

## Testing the wave packet approach to neutrino oscillations in future experiments

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When neutrinos propagate over long distances, the mass eigenstate components of a flavor eigenstate will become spatially separated due to their different group velocities. This can happen over terrestrial distance scales if the neutrino energy is of order MeV and if the neutrino is localized (in a quantum mechanical sense) to subatomic scales. For example, if the Heisenberg uncertainty in the neutrino position is below  $10^{-2}$  Å, neutrino decoherence can be observed in reactor neutrinos using a large liquid scintillator detector.

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Even though the existence of neutrino oscillations has been unambiguously proven experimentally, the theoretical description of this phenomenon is still occasionally disputed, see for example [1–11]. The reason is that neutrino oscillations, being a space- and time-dependent phenomenon, cannot be fully described in terms of infinitely delocalized energy and momentum eigenstates as is used in most other applications in high energy physics. The signature dependence of neutrino oscillations on the distance between the neutrino source and the detector obviously cannot be observed unless the source and the detector are localized. The uncertainty principle then implies that the source, and consequently the neutrinos that it produces, must be in a superposition of different momentum states. That is, the neutrino wave function cannot be a plane wave, but must be a wave packet [12–22]. For the purpose of this paper, it will be sufficient to consider Gaussian wave packets, but our results will apply also to more general wave packet shapes [22]. We write the neutrino wave function as

$$\langle \mathbf{x} | \nu_\alpha(t) \rangle \propto \sum_j U_{\alpha j}^* \exp \left[ - \frac{(\mathbf{x} - \mathbf{v}_j t)^2}{4\sigma_x^2} \right], \quad (1)$$

where  $\alpha$  and  $j$  are flavor and mass eigenstate indices, respectively,  $U_{\alpha j}$  are the elements of the leptonic mixing matrix,  $\sigma_x$  is the width of the wave packet, which depends on the properties of the neutrino source<sup>1</sup>, and  $\mathbf{v}_j$  is the group velocity corresponding to the  $j$ th neutrino mass eigenstate.

As the neutrino propagates, the wave packets corresponding to different neutrino mass eigenstates will become separated in space and time due to their different group velocities [12]. After a propagation distance larger than  $\sigma_x/|\mathbf{v}_j - \mathbf{v}_k|$ , the wave packets of the  $j$ th and  $k$ th mass eigenstates will have no significant overlap any more and their coherence will be lost. Coherence can be *restored* if the detection process is delocalized over distances larger than the wave packet separation [23]. If this is not the case, neutrino oscillations will be suppressed. Flavor change is still possible, but it will no longer depend on distance. Detailed calculations show that the  $\nu_\alpha \rightarrow \nu_\beta$  oscillation probability for ultra-relativistic neutrinos with an average energy  $E \gg m_j$  at long baseline  $L$  can

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<sup>1</sup> Clearly, the observability of oscillations depends on the localization of the detector in the same way as on that of the source. Detector localization is incorporated into the description of oscillations in [18, 19, 22]

be written as [24, 25]

$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq \sum_{j,k} U_{\alpha j}^* U_{\alpha k} U_{\beta k}^* U_{\beta j} \exp \left[ -2\pi i \frac{L}{L_{jk}^{\text{osc}}} - \left( \frac{L}{L_{jk}^{\text{coh}}} \right)^2 \right], \quad (2)$$

with the oscillation lengths  $L_{jk}^{\text{osc}} = 4\pi \bar{E} / \Delta m_{jk}^2$  and the coherence lengths

$$L_{jk}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{jk}^2|} \sigma_{x,\text{eff}} = 7347 \text{ km} \left( \frac{E}{\text{MeV}} \right)^2 \left( \frac{7.7 \times 10^{-3} \text{ eV}^2}{\Delta m_{jk}^2} \right) \left( \frac{\sigma_{x,\text{eff}}}{\text{\AA}} \right). \quad (3)$$

Here  $\sigma_{x,\text{eff}}$  is an effective wave packet width that depends on the spatial delocalization of the source and that of the detector, and is dominated by the larger of the two.<sup>2</sup>

Eq. (3) shows that neutrinos from astrophysical sources always arrive at the Earth as completely incoherent mixtures of mass eigenstates, while observation of decoherence effects in terrestrial experiments would require a long baseline, low neutrino energy, and small  $\sigma_{x,\text{eff}}$ .

