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A dark energy view of inflation

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Traditionally, inflationary models are analyzed in terms of parameters such as the scalar spectral index n_s and the tensor to scalar ratio r, while dark energy models are studied in terms of the equation of state parameter w. Motivated by the fact that both deal with periods of accelerated expansion, we study the evolution of w during inflation, in order to derive observational constraints on its value during an earlier epoch likely dominated by a dynamic form of dark energy. We find that the cosmic microwave background and large-scale structure data is consistent with $w_{\rm inflation} = -1$ and provides an upper limit of $1 + w \lesssim 0.02$. Nonetheless, an exact de Sitter expansion with a constant $w = -1$ is disfavored since this would result in $n_s = 1$.

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I. INTRODUCTION

The nature of the dark energy has been seen as one of the principal puzzles in cosmology, and in theoretical physics as a whole, ever since the supernova observations [1, 2] in 1998 confirmed the mounting suspicion that the expansion rate of the Universe is accelerating. One of the leading contenders is the cosmological constant, for which the equation of state w equals -1, both on theoretical grounds and because no confirmed deviations from $w = -1$ have come from cosmological observations.

However, the current phase of accelerated expansion is most likely not the only one in the history of the Universe: it is thought that a much earlier epoch of accelerated expansion called inflation created the initial fluctuations that led to large-scale structure and solved several problems of the standard Big Bang cosmology. The spectrum of fluctuations that we observe today, particularly in the cosmic microwave background (CMB) radiation, indicates that they were created by a mechanism that was able to act outside the normal causal horizon [3, 4]. It is commonly believed that the structure we see in the CMB and in the distribution of galaxies arose from quantum fluctuations that were stretched outside the Hubble horizon by a phase of accelerated expansion, not dissimilar to the one that is being observed today.

We know that inflation ended early in cosmic history, before the epoch of Big Bang nucleosynthesis: an inflating Universe is nearly empty of matter and does not form galaxies. As a consequence, inflation could not have been driven by a pure cosmological constant. Since the Universe apparently began to inflate again several billion years ago, it is natural to ask whether hypothetical observers present during primordial inflation would have been able to distinguish between a cosmological constant and an alternative model such as a scalar field by studying the expansion history quantified by w. In this paper, we will link the usual inflationary observables to w and provide constraints on w during the period when the observable scales left the horizon.

II. THE EQUATION OF STATE OF THE INFLATON

We assume that inflation started well before the observable scales left the horizon, i.e., that it lasted longer than about 60 e-folds of expansion, so that the only significant contribution to the energy density ρ is the one from the inflaton itself and that the Universe can be taken to be spatially flat. This implies that the Friedmann and energy conservation equations are

$$H^2 = \frac{\rho}{3M_{\rm Pl}^2}\,, \tag{1}$$

$$\dot{\rho} = -3H(1+w)\rho\,. \tag{2}$$

Here we used the reduced Planck mass, $M_{\rm Pl}^2 \equiv 1/8\pi G$ in our units where $c = \hbar = 1$, and the Hubble parameter $H \equiv \dot{a}/a$ where a is the scale factor. We can compute the equation of state parameter $w = p/\rho$ once we know the expansion rate H,

$$1 + w = -\frac{2}{3}\frac{\dot{H}}{H^2}\,. \tag{3}$$

It is of course equally possible to compute w directly from the pressure and the energy density of the inflaton. However, the form given above is especially useful in the case of single-field inflation, in which case the perturbations generated are linked to H as there is only a single degree of freedom present (exemplified by the potential of the

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inflaton field). This allows us to connect the expression for w directly to quantities related to the perturbations.

