.

Our result is based on: new, precise data from CLEO [2]; a QCD calculation to extrapolate the photon spectrum at low $z = \frac{2E_\gamma}{M_\Upsilon}$ [3], based on a combined use of the effective field theories NRQCD, pNRQCD and SCET; accurate estimates of the NRQCD color octet matrix elements in eq.(2) coming from lattice calculations [4] and from continuum calculations [3].

By equating the experimental value of R_γ to the NRQCD $O(v^2)$ theoretical computation we get

$$\alpha_s(M_{\Upsilon(1S)}) = 0.184^{+0.015}_{-0.014}\,, \qquad \alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}\,, \tag{3}$$

very close to the central value of the PDG with competitive errors. Our result is rather insensitive to the values of the octet matrix elements because they cancel in the ratio, apart from one of them ($O_8(\,^3S_1)$) which, however, turns out to be small from the lattice calculation and does not have a major impact in our results. The determination is valid at next-to-leading order in $\alpha_s(m_b)$ and in v^2. At this order, terms corresponding to new qualitative

features appear (radiative, relativistic, octet corrections), each of them of natural size, but whose sum is numerically of order one and hence large. Higher-order corrections $O(v^3)$ are expected to be smaller by one order in v since they do not introduce new qualitative features. We have used part of the available higher-order corrections to check that this is indeed the case [1]. At present, the main uncertainty in our extraction of $\alpha_{\rm s}$ comes from the systematic uncertainties in $R_\gamma^{\rm exp}$.

Concerning the extraction of $\alpha_{\rm s}$ in [5], which due to the small error is dominating the final value obtained from heavy quarkonium decays in the PDG2006, there are three main differences: (i) On the theoretical side, the color octet NRQCD matrix elements are ignored in $\Gamma(\Upsilon(1S) \to \gamma X)$, whereas we find that they contribute between 30% and 100%. (ii) Older data are used, which are fully consistent with, but not as precise as, the more recent ones, and an older analysis, which relies on the Field model for extrapolations to low z. (iii) The extraction is actually done from $\Gamma(\Upsilon(1S) \to \gamma X)/\Gamma(\Upsilon(1S) \to l^+l^-)$. We believe, contrary to a statement in [5], that the latter increases rather than decreases the theoretical uncertainties associated to color octet operators. Indeed, whereas the ratio radiative/total has the same color octet operators in the numerator and denominator except for one, the ratio radiative/leptonic (total/leptonic) has two (three) different color octet operators in the numerator and denominator. Furthermore, the leptonic width is known to suffer from large higher-order corrections in $\alpha_{\rm s}$, which introduce further uncertainties.

Concerning the extraction of the CLEO paper [2], there are two main differences: (i) On the theoretical side an old formula was used there, in which the NRQCD color octet operators were ignored. This introduces large theoretical uncertainties. In practice, however, we find that numerically they are not so important for the final result. (ii) For the total radiative width, two numbers are quoted depending on whether the so called Field model or the GS parameterization, which is in fact a QCD calculation, are used for the extrapolation of the photon spectrum at low z. The final number is given as the average of the two procedures. We believe that the use of the Field model, which uses a parton shower Monte Carlo technique to incorporate the effects of gluon radiation by the outgoing gluons in the decay, introduces an unnecessary model dependence that moves the actual central value and artificially increases the errors. Our final results are similar to the ones presented in [2] for the GS parameterization.

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α_s for New Physics

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Beyond its intrinsic importance for phenomenology and technological developments, knowing the precise value of α_s can provide important information on new physics beyond the standard model. Here we discuss two such examples where α_s may play a crucial role in testing new physics scenarios: (1) Grand Unification and (2) the Multiverse.