These requirements can be fulfilled for reactor neutrinos observed in a future large liquid scintillator detector like Hanohano [26] or LENA [27]. The reactor neutrino event rate peaks at  $E \sim 4$  MeV, and the distance from a multipurpose detector like Hanohano or LENA to the nearest nuclear power station will be of order  $L \gtrsim 100$  km to avoid large backgrounds in geo-neutrino, supernova relic neutrino, and proton decay studies.

It is more difficult to estimate the wave packet width  $\sigma_{x,\text{eff}}$  entering in eq. (3). Since it depends on the quantum mechanical localization of the neutrino in space, we have to ask how well we can *in principle* determine the position of the neutrino production and detection points, given perfect experimental equipment.

For neutrinos emitted from free particles in flight, the spatial uncertainty of the production process will be similar to the mean free path of the parent particle [28].

A neutrino emission or detection process in a solid or liquid is localized at least to interatomic distance scales of  $\sigma_{x,\text{eff}} \sim \mathcal{O}(1 - 10 \text{ \AA})$  because the emitting or absorbing atom is continuously interacting with its neighbors. The latter can be viewed as a thermal bath in the sense that their quantum states at different times are completely uncorrelated. An interaction of a particle with a thermal bath constitutes a measurement in the quantum mechanical sense that localizes the particle.

Another reason why  $\sigma_{x,\text{eff}}$  cannot be larger than an interatomic distance is that we can in principle measure the location of the production and detection vertices to that accuracy in an experiment. Whenever we detect that a neutrino has been produced or absorbed (for example by detecting the associated charged lepton), we can in principle quickly bombard the source with a high-energy beam of probe particles to determine which nucleus has undergone a transition in the process.

Finally, a spatial uncertainty much larger than a few  $\text{\AA}$  would imply that *all* particles participating in the process would have to be delocalized over many interatomic distances. Imagine an energetic charged particle like an outgoing charged lepton traveling through a solid or liquid material and interacting with the atoms. For concreteness, let us assume that it excites the atomic shells, which subsequently relax by emitting scintillation light. If the energetic particle was delocalized over many interatomic distances, it would be impossible to tell which atoms is excited. The resulting scintillation light would then behave as if it came from several atoms simultaneously and

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<sup>2</sup>  $\sigma_{x,\text{eff}}$  depends also on the temporal delocalization of the production and detection processes, which, however, is usually similar to the spatial delocalization.

would therefore exhibit interference patterns. We are not aware of any experimental evidence for such interference patterns.

For neutrino emission or detection in a solid state crystal, the upper limit on the delocalization scale can be even tighter than 1–10 Å. To be specific, consider neutrino production in a reactor fuel rod. Once we know the position of the production vertex to within one lattice spacing—which is possible according to the above arguments—we can make use of our knowledge of the crystal structure to pin down the exact lattice site where the process must have occurred. In a crystal at zero temperature, the emitting nucleus would be localized at its lattice site to within its own size of order  $\text{few} \times 10^{-15}$  m. At finite temperature  $T$ , it will oscillate about this position, with the amplitude of these vibrations being of order  $[T/m\Theta_D^2]^{1/2} \sim 0.01\text{--}0.1$  Å for Debye temperatures  $\Theta_D$  of few hundred Kelvin and nuclear masses  $m \sim 100$  GeV. To arrive at this estimate, we have assumed the nucleus to be bound in a harmonic oscillator potential  $V = \frac{1}{2}m\omega^2x^2$  with characteristic frequency  $\omega = \Theta_D$ , and the thermal excitation energy to be of order  $T$ . We conclude that the spatial uncertainty associated with a neutrino production process in a solid state source is roughly between  $10^{-5}$  Å and 0.1 Å, depending on the temperature. Similar arguments apply to solid state neutrino detectors.

At least for reactor neutrino experiments with liquid scintillator detectors, we can also set a *lower* limit on  $\sigma_{x,\text{eff}}$  from the fact that the KamLAND experiment has observed neutrino oscillation consistent with the results from solar neutrino experiments at baselines of  $\mathcal{O}(100)$  km [29].<sup>3</sup> This translates into the limit  $\sigma_{x,\text{eff}} \gtrsim 10^{-3}$  Å.

From the above considerations we conclude that, for reactor neutrino experiments

$$10^{-3} \text{ \AA} \lesssim \sigma_{x,\text{eff}} \lesssim 10^1 \text{ \AA} \quad (4)$$

or

$$100 \text{ km} \lesssim L_{21}^{\text{coh}} \lesssim 1\,000\,000 \text{ km} \quad (5)$$

These estimates show that it may well be possible to detect neutrino wave packet decoherence effects in terrestrial experiments; this would prove that the wave packet formalism is indeed well-suited to describe neutrino oscillations, and a measurement of the coherence length would provide interesting information about the quantum mechanics of the production and detection processes.

