OBSERVATION OF SINGLE TOP QUARK PRODUCTION
AT THE TEVATRON COLLIDER

A.P. HEINSON
Department of Physics and Astronomy, University of California, Riverside
Riverside, California 92521-0413, USA
ann.heinson@ucr.edu

Received: February 8, 2010

On March 4, 2009, the DØ and CDF collaborations at Fermilab’s Tevatron Collider
submitted papers to Physical Review Letters announcing observation of single top quark
production.1,2 This review paper describes the successful searches carried out independently
by the two collaborations, allowing the reader to see the similarities and differences
that led to the simultaneous discoveries. Both collaborations measured a cross section
σ(\(p\bar{p}\rightarrow t\bar{b} + X, t\bar{q} + X\)) consistent with the standard model prediction at 5.0 standard
deviation significance, and set a lower limit on the quark mixing matrix element |\(V_{tb}\)|
without assuming matrix unitarity with three quark generations.

Keywords: Single top quarks; electroweak production; Tevatron collider.

PACS Nos.: 14.65.Ha, 12.15.Hh, 12.15.Ji, 13.85.Qk

1. Introduction to the Top Quark

The top quark, an up-type quark, and the bottom quark, a down-type quark, together
form the third generation of quarks. Both top (t) and bottom (b) quarks have spin 1/2. The top quark has electric charge \(+2e/3\) and a mass of
173.1 ± 1.3 GeV.3 The bottom quark has charge \(−e/3\) and mass 4.20 \(+0.07\) GeV.4
All other quarks (u, d, c, s) are nearly massless in comparison. The top quark has a lifetime5 of 0.5 \(	imes 10^{-24}\) s that is much smaller than the strong interaction timescale,
and is thereby unique in the quark family, decaying before it can form a bound
state with another quark.6 Thus, the kinematics of the particles from the top quark
decay contain information about the bare top quark itself. The Cabibbo-Kobayashi-
Maskawa (CKM) matrix “V” describes quark mixing.7 When there are exactly three
quark generations, the matrix is unitary, and a global fit to all available precision
data constrains the element |\(V_{tb}\)| to be very close to one.8 Therefore, in the standard
model (SM) the top quark decays almost every time to a W boson and a \(b\) quark.
The tiny SM values for |\(V_{td}\)| and |\(V_{ts}\)| indicate that decays to \(Wd\) and \(Ws\) are
extremely rare.8 If there were a fourth quark generation (\(t', b'\)), then decays to light
quarks could occur more often since |\(V_{tb}|| would no longer be constrained to have a
value near one.
1.1. Production of Top Quarks at the Tevatron

Quarks are sensitive to both the strong and electroweak forces. The strong force is far more powerful than the electroweak force, and thus top quarks are produced most often at hadron colliders via the decay of a highly energetic virtual gluon to a top quark and a top antiquark ($\bar{t}$). The rate for this process is about 7 pb at the Tevatron,\(^9\)\(^{-12}\) where it was first observed by the CDF and DØ collaborations in 1995.\(^{13,14}\) Top quarks can also be produced without their antiparticle partner via the electroweak interaction.\(^15\)\(^{-44}\) In this case, a t-channel virtual $W$ boson and a highly energetic bottom quark combine and produce a top quark, or a far off-shell s-channel $W$ boson decays to produce a top quark and a bottom antiquark. A third process is predicted to exist that occurs via both the s-channel and t-channel, when a top quark is produced together with a $W$ boson.\(^{29,32,43}\) Charge-conjugate processes that produce top antiquarks are expected via the same mechanisms. Contrary to expectations based on the relative feebleness of the electroweak force, the rates for single top quark production are calculated to be quite high, at about 2 pb for the t-channel process and 1 pb for the s-channel process.\(^34,43\) This is because higher-order corrections to the tree-level calculations for the t-channel process are large. (The rate for $tW$ production is predicted\(^43\) to be about 0.3 pb and this process is not seen at the Tevatron.) Therefore, one might expect searches of the Tevatron data at the DØ and CDF experiments to observe s-channel and t-channel single top quark production rather easily, given that the current datasets are 50 times larger than those used to discover the top quark in 1995 using the pair production mode. The reason that it has been very difficult to observe single top quark production is not because the signal rate is too low, but because the background processes are over 30 times larger than for top quark pair events. The main leading order Feynman diagrams for strong and electroweak production of top quarks at the Tevatron are shown in Fig. 1.

2. The Search for Single Top Quarks at the Tevatron

2.1. The Tevatron Collider and the DØ and CDF Experiments

Fermi National Accelerator Laboratory is the home of the Tevatron Collider, a 6.3 km circumference proton-antiproton ($p\bar{p}$) accelerator with superconducting magnets. This machine began operating in collider mode (versus earlier provision of fixed target beams) in 1985 with only the Collider Detector at Fermilab (CDF) in operation to record the collisions. The DØ experiment started operation in 1992, at which time the collision energy was 1.8 TeV. This was raised in 2001 to 1.96 TeV.

The two large multipurpose detectors have similar structures:\(^{45,46}\) they consist of concentric layers of detectors tightly packed around the beampipe at a Tevatron collision region to a height of 9 m, with each detector layer having a different purpose. The inner layers are composed of silicon microstrip detectors, which provide the three-dimensional positions where charged particles pass through. DØ
Fig. 1. Representative leading order Feynman diagrams for strong production of top quark pairs from (a) quarks and (b) gluons, and for electroweak production of single top quarks from (c) s-channel “tW” production, (d) t-channel “tqb” production, and (e) “tW” production. Process (a) produces 85% of the tt rate at the Tevatron. Process (e) is not observed there as the cross section is too low. The notation “tb” refers to the t-bar and charge conjugate t-bar processes together; “tqb” refers to tqb and tQB, and “tW” refers to tW minus and tW plus.

has an outer tracking detector of scintillating fibers and CDF has an outer gaseous wire drift chamber. Each of these tracking systems is encased in a solenoid magnet with field-lines parallel to the beampipe; DØ’s field strength is 2.0 Tesla and CDF’s is 1.4 T. The magnetic fields curve the tracks of charged particles, which enables their transverse momentum to be measured. DØ’s magnet has a much smaller radius (60 cm) compared to CDF’s (150 cm) since it was retrofitted inside the calorimeter in 2001. It therefore does not allow as much space for the tracking detectors, resulting in far fewer hits per track, which makes pattern recognition difficult with resulting lower track reconstruction efficiency and higher fake track rates. The momentum resolution in DØ is poorer because of the shorter track length.

Outside the central magnets, each detector has layers of calorimetry used to measure the energy of particles and to distinguish between electromagnetic particles (electron and photons, with or without matching central tracks), and jets (from quarks and gluons). DØ’s liquid-argon/uranium calorimeter is more hermetic and covers a larger angular region (pseudorapidity |η| < 4.2 versus CDF’s |η| < 3.6, where η = −ln[tan(θ/2)] and θ is the polar angle), giving better acceptance for forward jets and missing transverse energy resolution.

Outside the calorimeter, DØ has a muon spectrometer with up to four layers of tracking detectors and a magnetic field strength of 1.9 T. Only muons (and invisible neutrinos) pass right through the calorimeter and their position and momentum is remeasured here, with wide pseudorapidity coverage to |η| < 2.0. CDF also has
several layers of muon detectors outside its calorimeters with coverage to $|\eta| < 1.6$. Each detector has a sophisticated multilevel trigger system used to select interesting events from the $p\bar{p}$ collisions, which occur every 396 ns.

2.2. Search History

Preparation for a search for single top quark production began at the DØ experiment in 1994\textsuperscript{47} (five months before the first observation of top quarks in pair production mode). DØ published the results of a search using simple kinematic event selection to set upper limits on the cross sections in 2000,\textsuperscript{48−50} with a follow-up analysis making first use of a multivariate analysis technique, neural networks, to separate signal from background in 2001.\textsuperscript{51} These analyses used 90 pb$^{-1}$ of data at $\sqrt{s} = 1.8$ TeV from Run I at the Tevatron (1992–1996), when DØ did not have a silicon vertex detector or central magnetic field and $b$ jets were identified via the presence of muons in jets from the $b$ decay. The CDF collaboration also searched Tevatron Run I data for single top quark production; they published upper limits on the cross sections from a cut-based selection in 2002\textsuperscript{52} and a follow-up analysis of the same dataset using neural networks in 2004.\textsuperscript{53} CDF’s analyses had the advantage of being able to use secondary vertex $b$-jet identification using the Silicon Vertex Detector.\textsuperscript{54} The limits on the cross sections for s-channel and t-channel production were about 10–20 times greater than the predicted values.

