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Quirks in supersymmetry with gauge coupling unification

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I investigate the phenomenology of supersymmetric models with extra vector-like supermultiplets that couple to the Standard Model gauge fields and transform as the fundamental representation of a new confining non-Abelian gauge interaction. If perturbative gauge coupling unification is to be maintained, the new group can be $SU(2)$, $SU(3)$, or $SO(3)$. The impact on the sparticle mass spectrum is explored, with particular attention to the gaugino mass dominated limit in which the supersymmetric flavor problem is naturally solved. The new confinement length scale is astronomical for $SO(3)$, so the new particles are essentially free. For the $SU(2)$ and $SU(3)$ cases, the new vector-like fermions are quirks; pair production at colliders yields quirk-antiquirk states bound by stable flux tubes that are microscopic but long compared to the new confinement scale. I study the reach of the Tevatron and LHC for the optimistic case that in a significant fraction of events the quirk-antiquirk bound state will lose most of its energy before annihilating as quirkonium.

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I. INTRODUCTION

Among the hurdles that must be cleared by any proposed extension of the Standard Model (SM) are the stringent limits on quantum corrections to the electroweak vector boson propagators due to new physics [1]-[6]. Low-energy supersymmetry [7] is generally safe in this regard, because of the fact that all of the new particles it introduces get their masses primarily from bare mass terms, not from their couplings to the Higgs vacuum expectation values (VEVs). This includes the Higgs chiral supermultiplets H_u and H_d themselves, which are vector-like, together forming a self-conjugate representation of the SM gauge group $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. It is therefore interesting to consider non-minimal supersymmetric models that maintain this feature by including more chiral supermultiplets transforming as vector-like representations of the gauge group.

Another well-known and appealing feature of the minimal supersymmetric standard model (MSSM) is the perturbative unification of running gauge couplings near $M_U \approx 2\times 10^{16}$ GeV. A sufficient (but not necessary) condition for extensions of the MSSM with extra vector-like supermultiplets to maintain gauge coupling unification is that the new fields come in complete multiplets of the $SU(5)$ global symmetry group that contains $G_{\rm SM}$. This paper studies the properties of models of this type that introduce a new non-Abelian gauge group G_X, under which the new chiral supermultiplets also transform but the MSSM fields are neutral. Models of this type have already been introduced by Babu, Gogoladze, and Kolda in [8], in the context of finding new large contributions to the lightest Higgs boson mass.[†] I will assume that the new chiral supermultiplets have masses at the TeV scale or below, and that the new gauge coupling g_X unifies with the $G_{\rm SM}$ couplings g_1, g_2, g_3 at M_U. In order to avoid a strong disruption of the running of the $G_{\rm SM}$ gauge couplings, it is necessary that the corresponding confinement scale Λ for the G_X interactions is below the masses of the new fermions and scalars that are also charged under $G_{\rm SM}$. This in turn implies an intriguing phenomenology studied first by Okun [15], later by Gupta and Quinn [16] and by Strassler and Zurek [17], and more recently in considerable depth by Kang and Luty in [18]. The new particles that transform non-trivially under the new gauge group (dubbed "theta particle" by Okun, and renamed "quirks" by Kang and Luty) can form exotic bound states with unusual signatures that depend strongly on the G_X confinement scale. When a heavy quirk-antiquirk pair is produced in a collider experiment, they fly apart but remain connected by a stable flux tube, which cannot break due to the large energy cost to produce an additional quirk-antiquirk pair. The maximum length of this flux tube is roughly of order $L \sim \Delta E/\Lambda^2$, where ΔE is the kinetic energy of the hard scattering production process. This length can range from microscopic to literally astronomical, but in any case it is much larger than the flux tube thickness $\sim \Lambda^{-1}$. The resulting collider signatures are potentially distinctive but also possibly quite difficult [18]-[27].

In this paper, I will study the basic properties of models that maintain perturbative unification of gauge couplings, and their renormalization group running, in Section II. The sparticle mass spectra are studied in Section III, and Section IV considers the impact on the

[†] Models with the same motivation, but without the new non-Abelian gauge group, have been studied in [9]-[13]. Other recent proposals for extra vector-like chiral supermultiplets are found in [14].

supersymmetric little hierarchy problem. Some salient aspects of the collider phenomenology of the quirks are discussed in Section V.

II. MSSM EXTENDED BY VECTOR-LIKE FIELDS COUPLED TO A NEW CONFINING NON-ABELIAN GAUGE INTERACTION

In order to maintain perturbative gauge coupling unification, the number of new particles transforming under the SM gauge group is limited to the equivalent of three copies of the $\mathbf{5}+\overline{\mathbf{5}}$ of the $SU(5)$ group that contains $G_{\rm SM}$, if they are not much heavier than 1 TeV. This assumes that $\alpha_i = g_i^2/4\pi$ $(i=1,2,3)$ are required to be perturbative (less than 0.3 or so) at and below the energy scale where they unify.[‡] (One could consider unification with larger couplings at and near the unification scale, but then both renormalization group (RG) running and threshold corrections will be necessarily out of control, and the low-energy manifestation of apparent unification must be considered merely accidental.) It follows that the new gauge non-Abelian group must be $G_X = SU(2)_X$ or $SO(3)_X$ or $SU(3)_X$. In the following, the new fields are taken to transform in the $N = 2$, 3, or 3 dimensional representations respectively for these three cases. Thus the new quirk chiral supermultiplets transform under $SU(2)_X \times G_{\rm SM}$ as:

$$D, \overline{D} = (\mathbf{2}, \mathbf{3}, \mathbf{1}, -\frac{1}{3}) + (\mathbf{2}, \overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \tag{2.1}$$

$$\overline{L}, L = (\mathbf{2}, \mathbf{1}, \mathbf{2}, \frac{1}{2}) + (\mathbf{2}, \mathbf{1}, \mathbf{2}, -\frac{1}{2}) \tag{2.2}$$

$$S, \overline{S} = (\mathbf{2}, \mathbf{1}, \mathbf{1}, 0) \times 2n_S, \tag{2.3}$$

or under $SU(3)_X \times G_{\rm SM}$ as:

$$D, \overline{D} = (\mathbf{3}, \mathbf{3}, \mathbf{1}, -\frac{1}{3}) + (\overline{\mathbf{3}}, \overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \tag{2.4}$$

$$\overline{L}, L = (\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{2}) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -\frac{1}{2}) \tag{2.5}$$

$$S, \overline{S} = [(\mathbf{3}, \mathbf{1}, \mathbf{1}, 0) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, 0)] \times n_S, \tag{2.6}$$

or under $SO(3)_X \times G_{\rm SM}$ as:

$$D, \overline{D} = (\mathbf{3}, \mathbf{3}, \mathbf{1}, -\frac{1}{3}) + (\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \tag{2.7}$$

$$\overline{L}, L = (\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{2}) + (\mathbf{3}, \mathbf{1}, \mathbf{2}, -\frac{1}{2}) \tag{2.8}$$

$$S \equiv \overline{S} = (\mathbf{3}, \mathbf{1}, \mathbf{1}, 0) \times n_S. \tag{2.9}$$

These are the main model frameworks considered below. With these assignments, $D, \overline{L}$ transform as a $\mathbf{5}$ and $\overline{D}, L$ transform as a $\overline{\mathbf{5}}$ of the usual Georgi-Glashow $SU(5)$, ensuring

[‡] It is crucial to use two-loop (or higher) beta functions to correctly implement this perturbativity requirement. This paper uses three-loop beta functions for supersymmetric gauge couplings and gaugino masses and two-loop beta functions for Yukawa couplings, scalar masses, and scalar cubic couplings. These can be found straightforwardly from general results in refs. [28, 29], and so are not listed explicitly here.

that the unification of $G_{\rm SM}$ gauge couplings persists. Also included are n_S $G_{\rm SM}$ singlets in the same representations of G_X. Note that since $SU(2)_X$ and $SO(3)_X$ have the same Lie algebra, the practical distinction between them is really whether the representations of the chiral superfields are doublets or triplets.

[There are some variations on the above models that are consistent with gauge coupling unification with the new fields at the TeV scale, which should be mentioned although they are inconsistent with an assignment of $D, \overline{D}, L, \overline{L}$ into $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$. First, for $SU(3)_X$ only, there is another, inequivalent, embedding in which $D, \overline{D}$ have the same assignments, but $\overline{L}, L = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}, \frac{1}{2}) + (\mathbf{3}, \mathbf{1}, \mathbf{2}, -\frac{1}{2})$ instead. Also, for $SU(3)_X$, one could put $D, \overline{D}$ into three singlets if $L, \overline{L}$ are in a triplet, or vice versa. Likewise, for $SU(2)_X$ or $SO(3)_X$, one could put $D, \overline{D}$ into any combination of n_1^D singlets, n_2^D doublets, and n_3^D triplets, and similarly for $\overline{L}, L$, provided that $n_1^D + 2n_2^D + 3n_3^D = n_1^L + 2n_2^L + 3n_3^L \leq 3$. Finally, it should be noted that the number and type of G_X representations of the SM singlets do not affect gauge coupling unification for $G_{\rm SM}$, and so are more generally arbitrary as long as they are anomaly-free under G_X. However, to keep the discussion below bounded, I will limit the discussion below to the models defined by eqs. (2.1)-(2.9).]

The supersymmetric mass parameters of the $D, \overline{D}, L, \overline{L}$ fields are assumed to arise by the same mechanism that gives the entirely analogous term $\mu H_u H_d$ in the superpotential of the MSSM. For example [30, 31], one may assume that the mass terms $H_u H_d$ and $D\overline{D}$ and $L\overline{L}$ and $S\overline{S}$ are forbidden at tree-level, and arise from non-renormalizable superpotential terms:

$$W = \frac{1}{M_P^2} X\overline{X} \left(\lambda_\mu H_u H_d + \lambda_D D\overline{D} + \lambda_L L\overline{L} + \lambda_{S_i} S_i \overline{S}_i \right) \tag{2.10}$$

(with an implied sum over $i = 1, \ldots, n_S$ if $n_S \neq 0$) when the fields $X, \overline{X}$ get VEVs roughly of order 10^{11} GeV. Here $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. These intermediate-scale VEVs are natural, for example [31], if there is also a superpotential

$$W = \frac{\lambda_X}{4M_P^2} X^3 \overline{X} \tag{2.11}$$

and soft terms

$$-\mathcal{L}_{\rm soft} = m_X^2 |X|^2 + m_{\overline{X}}^2 |\overline{X}|^2 + \left(\frac{a_X}{4M_P^2} X^3 \overline{X} + \text{c.c.} \right). \tag{2.12}$$

Non-trivial VEVs for $X, \overline{X}$ break a Peccei-Quinn symmetry, giving rise to an invisible axion solution to the strong CP problem [30]. There will be a non-trivial local minimum of the potential provided that $|a_X|^2 - 6|\lambda_X|^2(m_X^2 + m_{\overline{X}}^2) > 0$, and it will be a global minimum if $|a_X|^2 - 8|\lambda_X|^2(m_X^2 + m_{\overline{X}}^2) > 0$ [32]. This will give rise to the vector-like mass terms in the low-energy effective superpotential

$$W = \mu H_u H_d + \mu_D D\overline{D} + \mu_L L\overline{L} + \mu_{S_i} S_i \overline{S}_i. \tag{2.13}$$

with μ, μ_D, μ_L, μ_S of order 100 GeV to 1 TeV, provided that the corresponding couplings $\lambda_\mu, \lambda_D, \lambda_L, \lambda_S$ are not too small.

