Resource Letter QCD-1: Quantum Chromodynamics

Andreas S. Kronfeld and Chris Quigg

Theoretical Physics Department, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510 USA

(Dated: May 20, 2010)

This Resource Letter provides a guide to the literature on Quantum Chromodynamics (QCD), the relativistic quantum field theory of the strong interactions. Journal articles, books, and other documents are cited for the following topics: quarks and color, the parton model, Yang-Mills theory, experimental evidence for color, QCD as a color gauge theory, asymptotic freedom, QCD for heavy hadrons, QCD on the lattice, the QCD vacuum, pictures of quark confinement, early and modern applications of perturbative QCD, the determination of the strong coupling and quark masses, QCD and the hadron spectrum, hadron decays, the quark-gluon plasma, the strong nuclear interaction, and QCD’s role in nuclear physics.

The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of a somewhat more specialized nature, and the letter A indicates rather specialized or advanced material.

PACS numbers: 12.38.-t, 24.85.+p

FERMILAB-PUB-10-040-T

Contents

1. Introduction

II. QCD
   A. A gauge theory for the strong interactions
   B. First consequences
   C. Textbooks
   D. Antecedent physical theories
      1. Flavor symmetry and current algebra
      2. The original quark model
      3. Quarks with color
      4. Partons
      5. Gauge invariance and Yang-Mills theory

III. Theoretical Tools
   A. Symmetries
      1. Light quarks
      2. Anomalous chiral symmetries
      3. Heavy quarks
   B. Potential models
   C. Renormalization and factorization
   D. Unitarity and analyticity
   E. Effective field theories
      1. Chiral perturbation theory
      2. Heavy-quark effective theory and nonrelativistic QCD
      3. Soft collinear effective theory
   F. Lattice gauge theory
   G. The QCD vacuum and confinement
   H. Dyson-Schwinger Equations
   I. Perturbative amplitudes
   J. Parton-shower Monte Carlo programs
   K. Extensions of QCD
   L. String theory

IV. Confronting QCD with Experiment
   A. Running of $\alpha_s$
   B. Hadron spectrum

V. QCD in the Broader Context of Particle Physics

VI. Frontier Problems in QCD

Acknowledgments

A. Links to Basic Resources
   1. Journals
   2. Electronic archives
   3. Pedagogical web sites

References

I. INTRODUCTION

Quantum chromodynamics (QCD) is a remarkably simple, successful, and rich theory of the strong interactions. The theory provides a dynamical basis for the quark-model description of the hadrons, the strongly interacting particles such as protons and pions that are accessible for direct laboratory study. Interactions among the quarks are mediated by vector force particles called
QCD describes a wealth of physical phenomena, from the structure of nuclei to the inner workings of neutron stars and the cross sections for the highest-energy elementary-particle-collisions. The QCD literature is correspondingly extraordinarily vast. To arrive at a manageable number of cited papers in this Resource Letter, we have chosen works that should be useful to professors and students planning a course for classroom or independent study. We have included some classic contributions and all elementary presentations of which we are aware. For more advanced material we favor works that provide ambitious students an entrée to the contemporary literature. These include well-established highly-cited review articles (because a search of literature citing a review is a gateway to newer topics) and more modern treatments that portray the state of the art and document the preceding literature well. Wherever possible, we give links to digital versions of the articles we cite. Many published articles are available in electronic form through the World Wide Web sites of individual journals, or through the e-print archive; see Appendix A.

From the time of its development in the 1950s, quantum electrodynamics (QED), the relativistic quantum field theory of photons and electrons, was viewed as exemplary. In the late 1960s and early 1970s, with the development of the electroweak theory, it became increasingly attractive to look to relativistic quantum field theories—specifically gauge theories—for the description of all the fundamental interactions.


QCD represents the culmination of that search for the strong interactions. In some respects, it has supplanted QED as our “most perfect” theory.


An excellent summary of the foundations and implications of QCD is


Some encyclopedia articles on QCD are


For a book-length exposition of the wonders of QCD, see


The rest of this Resource Letter is organized as follows. We begin in Sec. II by reviewing the basics of the theory of QCD, giving its Lagrangian, some essential aspects of its dynamics, and providing a connection to earlier ideas. In Sec. III we cover literature on theoretical tools for deriving physical consequences of the QCD Lagrangian. Section IV covers the most salient aspects of the confrontation of QCD with experimental observations and measurements. Section V situates QCD within the broader framework of the standard model of particle physics. We conclude in Sec. VI with a brief essay on frontier problems in QCD. Appendix A gives links to basic online resources.

II. QCD

As a theory of the strong interactions, QCD describes the properties of hadrons. In QCD, the familiar mesons (the pion, kaon, etc.) are bound states of quarks and antiquarks; the familiar baryons (the proton, neutron, Δ(1232) resonance, etc.) are bound states of three quarks. Just as the photon binds electric charges into atoms, the binding agent is the quantum of a gauge field, called the gluon. Hadrons made of exclusively of gluons, with no need for valence quarks, may also exist and are called glueballs. Properties of hadrons are tabulated in


In this section we begin with the Lagrangian formulation of QCD. Readers who are not yet familiar with the Dirac equation may wish to skip this mathematical discussion and head straight to Sec. II.B for a résumé of the main themes of QCD, to Sec. II.C for a list of textbooks, or to Sec. II.D for resources on the ideas out of which the quantum field theory QCD emerged in the early 1970s.

A. A gauge theory for the strong interactions

Quantum chromodynamics is the theory of strong interactions among quarks derived from the color gauge
symmetry SU(3). It is advantageous for many purposes to express the theory in Lagrangian form. As in a classical theory, one can easily derive the equations of motion. In a quantum theory, the Lagrangian also provides a convenient framework for quantization and the development of perturbation theory, via Feynman rules, in a Lorentz-covariant fashion. The Lagrangian formalism lends itself particularly to the consideration of symmetry principles and their consequences. Invariance of the Lagrangian under a global, i.e., position-independent, symmetry operation implies a conservation law through Noether’s theorem. Requiring the Lagrangian to be invariant under local, i.e., position-dependent, transformations demands an interacting theory, in which spin-one force particles couple minimally to the conserved current of the global symmetry. Thus a global U(1) phase symmetry underlies QED. A symmetry related to conservation of the electromagnetic current, and symmetry. Thus a global U(1) phase symmetry is realized. A symmetry. For QCD based on SU(N) gauge symmetry, the quark and gluon color factors are N − 1, whereas the neutral photon does not couple directly to other photons.

The color matrices for the fundamental (quark) representation satisfy

\[ \sum_{i} \lambda_{ab}^{i} \lambda_{bc}^{j} = 4 C_{F} \delta_{ac}, \quad C_{F} = \frac{N^{2} - 1}{2N}, \]

while the color matrices for the adjoint (gluon) representation, \( T_{ij}^{k} = -if^{ijk}, \) obey

\[ \text{tr} (T^{k} T^{l}) = \sum_{ij} f^{ijk} f^{ijl} = C_{A} \delta^{kl}, \quad C_{A} = N. \]

For QCD based on SU(3) gauge symmetry, the quark and gluon color factors are

\[ C_{F} = \frac{4}{3}, \quad C_{A} = 3. \]

It is sometimes advantageous to carry out calculations for general values of \( C_{F} \) and \( C_{A} \), to test the non-Abelian structure of QCD (see Sec. IV D).


The Lagrangian \( \mathcal{L} \) in Eq. (1) is invariant under the transformations

\[ \psi(x) \rightarrow e^{i\omega(x)} \psi(x), \]

\[ \bar{\psi}(x) \rightarrow \bar{\psi}(x) e^{-i\omega(x)}, \]

\[ B_{\mu}(x) \rightarrow e^{i\omega(x)} [B_{\mu} + (ig)^{-1} \partial_{\mu}] e^{-i\omega(x)}, \]

where the matrix \( \omega(x) = \frac{i}{2} \omega^{I}(x) \lambda^{I} \) depends on the space-time coordinate \( x \). Generically the matrices \( \omega(x) \) and \( B_{\mu}(x) \) do not commute, a feature that again distinguishes QCD from QED.

If we recast the matter term in the Lagrangian (1) in terms of left-handed and right-handed fermion fields, the structure constants \( f^{ijkl} \) can be expressed as

\[ f^{ijkl} = (4i)^{-1} \text{tr} \{ \lambda^{i} \lambda^{j}, \lambda^{k} \}. \]

The nonvanishing structure constants distinguish QCD from QED: QCD is a non-Abelian gauge theory. The gluon field-strength tensor is

\[ G_{\mu\nu}^{i} = \partial_{\nu} B_{\mu}^{i} - \partial_{\mu} B_{\nu}^{i} + gf^{ijk} B_{\mu}^{j} B_{\nu}^{k}, \]

and the last term marks a fundamental dynamical difference between QCD and QED. Via \( \text{tr}(G_{\mu\nu} G^{\mu\nu}) \) in Eq. (1), it leads to three-gluon and four-gluon interactions that have no counterpart in QED. The gluon carries color charge and, thus, experiences strong interactions, whereas the neutral photon does not couple directly to other photons.