Grand Unification in Higher Dimensions — Grand unification provides a beautiful understanding of the gauge quantum numbers of the quarks and leptons. It also leads to a prediction of α_s through unification of the three gauge couplings at a high energy, allowing us to test the idea. In fact, in supersymmetric grand unified theories, the predicted value of α_s agrees with the experimental value at the 10% level. However, because of unknown threshold corrections at the unification scale, the prediction is somewhat model dependent; and in typical models, it is around $\alpha_s \simeq 0.130$, so that the agreement is not perfect [1]. Moreover, theories often suffer from severe phenomenological problems, such as doublet-triplet splitting, too fast dimension five proton decay, and unwanted mass relations between light generation fermions.

Grand unification in higher dimensions elegantly solves these problems [2]. In this framework, the unified gauge symmetry is broken due to a compact extra dimension(s) of order the unification scale. The gauge and Higgs fields propagate in the bulk of higher dimensional spacetime, leading to automatic mass splitting between the standard model and unified gauge bosons, as well as between doublet and triplet Higgs fields. The structure of higher dimensional mass terms automatically suppresses dangerous dimension five proton decay. Matter fields may either propagate in the bulk or localized on one of the branes, i.e. the "edge" of compact space, which can reproduce a complicated pattern of quark and lepton masses and mixing without contradicting unification at high energies. A schematic picture of this framework is depicted in Fig. 1.

An interesting aspect of the above framework is that threshold corrections at the unification scale are fixed in terms of only a few parameters. In the minimal model of supersymmetric $SU(5)$ in 5 dimensions, it is only the size of the extra dimension M_*L (in units of the cutoff/string scale) that we need to know to compute the threshold effects. Together with the assumption of strong coupling at the cutoff scale, which fixes $M_*L \approx O(100)$, we can predict α_s *without any free parameter*. The value obtained is

$$\alpha_s(M_Z) = 0.118 \pm 0.004 \pm 0.003, \tag{1}$$

where the first and second errors represent uncertainties from the superpartner spectrum and M_*L, respectively. The former can be reduced/eliminated if superpartners are discovered at the LHC. It is interesting that the minimal model indeed improves the prediction over conventional 4 dimensional supersymmetric grand unification.

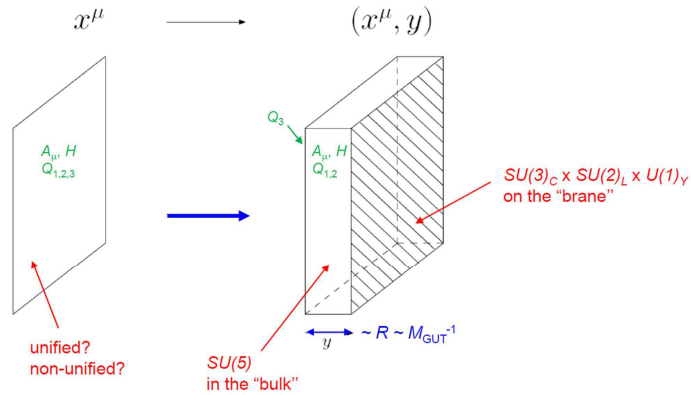

Figure 1: The basic framework of higher dimensional grand unification.

Precision Higgs Mass Prediction from the Multiverse — The origin of the electroweak scale is one of the major mysteries in the standard model. While its smallness (compared with the Planck scale) may be ensured by new physics at the TeV scale, it is possible that it arises simply as a result of environmental selection in the multiverse (as does the cosmological constant). The low energy theory may then be simply the standard model. Is there any experimental handle on such a scenario?