This turns out to be especially simple when working with the slow-roll parameters in the so-called Hamilton–Jacobi formalism, see e.g. Ref. [5] for detailed derivations. The first two slow-roll parameters are defined as

$$\epsilon_H = 2M_{\rm Pl}^2 \left(\frac{H'}{H}\right)^2, \tag{4}$$

$$\eta_H = 2M_{\rm Pl}^2 \frac{H''}{H}. \tag{5}$$

Here $'$ denotes a derivative with respect to the scalar field ϕ. Since $H' = \dot{H}/\dot{\phi}$ and $\dot{\phi} = -2M_{\rm Pl}^2 H'$, we find together with Eq. (3) that

$$1 + w = \frac{2}{3}\epsilon_H. \tag{6}$$

The equation of state during inflation is therefore directly given by the first slow-roll parameter. To lowest order in slow-roll this is also related to the tensor to scalar ratio by $r = 16\epsilon_H$. Without any further work we can deduce that, since primordial gravitational waves have not been observed, there is no observational requirement for a deviation from $w = -1$ during inflation. The upper limit on r from the five-year Wilkinson Microwave Anisotropy Probe (WMAP) data for a flat ΛCDM model without running is about 0.43 [6], corresponding to a maximum deviation from $w = -1$ of 0.02. We will derive precise numerical constraints in the next section.[1]

This result is at first glance a bit puzzling: An equation of state $w = -1$ leads to de Sitter expansion which in turn creates a scale-invariant Harrison–Zel'dovich (HZ) spectrum. However, the WMAP five-year data paper also claims a 2.5 sigma deviation from a HZ spectrum. The explanation is that the deviation of the scalar spectral index n_s from the HZ case ($n_s = 1$) can be caused by the second slow-roll parameter η_H, given to lowest order in slow roll by

$$2\eta_H = (n_s - 1 + 4\epsilon_H). \tag{7}$$

Thus even if at a given time $\epsilon_H \approx 0$, it is still possible to obtain $n_s \neq 1$ through a non-zero η_H.

A non-zero η_H implies that w will evolve away from -1. How quickly will it do that? Possibly fast enough to lead to measurable deviations during the observable number of e-foldings? We find

$$\frac{d\ln(1+w)}{dN} = \frac{d\ln\epsilon_H}{dN} = 2(\eta_H - \epsilon_H) \tag{8}$$

where $N = -\ln a$ is the number of e-foldings. Since the rate of change of ϵ_H is proportional to ϵ_H itself, it can become very small if ϵ_H is very small. Close to de Sitter the field freezes and moves only very slowly, but even this slow motion leads to observable effects in the power spectrum of the perturbations. This is unfortunately an observational channel that is not available for the contemporary dark energy. Indeed, the ways in which we probe inflation and today's dark energy are very different: we have no way to constrain directly the expansion history during inflation, but we can see the spectrum of the curvature perturbations generated during this epoch. On the other hand, while we can observe directly the recent expansion history of the Universe and infer the equation of the state of the dark energy, the fluctuations generated during the current bout of accelerated expansion are impossible to observe both because of their tiny predicted amplitude and because they become classical only when outside the current horizon.

The likelihood of a tiny value of ϵ_H has been hotly debated in the inflation literature (e.g. Refs. [7, 8]), since it would prevent direct detection of inflationary gravitational waves, e.g. by a CMB polarization satellite mission [9]. Within the framework of the early large-field inflation models, such as monomial potentials, a tiny ϵ_H and large η_H would look rather unnatural, and hence the observed $n_{\rm s} \approx 0.96$ would suggest $r \approx 0.1 - 0.2$ and $1 + w \approx 0.01$, both well within current experimental bounds. However by contrast the paradigm of small-field models, such as hybrid inflation, motivated by the need to keep the field values small in a supergravity context, does suggest that ϵ_H must be extremely small at horizon crossing, thus indicating w very close to -1.

III. NUMERICAL INVESTIGATION

In order to obtain numerical constraints on w during inflation, we need to link it to observational quantities. In this paper we will use the spectrum of the primordial fluctuations, as observed in the CMB. The link between w and H given in Eq. (3) is fundamental, failing to hold only if either the universe was very different from Friedmannian during inflation or if there were other contributions to the expansion rate present. The first would invalidate the whole inflationary framework, while in the second case our w would correspond to an effective total w.