The $S, \overline{S}$ fields do not couple to $G_{\rm SM}$ gauge fields, and so are not constrained by LEP2 or Tevatron or other direct production, nor do they affect the SM gauge couplings directly. So, some number n of them (with $0 \leq n \leq n_S$) could actually have current masses μ_{S_i} that are far below the electroweak scale. This would occur if the λ_{S_i} coupling(s) in eq. (2.10) are absent (perhaps replaced by terms of even higher dimensionality), or just small. Note that for $n > 0$, there will be no stable flux tubes for pair-produced particles charged under G_X, because then as the particles produced in the hard collision fly apart, the gauge string will break to form bound states with size of order Λ^{-1} just as in ordinary QCD. This is because the energy cost to produce an additional pair of light $S, \overline{S}$ after the hard collision would then be small.

The new fermion content of the theory consists of a color triplet charge $\pm 1/3$ Dirac fermion $(\psi_D, \overline{\psi}_D)$ with mass μ_D; a charge ± 1 Dirac fermion $(\psi^+_{\overline{L}}, \psi^-_L)$ with mass μ_L; and charge 0 fermions $\psi^0_{\overline{L}}, \psi^0_L, \psi^0_{S_i}, \psi^0_{\overline{S}_i}$. The scalar partners of these particles will have soft-supersymmetry breaking squared-mass terms:

$$
\begin{aligned}
-\mathcal{L} \;=\; & m^2_{\widetilde{D}}|D|^2 + m^2_{\widetilde{\overline{D}}}|\overline{D}|^2 + m^2_{\widetilde{L}}|L|^2 + m^2_{\widetilde{\overline{L}}}|\overline{L}|^2 + (m^2_{\widetilde{S}})_{ij} S^*_i S_j + (m^2_{\widetilde{\overline{S}}})_{ij} \overline{S}^*_i \overline{S}_j + \\
& + (b_D D\overline{D} + b_L L\overline{L} + (b_S)_{ij} S_i \overline{S}_j + \text{c.c.}),
\end{aligned}
\tag{2.14}
$$

where the scalar components are denoted by the same symbol as the chiral supermultiplets of which they are members. In the case $G_X = SU(2)$ or $SO(3)$, the fields S_i and $\overline{S}_i$ actually have the same quantum numbers, and so can mix with further soft mass terms $(m^2_{\widetilde{S}})_{ij} S^*_i \overline{S}_j$, etc., but for simplicity I assume that mixing between S_i and $\overline{S}_j$ chiral supermultiplets is absent. Also for simplicity, I will assume that the above S_i and $\overline{S}_j$ soft terms are diagonal in the same basis that the superpotential masses μ_{S_i} are diagonal. This is natural if the soft supersymmetry breaking arises in a flavor-blind framework such as gaugino mass dominance.

In the absence of Yukawa couplings involving the new chiral supermultiplets, the charge 0 fermions are unmixed, and form Dirac fermions with masses μ_L and μ_{S_i}. For $n_S > 0$, the new chiral supermultiplets can have Yukawa couplings in addition to their mass terms in eq. (2.13):

$$
W = k_i H_u L \overline{S}_i + k'_i H_d \overline{L} S_i. \tag{2.15}
$$

and corresponding soft scalar cubic terms,

$$
-\mathcal{L} = a_{k_i} H_u L \overline{S}_i + a_{k'_i} H_d \overline{L} S_i + \text{c.c.} \tag{2.16}
$$

As mentioned above, $H_u L S$ and $H_d \overline{L S}$ couplings for $SU(2)_X$ or $SO(3)_X$ are also possible, but are omitted here for simplicity. The superpotential Yukawa couplings produce mixing between the gauge eigenstate fermions, yielding Dirac fermions $(\psi^0_j, \overline{\psi}^0_j)$ which are mixtures of $\psi^0_L, \psi^0_{S_i}$ and of $\psi^0_{\overline{L}}, \psi^0_{\overline{S}_i}$, respectively. For example, if only one pair S and $\overline{S}$ has couplings

to the Higgs fields, then the new neutral fermion mass matrix becomes

$$-\mathcal{L} = \begin{pmatrix} \psi_{L^0} & \psi_S \end{pmatrix} \begin{pmatrix} \mu_L & kv_u \\ k'v_d & \mu_S \end{pmatrix} \begin{pmatrix} \psi_{\overline{L}^0} \\ \psi_{\overline{S}} \end{pmatrix} + \text{c.c.} \tag{2.17}$$

when the MSSM Higgs fields get their VEVs v_u, v_d with $\tan\beta = v_u/v_d$ and $v = \sqrt{v_u^2 + v_d^2} \approx$ 175 GeV. The couplings k, k' gives rise to 1-loop effects that can significantly raise the lightest Higgs scalar boson mass due to a lack of complete cancellation between scalar and fermion loops, especially for large k if $\tan\beta$ is not small. This was the motivation of [8], but as noted in similar contexts in [10, 11] and remarked on further below, it is doubtful whether this really ameliorates the supersymmetric little hierarchy problem.

In keeping with the idea that the apparent gauge coupling unification for $G_{\rm SM}$ is telling us something important about the underlying theory, I will assume that the new non-Abelian gauge coupling g_X unifies with g_1, g_2 and g_3 at a scale $M_U \gtrsim 2 \times 10^{16}$ GeV. In practice, I use three-loop RG equations to run up from the electroweak scale, and declare the scale where $g_1 = g'\sqrt{5/3}$ and g_2 meet to be M_U, and require g_X to be equal to them there. The QCD coupling g_3 typically misses this common value at M_U by a small amount that can be reasonably ascribed to threshold corrections. Now, RG running g_X from this scale, I require that it remains finite down to scales well below the masses of the quirks $D, \overline{D}, \overline{L}, L$. Otherwise, two-loop effects would strongly affect the running of the SM gauge couplings, rendering their apparent unification merely accidental. For $SU(2)_X$ and $SO(3)_X$, this requirement is automatically satisfied for all $n_S \geq 0$, but for $SU(3)_X$ it requires $n_S \geq 3$. I therefore consider $n_S = 3$ to be the minimal viable model for the $SU(3)_X$ case.

For illustration, the running of the gauge couplings is shown at three-loop order in Figure 1, for the three cases $SU(2)_X$ with $n_S = 0$ and $SO(3)_X$ with $n_S = 0$ and $SU(3)_X$ with $n_S = 3$. For simplicity, I have assumed vanishing Yukawa couplings and chosen a single scale $M_{\rm thresh} = 1$ TeV as the effective average mass of the new particles charged under G_X and the MSSM superpartners. The unification will have some dependence on the actual thresholds, which one might imagine is roughly comparable to the unknown threshold dependence due to high-scale particles. In the case of $SU(2)_X$ with $n_S = 0$, the gauge coupling g_X runs quite slowly, and is somewhat weaker than the QCD coupling at the TeV scale. In contrast, for the $SO(3)_X$ case with $n_S = 0$, g_X runs quickly to very small values in the infrared, due to a large positive beta function coefficient. For the minimal viable $SU(3)_X$ case with $n_S = 3$, the g_X beta function is even more negative than the QCD beta function, leading to a gauge coupling at the TeV scale that is larger, but still perturbative and not running very fast. For non-minimal models with n_S larger than these values, the TeV-scale values of α_X are smaller, because the g_X beta function is larger.

Below the masses of the quirks and their supersymmetric partners, the coupling g_X has a negative beta function, and diverges at some scale Λ when calculated at any particular loop order in a specified scheme. Given the $\overline{\rm MS}$ beta function for $\alpha_X = g_X^2/4\pi$ up to 4-loop

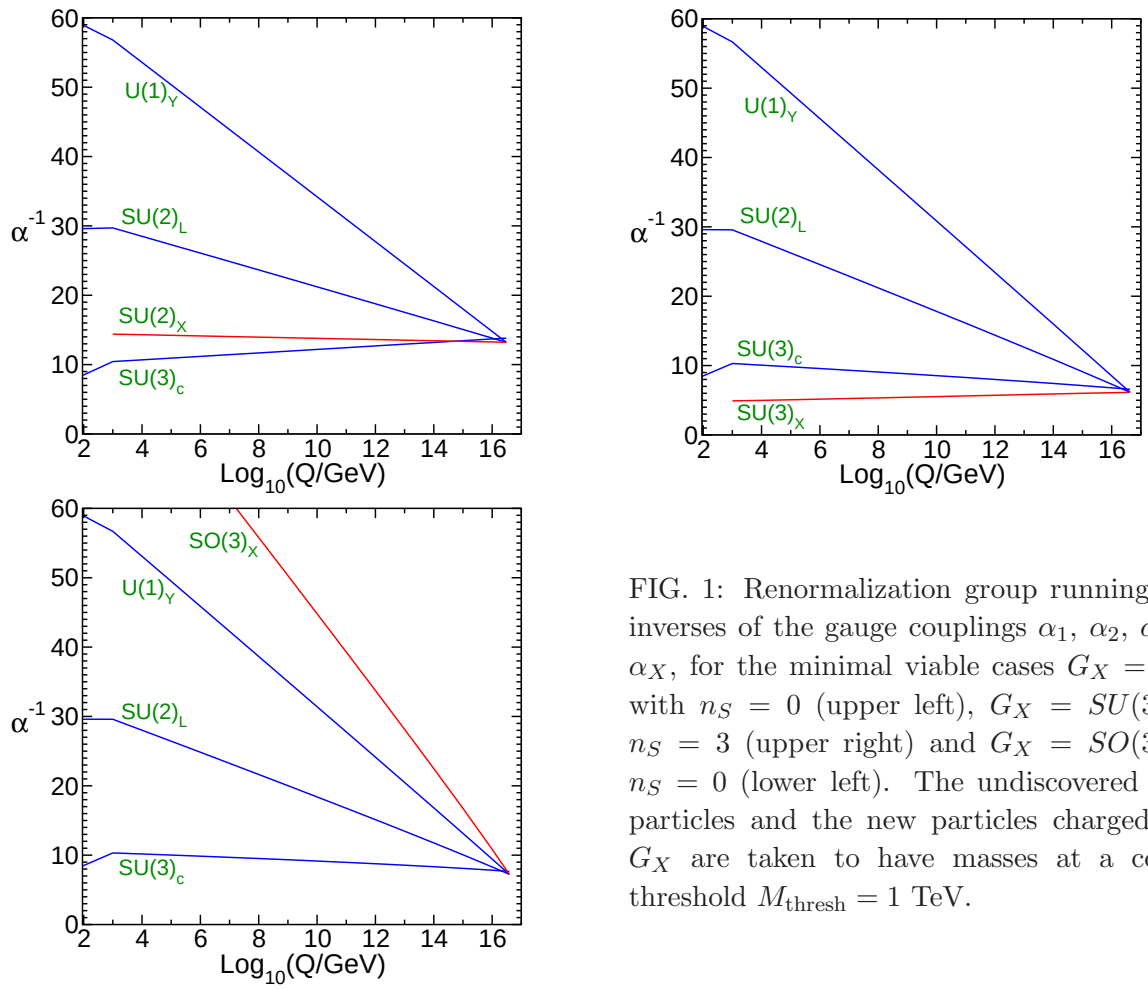

FIG. 1: Renormalization group running of the inverses of the gauge couplings α_1, α_2, α_3, and α_X, for the minimal viable cases $G_X = SU(2)$ with $n_S = 0$ (upper left), $G_X = SU(3)$ with $n_S = 3$ (upper right) and $G_X = SO(3)$ with $n_S = 0$ (lower left). The undiscovered MSSM particles and the new particles charged under G_X are taken to have masses at a common threshold $M_{\text{thresh}} = 1$ TeV.

order:

$$\beta_{\alpha_X} = Q\frac{d\alpha_X}{dQ} = -2\left(b_0\alpha_X^2 + b_1\alpha_X^3 + b_2\alpha_X^4 + b_3\alpha_X^5 + \ldots\right), \tag{2.18}$$

the scale Λ can be defined,[§] using any convenient $\alpha_X(Q_0)$ with $Q_0 \leq M_{\text{thresh}}$ as input, by an expansion in inverse powers of $t \equiv \ln(Q_0^2/\Lambda^2)$ [33]:

$$\begin{aligned}\alpha_X(Q_0) \;=\; \frac{1}{b_0 t}\Big(&1 - [b_1 \ln t]/b_0^2 t + [b_0 b_2 + b_1^2(\ln^2 t - \ln t - 1)]/b_0^4 t^2 \\ &+ [b_0^2 b_3 - b_1^3(2\ln^3 t - 5\ln^2 t - 4\ln t + 1) - 6 b_0 b_1 b_2 \ln t]/2 b_0^6 t^3 + \ldots\Big).\end{aligned} \tag{2.19}$$

It is common in rough estimates to only use the one-loop-order estimate $\alpha_X(Q_0) = 1/b_0 t$, with $b_0 = (11C_A - 2T_F)/12\pi$ where $(C_A, T_F) = (N, n)$ for $SU(N)_X$ and $(C_A, T_F) = (2, 2n)$ for $SO(3)_X$, with n denoting the number of the n_S SM singlet fields that have masses below

[§] Note that the definition for Λ used here corresponds to $\Lambda/4$ in ref. [18].