While supersymmetry is no longer needed at the TeV scale in this scenario, it may still exist at very high energies, e.g., as a consequence of string theory. Suppose supersymmetry exists at some high energy scale $\tilde{m}$, i.e. superpartners have masses of order $\tilde{m}$. In this case, the theory below $\tilde{m}$ is just the standard model, but with the Higgs quartic coupling determined by the weak gauge couplings: $\lambda = (g^2 + g'^2)/8$ in the limit where the low energy Higgs arises from one Higgs supermultiplet (large $\tan\beta$). This allows for predicting the Higgs boson mass through renormalization group equation. The result is surprisingly precise:

$$\begin{aligned} M_H &= 141.0 \text{ GeV} + 1.8 \text{ GeV} \left(\frac{m_t - 173.1 \text{ GeV}}{1.3 \text{ GeV}} \right) - 1.0 \text{ GeV} \left(\frac{\alpha_s(M_Z) - 0.1176}{0.002} \right) \\ &\quad + 0.14 \text{ GeV} \left(\log_{10} \frac{\tilde{m}}{10^{14} \text{ GeV}} \right) + 0.10 \text{ GeV} \left(\frac{\delta}{0.01} \right) \pm 0.5 \text{ GeV}, \end{aligned} \tag{2}$$

where δ represents possible threshold corrections, typically of $O(0.01)$. The uncertainty is dominated by experimental errors on the top quark mass and α_s, which can be reduced.

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The effect of α_S on Higgs Production at the LHC

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Reliable quantitative predictions for cross sections at the LHC require control over uncertainties arising from QCD. These errors take two primary forms: our inability to calculate to high enough order in the QCD perturbative expansion, and the imprecise determination of the non-perturbative input parameters of QCD such as the parton distribution functions (PDFs) and the strong coupling constant. Impressive progress has been made recently in calculating the perturbative cross sections needed in LHC analyses. Fixed-order cross sections for high-multiplicity scattering processes are becoming available through next-to-leading order (NLO) using powerful new calculational techniques [1]. Next-to-next-to-leading order (NNLO) calculations of benchmark processes such as Higgs and electroweak gauge boson production are available in the form of flexible numerical programs which are easy to incorporate into experimental analyses [2]. For several important LHC processes, the dominant uncertainties now come from determinations of PDFs and α_s.

The most striking such process is Higgs boson production through gluon fusion. This process is famously sensitive to large perturbative QCD corrections. The theoretical community has invested several decades into understanding the gluon-fusion mechanism. A simple effective-theory framework for this process exists, and the NNLO QCD corrections, NLO electroweak corrections, and the resummation of large threshold logarithms are available [3]. Recent predictions for the inclusive cross section used by the LHC collaborations have been given in Ref. [4], from which the following Table of $\sqrt{s} = 7$ TeV results has been taken.

M_H (GeV)	σ (pb)	Scale (%)	PDF4LHC (%)
120	16.63	$+7.2 - 7.9$	$+7.6 - 7.0$
150	10.52	$+6.6 - 7.4$	$+7.6 - 7.5$
180	6.76	$+6.2 - 7.0$	$+7.5 - 7.8$
210	4.74	$+6.0 - 6.7$	$+7.5 - 7.9$
240	3.59	$+5.9 - 6.4$	$+7.7 - 8.0$
300	2.42	$+5.8 - 6.0$	$+8.0 - 8.3$
400	2.03	$+5.9 - 5.4$	$+8.8 - 8.6$
600	0.336	$+6.1 - 5.2$	$+10.1 - 9.4$

The uncertainties arising from PDFs and the strong coupling constant, denoted as 'PDF4LHC' in the Table (named for the procedure by which they are obtained, described in Ref. [4]) dominate for all Higgs masses over the estimated errors arising from unknown higher-order QCD corrections denoted as 'scale.' The situation is potentially worse than indicated, as recently emphasized [5]. As the Higgs cross section is proportional to $\sigma \sim \alpha_S^2 f_g^2$, and receives large $\mathcal{O}(\alpha_S^3)$ corrections, it is sensitive to the myriad assumptions that enter extractions of α_S and the PDFs. The current PDF4LHC error estimate neglects PDF extractions for