The color matrices for the adjoint (gluon) representation satisfy

\[ \sum_{i} \lambda_{ab}^{i} \lambda_{bc}^{j} = 4 C_{F} \delta_{ac}, \quad C_{F} = \frac{N^{2} - 1}{2N}, \]

while the color matrices for the adjoint (gluon) representation, \( T_{ij}^{k} = -if^{ijk}, \) obey

\[ \text{tr} (T^{k} T^{l}) = \sum_{ij} f^{ijk} f^{ijl} = C_{A} \delta^{kl}, \quad C_{A} = N. \]

For QCD based on SU(3) gauge symmetry, the quark and gluon color factors are

\[ C_{F} = \frac{4}{3}, \quad C_{A} = 3. \]

It is sometimes advantageous to carry out calculations for general values of \( C_{F} \) and \( C_{A} \), to test the non-Abelian structure of QCD (see Sec. IV D).


The Lagrangian \( \mathcal{L} \) in Eq. (1) is invariant under the transformations

\[ \psi(x) \rightarrow e^{i\omega(x)} \psi(x), \]

\[ \bar{\psi}(x) \rightarrow \bar{\psi}(x) e^{-i\omega(x)}, \]

\[ B_{\mu}(x) \rightarrow e^{i\omega(x)} [B_{\mu} + (ig)^{-1} \partial_{\mu}] e^{-i\omega(x)}, \]

where the matrix \( \omega(x) = \frac{i}{2} \omega^{I}(x) \lambda^{I} \) depends on the space-time coordinate \( x \). Generically the matrices \( \omega(x) \) and \( B_{\mu}(x) \) do not commute, a feature that again distinguishes QCD from QED.

If we recast the matter term in the Lagrangian (1) in terms of left-handed and right-handed fermion fields, the structure constants \( f^{ijkl} \) can be expressed as

\[ f^{ijkl} = (4i)^{-1} \text{tr} \{ \lambda^{i} \lambda^{j}, \lambda^{k} \}. \]

The nonvanishing structure constants distinguish QCD from QED: QCD is a non-Abelian gauge theory. The gluon field-strength tensor is

\[ G_{\mu\nu}^{i} = \partial_{\nu} B_{\mu}^{i} - \partial_{\mu} B_{\nu}^{i} + gf^{ijk} B_{\mu}^{j} B_{\nu}^{k}, \]

and the last term marks a fundamental dynamical difference between QCD and QED. Via \( \text{tr}(G_{\mu\nu} G^{\mu\nu}) \) in Eq. (1), it leads to three-gluon and four-gluon interactions that have no counterpart in QED. The gluon carries color charge and, thus, experiences strong interactions, whereas the neutral photon does not couple directly to other photons.

The color matrices for the fundamental (quark) representation satisfy

\[ \sum_{i} \lambda_{ab}^{i} \lambda_{bc}^{j} = 4 C_{F} \delta_{ac}, \quad C_{F} = \frac{N^{2} - 1}{2N}, \]

while the color matrices for the adjoint (gluon) representation, \( T_{ij}^{k} = -if^{ijk}, \) obey

\[ \text{tr} (T^{k} T^{l}) = \sum_{ij} f^{ijk} f^{ijl} = C_{A} \delta^{kl}, \quad C_{A} = N. \]

For QCD based on SU(3) gauge symmetry, the quark and gluon color factors are

\[ C_{F} = \frac{4}{3}, \quad C_{A} = 3. \]
we see that it becomes highly symmetrical in the limit of vanishing quark mass, \( m \to 0 \).Absent the mass term, there is no coupling between the left-handed and right-handed quark fields, \( \psi_{L,R} \equiv \frac{1}{2} (1 \mp \gamma_5) \psi \), and so the Lagrangian is invariant under separate global phase transformations on the left-handed and right-handed fields. Generalizing to the case of \( n_f \) flavors of massless quarks, we find that the QCD Lagrangian displays an SU(\( n_f \)) \( \times \) SU(\( n_f \)) \( \times \) U(1) \( \times \) U(1) \( \times \) chiral symmetry. In nature, the up- and down-quark masses are very small (compared to the proton mass), and the strange-quark mass is also small. Therefore, \( n_f = 2 \) (isospin) and \( n_f = 3 \) (flavor SU(3)) chiral symmetries are approximate. We return to chiral symmetries in Sec. III A 1.

B. First consequences

As mentioned above, the three- and four-gluon interactions make the physics of QCD essentially different from the mathematically similar QED. In quantum electrodynamics, an electron’s charge is partially screened by quantum-mechanical effects, called anomalies, discussed in Sec. III A 2.

The U(1) factors may be rewritten U(1) \( \times \) U(1) \( \times \) chiral symmetry. The vector (V) symmetry applies the same phase factor to left- and right-handed fields; it leads via Noether’s theorem to a conserved charge, namely baryon number. The axial-vector (A) symmetry applies opposite phase factors to left- and right-handed fields; it is broken by certain quantum-mechanical effects, called anomalies, discussed in Sec. III A 2.

Asymptotic freedom points to the existence of a domain in which the strong interactions become sufficiently weak that scattering processes can be treated reliably in perturbation theory using techniques based on the evaluation of Feynman diagrams. The path to asymptotic freedom is described in the Nobel Lectures.

For another view of the historical setting, see

\[ \frac{1}{\alpha(Q)} = \frac{1}{\alpha(m_e)} - \frac{2}{3\pi} \log \left( \frac{Q}{m_e} \right), \quad (18) \]

where \( m_e \) is the electron’s mass, and the formula holds for \( Q > m_e \). Note the sign of the logarithm: at larger values of \( Q \), which is to say shorter distances, the effective charge increases.

In quantum chromodynamics, gluons can fluctuate into further quark-antiquark pairs, and this vacuum polarization exerts a similar screening effect, tending to increase the effective color charge at short distances. But this tendency is overcome by antiscreening effects that arise from the contributions of gluon loops to the vacuum polarization. The gluon loops are present because of the three-gluon and four-gluon vertices that arise from the non-Abelian nature of the SU(3) symmetry. To one-loop approximation, the strong-interaction analogue of the fine structure constant, \( \alpha_s \equiv g^2/4\pi \), evolves as

\[ \frac{1}{\alpha_s(Q)} = \frac{1}{\alpha_s(\mu)} + \frac{33 - 2n_f}{6\pi} \log \left( \frac{Q}{\mu} \right), \quad (19) \]

where \( \mu \) defines the reference, or renormalization, scale.

If the number of quark flavors \( n_f \leq 16 \), as it is in our six-flavor world, then the coefficient of the \( \log(Q/\mu) \) term is positive, and \( \alpha_s \) decreases at large values of \( Q \) or short distances. This is the celebrated property of asymptotic freedom, announced in


Asymptotic freedom points to the existence of a domain in which the strong interactions become sufficiently weak that scattering processes can be treated reliably in perturbation theory using techniques based on the evaluation of Feynman diagrams. The path to asymptotic freedom is described in the Nobel Lectures.

For another view of the historical setting, see

\[ \text{“The discovery of asymptotic freedom and the emergence of QCD,” D. J. Gross, Rev. Mod. Phys. 77, 837–849 (2005) [doi: 10.1103/physrevlett.77.1346]. (A) } \]

\[ \text{“Asymptotic freedom from paradox to paradigm,” F. Wilczek, Rev. Mod. Phys. 77, 857–870 (2005) [hep-ph/0502113]. (I) } \]


Asymptotically free theories are of special interest because they predict behavior very close to Bjorken scaling in deeply inelastic scattering (see Secs. III D 4 and IV E). No renormalizable field theory without non-Abelian gauge fields can be asymptotically free:

\[ \text{“When was asymptotic freedom discovered? Or the rehabilitation of quantum field theory,” G. ’t Hooft, Nucl. Phys. Proc. Suppl. 74, 413 (1999) [hep-th/9808154]. (A) } \]

The complementary behavior of QCD in the long-distance limit, known as infrared slavery, points to the confinement of quarks into color-singlet hadrons, as explained in

\[ \text{“The confinement of quarks,” Y. Nambu, Sci. Am. 235, 48–70 (November, 1976). (E) } \]

This picture leads to the crucial insight that most of the mass of hadrons such as the proton arises not from the masses of their constituents, the quarks, but from the quarks’ kinetic energy and the energy stored in the gluon field,
The development of lattice gauge theory has made possible a quantitative understanding of how these phenomena emerge at the low-energy scale associated with confinement.

C. Textbooks

Many books treat quantum chromodynamics, in whole or in part, from a modern point of view. Among books addressed to general readers and undergraduate students, see Ref. 7 and


Several excellent textbooks are addressed to graduate students and researchers:


Among many fine field-theory textbooks, is particularly inclined toward QCD and the issue of factorization.

The biannual Review of Particle Physics (Ref. 8) contains several concise reviews of QCD and other topics in particle physics. For a well-chosen collection of longer review articles providing an encyclopedic treatment of QCD, see


D. Antecedent physical theories

The modern gauge theory is the synthesis of several ideas. For completeness we provide some historical and review references here.