G_X	n_S	$\alpha_X^{-1}(1\text{ TeV})$	$\Lambda^{(1)}$	$\Lambda^{(2)}$	$\Lambda^{(3)}$	$\Lambda^{(4)}$
$SU(2)$	0	9.3	0.35 GeV	1.3 GeV	1.1 GeV	1.1 GeV
	1	14.4	4.4 MeV	19 MeV	17 MeV	17 MeV
	2	19.5	57 keV	280 keV	250 keV	250 keV
	3	24.5	0.76 keV	4.1 keV	3.7 keV	3.7 keV
	4	29.5	11 eV	60 eV	55 eV	55 eV
$SU(3)$	3	4.9	61 GeV	140 GeV	120 GeV	120 GeV
	4	9.9	3.5 GeV	11 GeV	9.3 GeV	9.5 GeV
	5	15.0	190 MeV	720 MeV	620 MeV	620 MeV
	6	20.1	10 MeV	44 MeV	38 MeV	38 MeV
	7	25.1	0.59 MeV	2.7 MeV	2.4 MeV	2.4 MeV
	8	30.2	32 keV	160 keV	140 keV	140 keV
	9	35.2	1.9 keV	9.7 keV	8.7 keV	8.8 keV
$SO(3)$	0	83	1.3×10^{-19} eV	1.1×10^{-18} eV	1.0×10^{-18} eV	1.0×10^{-18} eV
	1	103	4.7×10^{-27} eV	4.4×10^{-26} eV	4.2×10^{-26} eV	4.2×10^{-26} eV

TABLE I: The G_X confinement scale $\Lambda^{(\ell)}_{\overline{\rm MS}}$, computed at various loop orders ℓ by using eq. (2.19) keeping terms of order $1/t^\ell$. The new particles charged under G_X are taken to have an effective average decoupling mass scale of $Q_0 = M_{\rm thresh} = 1$ TeV, with no SM singlet quirks with current masses μ_{S_i} smaller than Λ.

Q_0 and are treated as non-decoupled below $M_{\rm thresh}$. However, it turns out that including the higher loop effects (with coefficients $b_{1,2,3}$ found in refs. [34]) are quite important for obtaining a stable value of the G_X confinement scale Λ. This is illustrated in Table I, which shows the results obtained for $\Lambda^{(\ell)}_{\overline{\rm MS}}$ at various loop orders ℓ, assuming again that the effective decoupling scale for particles charged under G_X is $M_{\rm thresh} = 1$ TeV. The point of carrying the calculation to 4-loop order is *not* because of the very slightly increased accuracy obtained (since there are threshold uncertainties here that are not known), but rather to demonstrate the stability of the results with respect to inclusion of higher-order terms. In fact, the 4-loop order results for Λ hardly differ at all from the 3-loop order ones, and only at the 10% level from the 2-loop order ones. However, they are notably larger than the 1-loop order estimate, which is therefore judged to be deprecated as an estimate of the physical G_X confinement scale.

Table I shows that the confinement scale Λ for $SO(3)_X$ is very small in energy units. In terms of length, the confinement scale for the minimal model $n_S = 0$ is of the order 10^{11} meters, very roughly of order the radius of the Earth's orbit around the Sun. For $n_S = 1$, the confinement length is of order 100 parsecs. Thus for all practical purposes, the quirks are actually free. Adding other SM singlets charged under $SO(3)_X$ will only decrease Λ, making the confinement length even larger.

For the minimal viable $SU(2)_X$ and $SU(3)_X$ models, the confinement energy scale is much larger. Increasing n_S leads to smaller Λ, as indicated in Table I. If n of the n_S SM

G_X	$n_S = n$	$\alpha_X^{-1}(1\text{ TeV})$	$\Lambda^{(4)}$
$SU(2)$	1	14.4	5.0 MeV
	2	19.5	6.3 keV
	3	24.5	1.3 eV
	4	29.5	2.0×10^{-5} eV
$SU(3)$	3	4.9	68 GeV
	4	9.9	1.5 GeV
	5	15.0	13 MeV
	6	20.1	38 keV
	7	25.1	30 eV
	8	30.2	0.0030 eV
	9	35.2	2.1×10^{-8} eV

TABLE II: As in Table I, but now taking all $n_S = n$ of the SM singlet fermions to have current masses much less than Λ, and showing only the 4-loop order result (which is nearly identical to the 3-loop order result in all cases).

singlets charged under G_X have current masses μ_{S_i} less than Λ, the confinement scale will be decreased. This is illustrated in Table II for the extreme case that all $n = n_S$ of the new singlets are lighter than Λ. Both Tables I and II take the effective average decoupling scale for the particles in the other new chiral supermultiplets (including both scalars and fermions) to be 1 TeV. More generally one can estimate:

$$\Lambda = \Lambda_{\text{tab}} \left(\frac{M_{\text{thresh}}}{\text{TeV}} \right)^{1-\Delta}, \qquad \Delta = \frac{3C_A - 5 - n_S}{(11C_A - 2n)/3}, \tag{2.20}$$

where now Λ_{tab} is the value given in Table I for $n = 0$, or Table II for $n = n_S$. Here, M_{thresh} is defined to be the effective average decoupling scale for the new supermultiplets, at which the threshold corrections to the gauge coupling g_X are small. As seen in Figure 1, in the minimal viable models the g_X coupling runs fairly slowly in the non-decoupled theory above $Q = M_{\text{thresh}}$. This means that in the minimal model for $SU(2)_X$, $\Lambda \approx 0.001 M_{\text{thresh}}$, while for the minimal viable $SU(3)_X$ model with $n_S = 3$, $\Lambda \approx 0.12 M_{\text{thresh}}$, if $n = 0$.

If the Yukawa couplings k, k' are present, there is a potentially important constraint from precision electroweak observables. The new contributions to the Peskin-Takeuchi S, T observables from the new fermions are:

$$\Delta T = \frac{N v^4}{480\pi s_W^2 M_W^2 M_F^2} [13(\hat{k}^4 + \hat{k}'^4) + 2(\hat{k}^3 \hat{k}' + \hat{k}\hat{k}'^3) + 18\hat{k}^2 \hat{k}'^2], \tag{2.21}$$

$$\Delta S = \frac{N v^2}{30\pi M_F^2} [4\hat{k}^2 + 4\hat{k}'^2 - 7\hat{k}\hat{k}']. \tag{2.22}$$

where $\hat{k} = k \sin\beta$ and $\hat{k}' = k' \cos\beta$ and $v \approx 175$ GeV, and for illustration purposes I have chosen $\mu_L \approx \mu_S \gg m_Z$ and assumed that the corresponding scalars are much heavier. The values of these Yukawa couplings are governed by infrared quasi-fixed points. For example, if k' is negligible, then the beta functions for k and the top Yukawa coupling are given at one-loop order by:

$$Q\frac{dy_t}{dQ} = \beta_{y_t} = \frac{y_t}{16\pi^2}\left[6y_t^2 + y_b^2 + Nk^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right], \tag{2.23}$$

$$Q\frac{dk}{dQ} = \beta_k = \frac{k}{16\pi^2}\left[(3+N)k^2 + 3y_t^2 - 4Cg_X^2 - 3g_2^2 - \frac{3}{5}g_1^2\right], \tag{2.24}$$

where $N = 2$, 3, or 3 and $C = 3/4$, $4/3$, or 2 for $G_X = SU(2)$, $SU(3)$, or $SO(3)$ respectively. The fixed points arise due to the balancing between the positive Yukawa and the negative gauge contributions [35]. Including two-loop effects, I find for the minimal viable models the infrared quasi-fixed-point values:

$$k_{\text{fixed}} = \begin{cases} 0.88 & [SU(2)_X, n_S = 0] \\ 0.76 & [SO(3)_X, n_S = 0] \\ 1.32 & [SU(3)_X, n_S = 3]. \end{cases} \tag{2.25}$$

at $Q = 1$ TeV. The resulting contributions to S, T can be used to put a lower bound on μ_L. Requiring the results to be within the current 95% CL ellipse from experimental results on m_t, m_W, and Z-peak observables using the same methodology as in [11], I estimate $\mu_L > 210, 225, 380$ GeV for the $G_X = SU(2), SO(3), SU(3)$ fixed point cases respectively. However, G_X confinement may play a significant role in modifying this estimate for $SU(3)$, because Λ in that case is larger than M_Z. For smaller Yukawa couplings $k \ll k_{\text{fixed}}$, there is no constraint as the vector-like particles decouple from precision electroweak observables.

III. SOFT SUSY-BREAKING MASSES AND THE SPARTICLE SPECTRUM

The presence of new vector-like supermultiplets has a profound effect on the spectrum of superpartner masses. They cause the G_{SM} gauge couplings to run to much larger values in the ultraviolet as they approach unification, resulting in bigger one-loop contributions to soft scalar squared masses from RG running, compared to the MSSM. The new supermultiplets also allow the G_X gaugino masses to contribute indirectly to MSSM gaugino and sfermion masses, through two-loop order effects. In this section, the patterns of soft supersymmetry breaking masses will be considered for these models. For simplicity, the discussion will be mostly limited to the scenario in which a unified gaugino mass parameter $m_{1/2}$ is much larger than the scalar masses and other sources of supersymmetry breaking at the RG scale where the gauge couplings unify. This gaugino mass dominated limit is motivated as a solution to the supersymmetric flavor problem, since it automatically produces flavor-blind soft terms.