which the Higgs production cross section is up to 25% lower than the sets utilized in the Table above. These sets leading to a lower production cross section typically also obtain lower values of $\alpha_s(M_Z)$ from their fit of the available data. For example, the ABKM set which gives the lowest predictions has $\alpha_S(M_Z) = 0.1135 \pm 0.0014$ [6], while the HERA-PDF set which gives the second lowest result has $\alpha_S(M_Z) = 0.1145$. Both α_S values obtained in these PDF extractions are many standard deviations away from the current world average$\alpha_S(M_Z) = 0.1184 \pm 0.0007$ [7]. A proposal to add an additional 'theoretical' uncertainty on the extraction of α_S leads to an increased error that partially resolves the differences between extractions at low-to-mid M_H [5]. The differences at high M_H are primarily due to differing gluons in the various sets, and are not accounted for by this procedure. Nevertheless, further exploration of this issue by the community, and the consideration of whether a 'theoretical' error on α_S should be added to the Higgs error budget at the LHC, would be welcome.

One may also ask if α_S may be constrained by measurements in other channels at the LHC. Event shapes in jet production may eventually permit an extraction with a 3% uncertainty, as discussed by Klaus Rabbertz in these proceedings. Studies of $V +$ jet production also appear promising. A conservative estimate of the theoretical error on the ratio of $W+1$ or more jets over the inclusive W production cross section is already at the few-percent level [8], with the prospect for further improvement once the NNLO corrections to the $W+1$ jet process are known. Luminosity and other systematic errors cancel in the ratio; the statistical and remaining systematic errors will improve as more data is collected. Further study of whether the strong coupling constant can be measured using these observables is warranted.

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α_s from the ILC

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The International Linear Collider (ILC) [1] is a proposal for an electron positron collider operating at a centre of mass energy of $Q = 500$ GeV (with an upgrade option to 1 TeV) and an luminosity of $2 \cdot 10^{34}$/cms. We illustrate the precision gain of an α_s measurement at 500 GeV by the reanalysis of JADE [2] and OPAL [3] event shape measurements, where α_s is determined to next-to-next-to-leading order, with the results $\alpha_s(m_{Z^0}) = 0.1210 \pm 0.0061$ ($\alpha_s(m_{Z^0}) = 0.1172 \pm 0.0051$) in the JADE analysis without (with) inclusion of resummed logarithms in the NLLA approximation, and $\alpha_s(m_{Z^0}) = 0.1201 \pm 0.0030$ ($\alpha_s(m_{Z^0}) = 0.1189 \pm 0.0041$) in the corresponding OPAL analyses. With a precision of 2.6%–5.0%, these are among the best measurements. In comparison to corresponding NLO analyses, the renormalisation scale uncertainty and the scatter from different variables are reduced.

We estimate the uncertainties of a corresponding $\alpha_s(m_{Z^0})$ measurement at 500 GeV as follows. The cited luminosity is about an factor 1000 higher than at LEP 1, the hadronic cross section is lower by this factor [5]. The selection efficiency is slightly lower than at LEP 2 [6]. Therefore a statistical precision of 0.0001 can be expected after taking data for a few years. The detector uncertainties in the JADE and OPAL analysis were of the order of 0.001. The ILC detector will be very hermetic and supply good tracking and calorimetry, and so this order of magnitude is expected to still hold [7]. The uncertainties induced by residual background after the JADE or OPAL selection cuts are of the order of 0.001. The effects from W and Z pairs rise with centre-of-mass energy. Above ∼350 GeV there will be additional background from electroweakly decaying top-pairs. The W contribution can be suppressed [8] by beam polarisation, and the properties of W and Z pairs have been measured properly at LEP. Therefore an uncertainty can be expected which is still of the same order [7]. The uncertainties of $\alpha_s(Q)$ related to the hadronisation correction are of the order of 10^{-2}...10^{-3} for JADE and OPAL. They fall steeply with centre-of-mass energy as can be understood from analytical power correction models. Their expected order in an measurement at 500 GeV is 0.0001 after evolving the α_s value back to 91 GeV. Estimates of the scale uncertainties from an NLO and NNLO measurement are given in Tab. 1. Measuring