The Tevatron collision energy was increased to 1.96 TeV in 2001, and the beam intensity was improved by a factor of about 15 over the course of the run (2002–present). The DØ and CDF detectors were significantly upgraded, with the addition amongst other things, of the central solenoid magnet to DØ and very large silicon tracking systems at both experiments.\textsuperscript{55,56} CDF analyzed 160 pb$^{-1}$ of Run II data using a cut-based selection and a maximum-likelihood fit to the variable “lepton charge × untagged jet pseudorapidity” and set 95% confidence level (CL) upper limits of 13.6 pb on s-channel production and 10.1 pb on t-channel production in 2005.\textsuperscript{57} DØ analyzed 230 pb$^{-1}$ of data using neural networks (NN) for signal-background separation and a Bayesian binned likelihood calculation using the NN output distributions, and set 95% CL upper limits of 6.4 pb in the s-channel and 5.0 pb in the t-channel in 2005.\textsuperscript{58,59}

The next step in the search led to a major improvement. The DØ collaboration increased its dataset by a factor of four to 0.9 fb$^{-1}$, switched the search to $tb+tqb$ combined (assuming the SM ratio of the two parts), loosened the selection cuts and used an improved $b$-jet identification algorithm to increase the signal acceptance by 13% over that obtained in the earlier analysis, and applied three multivariate methods to separate signal from background to reach 3.4 standard deviation ($\sigma$) significance for a single top quark signal. The measured cross section for $tb+tqb$ production combined was $4.9 \pm 1.4$ pb. The measurement significance represents a probability of 0.035% for the background to have fluctuated up and given a false measurement of signal with a cross section of at least 4.9 pb. A significance greater than $3\sigma$ is considered in the high energy physics community not to be sufficient for
a claim of discovery or first observation (which is set at 5σ), but is high enough to indicate that “evidence” for the process in question has been seen; it is a very exciting threshold to reach. The result was published in 2007 and has received well over 100 citations to date. Small improvements were made to the analysis and a slightly more significant result (3.6σ) was published in a long paper in 2008. The CDF collaboration performed a similar analysis on 2.2 fb$^{-1}$ of data and reached a significance for single top quark signal of 3.7σ, published in 2008. They measured the cross section for $tb+ tqb$ production to be $2.2 \pm 0.7$ pb.

3. Measurement Overview

After the “evidence” papers, the DO and CDF collaborations each worked to improve their analysis methods and apply them to larger datasets. Both collaborations select events with one isolated high transverse momentum ($p_T$) lepton (electron or muon) and large missing transverse energy ($E_T$, indicative of a leptonic $W$-boson decay, together with two, three, or four jets. One or two of the jets is identified as originating from a $b$ quark, which could be from the top quark decay or produced together with it. The CDF collaboration has an additional independent search channel that requires no identified charged lepton, which picks up events lost to electron or muon identification inefficiencies, and some $\tau$+jets events where the $\tau$ decayed hadronically (but there was no explicit $\tau$ reconstruction). This is the first time that the $E_T$+jets channel has been used in a single top quark measurement.

Both collaborations include signal and background events in their lepton+jets channels with $t\rightarrow Wb$, $W\rightarrow \tau \nu$, and $\tau \rightarrow e \nu \nu$ or $\tau \rightarrow \mu \nu \nu$. Neither includes events with $\tau$→hadrons in the signal acceptance since hadronic $\tau$ reconstruction is difficult. A new search based just on this decay channel has recently been completed by DO.

After event selection, the signal-to-background ratio is approximately 1:20. The backgrounds are mostly $W$+jets events (especially at low jet multiplicity), followed by $t\bar{t}$ pairs (especially at high jet multiplicity), with small contributions from $Z$+jets, dibosons ($WW$, $WZ$, $ZZ$), and multijets. Top pairs look like signal when one $W$ boson decays leptonically ($e\nu$ or $\mu\nu$) and the other decays hadronically ($u\bar{d}$, $c\bar{s}$, etc.) producing lepton+jets events, and also when both $W$’s decay leptonically and event reconstruction fails to identify one of the leptons. $Z$+jets events and some diboson processes also mimic single top quark signals when the $Z$ boson decays to a pair of leptons ($e^+ e^-$ or $\mu^+ \mu^-$) and one of the leptons is lost, generating fake $E_T$. Multijet events look like signal in the electron channel when a jet is misidentified as an electron and a jet’s energy is mismeasured, creating false $E_T$. In the muon channel, the multijet background comes mostly from $b\bar{b}$ events where one of the $b$’s decays to a muon that travels wide of its jet or the jet is not reconstructed (its energy is too low maybe). Example Feynman diagrams for the $W$+jets and multijets processes are shown in Fig. 2. After event selection, DO has 4,519 lepton+jets events and CDF has 3,315 lepton+jets events and 1,411 $E_T$+jets events.
Fig. 2. Representative leading order Feynman diagrams for the background processes: (a) $W$+jets with real $b$-jets, (b) $W$+jets with a light jet mistagged as a $b$-jet, (c) multijets with a jet misidentified as an electron, and (d) $b\bar{b}$ multijets with a nonisolated muon (from a $b$ decay) misidentified as an isolated one (from a $W$ decay).

4. Data Samples

For the observation analysis, DØ uses 2.3 fb$^{-1}$ of Run II data, collected from August 2002 until August 2007. The dataset is split in two parts ("Run IIA" and "Run IIB") to denote a significant upgrade to the detector, when a new layer of silicon microstrip detectors was added around the beampipe. This improved the tracking and $b$-tagging efficiencies. The Run IIB half dataset has higher instantaneous luminosity than the Run IIA half, which lowers the primary vertex identification efficiency and increases track multiplicities in the events. The CDF collaboration uses a 3.2 fb$^{-1}$ dataset for the lepton+jets analysis, which is not split like DØ’s. The $E_T$+jets analysis uses 2.1 fb$^{-1}$ of data.

DØ selects data that pass any reasonable trigger. This requirement is relaxed from earlier analyses where only lepton+jets triggers were used. The change increases the signal acceptance by 16% in the electron channel and 20% in the muon channel. It also increases the trigger efficiency to $\approx 100\%$, meaning that no correction functions are needed to model trigger turn-on curves for Monte Carlo events. CDF’s triggers include a high-$p_T$ electron trigger, a high-$p_T$ muon trigger, and one that requires high $E_T$ and either an energetic electromagnetic cluster or two jets.

5. Signal and Background Simulation

5.1. Single Top Quark Signal Models

The single top quark signal is modeled to reproduce next-to-leading order (NLO) kinematics using modified leading order (LO) generators. DØ uses SINGLETOP, a version of COMPTOT adapted by its authors for DØ, and CDF uses MADEVENT based on MADGRAPH with their own modifications. In fact, s-channel simulation at LO reproduces NLO kinematics without changes and it is only the t-channel that needs such attention. The transverse momentum distribution of the bottom antiquark in the $2\to2$ process $q'b\to tq$ (from SINGLETOP or MADEVENT) after back-propagation of the initial-state $b$ to $g\to b\bar{b}$ (from PYTHIA) is matched to that of the $\bar{b}$ in the $2\to3$ process $q'g\to tq\bar{b}$ (from SINGLETOP or MADEVENT). Simulated events
from the $2 \rightarrow 2$ calculations are kept if $p_T(\bar{b}) \leq 10$ GeV (DØ) or $\leq 20$ GeV (CDF) and ones from the $2 \rightarrow 3$ process are used if $p_T(\bar{b}) > 10$ GeV (DØ) or $> 20$ GeV (CDF). The $2 \rightarrow 2$ process is scaled by a $K$ factor to make the rates at the cut-off point match: $K = 1.21$ for DØ. There is another $t$-channel subprocess where a gluon produces a $t\bar{t}$ pair and the $\bar{t}$ combines with a radiated $W$ boson to produce a $\bar{b}$. This subprocess has a cross section only a few percent of the $tbg \rightarrow b\bar{b}$ subprocess, with a large negative interference between the two subprocesses. Both DØ’s SINGLETOP and CDF’s MADEVENT models include the $g \rightarrow t\bar{t}$ subprocess and the interference. The models also both have finite widths for the top quark ($\approx 1.5$ GeV) and $W$ boson ($\approx 2.0$ GeV). In all signal (and $t\bar{t}$ background) models, the top quarks and their daughter $W$ bosons are decayed at the time of production, before later processing with PYTHIA, so that all spin properties of the top quarks are preserved in the angular correlations of the final decay products. DØ uses a top quark mass of 170 GeV for signal simulation, CDF uses 175 GeV. Each value was chosen at a time when it was close to the world average value, which shifts slightly once or twice a year as the measurement is improved. This difference does not have a significant effect on the final results.