To go from H to the primordial power spectrum requires a specific model. Here we assume that inflation was due to a single canonical scalar field, though without making the common assumption of slow-roll. An interesting future project is to relax this condition by investigating a range of other models, for example K-inflation models with a different sound speed [10]. While this may change quantitative limits on w, we do not expect it to change the qualitative results. We also note that our model imposes the Dominant Energy Condition $w \geq -1$ by construction.

In order to compute H during the observable range of scales, we use the module provided by Lesgourgues and

[1] After we completed the calculations for this paper, the WMAP team released the 7-year data (WMAP7). It gives results very similar to WMAP5 and we would not expect qualitative, or even significant quantitative, changes. For example, the upper limit on r decreases only slightly to 0.37.

Valkenburg (LV) [11] which takes the slow-roll parameters at the pivot scale as an input. The pivot scale is fixed to be $k_* = 0.01/\mathrm{Mpc}$ (roughly in the center of the observable range); this is the scale at which the Hubble parameter H is expanded as a Taylor series in $(\phi - \phi_*)$, with the scale factor set to $a_* = k_*/H_*$. We then approach the problem from two slightly different angles. To reconstruct the evolution of w from the time when the observable scales left the horizon up to the end of inflation, we use the flow-equation formalism [12] to derive the evolution of ϵ_H from the end of inflation to the observable scales. In this we proceed similarly to Ref. [13] by selecting "initial" values for the first four slow-roll parameters at the end of inflation (in fact we only choose three of them since ϵ_H is always equal to 1) and flowing them back 60 e-foldings using the flow equations. The values obtained at $N = 60$ are then used to compute the observables using the LV module. The appropriate value of N changes with the inflationary energy scale, and may be smaller in low energy scales models. But this does not impact our conclusions, since we are interested in the experimental constraints on w around the scales that are directly probed by observations, and additionally, as our later results show, the constraints remain fairly constant over a range of N. This allows us to avoid more sophisticated approaches to treating this uncertainty, as given for instance in Ref. [14].

On the other hand, we do not really know what happened after the observable scales left the horizon, as we do not have any observations concerning that period. Based on this reasoning, Lesgourgues and Valkenburg [11] argued that considering only the observable scales makes it possible to work with a relatively low-order expansion of the scalar field potential without introducing artificial constraints. We use the module provided by LV to compute the observables in their framework and to compare the results with those from the flow-equation formalism.

In both cases we use CosmoMC [15] to perform a Markov Chain Monte Carlo exploration of the parameter space, which includes, depending on our method:

- the slow-roll parameters at $N = 0$, i.e. at the end of inflation, for our first approch; the useful parameters at the pivot scale (fixed at $N = 60$) are computed by solving numerically the flow equations;
- the slow-roll parameters at the pivot scale directly for the second method.

Here we used the first four slow-roll parameters ϵ_H, η_H, ${}^2\beta_H = \xi_H$ and ${}^3\beta_H$ [16] with the following ranges (at $N = 0$ for the first method, and at the pivot scale for the second): $\epsilon_H \in [0.0, 1.0]$ (fixed to 1.0 for the first method), and η_H, ξ_H, ${}^3\beta_H \in [-10.0, 10.0]$. CosmoMC works together with CAMB [17] to compute the CMB power spectrum and then uses the WMAP five-year likelihood code [6]. The inflationary power spectrum is calculated using the Lesgourgues–Valkenburg module which solves the perturbation mode equation. This setup allows us to compute chains of acceptable expansion histories during inflation. These were then mapped into chains of $w(\phi)$ (Fig. 1).