The experimental prospects for detecting neutrino decoherence have been studied previously in the context of atmospheric neutrinos [32], short-baseline ( $\mathcal{O}(\text{km})$ ) reactor experiments [33], superbeams [34, 35], and a neutrino factory [33]. Here, we will study the prospects of observing the predicted wave packet decoherence from eq. (4) by observing reactor neutrinos at very long baselines of at least a few hundred km in a large liquid scintillator detector like Hanohano or LENA. We have simulated this setup using GLOBES [36, 37], assuming a 5-year run of a LENA-like [27] detector with a fiducial mass of 45 kt and a Gaussian energy resolution of  $0.07 \times (E/\text{MeV} - 0.8)^{1/2}$  MeV. As systematic uncertainties, we include a 3% error on the reactor neutrino flux, a 50% uncertainty on the geo-neutrino flux, a 0.5% energy calibration error, and a 0.5% spectral error, uncorrelated between different energy bins. We study three different detector sites: The Pyhäsalmi mine in Finland, the Deep Underground Science and Engineering Laboratory (DUSEL) in South Dakota, USA, and a site in Hawaii. Unless otherwise noted, we take into account the neutrino flux from all nuclear power stations in the world [38], as well as the geo-neutrino background, which

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<sup>3</sup> The possibility cannot be ruled out that KamLAND is affected by neutrino decoherence at a subdominant level. In this case, a reanalysis would be required, leading to modifications of the global fit of the solar oscillation parameters. The experiment we propose in this paper would be able to determine if such a reanalysis is indeed necessary. Decoherence effects in KamLAND have been studied for example in [30, 31].

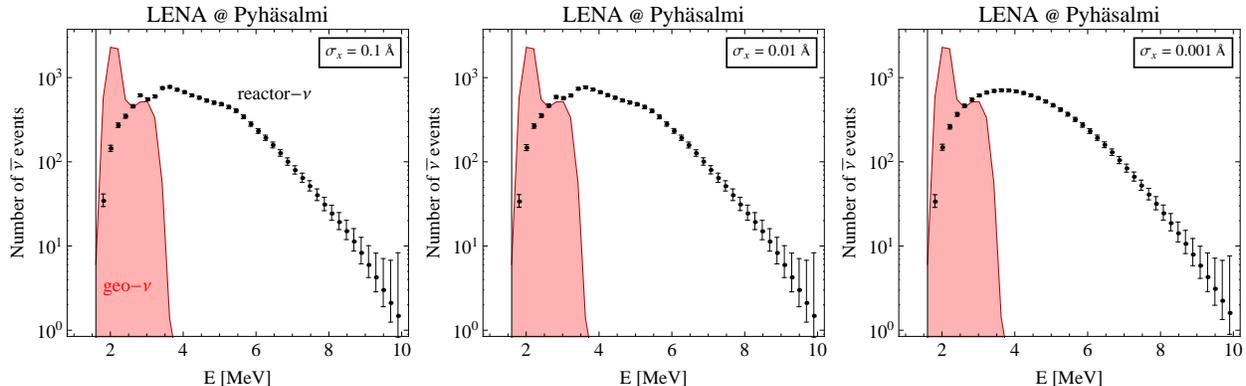


Figure 1: The neutrino spectrum in a 45 kt (fiducial) liquid scintillator detector located in the Pyhäsalmi mine in Finland for different values of the wave packet width  $\sigma_{x,\text{eff}}$ . The red curve is the geo-neutrino background, while the black dots are the reactor neutrino signal.

dominates the event rates below  $E \lesssim 3.3$  MeV.<sup>4</sup> The reactor neutrino spectrum we are using is based on [29, 39], while the geo-neutrino spectrum has been kindly provided to us in machine-readable form by the author of ref. [40]. The cross sections for neutrino detection in inverse beta decay are from ref. [41].

In fig. 1, we show the effect of decoherence on the neutrino spectrum expected at the Pyhäsalmi mine in Finland. For large  $\sigma_{x,\text{eff}}$ , where  $L^{\text{coh}} \gg L$ , an oscillation pattern is visible, even though it is smeared due to the overlap of signals from different reactors at various baselines. For small  $\sigma_{x,\text{eff}}$  oscillations are completely smoothed out by decoherence.