For modeling the parton kinematics in the protons and antiprotons, DØ uses the CTEQ6M next-to-leading-order parton distribution functions (PDF), CDF uses the CTEQ5L leading order PDFs. The $Q^2$ scale for the s-channel model is $M_{top}^2$ (DØ) or $\hat{s}$ (CDF), and for the $t$-channel model, $(M_{top}/2)^2$ (DØ) or $\hat{t} + M_{top}^2$ (CDF) are used. DØ’s values in SINGLETOP are chosen to make the LO and NLO cross sections be the same and CDF’s values in MADEVENT are chosen to closely match those used in the NLO ZTOP event generator. Both collaborations use PYTHIA to add the underlying event from the $\bar{p}p$ interaction, the initial-state and final-state radiation, and to hadronize and fragment the final state quarks and gluons into jets. They also both use TAUOLA to decay tau leptons. For $B$-hadron decay, DØ uses EVTGEN from the BaBar experiment and CDF uses QQ from the CLEO experiment. Events from multiple primary vertices are overlaid onto the primary MC event with a Poisson multiplicity distribution in order to simulate the high instantaneous luminosity. DØ uses zero-bias data events and CDF uses MC events generated with PYTHIA. The mean number of $p\bar{p}$ collisions per bunch crossing for this dataset is two for Run IIa and five for Run IIb.

5.2. Background Models

All background components except multijet events are simulated using Monte Carlo models. Both collaborations use the ALPGEN event generator, which has leading-log (LL) precision, coupled to PYTHIA to simulate $W$+jets events, including full modeling of events with massive $b$ and $c$ jets. The version of ALPGEN used includes parton-jet matching to avoid double-counting some regions of jet kinematics. The samples are generated in the following sets ($lp =$ light partons): $W$+0lp, $W$+1lp, $W$+2lp, $W$+3lp, $W$+4lp, $W$+5lp (this set includes $W$+single massless charm):
$Wc\bar{c}+0\ell p$, $Wc\bar{c}+1\ell p$, $Wc\bar{c}+2\ell p$, $Wc\bar{c}+\geq3\ell p$; and $Wb\bar{b}+0\ell p$, $Wb\bar{b}+1\ell p$, $Wb\bar{b}+2\ell p$, $Wb\bar{b}+\geq3\ell p$, which are summed weighted by the ALPGEN LL average cross section for each subset and then split to obtain $W+2\text{jets}$, $W+3\text{jets}$, and $W+4\text{jets}$ event sets. DØ uses this same version of ALPGEN and generation method to simulate $t\bar{t}$ events; CDF uses PYTHIA, which only adds extra jets through initial-state and final-state radiation (not from the hard scatter), but this is not critical since they do not include events with four jets in their analyses. Smaller backgrounds are modeled using ALPGEN and PYTHIA (DØ) and PYTHIA (CDF).

Some more details of the $W+$jets modeling are in order, since this background is critical in the most important 2-jets analysis channels. DØ uses the CTEQ6L1 PDF and scale $Q^2 = m_W^2 + \sum m_T^2$ (recommended by the ALPGEN authors), where $m_T$ is the transverse mass defined as $m_T^2 = m^2(\text{parton}) + p_T^2(\text{parton})$ and the sum $\sum m_T^2$ extends to all final state partons (including the heavy quarks, excluding the $W$ decay products). CDF uses CTEQ5L and $Q^2 = m_W^2 + \sum p_T^2(\text{jets})$ (possibly accidentally chosen due to an ALPGEN bug in labeling the options).

The multijet background is modeled by DØ using data with much looser lepton selection than for signal selection. They select events that pass all final cuts in the electron channel except that the electromagnetic object fails the electron identification cuts, including not requiring a track matched between the primary vertex and energy cluster in the calorimeter. This is a very loose selection, with a ten-fold increase in statistics compared to that used in the earlier evidence analysis (when a matching track was required). The reason for this change is to ensure sufficient statistics after $b$ tagging to make a proper measurement of this background. In DØ’s muon multijet data, the muon is not required to be isolated from a jet. In the previous analysis, a partial isolation requirement was applied, and removing this criterion increased the muon multijet background statistics by a factor of ten. In the electron channel, the ratio of electron+jets and photon+jets events is used to determine a reshaping weight as a function of the electron $p_T$ to make the background model sample better match the actual multijet events remaining after signal selection. The function boosts the fraction of low-energy events. In the muon multijet background dataset, any jets close to the muon are removed and the $E_T$ is recalculated in order to make the jets reproduce those in the signal data. No kinematic reshaping is needed. To obtain background samples that model the multijets backgrounds, the samples as described (with an electron or muon that fails final identification criteria) are scaled by functions that represent the probability for a failing lepton to pass the identification cuts.

CDF model the multijets background in the lepton+jets channels using a data sample with $E_T$ below the signal selection threshold of 25 GeV, and project it into the high-$E_T$ signal region using a fit to the shape of the $E_T$ distribution. They model the dominant multijets background in the $E_T+$jets channels using pretagged data that pass all selection cuts together with a tag-rate matrix calculated using an independent $E_T+$jets dataset.
5.3. Detector Simulation

After the MC samples are generated, they are processed through code that models the geometry and material of each subdetector system, and then through further code that generates digitized signals modeled to resemble those from the subdetectors. After this, the MC events look very like those from data and both are processed through event reconstruction software to identify the correct primary vertex, leptons, jets, and so on, ready for further analysis.

5.4. Background Normalization

The $t\bar{t}$, $Z$+jets, and diboson backgrounds are normalized to (N)NLO theory cross section values, with each collaboration using a $t\bar{t}$ cross section appropriate for the top quark mass chosen for its analysis.

Before the $W$+jets backgrounds can be normalized, corrections are applied to modify the leading log $\text{ALPGEN}$ fractions of heavy flavor jets ($c$, $\bar{c}$, and $b\bar{b}$) to account for missing higher order contributions and make them match what is seen in data. The DØ collaboration scales the $Wjj$, $Wcj$, $Wc\bar{c}$ and $Wb\bar{b}$ subprocesses to their NLO predictions ($K' = \sigma_{\text{NLO}}/\sigma_{\text{LL}}$ and $K'_{\text{HF}} = \sigma_{\text{NLO}}^{\text{HF}}/\sigma_{\text{NLO}}$ factors. This is also done for the small $Z$+jets background. The $K'$ factor is 1.30. $K'_{\text{HF}} = 1.47$ for $Wc\bar{c}$ and $Wb\bar{b}$, 1.67 for $Zc\bar{c}$, and 1.52 for $Zb\bar{b}$. These factors come from calculations using the NLO MC event generator $\text{mcfm}$.

For $Wcj$, $K'_{\text{HF}} = 1.38$, from a data measurement that agrees with NLO theory. The important $Wb\bar{b}$ and $Wc\bar{c}$ subprocesses are then checked against data after $b$ tagging and an empirical correction of $0.95 \pm 0.13$ is applied to get good data-background agreement. This factor accounts for contributions to the heavy flavor rate from Feynman diagrams at higher order than NLO not included in $\text{mcfm}$. The uncertainty on the empirical correction factor is the third largest component of the total systematic uncertainty on the cross section measurement. It includes a 9% statistical contribution from the variation of the correction when measured in different analysis channels ($e$, $\mu$, 1-tag, 2-tags, Run Ila, Run Iib), 8% from the $Wcj$ $K'_{\text{HF}}$ factor uncertainty (10%), and 7% from the uncertainty on the assumed single top cross section (40%, based on the difference between DØ and CDF’s published evidence measurements). CDF compresses these three steps into one, and, from a data-background comparison after $b$ tagging in $W$+1jet events, applies a scale factor of $1.4 \pm 0.4$ to $Wcj$, $Wc\bar{c}$ and $Wb\bar{b}$ relative to the LL $Wjj$ process. Converting DØ’s scale factors to allow a comparison gives $1.47 \times 0.95 = 1.40$, so things are consistent.

The $W$+jets and multijets backgrounds are normalized to data before $b$ tagging. DØ normalizes the sum of the two backgrounds using an iterative Kolmogorov-Smirnov procedure with the $p_T$(lepton), $E_T$, and $W$ boson transverse mass $M_T(W)$ variables. For the multijets background, CDF uses a fit to the $E_T$ distribution at low $E_T$, extrapolated to high $E_T$ and does not anticorrelate the two components. After subtracting all other background components, they normalize the $W$+jets background to the number of data events.
5.5. Model Corrections

Both collaborations need to correct the MC efficiency to reproduce the efficiency of the detector, event reconstruction, and particle identification. This is done for electrons, muons, and jets. All MC events are reweighted to make the instantaneous luminosity distribution (number of overlaid zero-bias events from multiple $p\bar{p}$ collisions) match that observed in the data. DØ also reweights the muon pseudorapidity $\eta$ distribution in $W+$jets events to better model the efficiencies of the regions between the central and forward muon systems.