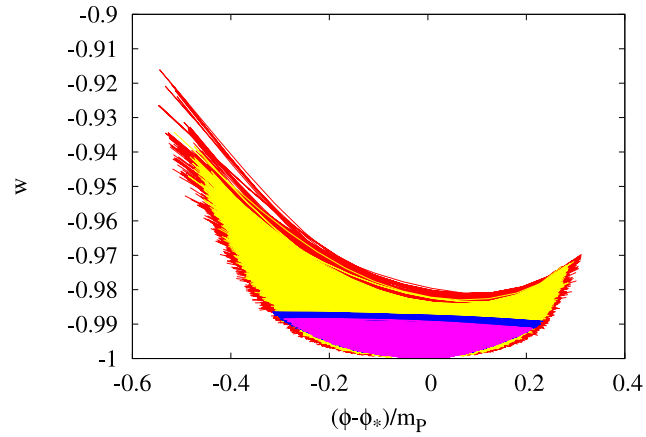

FIG. 1: The evolution of $w(\phi)$ for a range of cases accepted by the CMB likelihood, around the field value ϕ_* at the pivot scale $k_* = 0.01/\mathrm{Mpc}$. The red and yellow curves lie within the 95% and 68% confidence regions for the LV formalism, blue and purple give the same information for the flow-equation formalism. From the outside inward, the colored regions are red, yellow, blue, and purple.

In principle Fig. 1 already shows the constraints on the equation of state parameter during inflation. But, as is easily seen in the figure, ϕ moves more and more slowly as we approach $w = -1$, which makes the constraints difficult to interpret. A better representation is $w(N)$ in terms of the number of e-foldings N before the end of inflation, see Fig. 2. However, in the LV formalism the field is never evolved until the end of inflation, so that N is not defined. An alternative way to plot the results in this situation is to map them instead to the horizon scale at that epoch, $k = aH$. Since the perturbations freeze in outside the horizon and turn into conserved curvature perturbations, this scale corresponds to the one that they have when they re-enter the horizon. We plot our constraints in this way in Fig. 3.

¿From the full evolution in Fig. 2 we see that w approaches -1 rapidly as we move into the past. The precise rate at which -1 is approached depends on the range of models chosen at the end of inflation (see e.g. Ref. [18]). Nonetheless, as shown in the inset, strong deviations from $w = -1$ are expected in the last few e-folds. To illustrate the scales involved: if we were to arbitrarily place today at $N = 7$ (the right-hand limit of the inset) and reverse time then $N = 0$ would roughly correspond to last scattering ($z \approx 1100$).

The current experimental uncertainty on the dark energy w is about 0.1, comfortably enclosing $w = -1$, and in the future will reach a precision of 0.02 or better. We find that the current limits on w during inflation are comparable, with a 95% limit of $1 + w < 0.02$ at $k \approx 0.01/\mathrm{Mpc}$, see Fig. 3. This agrees well with the arguments in the previous section, but the figure shows also the precise shape of the constraints. There is no

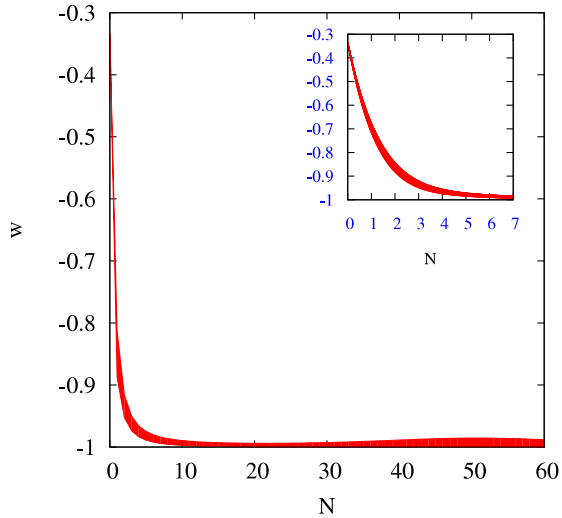

FIG. 2: The complete evolution of $w(N)$, from the flow-equation results accepted by the CMB likelihood. Inflation is made to end at $N = 0$ where $w(N = 0) = -1/3$ corresponding to $\epsilon_H(N = 0) = 1$. For our choice of priors on the slow-roll parameters at $N = 0$, we find that w decreases rapidly towards -1 (see inset) and stays close to it during the period when the observable scales leave the horizon ($N \approx 40 - 60$).