	α_s(91 GeV) measurement	α_s(500 GeV) estimate	α_s(91 GeV), evolved from α_s(500 GeV)
NLO, missing: α_s^3	0.1192 ± 0.0047	0.0959 ± 0.0024	0.1192 ± 0.0038
NNLO, missing: α_s^4	0.1205 ± 0.0027	0.0967 ± 0.0011	0.1205 ± 0.0017

Table 1: α_s measurements at 91 GeV with the scale uncertainty [3][4], their evolution to 500 GeV and estimates of the scale uncertainty at 500 GeV.

α_s at 500 GeV instead of 91 GeV reduces the scale uncertainty to 80% in the NLO case, and to 60% in the NNLO case.

To study the measurement of thrust by the SiD detector at the ILC, we process u, d, s, c, b - quark pairs, generated by whizard at 500 GeV with PYTHIA parton shower and hadronization. The effects of initial state radiation and beamstrahlung are included and a cut of $m > 475$ GeV is imposed on the reconstructed quark pair invariant mass. Comparing the thrust distribution from truth particles (with mean $\langle T \rangle = 0.953$) and reconstruction ($\langle T \rangle = 0.952$), no systematic deformations are seen. Studying the reconstructed thrust values versus the MC values, Fig. 1, the migrations in the range $T > 0.7$, which is measured by OPAL [4], are small.

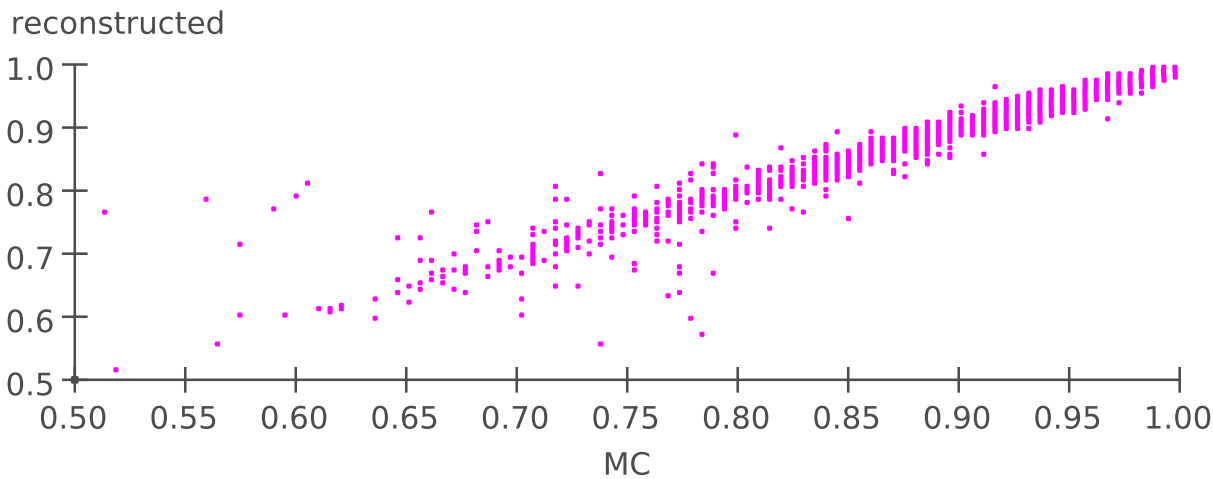

Figure 1: Thrust values of the reconstructed particles versus values of the MC input particles.

We would like to thank Norman Graf for help with the SiD Monte Carlo samples.