For $W+$jets events, both collaborations find the pseudorapidity distributions of the jets from the ALPGEN simulation do not match data well (there are presumed to be slightly different Feynman diagrams in the calculation compared with e.g., the SHERPA model which has wider jet $\eta$ distributions). The ALPGEN distributions are too narrow, and empirical reweightings are applied to these distributions ($\eta$(jet1), $\eta$(jet2), $\Delta\phi$(jet1,jet2), and $\Delta\eta$(jet1,jet2) for DØ, similarly for CDF) to make the background model match data before $b$ tagging. Since DØ’s reweighting uses binned functions derived in each analysis channel separately, it also takes account of imperfections of the detector model in the intercryostat regions.

6. Event Reconstruction and Particle Identification

6.1. Primary Vertices

There are several primary vertices in each event, on average, because of the high collision rate leading to multiple interactions. They are reconstructed at DØ by first clustering tracks according to their positions along the beamline, then the location and width of the beam is measured and used to refit the tracks. Finally, each cluster of tracks is associated with a vertex, and the one with the lowest probability of coming from a zero-bias collision is chosen as the primary vertex for that event.

6.2. Electrons

Electrons are defined as clusters of energy deposited in the electromagnetic section of the calorimeter that are consistent in shape and other properties with an electromagnetic shower. The cluster must be isolated from other energy in the event and have a track that points to it from the primary vertex.

6.3. Muons

Muons are identified by matching reconstructed tracks from the outer muon system to ones from the inner tracking system. The match is made spatially and (at DØ) in transverse momentum and muon charge. Muons must be isolated from nearby tracks and jets to show they are from $W$ (or $Z$) boson decay and not from heavy flavor ($b$ or $c$) decay inside a jet.
6.4. Jets

Jets are reconstructed using energy deposited in the calorimeters. DØ applies the midpoint cone algorithm\textsuperscript{83} in \((y, \phi)\) space, where \(y\) is the rapidity and \(\phi\) is the azimuthal angle, and the cone radius is 0.5. CDF uses a clustering algorithm\textsuperscript{84} in \((\eta, \phi)\) space with a cone radius of 0.4. There are several requirements on where the energy is deposited to reject noisy jets (whose energy would be mismeasured). The energy of each jet is corrected if there is a muon in the jet, to account for energy taken away by that muon and associated (invisible) neutrino from a heavy quark decay. The jet’s energy is also corrected using the jet energy scale calibration to ensure that the absolute value is correct. For most jets \((E_T, \eta)\), the uncertainty on the jet energy scale is between 1\% and 2\%.

6.5. Missing Transverse Energy

The missing transverse energy is computed by adding up vectorially the transverse energies in all cells of the electromagnetic and fine (inner) hadronic calorimeters (for DØ). Cells in the coarse (outer) hadronic calorimeter are only added if they form part of a good jet. This quantity is corrected for all the energy corrections applied to other objects in the event and for the momentum of isolated muons. CDF’s computation of \(\not{E_T}\) is similar.

7. Event Selection

The analyses start out with very large numbers of events in data and MC signal and background samples. For example, DØ uses data skims with one electron or one muon in them, which contain 1.2 billion events, and 85 million MC events. From these samples, the analyses first select events that look like signal and reject events that do not. That is, each collaboration devises selection cuts designed to keep as many MC signal events as possible while rejecting as much background as they can. The DØ collaboration chooses to maximize signal acceptance while allowing for a slightly worse signal-to-background ratio, whereas the CDF collaboration chooses tighter selection cuts that produce a lower signal acceptance but better signal-to-background ratio. Thus, although DØ starts the analysis with about 30\% less integrated luminosity to analyze than CDF, they end up with more expected signal events in the lepton+jets channel, and a similar number in total when considering also the \(\not{E_T}+\)jets channel after all selections are applied. DØ pursues this strategy because their studies show that the overall sensitivity of the analysis is proportional to the signal acceptance.

7.1. Kinematic Cuts

The kinematic cuts used in the analyses are shown in Table 1. For simplicity, only the cuts in the channels with exactly two jets are shown, since these channels contribute
most to the analysis sensitivity. For the lepton+jets analyses, both collaborations also use events with three jets, and DØ also uses events with four jets. These channels have slightly harder cuts for electron $p_T$, $E_T$, and total transverse energy $H_T$ than those shown in the table, to reject the higher multijets background.

Table 1. Kinematic selection cuts used in the 2-jets analysis channels to identify events that look like single top quark signal and reject backgrounds.

<table>
<thead>
<tr>
<th></th>
<th>DØ’s Selection</th>
<th>CDF’s Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lepton+2Jets</td>
<td>Lepton+2Jets $E_T+2$Jets</td>
</tr>
<tr>
<td>Electron</td>
<td>$p_T &gt; 15$ GeV</td>
<td>$p_T &gt; 20$ GeV</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>Muon</td>
<td>$p_T &gt; 15$ GeV</td>
<td>$p_T &gt; 20$ GeV</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>Neutrino</td>
<td>$E_T &gt; 15$ GeV</td>
<td>$E_T &gt; 25$ GeV</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>Jet1</td>
<td>$p_T &gt; 25$ GeV</td>
<td>$p_T &gt; 20$ GeV</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>Jet2</td>
<td>$p_T &gt; 15$ GeV</td>
<td>$p_T &gt; 20$ GeV</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>Total $E_T$</td>
<td>$H_T(jets,e,E_T) &gt; 120$ GeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_T(jets,\mu,E_T) &gt; 110$ GeV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\eta</td>
</tr>
</tbody>
</table>

Motivation for DØ’s choice of lower transverse energy thresholds and wider jet pseudorapidity distributions than used in, for example, a top pairs measurement can be seen in Fig. 3 for the t-channel single top quark process. The light quark that radiates the $W$ boson has a very wide $\eta$ distribution in both the forward and backward directions (shown by the red histograms in the plots). This is a very strong signature for single top quark production that will be used as a powerful variable to separate signal from background. The soft $\bar{b}$ produced from the gluon splitting has an even wider $\eta$ distribution (dark green histograms) and low $p_T$, and finding this jet increases the double-$b$-tagged signal acceptance.

Fig. 3. Distributions of (a) the transverse momentum and (b) the signed pseudorapidity of partons in t-channel single top quark events, from the COMPHEP-SINGLETOP simulation.
Observation of Single Top Quark Production

There are additional selection cuts not shown in Table 1. In the lepton+jets channels, events are rejected if there is a second isolated lepton, which rejects dilepton decays of $t\bar{t}$, $Z+$jets, and diboson events. DØ has an upper cut on $E_T$ of 200 GeV to reject misreconstructed events. Both collaborations throw out events with low $E_T$ just above the cut thresholds when it is aligned or back-to-back with one of the objects in the event, indicative of a misreconstructed event. The primary vertex must be clearly identified and near the center of the detector, and the lepton must originate from it. The regions between DØ’s central and end calorimeter cryostats are tricky to instrument and model accurately, and if the leading jet in the muon analysis channel points to this region, the threshold on it is raised to 30 GeV. Finally, DØ has cuts on muon track curvature significance designed to reject events where the muon has been misreconstructed. In CDF’s $E_T+$jets analysis, a neural network with 15 input variables is trained to separate the multijets background from signal and a cut is placed on the output distribution.

After the kinematic event selection, DØ’s background samples retain 4 million MC events and 0.8 million multijet data events, and there are 0.5 million single top quark signal MC events. The signal data contain 114,777 events, with predicted background components: $Wjj = 71\%$, $Wcj = 6\%$, $Wc\bar{c} = 6\%$, $Wb\bar{b} = 3\%$, $Z+$jets = 6\%, dibosons = 2\%, $t\bar{t} = 1\%$, and multijets = 5\%. The expected single top quark signal is $tb = 0.13\%$ and $tqb = 0.26\%$, with a signal-to-background ratio for $tb+tqb$ of 1:260. Clearly, an additional method is needed to select events for the analyses to stand any chance of finding the single top quark signal.