To quantify this observation and determine the discovery potential for decoherence effects, we have simulated the expected event spectrum for different values of  $\sigma_{x,\text{eff}}$ , and have then performed a fit assuming no decoherence. The resulting  $\chi^2$  is plotted in fig. 2. We find that prospects for observing decoherence are best at the Pyhäsalmi site due to the proximity of only few nuclear reactors. This keeps washout of oscillations due to the superposition of many reactor spectra at many different baselines small. On the other hand, there is no nearby reactor that would make the detector blind to events from distant reactors that could carry information about decoherence. In the DUSEL scenario, there are no nearby reactors either, but washout due to the larger number of nuclear reactors in the eastern United States is a problem. In Hawaii, on the other hand, the nearest nuclear reactors are thousands of kilometers away, so that the number of events is too small for the presence or absence of an oscillation pattern to be seen. Moreover, at such long baselines, consecutive oscillation maxima are very close in energy, so the detector resolution becomes an issue. Thus we conclude that the optimal detector site for a decoherence measurement is one that has a few (but not too many) nuclear reactors at baselines of several 100 km.

Fig. 3 shows that this condition is fulfilled in many locations around the world. In the top panel, we have mapped the locations of nuclear power stations [38] (as of 2000), while the bottom panel shows the significance at which wave packet decoherence can be detected if the wave packet width is 0.005 Å, close to the lower end of the allowed range from eq. (4). We have checked that a larger values  $\sigma_{x,\text{eff}} = 0.01$  can only be detected in few places in Japan, while even larger values cannot be detected anywhere in the world with the experimental setup we have simulated. Note that for each grid point in fig. 3, we have simulated only the 32 closest nuclear reactors to speed

<sup>4</sup> Note that the data on reactor sites available to us dates from 2000, so some recently commissioned stations may not be included. To partly compensate for this, we assume all stations that were under construction in 2000 to be operational at full power by now.

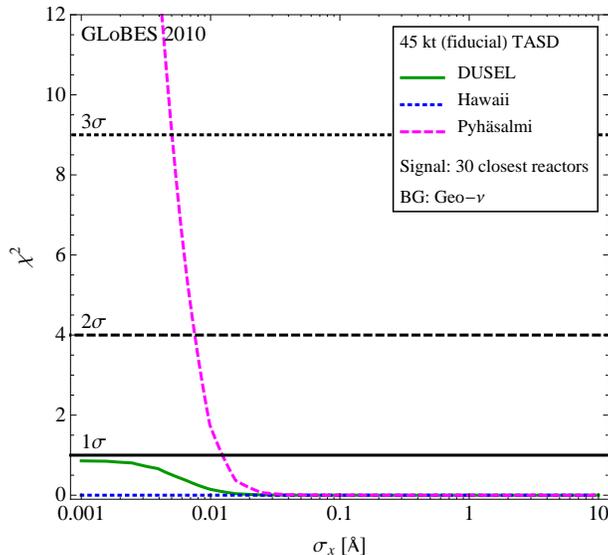


Figure 2: Discovery reach for neutrino decoherence in a 45 kt (fiducial) detector located at DUSEL, Hawaii, or Pyhäsalmi. Only the Pyhäsalmi site allows for good sensitivity to decoherence effects.

up the numerical computations.

In conclusion, we have presented theoretical arguments showing that neutrino wave packets in a reactor experiment should have a width between 0.001 Å and 10 Å. The lower limit is based on the observation of oscillations at KamLAND, while the upper limit has been estimated by considering the localization of typical neutrino production and detection processes. These numbers imply that wave packet decoherence due to different group velocities of different neutrino mass eigenstates can occur over distances of  $10^2$ – $10^6$  km for  $\mathcal{O}(\text{few MeV})$  neutrinos. In higher energy experiments or in experiments in which neutrinos are produced by decay in flight of some parent particle, the wave packet width can be much larger, while in a hypothetical experiment in which  $\mathcal{O}(\text{few MeV})$  neutrinos are emitted and detected by nuclei bound in solid state lattices at very low temperature, the localization can be of  $\mathcal{O}(\text{few} \times 10^{-5})$  Å, corresponding to a coherence length of order 1 km. We have then performed detailed numerical simulation to show that wave packet decoherence is observable in a future large liquid scintillator detector like Hanohano or LENA, but only if the coherence length is at the lower end of the expected range and if the detector site is far (few hundred kilometers), but not too far ( $\lesssim 1000$  km) from the nearest nuclear power station.