1. Flavor symmetry and current algebra

The idea of flavor symmetries underlying hadron masses and decay amplitudes predates QCD:


Early papers are collected in

43. The Eightfold Way, M. Gell-Mann and Y. Ne’eman (W. A. Benjamin, New York, 1964). (I)

Both weak and electromagnetic interactions of the strongly interacting particles (hadrons) are described by currents. The SU(3)$_{\text{flavor}}$ classification symmetry relates properties of the weak and electromagnetic interactions of hadrons. Gell-Mann proposed that the charges associated with weak-interaction currents could be identified with SU(3)$_{\text{flavor}}$ symmetry operators:

44. “The symmetry group of vector and axial vector currents,” M. Gell-Mann, Physics 1, 63–75 (1964). (I–A)

The current-algebra hypothesis states that the time components of the vector and axial-vector matrix elements satisfy quark-model equal-time commutation relations. Current algebra fixes the strength of the leptonic and hadronic parts of the weak current, and it proved immensely fruitful for interactions involving pseudoscalar mesons. In QCD, an SU(3)$_{\text{flavor}}$ symmetry appears in the limit that the quark masses can be neglected.

A useful early review of current algebra can be found in


and an early reprint volume with explanatory text is


A later, somewhat more mature assessment is


2. The original quark model

The notion of fractionally-charged quarks was introduced in


Zweig used the term “aces” for quarks. An early review of the quark model is in


With confinement in QCD, however, the search for isolatable fractional charges is a somewhat more subtle subject, perhaps explaining why searches for fractionally-charged particles have been to no avail.

3. Quarks with color

A second challenge to the quark model lay in the spin–statistics puzzle for the baryons. If the baryon \( J = \frac{1}{2} \) octet and \( J = \frac{3}{2} \) decuplet are taken to be composites of three quarks, all in relative s-waves, then the wave functions of the decuplet states appear to be symmetric in space×spin×isospin, in conflict with the Pauli exclusion principle. As explicit examples, consider the \( \Omega^- \), formed of three (presumably) identical strange quarks, sss, or the \( \Delta^{++} \), an isospin-\( \frac{3}{2} \) state made of three up quarks, uuu. To reconcile the successes of the quark model with the requirement that fermion wave functions be antisymmetric, it is necessary to hypothesize that each quark flavor comes in three distinguishable species, which we label by the primary colors red, green, and blue. Baryon wave functions may then be antisymmetrized in color. For a review of the role of color in models of hadrons, see

Further observational evidence in favor of the color-triplet quark model is marshaled in


For a critical look at circumstances under which the number of colors can be determined in \( \pi^0 \to \gamma \gamma \) decay, see


The cross section for the reaction \( e^+e^- \to \text{hadrons} \) (cf. Sec. IV.C) provides independent evidence that quarks are color triplets. As discussed above, color attains a deeper dynamical meaning in QCD.

4. Partons

Meanwhile, high-energy scattering experiments showed signs of nucleon substructure in the SLAC-MIT experiments on deeply inelastic electron-nucleon scattering. The structure functions that describe the internal structure of the target nucleon as seen by a virtual-photon probe depend in principle on two kinematic variables: the energy \( \nu = E - E' \) lost by the scattered electron and the four-momentum transfer, \( Q^2 \). At large values of \( \nu \) and \( Q^2 \) the structure functions depend, to good approximation, only on the single dimensionless variable, \( x = Q^2/2M\nu \) (where \( M \) is the nucleon mass), as anticipated by


Bjorken scaling implies that the virtual photon scatters off pointlike constituents; otherwise large values of \( Q^2 \) would resolve the size of the constituents. An early overview is


The first observations are reported in


The experiments and their interpretation in terms of the parton model, which regards the nucleon as a collection of quasifree charged scattering centers, are reviewed in the Nobel Lectures,


and in the narrative,

67. The Hunting of the Quark, M. Riordan (Simon & Schuster, New York, 1987). (E–I)

A theoretical framework called the “parton model” based on pointlike constituents of unknown properties was developed in


Complementary experiments in high-energy neutrino beams soon sealed the identification of the charged partons as quarks, and pointed to the importance of neutral partons later identified as the gluons of QCD.


See Sec. IV.E for references to works covering the recent experiments.

The parton-model interpretation of high-transverse-momentum scattering in hadron collisions was pioneered by


and implemented in practical terms in


5. Gauge invariance and Yang-Mills theory

The idea that a theory of the strong nuclear interactions could be derived from a non-Abelian symmetry such as isospin dates to the work of


The development of the notions of gauge invariance is detailed in


and many useful readings are compiled in


The concepts and consequences of local gauge invariance are recalled in


and the history of gauge theories is explored in


In particular, Shaw’s thesis is reprinted and discussed in Chapter 9 of Ref. 83. For an assessment of a half-century’s development of gauge symmetry, see
The massless states are called Nambu-Goldstone particles. When a small amount of explicit symmetry breaking arises, as with pions, these states acquire a small mass and are called pseudo-Nambu–Goldstone particles.

An informative toy model in which the nucleon mass arises essentially as a self-energy in analogy with the appearance of the mass gap in superconductivity was presented in

This construction, three years before the invention of quarks, prefigured our current understanding of the masses of strongly interacting particles in quantum chromodynamics. The pions arose as light nucleon-antinucleon bound states, following the introduction of a tiny “bare” nucleon mass and spontaneous chiral-symmetry breaking.

Spontaneous symmetry breaking is common in physics, and parallels to condensed-matter physics are drawn in

Meanwhile, QCD explains the origin of chiral symmetry via the smallness of the up-, down-, and strange-quark masses—recall the discussion following Eq. (17). The spontaneous breaking is driven by the formation of a condensate of the light quarks, measured by the vacuum expectation value $\langle 0 | \bar{q} q | 0 \rangle$. For a careful calculation see

yielding (in the $\overline{\text{MS}}$ scheme at 2 GeV)

$$\langle 0 | \bar{q} q | 0 \rangle = (242 \pm 4^{+19}_{-18} \text{ MeV})^3,$$

where the first error stems from Monte Carlo statistics and the second encompasses systematic effects, such as extrapolation to vanishingly small up- and down-quark masses.

2. Anomalous chiral symmetries

Among the chiral symmetries of light quarks, the flavor-singlet symmetry is special, because a quantum-
mechanical effect, called the anomaly, breaks the classical conservation law. This effect implies that the $\eta'$, unlike the pions and kaons, should not be a pseudo-Nambu–Goldstone particle with small mass.

“A global fit to determine the pseudoscalar mixing angle and the gluonium content of the $\eta'$ meson,” F. Ambrosino et al., JHEP 07, 105 (2009) arXiv:0906.3819 [hep-ph]. (I)

The details of how this arises are connected to the non-trivial vacuum structure of QCD (discussed in Sec. III G):


The phase of the quark mass matrix combines with the coefficient of $\varepsilon_{\mu\rho\sigma} \text{tr}(G^\mu G^\rho)$ in the Lagrangian to cause effects that violate CP symmetry. Curiously, this combination—the difference of two quantities with starkly distinct origins—is constrained by the neutron electric dipole moment to be $\sim 10^{-11}$. The strong CP problem was clearly posed, and a still-popular resolution proposed in


Further possible resolutions are explained in


The Peccei-Quinn solution requires a new particle, the axion, with several implications for particle physics and, possibly, cosmology. These connections, and the status of axion searches, are reviewed in


3. Heavy quarks

Hadrons containing heavy quarks exhibit simplifying features. In a bound state with one heavy quark, and any number of light quarks and gluons, the identity (flavor or spin) of the heavy quark alters the dynamics very little, because the heavy quark sits essentially at rest inside the hadron:


“On annihilation of mesons built from heavy and light quark and $B^0\bar{B}^0$ oscillations,” M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. 45, 292 (1987). (I–A)

The center of mass of the hadron and the heavy quark are essentially the same, with the light degrees of freedom in orbit around the heavy quark. A set of approximate symmetries emerge, the heavy-quark flavor and spin symmetries.


In a meson with a heavy quark and corresponding antiquark, the two orbit each other. The velocity depends on the heavy-quark mass, but the spin decouples (to leading order), in analogy with QED applied to atomic physics.


B. Potential models

The observation that asymptotic freedom suggests nonrelativistic atoms of heavy quarks and antiquarks is due to


The nonrelativistic description was elaborated in


A midterm review of potential models can be found in


and newer reviews include

More recently this line of research has been addressed further through effective field theories (see Sec. III E 2).

C. Renormalization and factorization

The renormalization group as a technique for summing to all orders in perturbation theory in electrodynamics was invented by


A clear statement of the algorithm and a thorough review of early applications appears in


The modern formulation of the renormalization group equations is due to


The power of renormalization group methods for a wide range of physical problems was recognized by


A fascinating survey with many references is


The theoretical apparatus required for a general analysis of quantum corrections and their implications for a running coupling constant is presented in


The ability to predict characteristics of high-energy reactions rests on parton-hadron duality and on separating short-distance hard-scattering matrix elements described by perturbative QCD from long-distance (nonperturbative) effects related to hadronic structure. Duality refers to the observation that inclusive hadronic observables may be computed in terms of quark and gluon degrees of freedom. These ideas are reviewed and confronted with recent experimental data in


updating the classic reference:


The distinction between short-distance and long-distance (or short and long time scales) is reminiscent of the Born-Oppenheimer approximation in molecular physics. The factorization of amplitudes and cross sections into parton distribution functions, elementary scattering amplitudes, and fragmentation functions that describe how partons materialize into hadrons, was an element of the exploratory studies reported in Ref. 75. Within the framework of QCD, factorization has been proved in many settings:


The short-distance behavior of quantum field theories, including QCD, is clarified by the operator-product expansion:
Techniques of factorization have been extended to exclusive processes in


and adapted to decays of hadrons containing a heavy quark in


The study of higher orders in perturbation theory, particularly the renormalization parts, can anticipate the pattern of nonperturbative effects. A standard review is


An intriguing feature of these effects makes the definition of quark masses somewhat subtle. The so-called “pole mass,” which corresponds closely to the classical notion of mass, is well-defined in perturbation theory:


yet the perturbative series signals the necessity of nonperturbative effects:


As a consequence, quark masses reported below are renormalized Lagrangian masses.