7.2. Heavy-Flavor Jet Tagging

The most powerful part of event selection is the identification of jets that originate from $b$ quarks. The algorithms use the long decay time of the $B$ hadrons (mean lifetime $\simeq 1.5 \times 10^{-12}$ s) which results in detached secondary vertices in the jets (> 1 mm between the primary and secondary vertices), together with other information about the tracks to find $b$ jets. The tagging algorithms are applied directly to jets in data and to most MC events at CDF, and are modeled with tag-rate functions for MC events at DØ together with taggability-rate functions to reproduce the detector geometric acceptance and operating efficiency. For $W+$light jets MC, CDF uses tag-rate functions measured in multijets data. $b$-jet identification is implemented at DØ by combining all the track and vertex information using a neural network. CDF uses the significance of the decay length of the secondary vertex in the $(r, \phi)$ plane for the lepton+jets and $E_T+$jets channels, and also a jet probability algorithm in the $E_T+$jets channel. Depending on where a cut is put on these variables, one can define looser or tighter $b$ tagging, where “loose” means higher probability to tag a $b$ jet ($58\%$ for $b$ jets within DØ’s Silicon Microstrip Tracker fiducial geometric acceptance) with associated higher probability to mistag a non-$b$ jet ($17\%$ for charm jets and $1.8\%$ for light quark and gluon jets), and “tight” means a lower $b$-tag probability ($47\%$ for $b$ jets at DØ) with associated lower fake tag rates ($10\%$ for
 jets and 0.5% for light jets). DØ requires one tight-tagged jet (and no loose-tagged jet) for its single-tagged analysis channels, and two loose-tagged jets for its double-tagged channels. CDF has one set point for both single-tagged and double-tagged lepton+jets channels, with efficiencies of 50% (b), 9% (c), and 1% (j) for fiducial jets within the Silicon Detector tracking system.

7.3. **Analysis Channel Separation**

To improve the sensitivity of the measurement, both DØ and CDF split their datasets into independent channels using the jet multiplicity (2, 3, and 4 for DØ), number of b-tagged jets (1 or 2), lepton flavor (DØ only, electron or muon), trigger type (CDF only, lepton, $E_T$ for muon+jets) and data-collecting period (DØ only, Run IIa and Run IIb), giving 24 independent lepton+jets analysis channels for DØ and eight for CDF. CDF’s $E_T$+jets channel with no isolated lepton is split by the number and type of $b$ tags (one SecVtx-tagged jet, two “SecVtx”-tagged jets, and one “SecVtx” and one “JetProb”-tagged jet). Measurements are made in each channel and combined at the end of the analysis. The signal-to-background ratios vary from 1:10 (2-jets/2-tags) to 1:37 (4-jets/2-tags) for DØ, with the most important 2-jets/1-tag channels having S:B = 1:20. CDF’s channels have S:B = 1:15 in the 2-jets channels (1-tag and 2-tag combined), S:B = 1:23 in the 3-jets channels, and S:B = 1:23 in the $E_T$+jets channels.

8. **Signal Acceptances and Event Yields**

After all event selections have been applied, the signal acceptances (percentage of total cross section) for DØ are (3.7 ± 0.5)% for the s-channel $tb$ process and (2.5 ± 0.3)% for the t-channel $tqb$ process. The t-channel process has a lower acceptance because the $b$ jet has low transverse momentum and is difficult to identify. CDF’s signal acceptances in the lepton+jets channels are 2.7% for the $tb$ process and 1.8% for the $tqb$ process. These values are lower than DØ’s because of the more restrictive trigger requirements, tighter kinematic selection, and tighter $b$ tagging in the double-tagged channel. In addition, CDF has the $E_T$+jets channel with a signal acceptance of 1.1% for $tb$+$tqb$ combined.

Table 2 shows the numbers of signal and background events expected, and the numbers of data events found. For simplicity here, all analysis channels have been combined. Four notes to understand the table are in order: (i) Remember that DØ uses $m_t = 170$ GeV and CDF uses 175 GeV for single top signal and $t\bar{t}$ background, with associated higher theory cross sections for the lower top quark mass. They each also use different theory calculations for these values: for single top, DØ uses Kidonakis 2006 values of 1.12±0.05 pb ($tb$) and 2.34±0.13 pb ($tqb$), and for $t\bar{t}$ they use the Kidonakis and Vogt 2003 value of 7.91$^{+0.61}_{-1.01}$ pb (where the $t\bar{t}$ uncertainty includes a component for the top quark mass). CDF uses for single top the Harris et al. 2002 values of 0.88 ± 0.12 pb ($tb$) and 1.98$^{+0.28}_{-0.22}$ pb ($tqb$), and for $t\bar{t}$ they use the Bonciani et al. 1998 value of 6.70 ± 1.32 pb. Thus, direct comparison of the
signals and $t\bar{t}$ backgrounds needs one or other experiment’s numbers to be rescaled to be valid. (ii) DØ’s analysis includes channels with four jets and CDF’s does not, so the fraction of $t\bar{t}$ events expected by DØ is higher than at CDF when showing yields with all channels combined. However, when one considers each jet multiplicity channel separately, then the relative fractions of $W$+jets, $t\bar{t}$, etc. are very similar between the two experiments. (iii) CDF’s $E_T$+jets channel $W$+jets yield does not include $Wjj$ where $j$ = a light jet. (iv) CDF’s $E_T$+jets channel multijets yield includes also the $Wjj$ events.

Table 2. Numbers of events after all selections have been applied. See comments in the text on how to compare the columns.

<table>
<thead>
<tr>
<th></th>
<th>DØ’s Yields</th>
<th>CDF’s Yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lepton+Jets, 2.3 fb$^{-1}$</td>
<td>Lepton+Jets, 3.2 fb$^{-1}$</td>
</tr>
<tr>
<td>$t\bar{t}$+jets signal</td>
<td>223 $\pm$ 30</td>
<td>191 $\pm$ 28</td>
</tr>
<tr>
<td>$W$+jets</td>
<td>2,647 $\pm$ 241</td>
<td>2,204 $\pm$ 542</td>
</tr>
<tr>
<td>$Z$+jets, dibosons</td>
<td>340 $\pm$ 61</td>
<td>171 $\pm$ 15</td>
</tr>
<tr>
<td>$t\bar{t}$ pairs</td>
<td>1,142 $\pm$ 168</td>
<td>686 $\pm$ 99</td>
</tr>
<tr>
<td>Multijets</td>
<td>300 $\pm$ 52</td>
<td>125 $\pm$ 50</td>
</tr>
<tr>
<td>Total prediction</td>
<td>4,652 $\pm$ 352</td>
<td>3,377 $\pm$ 505</td>
</tr>
<tr>
<td>Data</td>
<td>4,519</td>
<td>3,315</td>
</tr>
</tbody>
</table>

Figure 4 shows the reconstructed $W$ boson transverse mass distributions from DØ (all channels combined) and CDF (lepton+2jets channels). The transverse mass is defined as: $M_T(W) = M_T(l, \nu) = \sqrt{2p_T(l)E_T(1 - \cos(\phi(l) - \phi(E_T)))}$.

![DØ 2.3 fb$^{-1}$](image1.png)  ![CDF 3.2 fb$^{-1}$](image2.png)

Fig. 4. Distributions of the $W$ boson transverse mass for (a) DØ, with all analysis channels combined, and (b) CDF, with all lepton+2-jets/1-tag channels combined.
9. Background Model Checks

Any analysis of the data is only valid if the background models reproduce the data in all variables used for event selection and to separate signal from background. In addition to checking the background model agreement with data for these distributions for every particle in each analysis channel, extensive cross checks using other data samples have been performed to ensure the separate components of the background model are accurately modeled. Samples that pass all selection cuts are used before $b$ tagging to certify the shape of the $W$+light jets background model. The $W$+heavy flavor background model’s agreement between data and background model in both shape and normalization is checked using a sample with exactly two jets, one or two $b$ tags, and, for DØ only since they have softer object $E_T$ requirements, $H_T(\ell, E_T, \text{jets}) > 300$ GeV. Many distributions are checked using these three cross-check samples and good agreement between data and background model is found. Figure 5 shows the transverse mass of the $W$ boson as an example.

Fig. 5. Distributions of the $W$ boson transverse mass for several cross-check samples: (a) DØ’s pretagged events, with all analysis channels combined, (b) DØ’s $W$+jets cross-check sample, (c) DØ’s $t\bar{t}$ pairs cross-check sample, (d) CDF’s two-jets untagged sample, and (e) CDF’s $t\bar{t}$ cross-check sample.
10. Systematic Uncertainties

The uncertainties in all searches are dominated by the statistical uncertainty from the size of the data sample. However, once there is enough data to observe and measure a signal, then systematic contributions to the total uncertainty become important. The total uncertainty on the cross section measurement by DØ is $\pm 22\%$, and for CDF it is $+29\%, -24\%$ in the lepton+jets channel, $+52\%, -46\%$ in the $E_T$+jets channel, and $+26\%, -22\%$ with these channels combined. The contribution from the data statistics in DØ’s measurement is $\pm 18\%$, leaving $\pm 13\%$ from systematic components. Normalization systematic uncertainties and shape-dependent systematic uncertainties are considered separately for each signal and background source in each analysis channel. The overall background uncertainty varies between 7% and 15% for the individual channels in DØ’s measurement. Shape and normalization uncertainties combined result in 20% uncertainties on the background model for single-tagged channels and 40% uncertainties on background for double-tagged channels, for events most like signal. The uncertainties on the background model for events most like background about 10% for single-tagged channels and 15% for double-tagged channels. DØ measures systematic uncertainty contributions from 23 different sources. Others were considered but found to be negligible. The largest source of systematic uncertainty comes from the $b$-ID tag-rate functions, including both normalization and shape parts, followed by the jet energy scale calibration (also normalization and shape), and the heavy-flavor correction factor for the $Wb\bar{b}$ and $Wc\bar{c}$ fractions in the MC model. Smaller contributions (in descending order) come from the integrated luminosity, the jet energy resolution, initial-state and final-state radiation, $b$-jet fragmentation, the $t\bar{t}$ pairs cross section, and lepton identification. CDF’s analyses include normalization uncertainty terms from 16 sources, and shape terms from a subset of nine sources. The most important ones are the jet energy scale, the event detection efficiency, and the $Wb\bar{b}$, $Wc\bar{c}$, and $Wcj$ scale factor.