The modified running of the gaugino masses pushes them to be smaller near the TeV scale than they would be in the MSSM. Given an input unified gaugino mass $m_{1/2}$ at the

unification scale, one finds for the running gaugino masses at $Q = 1$ TeV:

$$(M_1,\ M_2,\ M_3)\ =\ m_{1/2} \times \begin{cases} (0.41,\ 0.77,\ 2.28) & [\text{MSSM}], \\ (0.21,\ 0.39,\ 1.16) & [SU(2),\ n_S = 0], \\ (0.112,\ 0.20,\ 0.57) & [SO(3),\ n_S = 0], \\ (0.080,\ 0.135,\ 0.40) & [SU(3),\ n_S = 3]. \end{cases} \tag{3.1}$$

Thus, in the extended models, to obtain the same physical gaugino masses, one must start with larger $m_{1/2}$ than one would in the MSSM. Since $m_{1/2}$ is not directly observable, it is also interesting to consider the ratios of these gaugino masses. They are also affected, but more mildly (being due to 2-loop effects):

$$(M_2/M_1,\ M_3/M_2,\ M_3/M_1)\ =\ \begin{cases} (1.87,\ 2.96,\ 5.53) & [\text{MSSM}], \\ (1.85,\ 2.95,\ 5.44) & [SU(2),\ n_S = 0], \\ (1.79,\ 2.84,\ 5.09) & [SO(3),\ n_S = 0], \\ (1.73,\ 2.85,\ 4.95) & [SU(3),\ n_S = 3], \end{cases} \tag{3.2}$$

where again unification of gaugino masses at the gauge coupling unification scale is assumed. The effect of the additional fields is thus to somewhat compress the gaugino mass spectrum compared to the MSSM case, with the ratio of gluino to bino masses decreased by about 10 per cent for $G_X = SO(3)$ and $SU(3)$. To obtain the physical masses, one must also include mixing with Higgsinos and the pole mass corrections, which are particularly important for the gluino [36–38].

In the extended models the squark and slepton masses are also relatively smaller (compared to $m_{1/2}$) at the TeV scale than in the MSSM. Taking a gaugino-mass dominated scenario (by assuming a vanishing common scalar squared mass $m_0^2 = 0$ at the unification scale), one finds for the first and second family squark and slepton masses at $Q = 1$ TeV:

$$(m_{\tilde{q}_1},\ m_{\tilde{u}_1},\ m_{\tilde{d}_1},\ m_{\tilde{\ell}_1},\ m_{\tilde{e}_1})\ =\ m_{1/2} \times \begin{cases} (2.15,\ 2.08,\ 2.07,\ 0.67,\ 0.37) & [\text{MSSM}], \\ (1.61,\ 1.55,\ 1.54,\ 0.57,\ 0.33) & [SU(2),\ n_S = 0], \\ (1.23,\ 1.18,\ 1.17,\ 0.48,\ 0.28) & [SO(3),\ n_S = 0], \\ (1.06,\ 1.02,\ 1.01,\ 0.43,\ 0.26) & [SU(3),\ n_S = 3]. \end{cases} \tag{3.3}$$

This shows that there is also a compression within the sfermion mass spectrum, as the ratio of the squarks to the slepton masses is decreased in the extended models compared to the MSSM in the $m_0^2 = 0$ limit, or more generally for any given value of m_0. This is because of the increased relative importance of the contribution to scalar masses from large renormalization scales where all of the gauge couplings and gaugino masses are larger.

Despite these compressions in the gaugino and sfermion sectors considered separately, the combined sparticle spectrum in the extended models is stretched rather than compressed compared to the MSSM. Comparing eqs. (3.1) and (3.3), one observes that in each of the extended models, the bino is much lighter than the lightest slepton, and so the lightest supersymmetric particle (LSP) will be a neutralino. This is in contrast to the well-known fact that the $m_0^2 = 0$ scenario in the MSSM problematically predicts a stau as the LSP. The

supersymmetry-breaking flavor problem thus can be naturally solved by taking $m_0^2 \ll m_{1/2}^2$ in the extended models without running into the difficulties found in the gaugino mass dominated MSSM.

Now consider the soft supersymmetry breaking masses for the new particles. Assuming gaugino mass unification, the G_X gaugino is heavier than all of the MSSM gauginos in the minimal $G_X = SU(2)$ and $SU(3)$ cases, but it is lighter than the MSSM gauginos if $G_X = SO(3)$. In terms of the unified gaugino mass parameter $m_{1/2}$, one finds at $Q = 1$ TeV:

$$M_{\widetilde{X}} \;=\; m_{1/2} \times \begin{cases} 1.30 & [SU(2),\, n_S = 0], \\ 0.051 & [SO(3),\, n_S = 0], \\ 0.65 & [SU(3),\, n_S = 3]. \end{cases} \tag{3.4}$$

[Compare eq. (3.1).] The scalar members of the $D, \overline{D}$ and $L, \overline{L}$ multiplets get RG contributions to their soft masses from both $G_{\rm SM}$ and G_X gaugino loops. Therefore, they are heavier than their MSSM counterparts with the same gauge quantum numbers. For the case where $m_{1/2}$ dominates, one finds approximately for the soft masses $m_{\widetilde{D}} = m_{\widetilde{\overline{D}}}$ and $m_{\widetilde{L}} = m_{\widetilde{\overline{L}}}$, again at $Q = 1$ TeV:

$$(m_{\widetilde{D}},\, m_{\widetilde{L}}) \;=\; m_{1/2} \times \begin{cases} (1.97,\, 1.40) & [SU(2),\, n_S = 0], \\ (1.22,\, 0.65) & [SO(3),\, n_S = 0], \\ (1.68,\, 1.51) & [SU(3),\, n_S = 3]. \end{cases} \tag{3.5}$$

Also, for the minimal $SU(3)$ case with $n_S = 3$, one finds that $m_{\widetilde{S}} = m_{\widetilde{\overline{S}}} = 1.48 m_{1/2}$. This is only slightly lower than $m_{\widetilde{D}}$ and $m_{\widetilde{L}}$, because most of the RG contribution to these soft masses comes from $\widetilde{X}$ loops in this case, which are the same for all of the new scalars.

The qualitative features of the above results are illustrated in Figure 2. The soft masses for the gauginos, the first-family sfermions, and the new scalars are shown for $Q = 1$ TeV. For purposes of comparison, $m_{1/2}$ is chosen so that the heaviest MSSM squark, $\tilde{q}$, has the same mass in each of the four cases. The mass spectra in the extended models are readily distinguishable from the usual "mSUGRA" case parameterized by $m_{1/2}, m_0, A_0$. This is because obtaining such a large ratio of scalar masses to gaugino masses in mSUGRA, would require a large m_0, which in turn would lead to a much more compressed scalar mass spectrum. In contrast, the extended models are characterized by relatively heavy scalars which nevertheless maintain a significant hierarchy between squarks, left-handed sleptons, and right-handed sleptons, especially in the $G_X = SO(3)$ and $SU(3)$ cases.

In order to keep the discussion bounded, I will not give detailed results on the extended models with more singlets [$n_S > 0$ for $G_X = SU(2)$ and $SO(3)$, and $n_S > 3$ for $G_X = SU(3)$]. However, the following qualitative features are notable. First, at one loop order, the presence of additional $G_{\rm SM}$ singlets does not affect the RG running of MSSM-field soft terms, so the effects are rather mild on the gluino, wino, bino, and MSSM squark and slepton masses. Second, increasing n_S will decrease both the g_X gauge coupling and the G_X gaugino soft mass at lower RG scales. Therefore, the G_X gaugino mass $M_{\widetilde{X}}$ will be smaller compared to the MSSM gaugino masses M_1, M_2, and M_3 than in the cases shown in Figure 2. Also, the

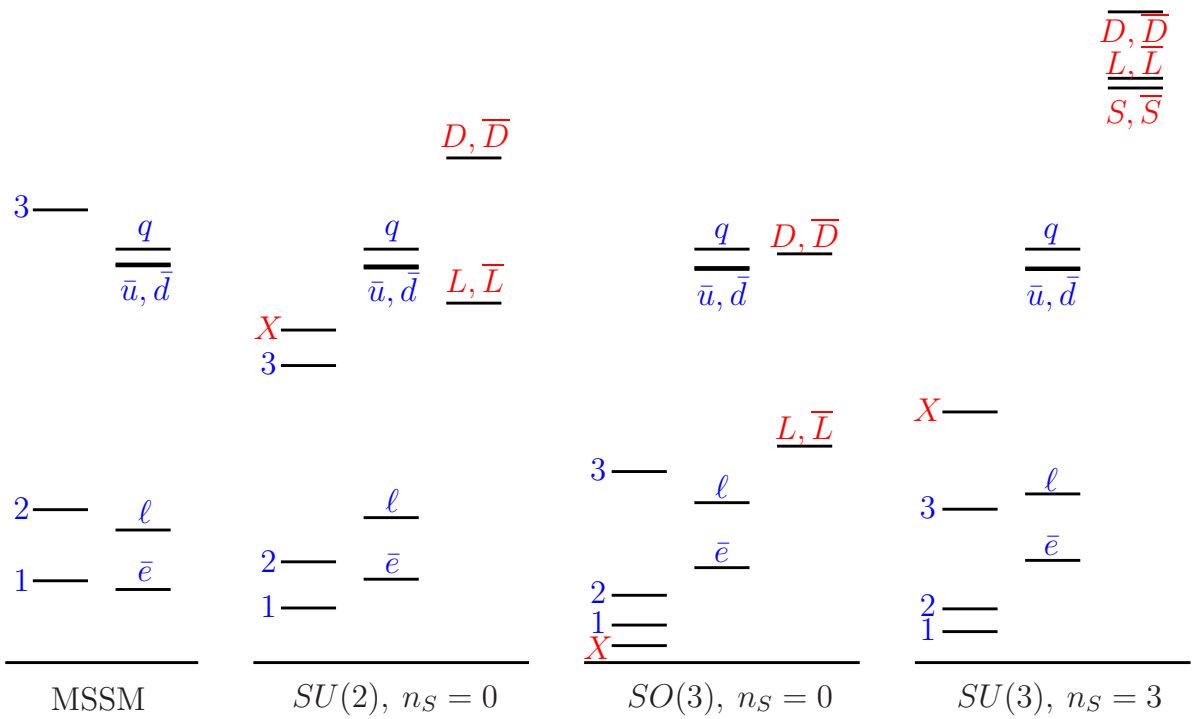

FIG. 2: Comparison of the soft supersymmetry-breaking mass spectra following from a unified input gaugino mass $m_{1/2}$ that dominates at the unification scale, for the MSSM and the minimal models with $G_X = SU(2), SO(3)$, and $SU(3)$. The labels $1, 2, 3, X$ refer to running gauginos masses for $U(1)_Y$, $SU(2)_L$, $SU(3)_c$ and G_X, respectively. The labels $q, \bar{u}, \bar{d}, \ell, \bar{e}$ refer to MSSM first family squark and slepton soft masses, and the soft scalar masses for vectorlike supermultiplets are labeled with symbols D, $\overline{D}$, L, $\overline{L}$, and (for the $G_X = SU(3)$ case with $n_S = 3$), $S, \overline{S}$. The value of $m_{1/2}$ is chosen so that the heaviest MSSM squark, $\tilde{q}$, has the same mass in each of the four cases.

soft masses $m_{\widetilde{D}} = m_{\widetilde{\overline{D}}}$ and $m_{\widetilde{L}} = m_{\widetilde{\overline{L}}}$ will become relatively smaller, tending towards the MSSM squark and slepton masses $m_{\tilde{d}}$ and $m_{\tilde{\ell}}$ respectively. The soft masses for $m_{\widetilde{S}} = m_{\widetilde{\overline{S}}}$ will also decrease for larger n_S, although they are always heavier than $M_{\tilde{X}}$, which decreases faster for larger n_S. For $n_S > 0$, the Yukawa couplings k, k' can also come into play, decreasing the $\widetilde{L}$, $\widetilde{\overline{L}}$, $\widetilde{S}$, and $\widetilde{\overline{S}}$ soft supersymmetry breaking masses.

IV. THE μ PARAMETER AND THE LITTLE HIERARCHY PROBLEM

The supersymmetric little hierarchy problem is a subjective but inspirationally important puzzle which questions the naturalness of viable model parameters. The essence of it is that once one applies constraints from the non-observation of a Higgs boson and of superpartners at both LEP2 and the Tevatron, the actual value of m_Z might be considered surprisingly low for generic soft supersymmetry breaking parameters.