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The Principle of Maximum Conformality

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A key problem in making precise perturbative QCD predictions is the uncertainty in determining the renormalization scale of the running coupling $\alpha_s(\mu^2)$. It is common practice to guess a physical scale $\mu = Q$ which is of order of a typical momentum transfer Q in the process, and then vary the scale over a range $Q/2$ and $2Q$. This procedure is clearly problematic since the resulting fixed-order pQCD prediction will depend on the renormalization scheme, and it can even predict negative QCD cross sections at next-to-leading-order [1]. Other heuristic methods to set the renormalization scale, such as the "principle of minimal sensitivity" [2], give unphysical results [3] for jet physics, sum physics into the running coupling not associated with renormalization, and violate the transitivity property of the renormalization group [4]. Such scale-setting methods also give incorrect results when applied to Abelian QED. Note that the *factorization scale* in QCD is introduced to match nonperturbative and perturbative aspects of the parton distributions in hadrons; it is present even in conformal theory and thus is a completely separate issue from *renormalization scale* setting.

Scales in QED: There is no ambiguity in setting the renormalization scale in quantum electrodynamics: In the standard Gell-Mann–Low scheme for QED, the renormalization scale is simply the virtuality of the virtual photon. For example, in electron-muon elastic scattering, the renormalization scale is the momentum transfer t; i.e., $\alpha(t) = \alpha(t_0)/(1 - \Pi(t, t_0))$ where $\Pi(t, t_0) = (\Pi(t) - \Pi(t_0))/(1 - \Pi(t_0))$ sums all vacuum contributions in the dressed photon propagator, proper and improper. Although the *initial* choice of renormalization scale t_0 is arbitrary, the *final* scale t is not. In the case of muonic atoms, the modified muon-nucleus Coulomb potential is precisely $\alpha(\vec{q}^{\,2})/\vec{q}^{\,2}$. One can use other renormalization schemes in QED, such as $\overline{MS}$ scheme, but the physical result will be the same after allowing for the displacement of scales. For example, if $Q^2 >> m_\ell^2$, $\alpha_{\overline{MS}}(e^{-5/3}t) = \alpha_{GM-L}(t)$. The same underlying principle for scale setting must hold in QCD since the n_F terms in the QCD β function have the same role as the lepton N_ℓ vacuum polarization contributions in QED.

PMC and BLM: The purpose of the running coupling in gauge theory is to sum all terms involving the β function; when the renormalization scale μ is set properly, all nonconformal $\beta \neq 0$ terms in a perturbative expansion arising from renormalization are summed into the running coupling. The remaining terms in the perturbative series are then identical to that of a conformal theory; i.e., the theory with $\beta = 0$. The divergent "renormalon" series of order $\alpha_s^n \beta^n n!$ does not appear in the conformal series. Thus as in QED, the renormalization scale μ is determined unambiguously by the "Principle of Maximal Conformality (PMC)". This is the principle underlying BLM scale setting [5] An important feature of PMC is that its QCD predictions are independent of the choice of renormalization scheme. The PMC procedure also agrees with QED in the $N_C \to 0$ limit. In the case of e^+e^- annihilation to three jets, the BLM/PMC scale is set by the gluon jet virtuality.

Global PMC Scale: Ideally, as in the BLM method, one should allow for separate scales for each skeleton graph; e.g., for to electron-electron scattering, one takes $\alpha(t)$ and $\alpha(u)$ for the t-channel and u-channel amplitudes, respectively. Setting separate scales can be a challenging task for complicated processes in QCD where there are many final-state

particles and thus many possible Lorentz scalars q_i^2. However, one can obtain a useful first approximation to the full BLM-PMC scale-setting procedure using a single *global* scale $\hat{\mu}$ which appropriately weights the individual BLM scales. The global scale [6] can be determined by varying the subprocess amplitude with respect to each invariant, thus determining the coefficients f_i of $\log q_i^2/\mu_0^2$ in the amplitude; the global PMC scale is then $\hat{\mu}^2 = C\Pi_i(q_i^2)^{w_i}$, where the weight $w_i = f_i/\sum_j f_j$. C is the scheme displacement; e.g., $C = e^{-5/3}$ for $\overline{MS}$.