11. Signal-Background Separation

The sensitivity to observe a signal with a large background is greatly improved by finding a variable that has a different shape for signal than for background. One can then keep only events in the maximal-signal region and measure a cross section by counting events if there is enough data, or, as in the case of single top quark production with only a few inverse femtobarns of data, one can perform a binned likelihood calculation comparing the shapes of the expected signal and background to data across the full distribution to further improve the sensitivity. Since the kinematics of single top quark events lie between those of the dominant lower-energy $W+$jets and higher-energy $t\bar{t}$ backgrounds, it is not possible to find a single simple variable with which to perform this calculation. Hence, DØ and CDF each combine many variables using several different methods to increase the signal-background separation power. DØ uses three discrimination methods and CDF uses
five in the lepton+jets channel, one in a separate s-channel $tb$ search, and one in the $E_T$+jets channel, which are briefly described here; more details are available elsewhere.\textsuperscript{61,90,91}

### 11.1. Discriminating Variables

DØ uses 97 discriminating variables in its final analysis, chosen from a much longer list to include only those variables with a different distribution for signal and at least one of the background components,\textsuperscript{17,28,38,40,92} and also to have good agreement between the shape of the background sum and data. The variables fall in five categories: object kinematics, event kinematics, jet reconstruction, top quark reconstruction, and angular correlations. The most powerful ones for separating single top quark signal from the $W$+jets and $t\bar{t}$ backgrounds in each category are shown in Table 3.

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>$W$+Jets</th>
<th>$t\bar{t}$ Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Object</strong></td>
<td>$E_T$</td>
<td>$p_T$(notbest2)</td>
</tr>
<tr>
<td><strong>Kinematics</strong></td>
<td>$p_T$(jet2)</td>
<td>$p_T$(jet4)</td>
</tr>
<tr>
<td></td>
<td>$E_T$(jet1,tag-(\mu))</td>
<td>$p_T$(light2)</td>
</tr>
<tr>
<td></td>
<td>$E$(light1)</td>
<td></td>
</tr>
<tr>
<td><strong>Event</strong></td>
<td>$M$(jet1,jet2)</td>
<td>$M$(alljets-tag1)</td>
</tr>
<tr>
<td><strong>Kinematics</strong></td>
<td>$M_T(W)$</td>
<td>Centrality(alljets)</td>
</tr>
<tr>
<td></td>
<td>$H_T(\text{lepton},E_T,\text{jet1,jet2})$</td>
<td>$H_T(\text{alljets-tag1})$</td>
</tr>
<tr>
<td></td>
<td>$H_T(\text{jet1,jet2})$</td>
<td>$H_T(\text{alljets-tag1})$</td>
</tr>
<tr>
<td></td>
<td>$H_T(\text{lepton},E_T)$</td>
<td>$H_T(\text{lepton},E_T,\text{alljets})$</td>
</tr>
<tr>
<td><strong>Jet</strong></td>
<td>Width$_\phi$(jet2)</td>
<td>Width$_\phi$(jet4)</td>
</tr>
<tr>
<td><strong>Reconstruction</strong></td>
<td>Width$_\eta$(jet2)</td>
<td>Width$_\eta$(jet4)</td>
</tr>
<tr>
<td></td>
<td>Width$_\eta$(jet2)</td>
<td>Width$_\eta$(jet2)</td>
</tr>
<tr>
<td><strong>Top Quark</strong></td>
<td>$M_{\text{top}}(W(S1),\text{tag1})$</td>
<td>$M_{\text{top}}(W(S2),\text{tag1})$</td>
</tr>
<tr>
<td><strong>Reconstruction</strong></td>
<td>$\Delta M_{\text{top}}^{\text{min}}$</td>
<td>$\Delta M_{\text{top}}^{\text{min}}$</td>
</tr>
<tr>
<td><strong>Angular</strong></td>
<td>$\cos(\text{light1,lepton})_{\text{btaggedtop}}$</td>
<td>$\cos(\text{lepton}_{\text{btaggedtop}},\text{btaggedtopCM})$</td>
</tr>
<tr>
<td><strong>Correlations</strong></td>
<td>$\Delta \phi(\text{lepton},E_T)$</td>
<td>$Q(\text{lepton}) \times \eta(\text{light1})$</td>
</tr>
<tr>
<td></td>
<td>$Q(\text{lepton}) \times \eta(\text{light1})$</td>
<td>$\Delta R(\text{jet1,jet2})$</td>
</tr>
</tbody>
</table>

Some comments on the notation are in order. The numbering $n$ of jet$n$, tag$n$, light$n$, etc. refers to the transverse momentum ordering of the jets, 1 is the highest $p_T$ jet of that type of jet, 2 is the second-highest $p_T$ jet, and so on. “tag” means a $b$-tagged jet. “light” means an untagged jet (it fails the $b$-tag criteria). “best” means the jet which, when combined with the lepton and missing transverse energy, produces a top quark mass closest to 170 GeV (the value at which
Observation of Single Top Quark Production

11.2. Boosted Decision Trees

A decision tree\(^\text{94}\) applies sequential cuts to the events but does not reject events that fail the cuts. The choice of variables and cuts at each level of the tree is made by training the trees on large sets of signal and background MC events. Boosting\(^\text{95}\) averages the results over many trees and improves the performance by about 20\%. DØ pioneered the use of boosted decision trees (BDTs) to separate signal from background in the single top search in 2006.\(^\text{60,61}\) They use custom code with 64 input variables from the total list of 97, and 50 boosting cycles with a separate set of BDTs for each of the 24 analysis channels.\(^\text{96,97}\) The same variables are used in every analysis channel, since the BDTs ignore ones that do not show sensitivity in any particular channel. With BDTs, there is also no need to split the signal and background samples by subcomponent to improve the sensitivity (which is beneficial with traditional neural networks\(^\text{51}\)), since they handle the varying kinematics without problem. The CDF collaboration also uses BDTs, recently included in the \textsc{tmva} package in \textsc{root}.\(^\text{98}\) They use 22 variables for 2-jet events and 29 for 3-jet events, with 400–600 boosting cycles.\(^\text{99}\) They train four sets of BDTs in total, since they combine electron and muon channels and the two trigger types. After boosting, the distributions of both signal and background are highly centralized between zero and one. In order to avoid using bins in the final calculation with predicted signal or data but no predicted background, DØ transforms its output distributions (from all three discriminant methods, not just BDTs) to ensure that every bin has at least 40 background events. This transformation clusters the
background events near zero and the signal events near one, and avoids instabilities in the final cross section measurement.

11.3. Traditional Neural Networks

DO made the first particle search using neural networks (NNs) to separate signal from background in 2001. The type used were multilayer feed-forward perceptrons from the mlpf1t package. CDF uses NNs in the observation analysis for the lepton+jets channels with 14 input variables, and for the $E_T$+jets channels with 11 variables. The networks in the lepton+jets channels come from the commercial neurobayes package. Despite its name, it is not a Bayesian NN package as described in the next subsection. The networks are trained on the same events as used with the BDTs to obtain the weights between nodes and thresholds at the nodes. An independent set of events is used to test the networks after each training cycle to avoid overtraining. Since NNs use all input variables (unlike BDTs, which ignore ones not found to be useful), care must be taken not to include variables with insufficient separation power uncorrelated from the other variables, otherwise noise is introduced into the system and the separation can decrease. This is the reason why far fewer variables are used with NNs than with BDTs.

11.4. Bayesian Neural Networks

DO introduced the use of Bayesian neural networks (BNNs) for signal-background separation in the 2006 single top evidence analysis. Like traditional NNs, a short list of input variables must be chosen, and DO uses the rulefit package to select between 18 and 28 variables per analysis channel. The networks have 20 hidden nodes. The Bayesian part of this technique is to average over many networks in each channel using the Markov-Chain MC sampling technique. DO uses 300 networks in each of the 24 analysis channels, with the final result in each channel being taken from an average of the last 100 networks in the chain. This averaging process makes the discrimination insensitive to details of which events are used in training, so it is not possible to overtrain the networks, although closure tests are performed using independent events to verify convergence. It is also not necessary to split the signal and background components with separate networks for optimal separation. The averaging process also improves the signal-background separation, since it is not dependent on the choice of starting parameters for the weights between nodes or thresholds at the nodes, which can lead to solutions at local minima which are not optimal without the averaging.