The largest loop correction to the h^0 mass in the MSSM is given by (in the decoupling limit $m_{A^0}^2 \gg m_{h^0}^2$):

$$m_{h^0}^2 \;=\; m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} \sin^2\beta \, y_t^2 \Big[m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2\right) + c_{\tilde{t}}^2 s_{\tilde{t}}^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)$$

$$+c_{\tilde{t}}^4 s_{\tilde{t}}^4 \Big\{ (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2 - \frac{1}{2}(m_{\tilde{t}_2}^4 - m_{\tilde{t}_1}^4) \ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2) \Big\} / m_t^2 \Big], \tag{4.1}$$

where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the top squark masses, and $c_{\tilde{t}}$ and $s_{\tilde{t}}$ are the cosine and sine of the top squark mixing angle $\theta_{\tilde{t}}$. Now in the models discussed in [8] and this paper, adding in the effects of the Yukawa coupling k one finds the further estimated correction in the case $\mu_L \approx \mu_S \approx M_F$ with heavier scalars with masses of order M_S [8]:

$$\Delta m_{h^0}^2 = \frac{N}{4\pi^2} k^4 v^2 \sin^4\beta \left[f(x) + \frac{X_k^2}{M_S^2}(1 - \frac{1}{3x}) - \frac{X_k^4}{12 M_S^4} \right]. \tag{4.2}$$

Here $x = M_S^2/M_F^2$ is the ratio of the average new scalar and new fermion masses in the $L, \overline{L}, S, \overline{S}$ sector, and $X_k = a_k/k - \mu \cot\beta$ is a mixing parameter for the scalars. The largest possible contributions come from the maximal (fixed-point) values of eq. (2.25). As was pointed out in ref. [8], for $G_X = SU(3)$ eq. (4.2) is enough to raise the Higgs mass by tens of GeV, depending on the details of the fermion and scalar masses in the new sector.

From the point of view of the supersymmetric little hierarchy problem, even raising the Higgs mass by a few GeV is potentially helpful. However, one must also consider the effect of the new sector on the scalar potential. The minimization of the Higgs potential in supersymmetry results in:

$$m_Z^2 = -2(|\mu|^2 + m_{H_u}^2) - \frac{1}{v_u}\frac{\partial}{\partial v_u}\Delta V + \mathcal{O}(1/\tan^2\beta), \tag{4.3}$$

where ΔV is the radiative part of the effective potential, with $\tan\beta = v_u/v_d = \langle H_u^0 \rangle / \langle H_d^0 \rangle$ and v_u treated as a real variable in the partial differentiation. In general, without further theoretical structure, μ and $m_{H_u}^2$ have no reason to be related, since μ is a supersymmetry-preserving parameter and $m_{H_u}^2$ is supersymmetry-breaking. In the MSSM with generic parameters, one finds that $-m_{H_u}^2$ tends to be much larger than m_Z^2, and eq. (4.3) seems to imply a percent-level fine-tuning of the difference between $|\mu|^2$ and $m_{H_u}^2$.

It is not possible to rigorously quantify fine tuning, since there can be no such thing as an objective measure on parameter space. Nevertheless, qualitative trends can be identified, and an obvious approach is to consider models with smaller predicted values of $-m_{H_u}^2$ at the weak scale to be more likely than those with very large $-m_{H_u}^2$, because then the fractional tuning required between it and $|\mu|^2$ will be less. This in turn means that smaller values of $|\mu|$ are more likely than very large values, since this is determined by eq. (4.3).

With this in mind, it is interesting to consider how the MSSM and its extensions, and variations of the most popular models of supersymmetry breaking, affect the weak-scale predictions for $-m_{H_u}^2$. For example, in the MSSM with $\tan\beta = 10$ and $m_t = 173.3$ GeV, one finds from RG running at $Q = 1$ TeV in terms of the GUT-scale input parameters $m_{1/2}$, A_0 and m_0:

$$-m_{H_u}^2 = 1.65 m_{1/2}^2 - 0.40 m_{1/2} A_0 + 0.11 A_0^2 - 0.022 m_0^2. \tag{4.4}$$

This formula shows that $-m_{H_u}^2$, and therefore $|\mu|^2$, and therefore the level of fine-tuning required, increase with the gaugino squared masses. In extended models the gaugino masses

at the unification scale have a varying relationship with the gaugino masses at the weak scale, which are more closely related to the physical masses, so it is useful to reformulate this in terms of the running gluino mass parameter M_3 also evaluated at $Q = 1$ TeV:

$$-m_{H_u}^2 = 0.32M_3^2 - 0.18M_3A_0 + 0.11A_0^2 - 0.022m_0^2, \qquad \text{[MSSM]}. \tag{4.5}$$

In fact, most of the dependence on the gaugino masses comes from the gluino mass [39], so this formula is approximately valid even for moderate deviations from gaugino mass universality. One can note that for a gluino mass of order 500 GeV, and small $|A_0|$, $-m_{H_u}^2$ is only of order $(280\text{ GeV})^2$, so that the tuning needed to get m_Z^2 in eq. (4.3) is of order 5%. The problem is that (although there is still considerable variation among models, particularly for large $|A_0|$) lower values of M_3 typically give a prediction for m_{h^0} that is smaller than 114 GeV, and higher values of M_3 require even more delicate cancellation between $-m_{H_u}^2$ and $|\mu|^2$. The "focus point" region [40, 41] occurs due to the small negative coefficient of m_0^2 in eq. (4.5), which allows a cancellation between the M_3^2 and m_0^2 terms for very large m_0^2, leading to a small value of $-m_{H_u}^2$ and therefore small $|\mu|^2$. However, this also can be judged to be fine-tuned, as the large value of m_0^2 has to be finely adjusted, given a value of the ostensibly independent parameter M_3.

We can now compare with the situation for the models in the present paper. In the minimal $SU(2)_X$ model with $n_S = 0$, one finds instead from the RG running:

$$-m_{H_u}^2 = 1.07M_3^2 - 0.44M_3A_0 + 0.12A_0^2 - 0.16m_0^2, \qquad [SU(2)_X,\ n_S = 0]. \tag{4.6}$$

again for $-m_{H_u}^2$ and M_3 evaluated at $Q = 1$ TeV, assuming gaugino and scalar mass universality, $\tan\beta = 10$ and $m_t = 173.3$ GeV. (The coefficients change, but not very radically, for larger n_S.) The larger coefficient of M_3^2 indicates that this is naively even more fine-tuned than the MSSM. However, this effect is not without compensation; as Figure 2 shows, one does not need as large a gluino mass to get large squark masses, which in turn lead to large positive contributions to $m_{h^0}^2$ from eq. (4.1). Also, the larger negative coefficient of m_0^2 means that the analog of the MSSM focus point region occurs at much smaller values of m_0^2 in this extended model. Since there is no such thing as an objective quantitative measure of fine-tuning, I choose not to attempt to make a definitive statement beyond observing the competing factors just mentioned.

For $SO(3)_X$ with $n_S = 0$, the analogous formula becomes:

$$-m_{H_u}^2 = 3.36M_3^2 - 0.91M_3A_0 + 0.13A_0^2 - 0.26m_0^2, \qquad [SO(3)_X,\ n_S = 0]. \tag{4.7}$$

Similarly, for the minimal viable $SU(3)_X$ model with $n_S = 3$ and small Yukawa couplings $k, k' \approx 0$, the analogous formula becomes:

$$-m_{H_u}^2 = 5.97M_3^2 - 1.28M_3A_0 + 0.13A_0^2 - 0.28m_0^2, \qquad [SU(3)_X,\ n_S = 3]. \tag{4.8}$$

In both of these cases, the situation again seems subjectively worse with respect to fine-tuning than the MSSM, due to the much larger coefficient of M_3^2. As shown in Figure 2, one does naturally get much larger squark masses for a given M_3, again leading to larger radiative corrections to $m_{h^0}^2$. However, with these large coefficients to M_3, the direct (but model

dependent) constraints on the gluino mass from Tevatron come into play in a significant way. Again, there is a chance for more cancellation between the gaugino and scalar contributions due to the negative coefficient of m_0^2.

It is also interesting to consider the situation for $n_S \geq 1$ with large Yukawa couplings near the fixed points of eq. (2.25). For $SU(2)_X$ with $n_S = 1$ and the fixed-point value $k = k_{\rm fixed} = 0.88$ at $Q = 1$ TeV, one finds:

$$-m_{H_u}^2 = 1.28 M_3^2 - 0.15 M_3 A_0 + 0.04 A_0^2 + 0.72 m_0^2, \qquad [SU(2)_X,\ n_S = 1,\ k = k_{\rm fixed}]. \quad (4.9)$$

Here the coefficient of the gaugino mass squared is even larger than for $k = 0$ [compare eq. (4.6)], and the coefficient of the scalar squared mass m_0^2 is large and positive, eliminating the possibility of cancellation to achieve a smaller $-m_{H_u}^2$. Similar results obtain for $SO(3)_X$ with $n_S = 1$ and $k = k_{\rm fixed} = 0.76$ at $Q = 1$ TeV:

$$-m_{H_u}^2 = 3.46 M_3^2 - 0.27 M_3 A_0 + 0.05 A_0^2 + 0.89 m_0^2, \qquad [SO(3)_X,\ n_S = 1,\ k = k_{\rm fixed}], \quad (4.10)$$

and for $SU(3)_X$ with $n_S = 3$ and one $k = k_{\rm fixed} = 1.32$ at $Q = 1$ TeV:

$$-m_{H_u}^2 = 17.0 M_3^2 - 0.27 M_3 A_0 + 0.02 A_0^2 + 0.71 m_0^2, \qquad [SU(3)_X,\ n_S = 3,\ k = k_{\rm fixed}]. \quad (4.11)$$

Therefore, even though the fixed-point Yukawa coupling can give large positive contributions to $m_{h^0}^2$, there is a quite detrimental effect on the fine-tuning needed to obtain the observed m_Z in models that have heavy enough gluinos (and charginos) to have evaded discovery at the Tevatron and LEP2. Similar effects have been noted before in the case of vector-like fermions without an additional gauge group in refs. [10], [11], [13].

Qualitatively, the model with $SU(2)_X$ and no new Yukawa coupling seems to be the least fine-tuned of the extended models. Adding new Yukawa couplings, despite increasing m_{h^0}, does not clearly alleviate the little hierarchy problem, and arguably make it much worse, especially in the cases of $SO(3)_X$ and $SU(3)_X$.

V. COLLIDER PHENOMENOLOGY OF THE QUIRKS

In this section, I will consider some features of the phenomenology of the quirks in the models discussed above, following for the most part general ideas and results from refs. [18], [19], [20], and [21]. For simplicity, I will consider only the fermions from the $D, \overline{D}$, $L, \overline{L}$, multiplets, and not their scalar partners. This is because supersymmetry breaking effects provide for the scalars ("squirks") and the G_X gaugino to have much larger masses, making them less immediately relevant for collider searches. (Even if they had the same masses, squirks would have much smaller production cross-sections than fermionic quirks. The G_X gauginos will not be produced directly in tree-level processes at colliders at all.) When produced, the squirks will decay promptly to quirks and MSSM gauginos. The G_X gaugino can undergo a three-body decay to a quirk, antiquirk and MSSM gaugino, if kinematically allowed. In this section, I will use the same symbols for the fermions as for the chiral supermultiplets to which they belong.