Commensurate Scale Relations (CSR) [7]: Relations between observables must be independent of the choice of scale and renormalization scheme. CSRs are thus fundamental tests of theory, devoid of theoretical conventions. For example, the PMC relates the effective charge $\alpha_{g_1}(Q^2)$ determined by measurements of the Bjorken sum rule, to the effective charge $\alpha_R(s)$ measured in the total e^+e^- annihilation cross section: $[1-\alpha_{g_1}(Q^2)/\pi]\times[1+\alpha_R(s^*)/\pi] = 1$. Because all $\beta \neq 0$ nonconformal terms are absorbed into the running couplings using PMC, one recovers the conformal prediction [8]; in this case, it is the Crewther relation [9]. The ratio of PMC scales $\sqrt{s}^*/Q \simeq 0.52$ is set by physics; it guarantees that each observable goes through each quark flavor threshold simultaneously as Q^2 and s are raised . Thus by applying PMC, the conformal commensurate scale relations between observables, such as the Crewther relation, become valid for non-conformal QCD at leading twist.

Conclusions: The PMC provides a consistent method for determining the renormalization scale in pQCD. The PMC scale-fixed prediction is independent of the choice of renormalization scheme, a key requirement of renormalization group invariance. The results avoid renormalon resummation and agree with QED scale-setting in the Abelian limit. The PMC global scale can be derived efficiently at NLO from basic properties of the PQCD cross section. The elimination of the renormalization scheme ambiguity using the PMC will not only increases the precision of QCD tests, but it will also increase the sensitivity of colliders to new physics beyond the Standard Model.

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Workshop on α_s 2011: Summary

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At this workshop, a large number of recent and newest developments in the field of precision measurements of α_s have been presented, and many in-depth discussions about specific topics which arose before and during this meeting were performed. At the end of this workshop, the status can be briefly summarised as follows:

- A selection of the **newest significant results**, not included in the 2009 world summary (c.f. the Introduction to this workshop, these proceedings), with *potential* to be included in a new world average 2011, gives:

  1. R. Abbate et al., "Thrust at N^{3}LL with power Corrections and a Precision Global Fit for $\alpha_s(M_{Z^0})$", arXiv:1006.3080, Phys. Rev. **D83**, 074021 (2011): $\alpha_s(M_{Z^0}) = 0.1135 \pm 0.0011$. Hadronization power corrections treated with field theoretic matrix elements fit to experimental data.
  2. G. Disssertori et al., "Precise determination of the strong coupling constant at NNLO in QCD from the 3-jet rate in e^+e^- annihilation at LEP", arXiv:0910.4283, Phys.Rev.Lett 104(2010) 072002: $\alpha_s(M_{Z^0}) = 0.1175 \pm 0.0025$.
  3. D0 Collaboration, V.M. Abazov et al., "Determination of the strong coupling constant from the inclusive jet cross section in $p\overline{p}$ collisions at $\sqrt{s} = 1.96$ GeV", arXiv:0911.2710, Phys.Rev.D80 (2009) 111107: $\alpha_s(M_{Z^0}) = 0.1161 \pm 0.0045$ (QCD in NLO).
  4. OPAL collaboration, G. Abbienti et al., "Determination of α_s using OPAL hadronic event shapes at $\sqrt{s} = 91 - 209$ GeV and resummed NNLO calculations", subm. to Eur.Phys.Jour.C: $\alpha_s(M_{Z^0}) = 0.1189 \pm 00041$.
  5. A. Pich, "α_s Determination from τ Decays: Theoretical Status", arXiv:1001.0389, Acta Phys. Polon. Supp. **3**, 165-170 (2010): $\alpha_s(M_{Z^0}) = 0.1213 \pm 0.0014$.