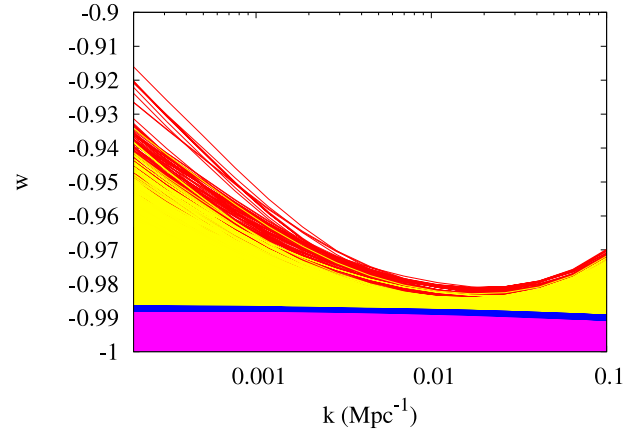

FIG. 3: The evolution of w as a function of the comoving scale k, using only the 5-year WMAP CMB data. Red and yellow are the 95% and 68% confidence regions for the LV formalism. Blue and purple are the same for the flow-equation formalism. From the outside inward, the colored regions are red, yellow, blue, and purple.

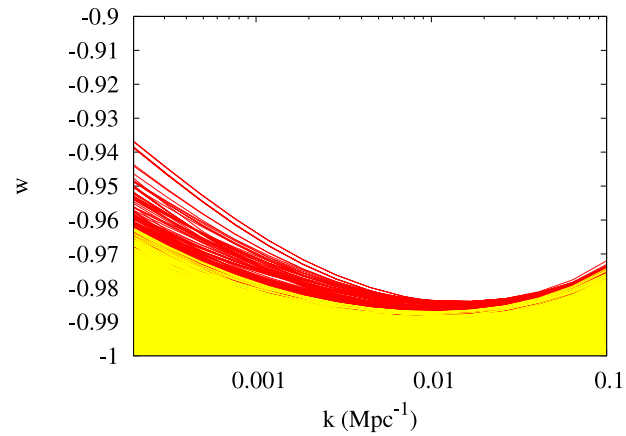

FIG. 4: The evolution of w as a function of the comoving scale k, as in Fig. 3, but using in addition the SDSS DR7 LRG data. We only show the LV limits.

lower limit on w (apart from $w \geq -1$ enforced by the model construction). However, the tentative observation of a deviation from a scale-invariant primordial power spectrum implies through Eq. (7) that ϵ_H and η_H cannot both be zero. Together with Eq. (8) this disfavours a constant $w = -1$. But as discussed in Section II, $(1 + w)$ can remain small over the observable range of scales and we find that this deviation is not visible in the figures.

The limits on w can be improved by extending the lever arm of the measurements, for example by adding galaxy survey data on smaller scales. We show the impact of using both WMAP 5-year CMB data and Sloan Digital Sky Survey (SDSS) Data Release 7 Luminous Red Galaxy data (DR7 LRG) [19] in Fig. 4. The shape of the constraints have not changed by much, but the limits have become somewhat tighter. We can achieve another small increase in precision by adding further CMB data on smaller scales, but again the improvement is small so we do not show those constraints.

We also notice that the prescription of LV allows for a stronger variation of w. The flow-equation formalism with the number of parameters and priors used here leads to very little evolution of w during the observable period. This does not mean that one of the two approaches is wrong, but rather that they impose different additional conditions. As always, it is important to be aware of these effective (and somewhat hidden) priors.