I will also assume for simplicity, and motivated by the results of the previous section, that the mixing of the $G_{\rm SM}$ singlets $S, \overline{S}$ with the doublets $L, \overline{L}$ due to the Yukawa couplings k, k' is small in most of the following. This implies that $S, \overline{S}$ decouple from collider phenomenology. The charged $(L^-, \overline{L}^+)$ and neutral $(L^0, \overline{L}^0)$ fermions form two Dirac fermion-antifermion pairs, each with tree-level mass μ_L. However, radiative corrections split the masses slightly, with $\Delta m \equiv m_{L^-} - m_{L^0}$ always positive. (If present, the Yukawa interactions k, k' that cause mixing with $S, \overline{S}$ would increase this splitting, so the lightest non-colored fermion is always neutral.) One finds $\Delta m > 270$ MeV for $\mu_L > 100$ GeV, with Δm approaching 355 MeV asymptotically for large μ_L [42]. This means that the decays

$$\overline{L}^+ \to \overline{L}^0 \ell^+ \nu_\ell, \;\; \overline{L}^0 \pi^+, \tag{5.1}$$

$$L^- \to L^0 \ell^- \overline{\nu}_\ell, \;\; L^0 \pi^-, \tag{5.2}$$

(and decays to more pions or other SM hadrons if non-zero k, k' increase Δm) mediated by the W boson are always kinematically allowed, and will occur with decay lengths of order centimeters [42] due to the small available kinematic phase space. Therefore, the lifetime of $\overline{L}^+, L^-$ is large compared to other processes to be discussed below; in particular they will form quirk-antiquirk bound states and annihilate before they decay. In the simplest scenario (barring additional couplings to be described in the next paragraph), the neutral quirk Dirac fermions $L^0, \overline{L}^0$ are completely stable, as are the colored quirks $D, \overline{D}$ with charges $\pm 1/3$. Note that none of the quirks can mix with the Standard Model fermions because of G_X conservation, so the lightest quirk is always stable. Such stable fermions could present a challenge for the standard cosmology with a high reheat temperature but need not be a disaster [43],[44],[18].

If $n_S \geq 1$ and the pairs $L, \overline{L}$ and $S, \overline{S}$ have the opposite matter parity from each other, then the Yukawa couplings k, k' are forbidden, but the superpotential term

$$W = \lambda_\ell \overline{S} \overline{L} \ell \tag{5.3}$$

is allowed, with ℓ an MSSM $SU(2)_L$ doublet lepton. This provides additional possible decay modes $\overline{L}^+ \to S \tilde{\ell}_L^+$ and $L^- \to \overline{S} \tilde{\ell}_L^-$, if $|\mu_L| > |\mu_S| + m_{\tilde{\ell}_L}$ so that these decays are kinematically allowed, or alternatively with the sleptons off-shell. These decays are not automatically kinematically suppressed, and so could happen promptly before quirk-antiquirk annihilation occurs. The fermions D and $\overline{D}$ may also have additional decay modes, if the possible superpotential terms

$$W = \lambda_q L \overline{D} q + \lambda_{\overline{d}} \overline{S} D \overline{d}, \tag{5.4}$$

are present. The first of these terms is only allowed if G_X is either $SU(2)$ or $SO(3)$, and if $L, \overline{L}$ have the opposite matter parity of $D, \overline{D}$ (if matter parity is conserved). It permits squark exchange to mediate the decays

$$D \to L^0 \tilde{d}_L, \quad L^- \tilde{u}_L, \tag{5.5}$$

$$\overline{D} \to \overline{L}^0 \tilde{d}_L^*, \quad \overline{L}^+ \tilde{u}_L^*, \tag{5.6}$$

with the MSSM squarks possibly off-shell due to kinematics. The second term in eq. (5.4) is only allowed if $n_S \geq 1$ and $S, \overline{S}$ have the opposite matter parity of $D, \overline{D}$. Then MSSM squark exchange can mediate the decays

$$D \to S\tilde{d}_R, \qquad \overline{D} \to \overline{S}\tilde{d}^*_R, \tag{5.7}$$

again with the squarks possibly off-shell. Whether these decays can be important depends on the kinematics as well as the size of λ_ℓ, λ_q, and $\lambda_{\overline{d}}$. For simplicity, they will be assumed to be absent or at least too small to make a difference below, except where noted otherwise.

The important direct pair-production processes for the new fermions are

$$pp \to D\overline{D}, \tag{5.8}$$
$$pp \to Z^{(*)}, \gamma^{(*)} \to \overline{L}^+ L^-,\ \overline{L}^0 L^0, \tag{5.9}$$
$$pp \to W^{+(*)} \to \overline{L}^+ L^0, \tag{5.10}$$
$$pp \to W^{-(*)} \to \overline{L}^0 L^-, \tag{5.11}$$

for the LHC, with the obvious substitution of $p\overline{p}$ for the Tevatron. Pair-produced quirks with masses much larger than Λ will move apart from each other with typically semi-relativistic speeds, and as described in [18], will be connected by G_X flux strings with tension σ. From the lattice, there is an estimate (see Table 7 and eq. (11) of [45]):

$$\sqrt{\sigma} = \Lambda_{\overline{\rm MS}} \times \begin{cases} 1.73 & SU(2)_X, \\ 1.86 & SU(3)_X, \end{cases} \tag{5.12}$$

so that the maximum string length in a given hard scattering event with kinetic energy ΔE in the center-of-momentum frame is

$$L = \frac{\Delta E}{\sigma} \approx 6\,\text{mm} \left(\frac{\Delta E}{100\ \text{GeV}}\right) \left(\frac{\text{keV}}{\Lambda_{\overline{\rm MS}}}\right)^2. \tag{5.13}$$

Therefore, the lengths of such strings, although much larger than Λ^{-1}, will typically be less than 1 mm for Λ greater than a few keV. From Table I, one finds that the quirky flux strings will be microscopic for $SU(2)_X$ with $n_S \leq 3$ and for $SU(3)_X$ with $n_S \leq 9$, assuming that g_X is unified with the SM gauge couplings and all singlets charged under $SU(N)_X$ are heavier than Λ.

For the case of $G_X = SO(3)$, the situation is quite different, because from Table I the confinement distance scale Λ^{-1} is literally astronomical, at least of order the Earth's orbit around the Sun even in the minimal model. The quirks in this case are essentially free particles with multiplicity 3 times larger than expected from their SM quantum numbers. Note that even if one rejected the unification of g_X with the SM gauge couplings in this model to arrive at a much larger Λ, the fact that the supermultiplets are in the adjoint representation of the Lie algebra means that they would not form stable flux tubes of the type discussed in [18] when pair-produced, even if any $S, \overline{S}$ fields are heavier than Λ (so $n = 0$). Instead, pair-produced particles charged under $SO(3)_X$ would each bind to a gauge

boson to form two stable G_X-singlet states with size of order Λ^{-1}, allowing the flux tube to break. Although the new fermions behave like free stable particles when pair-produced at colliders, the fact that they will come in three-fold exactly degenerate multiplets will in principle allow a determination of their nature from their production cross-sections. In the simplest case, $D, \overline{D}$ and $\overline{L}^0, L^0$ will be absolutely stable, with $\overline{L}^+, L^-$ having decays to $\overline{L}^0, L^0$ via soft pion or lepton emission as discussed above. The $\overline{L}^0, L^0$ are only weakly interacting and thus invisible, but known collider search strategies [46] for stable strongly interacting particles apply for $D, \overline{D}$. However, as noted above, $D, \overline{D}$ may be able to promptly decay according to eqs. (5.5)-(5.7), depending on both kinematics and the allowed superpotential terms. If so, then the signatures will always contain $E_T^{\rm miss}$, and will resemble those for ordinary MSSM squarks.

For the remainder of this section, consider the cases of $SU(2)_X$ and $SU(3)_X$, with the G_X confinement scale less than the masses of the quirks that have SM gauge interactions, and stable microscopic flux strings joining the quirk-antiquirk pairs. The quirk-antiquirk pair will then form an exotic bound state with invariant mass given approximately by the total center-of-momentum energy of the hard partonic scattering that produced them. This quirk-antiquirk string state can lose energy either by G_X-glueball emission, by radiation of many soft photons, or in the case of the $D\overline{D}$ state by radiation of numerous soft pions, a "hadronic fireball" [18]. The large multiplicity of soft pions or photons may be detectable as anomalous "underlying events" [18, 21] that accompany the hard scattering production, and may be used as an additional tag to dramatically reduce backgrounds.

If the quirk and antiquirk lose most of their initial relative kinetic energy before annihilating, they will briefly form a "quirkonium" bound state which then decays to two or three hard partons with invariant mass peaked at twice the mass of the quirk [18]. Alternatively, however, the neutral and colorless quirk and antiquirk states might [18] have a prompt annihilation before they can lose enough energy to form a low-lying quirkonium state. In that case, the final states will have a broad distribution of annihilation products, which will therefore be much harder to discern above hadron collider backgrounds. It is difficult to estimate in advance what proportion of the events will fall into these two categories, due to the non-perturbative nature of the energy loss mechanisms, which do not have direct analogs in experimentally known hadronic physics.

For the weakly interacting quirks, and for the strongly interacting quirks if $\Lambda > \Lambda_{\rm QCD}$, one might suspect the non-perturbative interactions by which the quirk-antiquirk string state loses energy to be dominated by G_X-glueball emission. However, this is quite uncertain, and can be suppressed or even eliminated by kinematics if $\Lambda \gg \Lambda_{\rm QCD}$. The masses of the G_X glueballs have been estimated by lattice computations [47]-[49], [45], with the results for the lightest two glueball states with $J^{PC} = 0^{++}$ and 2^{++}:

$$m_{0^{++}} = 6.7\Lambda_{\overline{\rm MS}}, \tag{5.14}$$

$$m_{2^{++}} = 9.6\Lambda_{\overline{\rm MS}}. \tag{5.15}$$

There are other heavier glueball states 0^{++*}, 3^{++}, 0^{-+}, 2^{-+}, 0^{-+*}, 2^{-+*}, and, for $SU(3)_X$ only there are also states with odd C, 1^{+-}, 3^{+-}, 2^{+-}, 0^{+-}, 1^{--}, 2^{--} 3^{--}, with masses ranging up to about $3m_{0^{++}}$. As can be seen from Tables I and II, in the case of $SU(3)_X$

these glueballs should have masses in the hundreds of GeV range for the minimal case of $n_S = 3$ and so could be comparable in mass or even heavier than the lighter quirks, and should be in the tens of GeV range for $n_S = 4$. This would prohibit energy loss of the quirk-antiquirk flux string states into G_X-glueballs. For the other cases listed in Table I, decays of the flux strings to G_X-glueballs should be allowed, but perhaps kinematically suppressed, leading to considerable uncertainty in the number of G_X-glueball states emitted and the likelihood of the quirk-antiquirk string state to lose most of its energy before annihilating. It is also possible that a few G_X-glueballs will be produced in the original hard scattering production.