11.5. Matrix Elements

Both DO and CDF use matrix elements (MEs) to separate signal from background. DO developed the method to measure the top quark mass in 2004, and was the first to apply them to signal-background separation in the
Observation of Single Top Quark Production

2006 single top evidence analysis. The matrix elements correspond to signal and background probability densities. DØ calculates matrix elements for three signal processes in the 2-jets channel and five in the 3-jets channel, together with eight background processes in the 2-jets channel and three in the 3-jets channel. The proton and antiproton are modeled using parton distribution functions and detector resolutions are taken into account using jet resolution transfer functions. The calculations are extremely CPU-intensive, and take many months to complete. To improve the sensitivity, DØ splits the analysis into events with $H_T < 175$ GeV (mainly $W+\text{jets}$ background) or $H_T \geq 175$ GeV (mainly $t\bar{t}$ and hard $W+\text{jets}$ background).

11.6. Likelihood Functions

CDF has an analysis that uses likelihood functions for signal-background separation with $tb+tqb$ as signal. They also search separately for only the s-channel $tb$ process, using different likelihood functions and input variables. Likelihoods are much simpler than NNs, they need no training on signal or background event samples, and do not take correlations between the variables into account. In the 2-jet channels, CDF’s likelihoods combine seven variables, including two powerful ones: the logarithm of the matrix element, and the jet flavor separator. In the 3-jet channels, 10 variables are combined.

11.7. Combining the Discriminant Outputs

The measurements from each discrimination method are correlated, but by less than 100%, and the discriminant outputs may thus be combined to improve the precision of the final measurement. DØ measures the correlation between its three analysis methods (BDT, BNN, ME) using an ensemble of pseudodatasets containing background and SM signal, and finds the correlation to be 74% between BDT and BNN, 60% between BDT and ME, and 57% between BNN and ME. To combine the three measurements in each of the 24 analysis channels, DØ uses an additional set of BNNs, each with three inputs and six hidden nodes. CDF uses an innovative method to combine its lepton+jets measurements: neuro-evolution of augmenting technologies (NEAT). This is a method for evolving neural networks with a genetic algorithm. Evolution starts with small simple networks that become increasingly complex over sequential generations. The networks are trained to give the best expected p-value (significance) for the result. This is unlike how traditional NNs are optimized during training, when the error function (signal-background similarity) is minimized. The NEAT networks are also used to optimize the binning for the measurement. Figure 6 shows the final output distributions for all analysis channels combined.
12. Cross Section Measurements

12.1. Bayesian Binned Likelihoods

The distributions from the combination discriminants from 24 independent lepton+jets analysis channels at DØ and eight lepton+jets plus three $E_T$+jets channels at CDF are used in a Bayesian binned likelihood calculation to extract the single top quark cross section. A flat nonnegative prior is used for the signal cross section. All systematic uncertainties on background normalization and shape and signal acceptance and their correlations are taken into account. The shape uncertainties from the jet energy scale are smoothed from bin to neighboring bin during the calculation. Using the full range of the discriminant outputs for this calculation means that the high statistics background-dominated region (near zero) is used to constrain the uncertainties on the much smaller background in the expected-signal-dominated region (near one). The signal cross section central value is taken from the position of the peak of the posterior density distribution, and the uncertainty on the cross section (statistical and systematic components combined) comes from the width of the distribution about the peak that encompasses 68% of its area ($\pm 1\sigma$). The cross section calculations are also performed using the outputs from each discriminant method separately, and using subsets of the data (all electron+jets channels, all 2-jets channels, all 1-tag channels, and so on) to check for consistency, which is found within the statistical uncertainty on the measurements.

12.2. Ensembles and Linearity Studies

To check that the discriminants do not introduce a bias into the measured cross section, DØ generates eight ensembles of pseudodatasets and runs them through the entire analysis chain. Each ensemble contains about 7,000 sets of events, where
the sets are constructed to each reproduce DØ’s 2.3 fb$^{-1}$ real dataset. Signal and background events are sampled from the MC event sets after all event selection cuts such that the numbers of each background component match the measured yields, smeared by Poisson statistics. All systematic uncertainties and their correlations between background and signal subcomponents are included in the calculations. The single top quark signal cross section is set at a different value spanning the range from 2 pb to 10 pb for each ensemble. For the three discrimination methods and for their combination, a plot is produced with the measured signal cross section versus the input signal cross section, and a fit made to the eight points. For all cases, the slope of the fitted relation is close to one and the intercept is close to the origin, which shows that the measured cross section, if it lies in this range, accurately represents the signal cross section in the data.

### 12.3. Single Top Quark Production Cross Sections

The measured single top quark cross sections are shown in Table 4. The expected and measured significances of each measurement are also shown; these are explained in the next section.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Single Top Cross Section</th>
<th>Uncertainty [%]</th>
<th>Expected Significance</th>
<th>Measured Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DO</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boosted Decision Trees</td>
<td>$3.74^{+0.95}_{-0.79}$ pb</td>
<td>$4.3\sigma$</td>
<td>$4.6\sigma$</td>
<td></td>
</tr>
<tr>
<td>Bayesian Neural Networks</td>
<td>$4.70^{+1.18}_{-0.95}$ pb</td>
<td>$4.1\sigma$</td>
<td>$5.4\sigma$</td>
<td></td>
</tr>
<tr>
<td>Matrix Elements</td>
<td>$4.30^{+0.99}_{-1.26}$ pb</td>
<td>$4.1\sigma$</td>
<td>$4.9\sigma$</td>
<td></td>
</tr>
<tr>
<td><strong>Combination (170 GeV)</strong></td>
<td>$3.94 \pm 0.88$ pb</td>
<td>$\pm 22%$</td>
<td>$4.5\sigma$</td>
<td>$5.0\sigma$</td>
</tr>
<tr>
<td><strong>CDF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boosted Decision Trees</td>
<td>$2.1^{+0.7}_{-0.6}$ pb</td>
<td>$5.2\sigma$</td>
<td>$3.5\sigma$</td>
<td></td>
</tr>
<tr>
<td>Neural Networks</td>
<td>$1.8^{+0.6}_{-0.6}$ pb</td>
<td>$5.2\sigma$</td>
<td>$3.5\sigma$</td>
<td></td>
</tr>
<tr>
<td>Matrix Elements</td>
<td>$2.5^{+0.7}_{-0.6}$ pb</td>
<td>$4.9\sigma$</td>
<td>$4.3\sigma$</td>
<td></td>
</tr>
<tr>
<td>Likelihoods</td>
<td>$1.6^{+0.8}_{-0.7}$ pb</td>
<td>$4.0\sigma$</td>
<td>$2.4\sigma$</td>
<td></td>
</tr>
<tr>
<td>Likelihoods, s-channel</td>
<td>$1.5^{+0.9}_{-0.9}$ pb</td>
<td>$1.1\sigma$</td>
<td>$2.0\sigma$</td>
<td></td>
</tr>
<tr>
<td>Combination, lepton+jets</td>
<td>$2.1^{+0.6}_{-0.6}$ pb</td>
<td>$+29%, -24%$</td>
<td>$1.4\sigma$</td>
<td>$2.1\sigma$</td>
</tr>
<tr>
<td>Neural Networks, $E_T$+jets</td>
<td>$4.9^{+2.6}_{-2.0}$ pb</td>
<td>$+52%, -46%$</td>
<td>$1.4\sigma$</td>
<td>$2.1\sigma$</td>
</tr>
<tr>
<td><strong>Combination (175 GeV)</strong></td>
<td>$2.3^{+0.6}_{-0.5}$ pb</td>
<td>$+26%, -22%$</td>
<td>$&gt; 5.9\sigma$</td>
<td>$5.0\sigma$</td>
</tr>
<tr>
<td><strong>Combination (170 GeV)</strong></td>
<td>$2.35^{+0.56}_{-0.56}$ pb</td>
<td>$+24%, -21%$</td>
<td>$1.4\sigma$</td>
<td>$2.1\sigma$</td>
</tr>
<tr>
<td><strong>Tevatron Combination (170 GeV)</strong></td>
<td>$2.76^{+0.58}_{-0.47}$ pb</td>
<td>$+21%, -17%$</td>
<td>$5.9\sigma$</td>
<td>$5.0\sigma$</td>
</tr>
<tr>
<td>Theory ($M_{top} = 170$ GeV)</td>
<td>$3.46 \pm 0.18$ pb</td>
<td>$-5%$</td>
<td>$5.9\sigma$</td>
<td>$5.0\sigma$</td>
</tr>
</tbody>
</table>