D. Unitarity and analyticity

Underlying the notion of parton-hadron duality, which enters into many applications of factorization, are unitarity and analyticity. Unitarity means merely that quantum mechanics (and, hence, quantum field theory) preserves probability, thereby imposing limits on scattering amplitudes and related quantities. Analyticity means that scattering amplitudes are analytic functions of kinematic variables, apart from poles or branch cuts, which correspond to stable particles and resonances or multiparticle thresholds, respectively.

These ideas and the formalism of quantum field theory can be used to derive semi-quantitative and, sometimes,
quantitative dynamical information. This approach goes under the name “QCD sum rules” and started with


An early, well-regarded review is


A review and reprint volume is


A more recent monograph explaining QCD sum rules is

150. QCD as a Theory of Hadrons (from Partons to Confinement), S. Narison (Cambridge University Press, Cambridge, 2002). (E–I–A)

and several pedagogical reviews of applications can be found in Ref. 39.

E. Effective field theories

Effective field theories isolate important low-energy degrees of freedom, absorbing the effects of highly virtual processes, such as those of high-mass particles, into coupling strengths of interactions. For capsule reviews, see


Two classes of effective field theories are employed to study QCD, one in which (light) quarks and gluons remain the basic degrees of freedom, and another treating hadrons as fundamental. In both cases, the power of the method is to retain and respect symmetry, renormalization, unitarity, analyticity, and cluster decomposition.

1. Chiral perturbation theory

The consequences of spontaneously broken symmetries are encoded in current algebra (see Sec. 11D1) and can be summarized in an effective Lagrangian for pions:


The formalism was extended to general patterns of spontaneous symmetry breaking in


For hadron dynamics chiral Lagrangians were developed further in


An early review is


The connection with the quark model is developed in


Chiral Lagrangians were then exploited to develop a systematic low-energy expansion, called chiral perturbation theory (χPT):


An excellent place to start learning the modern perspective is


This is now a subject with broad applications, describing, for example, the pion and kaon clouds surrounding a nucleon. This material is pedagogically reviewed in


States with nucleonic properties can also arise from soliton configurations of the pion field, which was first noticed before the advent of QCD:


The so-called Skyrminon approach to the nucleon enjoyed a renaissance in the 1980s, reviewed in


2. Heavy-quark effective theory and nonrelativistic QCD

The simpler dynamics of heavy-quark systems lend themselves to effective field theories. For heavy-light hadrons (those with one heavy quark), this insight led to the development of the heavy-quark effective theory (HQET) in


Some pedagogical reviews are


and a textbook is


In quarkonium, a heavy quark’s velocity is larger than in a heavy-light hadron. The appropriate effective field theory has the same Lagrangian as HQET, but the relative importance of various interactions is different. This field theory is called nonrelativistic QCD (NRQCD) and was first developed for bound-state problems in Ref. 107 and


The classification of NRQCD interactions, focusing on the quarkonium spectrum, was further elucidated in


NRQCD was extended to encompass decay, production, and annihilation in


In some applications, the QCD coupling $\alpha_s$ is small at both the heavy-quark mass and heavy-quark momentum scales:

Then the appropriate effective field theory is potential NRQCD (PNRQCD):


PNRQCD provides a field-theoretic basis for understanding the success of the potential models of Sec. III.B. For a review, consult


NRQCD and PNRQCD have also been used to understand top-quark pair production at threshold. Top quarks decay before toponium forms:


3. *Soft collinear effective theory*

In high-energy amplitudes, one often considers a jet of particles, the details of which are not detected. The semi-inclusive nature of jets circumvents issues of infrared and collinear divergences, much like the Bloch-Nordsieck mechanism in QED:


The infrared and collinear degrees of freedom can be isolated in the soft collinear effective theory (SCET), first established for decays of $B$ mesons:


Meanwhile, SCET has been applied to many high-energy scattering processes, starting with


and more recently to many aspects of jets:


F. *Lattice gauge theory*

With an explicit definition of its ultraviolet behavior, lattice gauge theory lends itself to computational methods, essentially integrating the functional integral of QCD numerically:

The first study connecting the confining regime to asymptotic freedom appeared in


A useful reprint collection of early work is


There are several good textbooks on lattice gauge theory, including


Lattice gauge theory is also the foundation of attempts at rigorous construction of gauge theories:

Gauge Theories as a Problem of Constructive Quantum Field Theory and Statistical Mechanics, E. Seiler (Springer, Berlin, 1982). (I–A)


For many years, numerical lattice-QCD calculations omitted the computationally very demanding contribution of sea quarks (quark-antiquark pairs that fluctuate out of the vacuum), leading to uncontrolled uncertainties. The first demonstration that incorporation of sea-quark effects brings a wide variety of computed hadron properties into agreement with experiment is


The maturation of numerical lattice QCD is discussed in


With these developments it is now possible to compute the hadron masses with a few percent precision:


and make predictions of hadronic properties needed to interpret experiments:


A more detailed comparison of lattice-QCD calculations with experiment is given in Sec. IV

Numerical lattice QCD is not merely a brute-force approach, but a synthesis of computation and effective field theories. Errors from nonzero lattice spacing are controlled with Symanzik’s effective theory of cutoff effects:


work that grew out of Ref. 117. Errors from finite volume can be controlled with general properties of massive field theories on a torus:


The light quarks in computer simulations often have masses larger than those of the up and down quarks,
but the extrapolation in quark mass can be guided by adapting chiral perturbation theory:


The charmed and bottom quarks often have masses close to the ultraviolet cutoff (introduced by the lattice), but the effects can be understood with HQET and NRQCD:


The idea that lattice QCD is a synthesis of computational and theoretical physics is explored in


Lattice gauge theory and chiral symmetry coexist unconventionally:


The efforts to understand and overcome these difficulties, for theories like QCD, is reviewed in


Lattice gauge theory, with its rigorous mathematical definition, is a suitable arena for deriving mass inequalities:


These related developments have been reviewed in


G. The QCD vacuum and confinement

The space of all non-Abelian gauge fields is not simply connected, but consists of sectors labeled by an integer n. The sectors arise when trying to satisfy a gauge condition, namely to specify \( \omega(x) \) in order to choose one representative field \( B_\mu(x) \) among all those related by Eq. 16.

In some cases it is necessary to specify different conditions in different regions of spacetime, and then it turns out that \( \omega(x) \) on the overlaps of the regions is an n-to-one mapping onto \( SU(n_c) \). In the quantum theory, tunneling can occur between the different sectors, and the tunneling events are called “instantons.” A classic discussion can be found in


Some further features appear at nonzero temperature:


Because of these sectors, the QCD Lagrangian, Eq. [1], can contain a term proportional to \( \varepsilon_{\mu\nu\rho\sigma} \text{tr}(G^{\mu\nu}G^{\rho\sigma}) \). The physical implication of this term is a possible violation of CP symmetry, as is discussed further in Sec. III.A.2.

It is widely believed that the nontrivial vacuum structure is connected to the special features of QCD, notably confinement. Opinion is divided whether instantons, i.e., the tunneling events, play the principal role, or whether strong quantum fluctuations do. The case for instantons can be traced from


and the case for fluctuations from


Another approach to confinement starts with the observation that any gauge condition has more than one solution:


In the Coulomb gauge one demands \( \nabla \cdot A = 0 \); further demanding a unique resolution of the Gribov ambiguity, one finds, with some assumptions, a confining potential:

243. “Renormalization in the Coulomb gauge and order parameter for confinement in QCD,” D.
A string picture of confinement emerges naturally from the perturbative properties of QCD. The energy required to separate a quark and antiquark,

\[ E = \sigma R, \]

is proportional to the string tension \( \sigma \) and the separation \( R \). Furthermore, the property of asymptotic freedom means that the “dielectric constant” of the QCD vacuum is \( \varepsilon_{\text{QCD}} < 1 \), in contrast to the familiar result for a dielectric substance, \( \varepsilon > 1 \). The QCD vacuum is thus a dia-electric medium. An electrostatic analogy leads to a heuristic understanding of confinement. It is energetically favorable for a test charge placed in a very effective dia-electric medium to carve out a bubble in which \( \varepsilon = 1 \). In the limit of a perfect dia-electric medium, the bubble radius and the energy stored in the electric field tend to infinity. In contrast, the radius of the bubble surrounding a test dipole placed in the medium occupies a finite volume, even in the perfect dia-electric limit, because the field lines need not extend to infinity.