If produced, the detection of G_X-glueballs is problematic. Their decay widths can be estimated for $SU(3)_X$ using eqs. (17), (23) and (30) of ref. [23] (see also ref. [26]) with matrix elements from eqs. (38) and (62) of ref. [50]:

$$\Gamma(0^{++} \to gg) = 360\, \alpha_S^2 \Lambda^9/\mu_D^8, \tag{5.16}$$
$$\Gamma(2^{++} \to gg) = 0.12\, \alpha_S^2 \Lambda^9/\mu_D^8, \tag{5.17}$$
$$\Gamma(0^{-+} \to gg) = 24\, \alpha_S^2 \Lambda^9/\mu_D^8. \tag{5.18}$$

The results for $SU(2)_X$ should be comparable and slightly smaller. This leads to proper decay lengths for G_X-glueballs of order

$$c\tau = \left(\frac{0.2}{\alpha_S}\right)^2 \left(\frac{\mu_D}{100\ \text{GeV}}\right)^8 \left(\frac{\text{GeV}}{\Lambda}\right)^9 \times \begin{cases} 0.14\,\text{meters} \quad (\text{for } 0^{++}), \\ 400\,\text{meters} \quad (\text{for } 2^{++}), \\ 2\,\text{meters} \quad (\text{for } 0^{-+}). \end{cases} \tag{5.19}$$

If $\Lambda \approx 1$ GeV as expected for the minimal $SU(2)_X$ model, a sizable fraction of the 0^{++} decays might occur within the detector, but only if μ_D is less than roughly 150 GeV, which may be ruled out already by Tevatron data (see below). For larger μ_D or smaller Λ, the decays of the G_X-glueball will occur outside of the detector and will be invisible. For much larger Λ as occurs in the $SU(3)_X$ model with $n_S = 3$ or 4, the decays may occur within the detector for any μ_D, but then the production of G_X glueballs in the flux-tube energy loss processes is likely irrelevant anyway due to kinematic suppression or prohibition. Even if the G_X glueballs are produced and decay promptly, the main decay is likely to a pair of gluons, and the resulting dijet mass peak signal from these decays will have to compete with a huge background from QCD. To have a significant branching fraction to $\gamma\gamma$, which has much smaller backgrounds, one can take $\mu_L < \mu_D$, with a leading-order estimate [23, 26]:

$$\frac{\text{BR}(0^{++} \to \gamma\gamma)}{\text{BR}(0^{++} \to gg)} = \frac{8\alpha^2}{9\alpha_S^2} \left(\frac{1}{4} + \frac{3\mu_D^4}{4\mu_L^4}\right)^2. \tag{5.20}$$

For example, with $\alpha_S \approx 0.2$, this ratio is of order 0.001 for $\mu_L = \mu_D$, but it rises to about 0.18 if $\mu_L/\mu_D = 0.5$, and is greater than 1 if μ_L/μ_D is less than 0.4. The non-perturbative nature of the G_X glueball production mechanisms means that the diphoton signal strengths, if any,

are extremely difficult to estimate even roughly, but to have even a hope of observation would seem to require $\mu_L < \mu_D$ and Λ of order a few GeV (not too small for $c\tau$ to be large, but not too large for G_X-glueball production to be kinematically suppressed). Nevertheless, given the uncertainties involved, this possibility highlights the general importance of searching for narrow diphoton peaks at large invariant masses at the LHC; this type of signal could also arise not only for the classic diphoton signal for a low-mass Higgs scalar boson, but also for stoponium [51, 52] or for Kaluza-Klein gravitons in theories with low-scale gravity [53].

Probably the most optimistic scenario for detecting the quirks occurs in the case that $D\overline{D}$ are strongly produced at a hadron collider and manage to lose most of their initial relative kinetic energy stored in the G_X flux tube by radiating soft pions and/or G_X glueballs, arriving at a low-lying quirkonium state with mass $\approx 2\mu_D$ before finally annihilating in a color-singlet S-wave ${}^{2S+1}L_J = {}^1S_0$ (η) or ${}^3S_1$ (ψ) state. The most promising channel for detecting the quirkonium peak is $\mu^+\mu^-$. The relevant annihilation decay widths for a ${}^3S_1$ state can be inferred from refs. [54] and [20]:

$$\Gamma(\psi \to f\overline{f}) = 4\alpha^2 e_D^2 N_c^f \beta_f \left[(1+2R_f)\Big(e_f - \frac{g_f^V}{c_W^2(1-R_Z)}\Big)^2 + \Big(\frac{\beta_f g_f^A}{c_W^2(1-R_Z)}\Big)^2\right]\Gamma_0, \quad (5.21)$$

$$\Gamma(\psi \to W^+W^-) = \frac{\alpha^2 e_D^2 \beta_W^3}{4c_W^4}\frac{1+20R_W+12R_W^2}{(1-R_Z)^2}\Gamma_0 \quad (5.22)$$

$$\Gamma(\psi \to ggg) = \frac{40\alpha_S^3}{81\pi}(\pi^2-9)\Gamma_0 \quad (5.23)$$

$$\Gamma(\psi \to XXX) = \frac{\alpha_X^3}{3\pi}\frac{(N^2-1)(N^2-4)}{N^3}(\pi^2-9)\Gamma_0 \quad (5.24)$$

where $e_f = (2/3, -1/3, -1, 0)$ and $N_c^f = (3,3,1,1)$ and $g_f^A = (1/4, -1/4, -1/4, 1/4)$ for $f = (u, d, e, \nu)$ respectively, and $g_f^V = g_f^A - e_f s_W^2$, and s_W and c_W are the sine and cosine of the weak mixing angle, and $e_D = -1/3$, and $R_i = m_i^2/M^2$, where $M \approx 2\mu_D$ is the quirkonium mass, and $\beta_i = \sqrt{1-4R_i^2}$, and Γ_0 is a common normalization proportional to the square of the wavefunction at the origin. The G_X gluon is represented by X. Note that final states gg, ZZ, $Z\gamma$, $\gamma\gamma$ and XX do not occur in ψ decays. The final states Zgg, γXX, ZXX do occur, but with branching ratios that turn out to be very small. For $N = 2$, the decay to three $SU(2)_X$ gauge bosons vanishes due to the N^2-4 factor, and for $SU(3)_X$ with $N = 3$ the XXX decay will be kinematically forbidden or at least highly suppressed by the large masses of the G_X glueballs that would have to be the final result of G_X-hadronization. Therefore final states involving G_X glueballs should not play a significant role in quirkonium decays.

For ${}^1S_0$ states, the dominant decay is to gg or XX, and $f\overline{f}$ does not occur at all at leading order. If we assume that the spin state is randomized by the non-perturbative processes that lose the initial relative kinetic energy, so that ψ and η states are populated in the ratio of 3 to 1, then the branching ratio of quirkonium to leptons should be given by 3/4 of the branching ratio indicated by eqs. (5.21)-(5.23). Numerically this yields[†] $\mathrm{BR}(D\overline{D} \to \mu^+\mu^-) = 0.093$

[†] The estimate in section 5.6 of [18] is parametrically different, and numerically smaller by a factor ∼5.

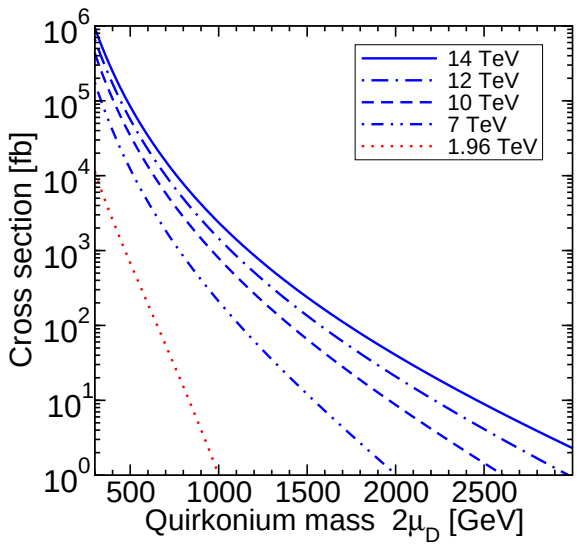

FIG. 3: The production cross-section for $D\overline{D}$ in pp collisions at $\sqrt{s}$ = 14, 12, 10, and 7 TeV, and in $p\overline{p}$ collisions at $\sqrt{s}$ = 1.96 TeV, obtained at leading order using CTEQ5LO [57] parton distribution functions (PDFs) with $Q = \mu_D$.

very nearly independent of the mass. It should be noted that this branching ratio does not apply to prompt annihilation of the quirks before they have settled into a color- and G_X-singlet quirkonium state; that branching ratio will be much smaller, and will not lead to a sharp dimuon peak, and so leads to a more pessimistic case.

In the most optimistic case that most of the $D\overline{D}$ states annihilate after losing most of their excess energy, there are good prospects for detection at hadron colliders, because the signal production is strong and peaked in invariant mass, while the dominant background is electroweak (Drell-Yan) and diffuse. The total production cross section at the Tevatron and at various LHC energies is shown in Figure 3. The CDF collaboration has published [55] a limit on cross-section times branching ratio for new states that decay to $\mu^+\mu^-$, based on 2.3 fb^{-1} of $p\overline{p}$ collisions at the Tevatron. Comparing the relevant spin-0 limit from Figure 3 in [55] to the results shown in Figure 3 of the present paper and using the estimate BR$(\mu^+\mu^-)$ = 0.093 from above, I obtain the lower mass bound $\mu_D > 375$ GeV in this optimistic case.

At the LHC, the invariant mass resolution for high-mass dimuons should be of the order of 5% [56] for the CMS detector. Therefore, as a rough estimate of the discovery reach, I consider a mass window from $0.9M$ to $1.1M$ where $M \approx 2\mu_D$ is the quirkonium mass, and require that $S/\sqrt{B}$ exceeds 5 in that window, where S is the number of signal events (which is also required to exceed 10) and B is the expected number of Drell-Yan background events. The Drell-Yan background cross-section is shown in Figure 4. Trigger and detector efficiencies are not included, but these are expected to be very high for high-mass dimuon events, and the QCD K-factor for the signal is not included. Dimuon backgrounds from sources other than Drell-Yan can be suppressed by requiring no extra hard jets or missing energy. In the following, I will again assume a spin-averaged BR$(\mu^+\mu^-) = 0.093$ for the signal. There is also a potential confirming signal from annihilation to e^+e^-, with an invariant mass peak that is similar but wider and smaller due to detector resolution and efficiency effects.

For a 1 fb^{-1} LHC run at $\sqrt{s}$ = 7 TeV, the signal cross-section in Figure 3 yields 20 expected dimuon events for μ_D = 500 GeV, and as shown in Figure 4 there is about 1 background event expected in the corresponding mass window $M(\mu^+\mu^-) = 1000 \pm 100$ GeV. Requiring 10 signal events, the discovery reach is up to about $\mu_D = 550$ GeV.

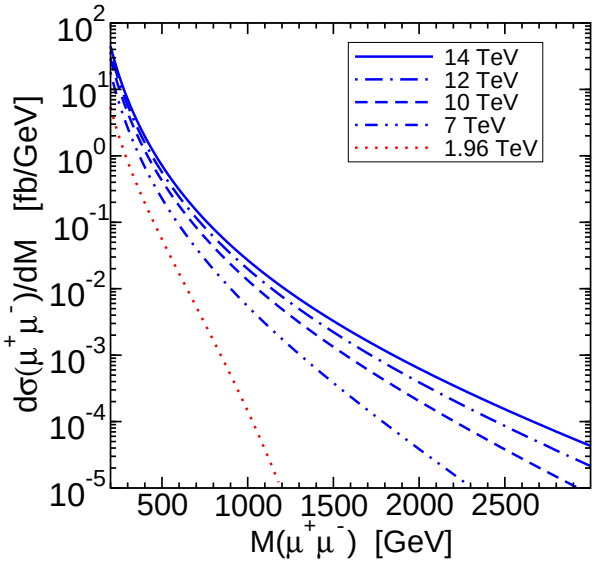

FIG. 4: The leading-order differential production cross-section for the $\mu^+\mu^-$ background, through γ^* and Z^*-mediated (Drell-Yan) processes in pp collisions at $\sqrt{s} = 14$, 12, 10, and 7 TeV, and in $p\overline{p}$ collisions at $\sqrt{s} = 1.96$ TeV, as a function of the invariant mass of the $\mu^+\mu^-$ pair. CTEQ5LO [57] PDFs were used with $Q = \sqrt{\hat{s}}$.