IV. CONCLUSIONS

It seems very likely that there have been at least two periods of accelerated expansion during the evolution of the Universe. During the first period, called inflation, the perturbations that led to today's structure were generated, while the second one has started only recently and is attributed to a mysterious dark energy. In this paper we ask what a similar physical origin would imply for the dark energy.

One point that is immediately clear is that since inflation ended, there is reason to assume that it was not due to a cosmological constant. This is supported by the tentative detection of a deviation from an exact Harrison-Zel'dovich spectrum with $n_s = 1$ [6]: a period dominated by a (possibly effective) cosmological constant would either result in no perturbations at all or in perturbations with an exactly scale-invariant spectrum, depending on how precisely the de Sitter state is reached. The former possibility is clearly excluded, and while current observations are not yet conclusive on whether $n_s = 1$ is excluded, the Planck satellite should settle the question

within the next few years, since it is expected to reach a precision of $\sigma_{n_s} \lesssim 0.005$ [20]. A clear detection of $n_s \neq 1$ would require either $w' \neq 0$ or $w \neq -1$, with a constant $w = -1$ being ruled out in both cases.

However, there is also no requirement for the equation of state parameter w to differ appreciably from -1 during inflation as w is directly proportional to the ratio of tensor to scalar perturbations, and no primordial gravitational waves have been detected so far. Thus, even though it may be possible that Planck demonstrates that inflation was *not* due to a cosmological constant, this does not imply that w was measurably different from -1. Indeed, we find that current data allows w to be arbitrarily close to -1 as long as it changes just slightly during its evolution. This direct link between w and the gravitational wave background reinforces the importance of the latter as a probe of early Universe physics: if it is detected then we know immediately that w was measurably different from -1 during inflation.

We have also found that the current experimental limits on w during inflation imply $1 + w < 0.02$ at a scale of $k \approx 0.01/\mathrm{Mpc}$. If we take seriously the idea that early and late-time acceleration are based on similar mechanisms, then this might suggest that dark energy probes need to reach at least this precision in order to have a reasonable chance of detecting any deviation from Λ. Following the arguments from the end of section II, one could argue for a target precision of about 0.01 for measuring w, beyond which there may well be a "w desert" extending to very low values of $(1 + w)$. This precision also roughly leads to a decisive Bayes factor in favour of ΛCDM if no deviation from -1 is detected (when looking at constant w, see e.g. Ref. [21] for the methodology).

However, the absence of an observational lower limit on w during inflation should not be taken as argument against measuring the recent expansion history and evolution of perturbations. Firstly, there is no direct evidence that the two periods of accelerated expansion are due to the same underlying physical mechanism. Secondly, even if that is so, it is likely that we are observing a very different epoch of the inflationary phenomenon today than in the early Universe. The acceleration became observationally relevant only very recently, less than one e-folding ago. If the onset of acceleration coincides with it becoming visible, then we could expect strong deviations from $w = -1$, since also at the end of inflation w deviated strongly from -1, see Fig. 2. On the other hand, it is also possible that the dark energy has been present much longer but was buried beneath the matter and has but surfaced recently. In this case inflation indicates that it is natural for a scalar field dark energy to have an equation of state close to $p = -\rho$.

Finally, inflation and the current epoch are accessible in very different ways: from inflation we observe the curvature perturbations generated out of quantum fluctuations, while for the recent history of the Universe we instead observe directly the evolution of the expansion history as well as possibly the impact of the dark energy perturbation or of deviations from General Relativity onto light deflection and the distribution of galaxies. If the physics underlying the accelerated expansion of inflation and dark energy are related, then the two sets of observations are complementary and mutually reinforcing, and observational results for either period of accelerated expansion may help to shed light on the other one as well.

Acknowledgments

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