After the two collaborations submitted their independent measurements for publication, they worked together to combine them into one Tevatron result.$^{112}$ The systematic uncertainty terms are classified to map between the two measurements so that correlations of some terms between the two measurements are properly taken into account. The combination is calculated using the Bayesian binned likelihood calculation on all input distributions simultaneously. The Tevatron combined result is also shown in Table 4.
13. Measurement Significance

The measured significance is defined from the p-value, which is the probability that the background fluctuated up to give a cross section measurement at least as large as the measured value. The expected significance comes from the p-value which is the probability that the background fluctuated up to give a cross section at least as large as the standard model theory value. The p-values are converted to significances in standard deviations ($\sigma$) assuming a Gaussian distribution. DØ measures these p-values using an ensemble of about 70 million pseudodatasets, each consisting of only background events with no signal events, by determining the fraction of pseudodatasets with a high enough cross section. CDF calculates the p-values by finding when the quantity $-2 \ln(\text{Prob(data|}S+B)/\text{Prob(data|}B))$ is less in pseudodatasets than in real data. CDF’s pseudodatasets are generated differently to DØ’s. Instead of sampling MC and multijet background events to generate each pseudodataset, they sample from the histograms of each distribution to perform the significance calculation. DØ finds a measured p-value of $2.5 \times 10^{-7}$ and CDF finds a measured p-value of $3.1 \times 10^{-7}$. The associated significances are shown in Table 4.

Both experiments have a measured significance of 5.0 $\sigma$, meeting the standard to claim “observation.”

14. Measuring the CKM Matrix Element $|V_{tb}|$

The Cabibbo-Kobayashi-Maskawa matrix describes the mixing between quarks to get from the strong interaction eigenstates to the weak-interaction ones. The term relating top quarks to bottom quarks is known as $V_{tb}$. The single top quark production cross section is proportional to $|V_{tb}|^2$ and can thus be used to measure the amplitude of $V_{tb}$. To make this measurement, the collaborations assume the standard model for top quark decay (i.e., mostly to $Wb$ and not much to $Wd$ or $Ws$) and that the $Wtb$ coupling is left-handed and CP-conserving. They do not, however, assume that there are exactly three quark generations for this measurement. That is, they do not assume CKM matrix unitarity, unlike measurements of $|V_{tb}|$ made using top quark decays in $t\bar{t}$ pairs. The measurements include uncertainties from the $tb+tqb$ theory cross section as well as those included in the cross section measurement. The theory cross section uncertainty from the top quark mass uncertainty is 4.2%, with 3.0% from the PDFs, 2.4% from the factorization scale, and 0.5% from the strong coupling constant $\alpha_s$. Two measurements of $|V_{tb}|$ are made: the first does not constrain the strength of the left-handed scalar coupling constant $f_L^t$ (where a nonnegative prior is used, with no upper bound), and the second sets $f_L^t = 1$ (when the prior is bounded between zero and one). The results of this measurement from DØ, CDF, and the Tevatron combination, are shown in Table 5.

15. Separate s-Channel and t-Channel Measurements

Both collaborations have made measurements of the s-channel $tb$ and t-channel $tqb$ single top quark cross sections separately. DØ re-trained the three sets of discriminants
Table 5. Measurements of the CKM matrix element $|V_{tb}|$.

| Experiment and Theory | $|V_{tb}|$ | $|V_{tb}| (f^{L}_{t} = 1)$ |
|-----------------------|----------|--------------------------|
| DØ $M_{top} = 170$ GeV Kidonakis 2006 | $1.07 \pm 0.12$ | $0.78 < |V_{tb}| \leq 1$ |
| CDF $M_{top} = 175$ GeV Harris et al. 2002 | $0.71 < |V_{tb}| \leq 1$ |
| Tevatron Combination $M_{top} = 170$ GeV Kidonakis 2006 | $0.77 < |V_{tb}| \leq 1$ |

with just t-channel single top as the signal, instead of $tb+tqb$ as in the observation analysis. CDF uses measurements obtained during their main analysis. Neither of these measurements assume the SM ratio for the s-channel and t-channel cross sections, unlike in the observation analysis. The results are shown in Fig. 7. DØ’s t-channel cross section measurement has a significance of $4.8\sigma$.

Fig. 7. Plots showing the separate s-channel $tb$ and t-channel $tqb$ cross section measurements from (a) DØ and (b) CDF, together with theory values and some beyond-the-SM model predictions. DØ’s measurements are for $M_{top} = 170$ GeV and CDF’s for 175 GeV. The theory cross sections shown are Kidonakis 2006 for DØ, Harris et al. 2002 for CDF (“NLO”) and Kidonakis 2006 for CDF (“NNNLO”).

16. Summary

The DØ and CDF collaborations have searched large Tevatron datasets and observed single top quark production for the first time, with $5\sigma$ significance for each of the measurements. The measured cross sections are consistent with NLO theory predictions. The analyses have been improved in many ways to achieve this, in particular over the years of the search:
DO’s Innovations

• Next-to-leading order simulation of signals with full spin information included
• Very loose kinematic cuts and use of all possible triggers to select more signal-like data and increase signal acceptance
• Multivariate techniques: BDTs, BNNs, and MEs used to improve signal-background separation
• Very large number of variables, including many not used at the Tevatron before such as jet widths, to separate signal from background
• Rebinning the discriminant outputs to ensure no bin has data or expected signal and no background events, which stabilizes the cross section measurement

CDF’s Innovations

• Including data with no identified lepton to extend the signal acceptance
• Jet flavor separator variable to discriminate $b$ jets from mistagged charm, light quark, and gluon ones after $b$ tagging
• Combining different measurements using the NEAT algorithm optimized to maximize the expected signal significance
• Binned likelihood fit to a discriminating variable shape to improve the measurement sensitivity

As the reader can see from these lists and the previous analysis descriptions, each collaboration has learned from the innovations of the other one, and the outcome is a deep understanding of the signals and multicomponent backgrounds in many analysis channels, with powerful new analysis techniques developed to extract a small signal from a very large background. Many of these techniques are now being applied to the search for the Higgs boson at the Tevatron, and the single top datasets are providing a unique place in which to test various aspects of the standard model and search for new physics.\textsuperscript{115–123}

Acknowledgments

I have worked with a large number of people from the DØ collaboration on this measurement and previous iterations over the past 15 years. Cecilia Gerber and Reinhard Schwienhorst co-led the team with me that made the observation measurement, and Arán García-Bellido co-led with me the evidence analysis in 2006. Fourteen students have obtained their Ph.D.s on the research at DØ, seven students have graduated with master’s degrees, nine Ph.D. students are still in the pipeline, and six postdocs have become tenure-track professors or lab staff scientists. It has been a pleasure to collaborate with all of the single top working group members and I have learned a lot from them. I have also enjoyed interactions with members of CDF’s single top working group over the years, and with many theorists who generously shared their perspectives and insights on single top quark physics. This paper is based on a plenary talk I presented at the 21st Rencontres de Blois: Windows on the Universe, Blois, France, June 2009, and associated proceedings.
For this review paper, Reinhard Schwienhorst and Liang Li from DØ and Tom Junk and Jan Lück from CDF helped answer my questions. This research has been supported by grants from the U.S. Department of Energy’s Office of Science.

References

Observation of Single Top Quark Production


71. T. Sjöstrand, S. Mrenna, and P. Skands, J. High Energy Phys. 05, 026 (2006). DØ used pythia v.6.409 and CDF used v.6.325. CDF used “Tune A” for the underlying event model and DØ used a modified version of Tune A. Both DØ and CDF use the $m^2$-ordered showering model (not the newer $p_T$-ordered one).


76. http://www.lns.cornell.edu/public/CLEO/soft/QQ.


83. DØ uses jets defined by an iterative seed-based cone algorithm with radius $R = \sqrt{\Delta \phi^2 + \Delta \eta^2}$ including midpoints as described in G.C. Blazey et al., Proceedings of the Workshop on QCD and Weak Boson Physics in Run II, edited by U. Baur, R.K. Ellis, and D. Zeppenfeld, pp. 47–77, (Fermilab, Batavia, 2000), FERMILAB-PUB-00-297.

84. CDF uses jets defined by a clustering algorithm with radius $R = \sqrt{\Delta \phi^2 + (\Delta \eta)^2}$ as described in G.C. Blazey and B.L. Flaugher, Ann. Rev. Nucl. Part. Sci. 49, 633 (1999).


90. http://www-d0.fnal.gov/Run2Physics/top/singletop/observation/


98. Toolkit for Multivariate Data Analysis with ROOT:
100. MLPfit: a Tool for Multi-Layer Perceptrons,
http://schwind.home.cern.ch/schwind/MLPfit.html