For LHC pp collisions at $\sqrt{s} = 14$ TeV, the signal cross-section times dimuon branching ratio for $\mu_D = 800$ GeV is 15 fb, with a background level in the mass window $M(\mu^+\mu^-) = 1600 \pm 160$ GeV of 0.8 fb. Therefore, discovery may be possible in this case with 1 fb^{-1}. The mass reach is essentially determined by the number of signal events, since the background levels in the high-mass windows are small. In the same way, with 10 fb^{-1}, I estimate the 10-event discovery reach to be up to $\mu_D = 1200$ GeV, and for 100 fb^{-1} up to about $\mu_D = 1600$ GeV.

In a more pessimistic scenario, the quirk and antiquirk may usually annihilate before they can settle into a low-lying color-singlet quirkonium state. The branching ratio to dileptons will be severely reduced in that case because there are color octet as well as color singlet decay states available, and the remaining dimuons will be distributed over larger invariant masses. If one supposes that only 10% of the $D\overline{D}$ pairs that are produced will settle into a low-lying color-singlet quirkonium state before annihilation, and uses only the dimuon events from this quirkonium peak, then the signal cross-section before BR$(\mu^+\mu^-)$ is effectively ten times smaller than shown in Figure 3. The limit from comparing to the CDF bound on cross-section times branching ratio (Figure 3 in [55], based on 2.3 fb^{-1}) results in $\mu_D > 180$ GeV. I estimate that the expected reach from a 1 fb^{-1} LHC run at $\sqrt{s} = 7$ TeV in this more pessimistic case is roughly $\mu_D = 350$ GeV, for which about 17 dimuon signal events and 6 background events would be expected in a mass window $M(\mu^+\mu^-) = 700 \pm 70$ GeV. For LHC runs at $\sqrt{s} = 14$ TeV with (1, 10, 100) fb^{-1}, I similarly estimate that the discovery reach for $D\overline{D}$ that annihilate at least 10% of the time from color-singlet S-wave quirkonium would extend to about $\mu_D = (500, 800, 1100)$ GeV.

In the case of the non-colored quirks $\overline{L}^+, L^-, \overline{L}^0, L^0$, the production rates are electroweak, and the energy loss rate for the quirk-antiquirk bound by the flux string is much lower [18]. The most promising signal may come from the production of the quirk-antiquirk states with a net ± 1 charge, as in eqs. (5.10) and (5.11), because charge conservation then prohibits the subsequent prompt annihilation to invisible G_X glueballs that may occur in the case of neutral bound states. The analogous case for fractionally charged squirks in "folded supersymmetry" was proposed and studied in [19]. The excess energy from the hard production will be radiated away in the form of G_X glueballs or soft photons, hopefully

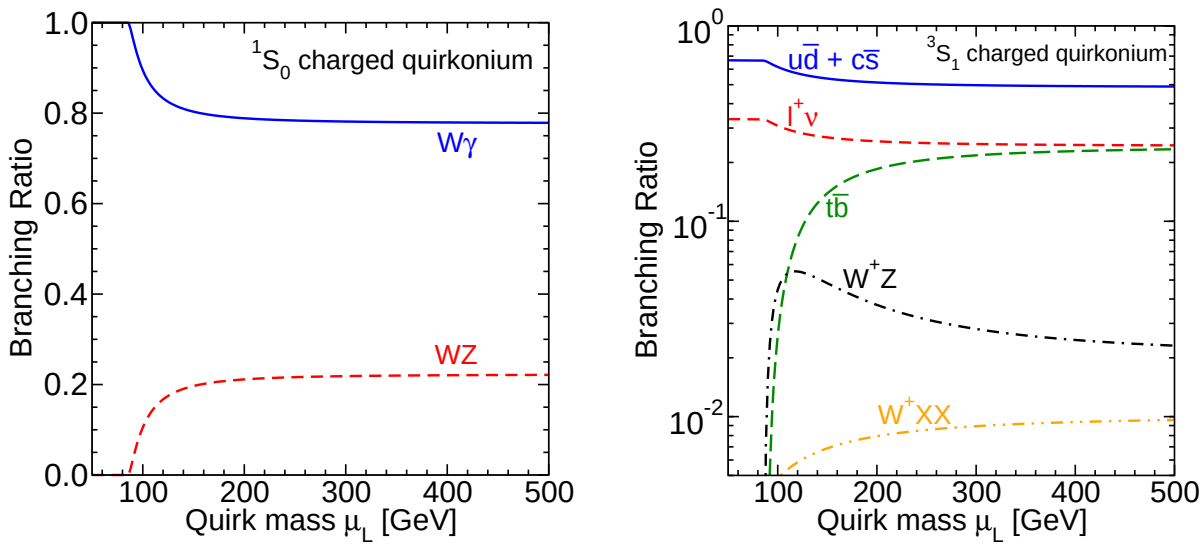

FIG. 5: Branching ratios for charged quirkonium (a low-lying $\overline{L}^+L^0$ or $\overline{L}^0L^-$ bound state) in a $^1S_0$ (left panel) or a $^3S_1$ (right panel) state.

allowing the quirk and antiquirk to finally annihilate when nearly at rest in a charged quirkonium bound state. The annihilation is strongest in an S-wave state. I will again assume that spins are randomized by the energy loss process, so that $^3S_1$ and $^1S_0$ states are populated in the ratio of 3 to 1. The branching ratios for such states have been computed in ref. [20], and are shown in Figure 5 for the present case of constituent quirks with charges ± 1 and 0. The $^1S_0$ state decays predominantly into $W\gamma$, with an invariant mass of nearly $2\mu_L$, and therefore a hard photon. (A somewhat smaller branching ratio to $W\gamma$ was obtained in ref. [20] for a case with fractionally charged constituent quirks.) This state may therefore be searched for in the $\ell^\pm\gamma + E_T^{\rm miss}$ channel at hadron colliders, as suggested in the similar squirk case of ref. [19].

The combined[‡] production cross-sections at the Tevatron and at various possible LHC energies for the charged quirk-antiquirk combination are shown in Figure 6. These cross-sections are about an order of magnitude larger than for fractionally-charged scalar quirks (as studied in ref. [19]) of the same mass. Partly counteracting this, one might expect that only about 1/4 of fermionic quirk-antiquirk production will end up in a $^1S_0$ state that can annihilate to $W\gamma$, rather than a $^3S_1$ state that decays mostly to jets or a single lepton plus neutrino. Thus, the effective branching ratio of charged quirkonium should be about 0.2 for $W\gamma$, a factor of 3-4 smaller than used in [19]. The net effect is that the total production cross-section times branching ratio for $W\gamma$ should be a factor of 2-3 times larger, for a given quirkonium mass, than in the study of ref. [19].

The largest background is from Standard Model $W\gamma$ production, which features a rapidly falling tail at high photon p_T. In contrast, the signal from $^1S_0$ quirkonium decaying to $W\gamma$ should have a photon p_T distribution that is approximately flat, with an endpoint near $\mu_L - m_W^2/4\mu_L$ in the idealized case that the transverse kick to the quirkonium is small. The

[‡] The charge +1 combination is produced more often than the charge −1 one at the LHC, as usual.

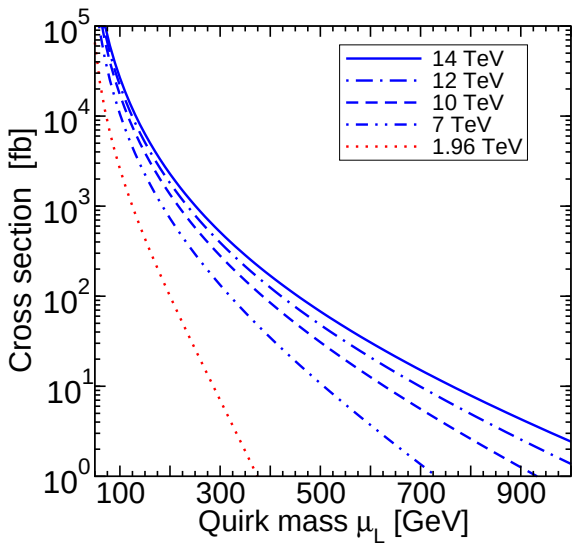

FIG. 6: Combined leading-order production cross-section for $\overline{L}^+L^0$ and $\overline{L}^0L^-$ in pp collisions at the LHC for $\sqrt{s}$ = 14, 12, 10, and 7 TeV, and in $p\overline{p}$ collisions at the Tevatron with $\sqrt{s}$ = 1.96 TeV. CTEQ5LO [57] PDFs were used with $Q = \mu_L$.

relevant photon p_T distribution has been studied at Tevatron by both CDF [58] and DØ [59], where it was found that the data is described well by the SM $W\gamma$ and other subdominant backgrounds including Wj with the jet faking a photon and $Z\gamma$ with one lepton from the Z missed. At hadron colliders, a $W\gamma$ mass peak can in principle be reconstructed if one assumes that the observed $E_T^{\rm miss}$ in the event is due to the neutrino from the leptonic W decay, but this is subject to the considerable uncertainty in how much missing energy is actually due to missing G_X-glueballs radiated from the initial state, as well as from the underlying event, additional jets, or from mismeasurement. However, the discovery potential may be greatly enhanced because one can also look for a large number of soft photons radiated as the quirk-antiquirk flux string loses energy, forming an anomalous "underlying event" with distinctive character. The resulting complications are beyond the scope of the present paper, but have been discussed in the analogous case of fractionally charged colorless squirks in [19, 21]. The search for $W\gamma$ candidates a with large photon p_T and a possible peak in invariant mass or transverse mass, in combination with an anomalous underlying event used as a background-reducing tag, may well be the best hope to detect the quirks in these models.

VI. CONCLUSIONS

Extensions of minimal supersymmetry with an extra non-Abelian gauge group and quirk supermultiplets maintain two of the hallmark successes of the MSSM: compatibility with perturbative gauge coupling unification and with constraints on precision electroweak observables. Natural mechanisms can put the quirk fermion masses at the TeV scale or below. It follows that requiring the unified gauge couplings to be perturbative, so that low-scale predictivity is not lost and the apparent unification of gauge couplings is not just an accident, the gauge group under which the new vector-like particles transform in the fundamental representation must be either $SU(2)$, $SU(3)$, or $SO(3)$. The presence of a new non-Abelian gauge group has a dramatic effect on the superpartner mass spectrum, and allows the soft supersymmetry breaking parameters to be dominated by the gaugino masses at the unification scale, while still having a neutralino LSP.

In the $SO(3)$ case, the confinement length scale is so large as to make the new particles essentially free in collider experiments. In contrast, in the minimal versions of the $SU(2)$ and $SU(3)$ cases, the quirk-antiquirk bound states produced at colliders will be microscopic. If a significant fraction of the colored quirk-antiquirk pairs produced at hadron colliders will lose most of their excess energy before annihilating as quirkonium, then there is significant reach in the dilepton mass peak channel. Even if this fraction is only 10%, Tevatron data that has already been analyzed should allow a limit of 180 GeV for the quirk mass to be set from the search for a $\mu^+\mu^-$ resonance. In the optimistic idealized case that all of the quirk-antiquirk pairs annihilate from quirkonium, this limit should be about 375 GeV. In a 1 fb^{-1} LHC run at $\sqrt{s} = 7$ TeV, the discovery reach could be as high as 550 GeV, and should extend well above 1 TeV with 100 fb^{-1} at $\sqrt{s} = 14$ TeV. The color singlet quirks in these models can also be searched for as quirkonium $W\gamma$ resonances. In all cases, the quirkonium peak can be accompanied by an anomalous underlying event consisting of many soft pions or photons, which can significantly aid in making a discovery [18, 21]. If low-energy supersymmetry is realized in nature, then it will be important to test the possibility that it is not minimal by searching for these events.

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