The dia-electric analogy is reviewed in Sec. 8.8 of


The physical picture is highly similar to MIT bag model:


The exclusion of chromoelectric flux from the QCD vacuum is reminiscent of the exclusion of magnetic flux from a type-II superconductor. In a dual version of the Meissner effect, with the roles of electric and magnetic properties swapped, the chromoelectric field between a separating quark and antiquark takes the form of an Abrikosov flux tube. For an introduction and tests of the picture, see


Lattice gauge theory (Ref. 22) was originally invented to understand confinement. Reviews of more recent analytical and numerical work can be found in


The connection between QCD potentials, spectroscopy, and confinement is reviewed in


An important theme in Ref. 250 is the lattice-QCD computation of the potential energy between static sources of color. As shown in Fig. 2, the potential looks Coulombic at short distances, in accord with asymptotic freedom, and linear at long distances, in accord with Eq. (21).

A series of conferences is devoted to the confinement problem. Their agendas and proceedings can be traced from


H. Dyson-Schwinger Equations

A fruitful continuum approach to nonperturbative dynamics is based on the infinite tower of Dyson-Schwinger equations, coupled integral equations that relate the Green functions of a field theory to each other. Solving these equations provides a solution of the theory, in that a field theory is completely defined by all of its \( n \)-point Green functions. A good starting point is

FIG. 2: The potential energy \( V(r) \) between static sources of color (in an approximation without sea quarks). The zero of energy and the units are set by a conventional distance \( r_0 \), defined by \( r_0^2 dV/dr = 1.65 \). The data points are from lattice QCD, generated at several values of \( \beta = 6/g^2 \), which—via dimensional transmutation—corresponds to varying the spacing between lattice sites. The black curve is a fit of these data to the potential model of Refs. 109, 110. From Ref. 250.
I. Perturbative amplitudes

A key consequence of factorization is to relate amplitudes for (some) hadronic processes to underlying processes of quarks and gluons. Parton amplitudes can be computed via Feynman diagrams, as discussed in Refs. [32,34,38]. As the complexity of the process increases, however, this approach becomes intractable. Remarkably, QCD amplitudes are simpler than the individual diagrams might suggest:


Perturbative QCD amplitudes also are related by recursion in the number of scattered gluons:


For an older review that remains useful for graduate students, see


The simplifications can be related to deep connections between Yang-Mills theories and string theories:


A parallel, and perhaps, even more fruitful, alternative to Feynman diagrams starts with constraints of unitarity:


The first decade of the 2000s witnessed rapid conceptual and technical development of these two sets of ideas, by many researchers, too many to list here. The review


contains a comprehensive set of references, and the most recent developments are discussed in


J. Parton-shower Monte Carlo programs

In a high-energy collision, although the parton-scattering can be factorized and computed in perturbation theory, a description of the full event is complicated first by radiation of gluons and $q\bar{q}$ pairs and later by the formation of hadrons. Several computer codes have been developed to automate the calculation of the initial parton scatter, treat the shower of partons, and model the hadronization. Useful reviews to the concepts can be found in


and a hands-on guide is

K. Extensions of QCD

QCD belongs to a class of Yang-Mills theories, and further information can be gleaned by varying the number of colors, $N_c$, and the number of flavors, $n_f$. Some classic and useful references on QCD as $N_c \to \infty$ are


Supersymmetry is a spacetime symmetry connecting bosonic and fermionic representations of the Poincaré group. Gauge theories with supersymmetry enjoy some simplifying features:


leading to interesting relations between strongly-coupled gauge theories of certain $(N_c, n_f)$ and weakly-coupled dual gauge theories with $(N_c', n_f')$.

L. String theory

String theory is a mathematical description of particles as vibrational modes of one-dimensional objects, instead of as points. First developed as a model of hadrons, string theory fell out of favor after the rise of QCD. But it has enjoyed a tremendous interest as a unifying theory of quantum mechanics and gravity, spurring a vast literature in mathematical physics. Now string theory has come full circle, with string techniques applied to gauge theories, starting with


An excellent pedagogical introduction is given in

277. “Introduction to the AdS/CFT correspondence,” I. R. Klebanov in Strings, Branes, and Grav...
FIG. 3: Measurements of the strong coupling $1/\alpha_s(Q)$ as a function of the energy scale $\ln Q$. In addition to hadronic $\tau$-decay, quarkonium, $\Upsilon$ decay, and $Z^0$-pole values from Ref. 284 we display black crosses: $e^+e^-$ collisions (Ref. 259); red squares: $e^+e^-$ collisions (Ref. 258); green diamonds: $e^+p$ collisions (Ref. 257); barred purple circles: $\bar{p}p$ collisions (Refs. 283–285); cyan crosses: $\bar{p}p$ collisions (Ref. 286); average value of $\alpha_s(M_Z)$, in 4-loop approximation and using 3-loop threshold matching at the heavy-quark pole masses $m_c = 1.5$ GeV and $m_b = 4.7$ GeV.

which draws particular attention to the high level of recent activity in the area of hadronic $\tau$ decays.

A representative selection of experimental determinations is shown, together with the evolution expected in QCD, in Fig. 3. We have drawn the displayed values from Ref. 284 together with determinations from $e^+e^-$ event shapes reported in


from jet studies in $e^+p$ scattering reported in


and the running coupling constant inferred from inclusive jet production in $\bar{p}p$ collisions,


The trend toward asymptotic freedom is clear, and the agreement with the predicted evolution is excellent, within the uncertainties in the measurements. An interesting challenge for the future will be to measure $\alpha_s(Q)$ with precision sufficient to detect the expected change of slope at the top-quark threshold.

It is conventional, and enlightening, to rewrite the evolution equation (19) in the form

$$\frac{1}{\alpha_s(Q)} = \frac{33 - 2n_f}{6\pi} \log \left( \frac{Q}{\Lambda_{\text{QCD}}} \right),$$

(22)

where $\Lambda_{\text{QCD}}$ is the QCD scale parameter, with dimensions of energy. (A generalization beyond leading order is given in Sec. 9 of Ref. 8.) Several subtleties attend this simple and useful parametrization. First, if we enforce the requirement that $\alpha_s(Q)$ be continuous at flavor thresholds, then $\Lambda_{\text{QCD}}$ must depend on the number of active quark flavors. Second, the value of $\Lambda_{\text{QCD}}$ depends on the renormalization scheme; the canonical choice is the modified minimal subtraction (MS) scheme introduced in


The $n_f$ and scheme dependence is given via labels on $\Lambda$, e.g., $\Lambda^{(n_f)}_{\text{MS}}$. Representative estimates of the QCD scale are $\Lambda^{(5)}_{\text{MS}} = 213$ MeV, $\Lambda^{(4)}_{\text{MS}} = 296$ MeV, and $\Lambda^{(3)}_{\text{MS}} = 338$ MeV (Ref. 284). The appearance of a dimensional quantity to parametrize the running coupling is sometimes called “dimensional transmutation.”

The theoretical underpinnings of high-precision determinations of $\alpha_s$ in deeply inelastic scattering are presented in


which provides a detailed analysis of remaining uncertainties in the QCD scale.

For another critical assessment of the theoretical analyses that underlie determinations of $\alpha_s$, see...
FIG. 4: Determinations of $\alpha_s(M_Z)$ from several processes. In most cases, the value measured at a scale $\mu$ has been evolved to $\mu = M_Z$. Error bars include the theoretical uncertainties. Adapted from Ref. 284.


When evolved to a common scale $\mu = M_Z$, the various determinations of $\alpha_s$ lead to consistent values, as shown in Fig. 4. A representative mean value (Ref. 284) is

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007. \quad (23)$$

The agreement of the determination of $\alpha_s$ from the hadron spectrum, via lattice QCD, and from high-energy scattering processes, via perturbative QCD (and factorization for deeply inelastic scattering), indicates that QCD describes both hadron and partons. In other words, a single theory accounts for all facets of the strong interactions.

**B. Hadron spectrum**

Soon after the conception of quantum chromodynamics, theorists formulated QCD-inspired models to open a dialogue with experiment. Simple notions about the order of levels, augmented by an effective color-hyperfine interaction were put forward in


The extension to excited baryons was given by


An extensive analysis of the meson spectrum in a QCD-inspired quark model is


Massless quarks were confined within a finite radius by fiat in the MIT bag model, which is explained in


Lattice QCD provides a way to compute the hadron mass spectrum directly from the QCD Lagrangian. The state of the art for light hadrons is shown in Fig. 5 and described in


With $\alpha_s$, the quark masses are the fundamental parameters of QCD. Hadron masses depend on the quark masses, so these calculations yield as by-products the best estimates of the light-quark masses (Ref. 301)

$$m_u = 1.9 \pm 0.2 \text{ MeV},$$
$$m_d = 4.6 \pm 0.3 \text{ MeV},$$

(24)
\[ m_s = 88 \pm 5 \text{ MeV}; \]

or, defining \( \hat{m} = (m_u + m_d)/2 \),
\[ \hat{m} = 3.54^{+0.64}_{-0.35} \text{ MeV}, \]
\[ m_s = 91.1^{+14.6}_{-6.2} \text{ MeV}, \]  

(25)

from


Both groups use \( 2 + 1 \) flavors of sea quarks. The quoted masses are in the \( \overline{\text{MS}} \) scheme at 2 GeV. Ratios of these results agree with chiral perturbation theory:


The estimates Eqs. (24) and (25) show that the up- and down-quark masses account for only \( 3\hat{m} \approx 10 \) MeV out of the nucleon mass of 940 MeV. Accordingly, to percent-level accuracy, nearly all the mass of everyday matter arises from chromodynamic energy of gluons and the kinetic energy of the confined quarks.