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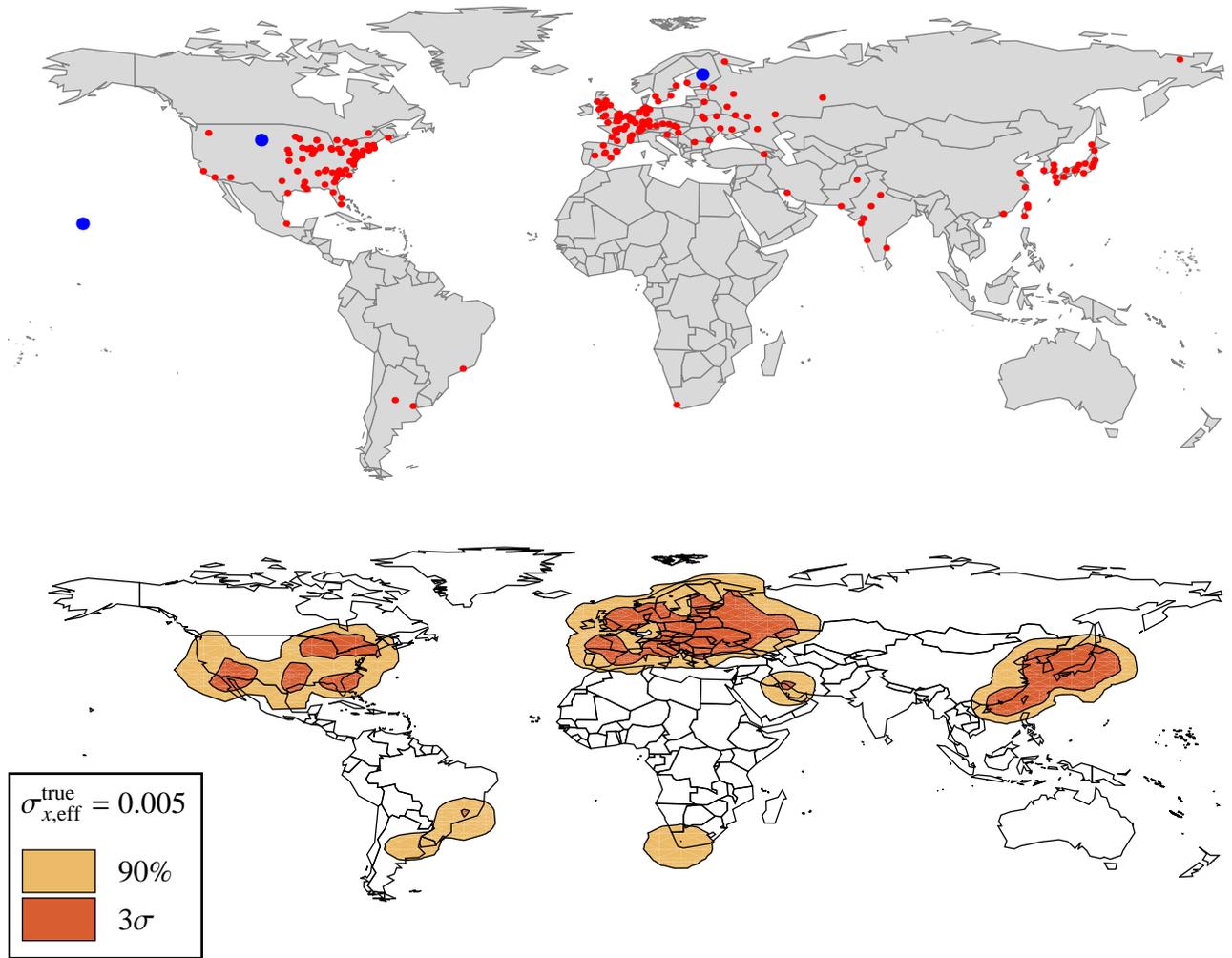


Figure 3: Top: A map of all nuclear reactors running or under construction as of 2000 (red dots), together with the proposed detector sites at Hawaii, Pyhäsalmi, and DUSEL (blue dots). Data on nuclear reactors is from [38]. Bottom: Discovery reach for wave packet decoherence with  $\sigma_{x,\text{eff}} = 0.005 \text{ \AA}$  at different geographic locations. Note that, due to limited resolution, the plot hides the fact that very close to any nuclear reactor, the sensitivity greatly decreases because the detector is blinded by short-baseline neutrinos in that case.

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