- The most imminent **problems remaining after all discussion** are:

  (a) The new method and result from e^+e^- Thrust distribution (result no. 1 above), using full NNLO perturbative QCD plus N^3LL power corrections from a field theory ansatz provides a rather small value of $\alpha_s(M_{Z^0})$ and - moreover - a very small overall error. This, if included in the 2009 world average (replacing the standard e^+e^- results from event shapes; c.f. the "Welcome and Introduction" article to this workshop), results in an average of $\alpha_s(M_{Z^0}) = 0.1174 \pm 0.0006$, with a rather bad overall χ^2 of 17 for 6 d.o.f. This single measurement then deviates by more than 4 standard deviation from the exclusive mean (of 0.1185 ± 0.0007), indicating that either this determination suffers from a large and so far unknown systematic shift, and/or its assigned error is largely underestimated.

  (b) The situation in the sector of α_s from τ-decays is still unsolved. Even after intensive discussion throughout this workshop, no agreement on a possible preference for one of the different perturbative approaches (FOPT vs. CIPT) was achieved.

(c) There was some scepticism on the various results from lattice gauge theory presented at this workshop, especially on the reliability of the very small quoted uncertainties. It was suggested to include more than the one result which was part of the 2009 summary.

(d) The reliability of the "standard" α_s determinations from hadronic event shapes and jet rates in e^+e^- annihilation final states, based on hadronisation corrections derived from Monte-Carlo models, was also discussed and criticized.

(e) General concern was raised about continuously shrinking systematic (and theoretical) uncertainties assigned to some of the results. Defining theoretical uncertainties by varying the QCD renormalisation scale within a factor of two, up and down, of the canonical scale - often the c.m. energy of the collision, may be a gross under-determination in some cases, especially in case of QCD in NLO only, but also in NNLO, where a small dependence on the scale may be accidental, and modifications of the renormalisation scheme would also be mandatory.

A new attempt to extract an updated world average value of $\alpha_s(M_{Z^0})$ will therefore have to rely on a careful selection and treatment of the results to be included. As a very **preliminary** first look into the new situation in 2011, the results listed in table 1 are selected to calculate a new world average. Compared to the previous study in 2009, results no. 3 and 4 as listed above have been included, while results no. 1, 2 and 5 were not (yet) accounted for due to the reasons given above. The weighted average is $\alpha_s(M_{Z^0}) = 0.1183 \pm 0.0006$ with $\chi^2 = 5.0/8\ d.o.f.$ Assuming an overall correlation factor of 0.37 to adjust χ^2 to $1/d.o.f.$ then gives as **very preliminary** world average 2011:

$$\alpha_s(M_{Z^0}) = 0.1183 \pm 0.0010\ .$$

Process	Q [GeV]	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
τ-decays	1.78	0.1197 ± 0.0016	0.11809 ± 0.00109	0.8
DIS [F_2]	2 - 170	0.1142 ± 0.0023	0.11866 ± 0.00132	1.7
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1198 ± 0.0032	0.11827 ± 0.00097	0.5
Lattice QCD	7.5	0.1183 ± 0.0008	0.11838 ± 0.00164	0.0
Υ decays	9.46	$0.119^{+0.006}_{-0.005}$	0.11832 ± 0.00094	0.1
e^+e^- [jets & shps]	14 - 44	0.1172 ± 0.0051	0.11835 ± 0.00094	0.2
$p\overline{p}$ incl. jets	50 - 145	0.1161 ± 0.0045	0.11831 ± 0.00097	0.5
e^+e^- [ew prec. data]	91.2	0.1193 ± 0.0028	0.11829 ± 0.00095	0.3
e^+e^- [jets & shps]	91 - 208	0.1208 ± 0.0038	0.11826 ± 0.00099	0.7

Table 1: Preliminary summary of recent measurements of $\alpha_s(M_{Z^0})$ (Feb. 2011). The rightmost two columns give the exclusive mean value of $\alpha_s(M_{Z^0})$ calculated *without* that particular measurement, and the number of standard deviations between this measurement and its exclusive mean.