In the elementary quark model, mesons are \( q\bar{q} \) color singlets, whereas baryons are \( qqq \) color singlets. Although QCD favors these configurations as the states of lowest energy, it also admits other body plans: quarkless mesons called glueballs, \( q\bar{q}\bar{q} \) mesons called hybrids, \( qq\bar{q} \) mesons called tetraquarks, \( qq\bar{q}\bar{q} \) baryons called pentaquarks, etc. At this time, there are no credible reports of non–quark-model baryons. The rich body of experimental information on non–quark-model mesons is reviewed in


Strong theoretical evidence for glueballs comes from lattice-QCD calculations in an approximation to QCD without quarks:


309. “Numerical evidence for the observation of a scalar glueball,” J. Sexton, A. Vaccarino, and D.

![FIG. 6: World data on the ratio \( R \) from Eq. (28), compared with predictions of the quark-parton model (dashed curve) and perturbative QCD at three loops (solid line), from Ref. 8](image_url)

C. The reaction \( e^+e^- \rightarrow \text{hadrons} \)

In the framework of the quark-parton model, the cross section for hadron production in electron-positron annihilations at center-of-momentum energy \( \sqrt{s} \) is given by

\[
\sigma_{\text{qpm}}(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi\alpha^2}{3s} \left[ 3 \sum_q e_q^2 \theta(s - 4m_q^2) \right],
\]

(26)

where \( e_q \) and \( m_q \) are the charge and mass of quark flavor \( q \) and the step function \( \theta \) is a crude representation of kinematic threshold.

The factor 3 preceding the sum over active flavors is a consequence of quark color. The rough agreement between measurements of the ratio of hadron production to muon-pair production and the prediction (26), shown as the dashed line in Fig. 6, is powerful evidence that quarks are color triplets.

The parton-level prediction is modified by real and virtual emission of photons, much as the quantum electro-dynamics prediction for \( \sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/3s \) is changed by real and virtual emission of photons. To leading order in the running coupling \( \alpha(s) \), the result is

\[
\sigma_{\text{QCD}}(e^+e^- \rightarrow \text{hadrons}) = \sigma_{\text{qpm}} \left[ 1 + \frac{\alpha(s)}{\pi} + \mathcal{O}(\alpha(s)^2) \right],
\]

(27)

The QCD prediction for

\[
R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},
\]

(28)

now known through order \( \alpha(s)^3 \), is shown as the solid line in Fig. 6.

Moments of the cross section

$$M_n \equiv \int_{4m_Q^2}^{\infty} ds \ s^{-(n+1)} R_Q(s), \quad (29)$$

where $R_Q$ is the part of $R$ [Eq. (28)] due to $QQ$, are useful for determining the masses of the charmed and bottom quarks. The most recent results are

$$m_c(3 \, \text{GeV}) = 986 \pm 13 \, \text{MeV}, \quad (30)$$

$$m_b(10 \, \text{GeV}) = 3610 \pm 16 \, \text{MeV}, \quad (31)$$

where the values are again in the $\overline{\text{MS}}$ scheme and the argument indicates the renormalization point. These results are taken from 313. “Charm and bottom quark masses: an update,” K. G. Chetyrkin et al., Phys. Rev. D80, 074010 (2009) [arXiv:0907.2110 [hep-ph]]. (A) which also serves as a useful entrée to the literature.

D. Jets and event shapes in $e^+e^- \rightarrow$ hadrons

A hadron jet is a well-collimated cone of correlated particles produced by the hadronization of an energetic quark or gluon. Evidence that hadron jets produced in the electron-positron annihilation into hadrons follow the distributions calculated for $e^+e^- \rightarrow q\bar{q}$ was presented in 314. “Azimuthal asymmetry in inclusive hadron production by $e^+e^-$ annihilation,” R. Schwitters et al., Phys. Rev. Lett. 35, 1320–1322 (1975) [doi: 10.1103/PhysRevLett.35.1320]. (I–A)


The notion that gluon radiation should give rise to three-jet events characteristic of the final state $q\bar{q}g$ was made explicit by 316. “Search for gluons in $e^+e^-$ annihilation,” J. R. Ellis, M. K. Gaillard, and G. G. Ross, Nucl. Phys. B111, 253 (1976) [doi: 10.1016/0550-3213(76)90452-3]. (I–A) and confirmed in experiments at the PETRA storage ring at the DESY Laboratory in Hamburg:


The definition of a three-jet cross section corresponding to the quark-antiquark-gluon final state is plagued by infrared difficulties—as is the specification of any final state with a definite number of partons. It is, however, possible to define infrared-safe energy-weighted cross sections that are calculable within QCD, as shown in 322. “Jets from quantum chromodynamics,” G. Sterman and S. Weinberg, Phys. Rev. Lett. 39, 1436 (1977) [doi: 10.1103/PhysRevLett.39.1436]. (I–A)


Definitions within SCET are discussed in Refs. 199, 204. Various observables are sensitive to different combinations of the quark and gluon color factors, $C_F$ and $C_A$, and so an ensemble of measurements may serve to test the QCD group-theory structure via Eq. (13). The constraints from a number of studies at LEP are compiled in Fig. 7. The combined result, presented in 324. “Tests of quantum chromodynamics at $e^+e^-$ colliders,” S. Kluth, Rept. Prog. Phys. 69, 1771–1846 (2006)


According to the parton model, a hadron is a collection of quasifree quarks, antiquarks, and gluons. In the "infinite momentum frame," in which the longitudinal momentum of the hadron is very large, each parton carries a fraction \( x \) of the hadron’s momentum. A parton distribution function \( f_i(x_i) \) specifies the probability of finding a parton of species \( i \) with momentum fraction \( x_i \). A highly intuitive formalism that generalizes the parton distributions to \( f_i(x_i, Q^2) \) and stipulates the evolution of parton distributions with momentum transfer \( Q^2 \) was given in


The Altarelli-Parisi prescription is appropriate for moderate values of \( x \) and large values of \( Q^2 \). The extension to higher-order corrections in the \( \overline{\text{MS}} \) scheme is presented in Ref. 294 and reviewed in


An early quantitative test appears in


Increasingly comprehensive data sets deepened the dialogue between theory and experiment. For an informative sequence of reviews, see


The series of annual workshops on deeply inelastic scattering and QCD may be traced from

In addition to its “valence” components, a hadron contains quark-antiquark pairs and gluons, by virtue of quantum fluctuations. In the extreme limit $Q \to \infty$, for any hadron, the momentum fraction carried by gluons approaches 8/17, and that carried by any of the six species of quark or antiquark approaches 3/68. The asymptotic equilibrium partition reflects the relative strengths of the quark-antiquark-gluon and three-gluon couplings, as well as the number of flavors. The current state of the art for parton distributions (at finite $Q$) is comprehensively documented in


A library providing a common interface to many modern sets of parton distributions is


The sets of parton distributions currently in wide use may be traced from


“Dynamical Parton Distribution Functions,” P. Jimenez-Delgado et al., http://doom.physik.uni-dortmund.de/pdfserver (A)

“Combined measurement and QCD analysis of the inclusive $ep$ scattering cross sections at HERA,” F. D. Aaron et al., H1 and ZEUS Collaborations, arXiv:0911.0884 [hep-ex]. (A)


It is conventional to separate quark (and antiquark) distributions into “valence” components that account for a hadron’s net quantum numbers and “sea” contributions in which quarks balance antiquarks overall. Neither a symmetry nor QCD dynamics demand that $q(x) = \bar{q}(x)$ locally, and experiment has now revealed a flavor asymmetry in the light-quark sea of the proton.


Sum rules that parton distributions must respect in QCD are reviewed in


The number densities $q(x, Q^2)$, $\bar{q}(x, Q^2)$, and $g(x, Q^2)$, of quarks, antiquarks, and gluons within a hadron can be calculated at large $Q^2$ by Altarelli-Parisi evolution (Ref. 327) from initial distributions determined at $Q_0^2$. However, at small values of the momentum fraction $x$, the resulting densities may become large enough that the partons overlap spatially, so that scattering and recombination may occur, as argued in


Recombination probabilities were computed in


and expectations for lepton-nucleon scattering at very small values $x$ are developed in

“Small $x$ physics in deep inelastic lepton hadron scattering,” B. Badelek, M. Krawczyk, K. Charchula, and


Experiments at the $e^+p$ collider HERA, which operated at c.m. energies up to $\sqrt{s} = 320$ GeV, probed the small-$x$ regime and established a rapid rise in the parton densities as $x \to 0$, as reviewed in Refs. 356,358. However, recombination phenomena have not yet been demonstrated. Implications of the HERA observations for future experiments are explored in


Our knowledge of the spin structure of the proton at the constituent level is drawn from polarized deeply inelastic scattering experiments, in which polarized leptons or photons probe the structure of a polarized proton and polarized proton-proton collisions. How current understanding developed, and what puzzles arose, can be traced in


Progress in making spin-dependent measurements can be traced through the spin physics symposia; the latest in the series is


For a set of spin-dependent parton distribution functions, with extensive references to the underlying measurements, see


Standard parton distribution functions provide detailed information about how spin and longitudinal momentum and spin are partitioned among the quarks, antiquarks, and gluons in a fast-moving hadron, but the information is integrated over transverse degrees of freedom. The role of orbital angular momentum of the partons in building a spin-$\frac{1}{2}$ proton is obscured. Generalized parton distributions inferred from exclusive scattering processes provide a tool for probing such subtleties of hadron structure.


An important undertaking of modern hadron physics is to understand how hidden flavors (e.g., virtual $s\bar{s}$ pairs) contribute to the structure of the nucleon. Recent experimental and theoretical progress toward unravelling the role of strange quarks in the nucleon can be traced in


In analogy to the hidden flavors of light quarks, hadrons could have an intrinsic component of charm-anticharm pairs:


F. Quarkonium

An early opportunity for QCD-inspired models of hadrons came with the discovery of the $J/\psi$ particle and other bound states of charmed quarks and antiquarks,


For an account hard on the heels of the discovery, see


Quarkonium spectroscopy was enriched by the discovery of the $\Upsilon$ family of $b\bar{b}$ bound states:


An accessible account of these discoveries is


For a summary of early comparisons between the $c\bar{c}$ and $b\bar{b}$ families, see


These discoveries spurred the development of potential models (see Sec. III B). Reviews of this work from the experimental perspective are in


Calculations of the quarkonium spectrum, once the exclusive province of potential models (cf. Sec. III B), are an important theme in lattice QCD. Three of the first papers on calculations with $2 + 1$ flavors of sea quarks are

“The $\Upsilon$ spectrum and $m_b$ from full lattice QCD,” A. Gray et al., Phys. Rev. D72, 094507 (2005) [hep-lat/0507013]. (A)

“Highly improved staggered quarks on the lattice, with applications to charm physics,” E. Pollana et al., HPQCD Collaboration, Phys. Rev. D75, 054502 (2007) [hep-lat/0610092]. (A)


The breadth of quarkonium physics—experimental, theoretical, and computational—is surveyed in


A novel form of quarkonium arises from binding a bottom quark and a charmed antiquark. The first observation of the pseudoscalar $B_c$ meson is reported in

“Observation of the $B_c$ meson in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV,” F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 81, 2432–2437 (1998) [hep-ex/9805034]. (I–A)

Precise measurements of the mass did not appear until later:

“Evidence for the exclusive decay $B_c^+ \rightarrow J/\psi \pi^+$ and measurement of the mass of the $B_c$ meson,” A. Abulencia et al., CDF Collaboration, Phys. Rev. Lett. 96, 082002 (2006) [hep-ex/0505076]. (I–A)

The mass of the $B_c$ was correctly predicted by PNRQCD:


and lattice QCD:
Recently, a new set of states has appeared in the charmonium spectrum that presents new challenges to hadron dynamics. Some of these may be (mostly) charm-anticharm states above the threshold for decay into charmed-meson pairs. Others cannot readily be identified in the same way. For a recent survey, see [400]. “The exotic XYZ charmonium-like mesons,” S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. 58, 51–73 (2008) [arXiv:0801.3867 [hep-ph]]. (I–A)

G. Jets in hadron collisions


Incisive comparisons with QCD were made in experiments at the SPS Collider, at energies up to 630 GeV:


Extensive studies have been carried out at the Tevatron Collider, at energies up to $\sqrt{s} = 1.96$ TeV. We show in Fig. 8 that perturbative QCD, evaluated at next-to-leading order, accounts for the transverse-momentum spectrum of central jets produced in the reaction

$$\bar{p}p \rightarrow \text{jet}_1 + \text{jet}_2 + \text{anything}$$

over more than eight orders of magnitude:


Similar results from the CDF experiment are reported in


For a summary of recent QCD studies at the Tevatron, see


The current state of the art is presented in


Jet phenomena in relativistic heavy-ion collisions are summarized in


For a discussion of jet definitions and their interplay with measurements, see Ref. [323].
H. Photon structure function

The proposal to determine the constituent structure of the photon by studying the scattering of a highly virtual photon on a real photon is due to


To the extent that a photon behaves as a vector meson, the momentum-fraction ($x$) and momentum-transfer ($Q^2$) dependences of its structure function should roughly resemble those of the proton structure function. But a parton-model calculation reveals that a pointlike contribution that arises when the photon fluctuates into a quark-antiquark pair should dominate over the vector-meson component at high $Q^2$.


Remarkably, the $x$-dependence of the photon structure function is fully calculable at large $Q^2$, in contrast to the proton structure function, for which the $x$-dependence at fixed $Q^2$ results from nonperturbative effects and, in practice, is taken from the data or, in the approach of Ref. 346, from a simple Ansatz.

QCD confirms the calculability of the photon structure function at large $Q^2$, and differs from the parton-model result, particularly as $x \rightarrow 1$. In leading logarithmic approximation, the result is reported in


The next-to-leading-order calculation improves the reliability of the predicted shape of the photon structure function:


also enabling a determination of the strong coupling $\alpha_s$, now at the 5% level.

For an excellent short review, see


Extensive experimental summaries appear in


A useful digest appears in Fig. 16.14 of Ref. 8.

I. Diffractive Scattering

The Pomeranchuk singularity, or Pomeron, designates the Regge pole with vacuum quantum numbers that controls the asymptotic behavior of elastic and total cross sections. The Regge intercept of the Pomeron, the location of the pole in the complex angular-momentum plane at zero momentum transfer, would be $\alpha_p = 1$ if total cross sections approached constants at high energies. Comprehensive modern fits to meson-baryon and especially proton-(anti)proton total cross sections initiated in


indicate that $\alpha_s \approx 1.08$.

With the advent of quantum chromodynamics, it was natural to begin searching for a dynamical description of the Pomeron’s origin. The idea that the Pomeron somehow emerged from the exchange of color-octet gluons between color-singlet hadrons, first articulated in


has great resonance today. A concrete realization of the Pomeron in QCD as a composite state of two Reggeized gluons was developed (in leading logarithmic approximation) by


The dynamics underlying the “BFKL Pomeron” also entail a resummation of leading $1/x$ corrections to structure functions.

In spite of much productive effort, the origin of the Pomeron and the details of its structure are still not entirely clear:

423. Quantum Chromodynamics and the Pomeron, J. R. Forshaw and D. A. Ross (Cam-


Although the Pomeron was conceived to account for "soft" scattering, the QCD interpretation implies that it should have a partonic structure, and thus a "hard" component. For a survey of evidence for a hard component from $e^+p$ collisions at HERA, see


The suggestion that Pomeron exchange should result in large rapidity gaps between jets crystallized in


Evidence for events of the suggested character was presented in


If the Pomeron can be exchanged between color-singlets, then, in analogy with two-photon physics or the multi-peripheral model, two Pomerons can collide and produce collections of particles with net vacuum quantum numbers, isolated by large rapidity gaps from other particle production in the event, as observed in

**429. "Observation of exclusive charmonium production and $\gamma\gamma \rightarrow \mu^+\mu^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," T. Aaltonen et al., CDF Collaboration, Phys. Rev. Lett. 102, 242001 (2009) [arXiv:0902.1271 [hep-ex]]. (I–A)**

A tantalizing possibility is that the Higgs boson might be discovered at the LHC in very quiet events:


### J. Weak boson production

In hadron colliders, an electroweak vector boson can be produced directly via fusion of a quark and antiquark. The cross section depends on the parton distributions discussed in Sec. [IV.E]. This extension of the parton model was first noted in


This process provided the basis for the discovery of the $W$ and $Z$ bosons:


What was then a QCD-guided discovery is now one of the most precise tests of perturbative QCD. The production cross sections and rapidity distributions for the Tevatron and the LHC have been carried out to the next-to-next-to-leading order in $\alpha_s$ in


These calculations have been validated (except at the largest accessible values of rapidity) in measurements performed at the Tevatron:


Production of electroweak bosons together with jets is discussed in Sec. [IV.K]

This history is set to repeat itself in the search for the Higgs boson, which relies on the next-to-next-to-leading order QCD calculation:


The Higgs-boson searches at the Tevatron and the LHC...
rily on these results, and on comparably precise calculations of background processes, in an essential way.

K. Heavy-quark production

Another probe of the short-distance dynamics of QCD is the production of heavy quark-antiquark pairs in hadron collisions:


An important application is the production of the top quark at the Tevatron:


Measurements of the top quark mass have been combined into an average:

144. “Combination of CDF and D0 results on the mass of the top quark,” Tevatron Electroweak Working Group, arXiv:0903.2503 [hep-ex]. (A)

yielding the result

\[ m_t = 173.1 \pm 1.3 \text{ GeV}, \]

where this mass has a more conventional definition (similar to that of the electron). The top-quark mass is now precise enough that the ambiguities raised in Refs. [141,142] are becoming quantitatively important.

QCD calculations are important not only to gain an understanding of the experimental signal, but also to understand the background, which stems from W production:


L. Inclusive B decays

Another useful application of perturbative QCD is to inclusive decays of hadrons containing a heavy quark. In practice, this approach applies to hadrons with the bottom quark. One again appeals to quark-hadron duality and applies the operator-product expansion to factorize the differential rate into short- and long-distance contributions. This rich subject launched with


The arc of this research is explained pedagogically in Ref. [178] and in further detail in


This formalism has several applications, using the experimental data to gain insight into long-distance QCD on the one hand, and to determine the bottom quark’s flavor-changing weak couplings. Both perspectives are treated in a thorough analysis of the then-current theory and data:

151. “Fits to moment measurements from \( B \to X_s l \nu \) and \( B \to X_c \gamma \) decays using heavy-quark expansions in the kinetic scheme,” O. Buchmüller and H. Flächer, Phys. Rev. D73, 073008 (2006) [hep-ph/0507253]. (A)

A by-product of these analyses is another determination of the bottom-quark mass. The status is summarized in


This review covers all of flavor physics, including aspects pertaining to this and the next subsection, and well beyond.

M. Exclusive meson decays

Pseudoscalar mesons can decay via the weak interaction to a charged lepton and its neutrino, and the rate can be compared with lattice-QCD calculations of the transition amplitudes. For \( \pi \) and \( K \) mesons, the calculations and measurements agree well. For mesons with heavy
quarks, the measurements lag the calculations somewhat, and the agreement is good but not spectacular. The current status is thoroughly discussed in Ref. 43.

Pseudoscalar mesons can also decay via the weak interaction to a lighter hadron in association with the lepton-neutrino pair. These three-body decays are called semileptonic. Lattice-QCD calculations predicted the normalization and kinematic distribution of semileptonic $D$ decays. A good place to start is


from which a comparison of a QCD calculation with measurements from several experiments is reproduced in Fig. 9. Similar comparisons can be made for semileptonic kaon and $B$-meson decays.

Nonleptonic kaon decays are too computationally challenging for lattice QCD and, apart from constraints from chiral perturbation theory, too conceptually challenging via other approaches. Numerous nonleptonic $B$ decays are kinematically allowed, posing conceptual challenges for lattice QCD. The high scale of the bottom-quark mass, however, allows a treatment in perturbative QCD, at least to leading order in $1/m_b$ (Refs. 141, 195). Broad studies provide information on flavor-changing couplings of the Standard Model:


455. “SCET analysis of $B \to K \pi$, $B \to K \bar{K}$, and $B \to \pi \pi$ decays,” C. W. Bauer, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D74, 034010 (2006) hep-ph/0510241. (A)

For a comprehensive set of references to the measurements and a comparison with calculations at the third order in $\alpha_s$, see Table 3 of


For an alternative approach, see


N. Heavy-ion collisions and the quark-gluon plasma

One of the goals of relativistic heavy-ion collisions is to investigate the quark-hadron phase transition that presumably occurred in the early universe.


By creating small volumes with high energy density or high particle density, heavy-ion collisions open a window on new phases of matter.


Experiments at Brookhaven National Laboratory’s Relativistic Heavy-Ion Collider (RHIC) imply the existence of a “perfect fluid” of quarks and gluons.


A series of conference on quark matter may be traced starting at


The phase diagram is thought to be much richer beyond the region explored by heavy-ion collisions. Figure 10 shows a current conception of QCD thermodynamics (Ref. 465). The region of Fig. 10 with $\mu \approx 0$ has been demonstrated with lattice QCD:


In addition to providing detailed information that is useful for interpreting heavy-ion collisions, these calculations have shown that QCD contains a phase in which (quasi-particle guises of) quarks and gluons are no longer confined, and the chiral symmetry of the quarks is restored. As shown in Fig. 11 the transition is smooth, but order parameters for deconfinement and for chiral symmetry restoration change qualitatively and quantitatively at essentially the same temperature.


Equation of state and QCD transition at finite temperature, A. Bazavov et al., HotQCD Collaboration, Phys. Rev. D80, 014504 (2009) arxiv:0903.4379 [hep-lat]. (A)

The transition temperature near 190 MeV corresponds to $2 \times 10^{12}$ K.

An intriguing result of these lattice-QCD calculations is how the order of the phase transition depends on the light- and strange-quark masses. If they were around half the size needed to explain the nonzero pion and kaon masses, then the transition would be first order, instead of a smooth crossover.


The chiral critical line of $N_f = 2 + 1$ QCD at zero and nonzero baryon density, P. de Forcrand and O. Philipsen, JHEP 01, 077 (2007) hep-lat/0607017.

FIG. 10: Phase diagram of QCD in the $\mu$-$T$ plane. Here $\mu$ denotes baryon chemical potential and $T$ temperature. At low $\mu$, there is a smooth transition with varying $T$, probed by heavy-ion collisions and lattice-QCD calculations. At higher $\mu$ the phases are informed by models and other theoretical considerations. Hadronic matter denser than neutron stars is thought to exhibit “color superconductivity,” first without and eventually with “color-flavor locking.” Adapted from Ref. 465.
That would expose the early universe to a latent heat as it cools below the critical temperature. Ramifications of the QCD phase transition on the early universe are discussed in


Numerical lattice QCD is for now limited to baryon chemical potential $\mu \approx 0$, with obstacles to the regime relevant to neutron stars.

Ordinary nuclei consist of protons and neutrons, which are composed of up and down quarks. Because the most stable nuclei have exceedingly long lifetimes—greater than the age of the universe—it is natural to idealize them as absolutely stable, up to the conjectured nucleon decay that arises in unified theories of the strong, weak, and electromagnetic interactions. If the strange-quark mass were comparable to the up- and down-quark masses, then the Pauli principle would be less restrictive, and the ground state of matter would be a mixture of $u$, $d$, and $s$ quarks. It has been conjectured that such strange matter is the true ground state in the real world, so that nuclear matter is metastable, as elaborated in


Small nuggets of strange matter are called strangelets.

According to the strange-matter hypothesis, compact stars might be strange stars, rather than neutron stars, as reviewed in


which summarizes strange-matter searches. Conferences on strange quark matter may be traced from


Using techniques of gauge-string duality (Ref. 277), theorists have attempted to infer characteristics of QCD in the strong-coupling regime from analogue theories that possess some degree of supersymmetry. Applications to heavy-ion collisions and confinement are reviewed in


One should bear in mind, however, that the archetype of the analogue theories, supersymmetric Yang-Mills theory with four supercharges, does not share some of the essential features of QCD: the coupling that corresponds to $\alpha_s$ does not run, and the theory does not confine.

O. QCD and nuclear physics

In principle, all of nuclear physics follows from QCD:


A recent review of QCD-based nuclear theory, emphasizing symmetries and effective field theories can be found in
V. QCD IN THE BROADER CONTEXT OF PARTICLE PHYSICS

Quantum chromodynamics is part of the extremely successful “Standard Model of Elementary Particles.” Some resources that help put QCD in the broader context of the Standard Model are given here.

A comprehensive source of general knowledge about particle physics, including many aspects of QCD, is the biannual review by the Particle Data Group (Ref. 8).

Many of the themes that came together in quantum chromodynamics may be traced in the contributions to two symposia on the history of particle physics:


Experimental steps that led to today’s standard model of particle physics are surveyed in the well-chosen collection,


Also see


507. “Gauge theories of the forces between elementary particles,” G. ’t Hooft, Sci. Am. 242, 104–138 (June,
Like quantum chromodynamics, the electroweak theory is a gauge theory, based on weak-isospin and weak-hypercharge symmetries described by the gauge group $SU(2)_L \times U(1)_Y$. For a look back at the evolution of the electroweak theory, see the Nobel Lectures by some of its principal architects:


Experiments (and the supporting theoretical calculations) over the past decade have elevated the electroweak theory to a law of nature. The current state of the theory is reviewed in


For general surveys of the standard model of particle physics, and a glimpse beyond, see


The common mathematical structure of QCD and the electroweak theory, combined with asymptotic freedom, encourages the hope that a unified theory of the strong, weak, and electromagnetic interactions may be within reach. The unification strategy, with some consequences, is presented in


VI. FRONTIER PROBLEMS IN QCD

Four decades after the synthesis of quarks, partons, and color into the QCD Lagrangian (Ref. 9)—and the essentially immediate discovery of asymptotic freedom (Ref. 1011)—QCD has been tested and validated up to energies of 1 TeV. Tests are poised to continue at even higher energies, as operations at the Large Hadron Collider (LHC) commence. It is fair to say, however, that most physicists do not expect big surprises at the LHC in the structure of QCD. Instead, QCD will be treated as basic knowledge, much like electrodynamics, enabling discoveries beyond the realm of the standard model of elementary particles (Ref. 42).

In this arena, future research will focus on techniques for evaluating parton amplitudes with increasingly many real and virtual particles, for both signals and backgrounds. The higher energies of the scattering processes will continue to entail many scales (several TeV compared to the top-quark mass, for example) and, hence, will need tools, such as the soft-collinear effective theory discussed in Sec. 5.13. Future experiments with $B$ decays will also continue to rely on QCD, at moderately high energies, to pin down the weak and any new interactions of quarks (or other particles carrying color).

The strong interactions comprise a richer field than the set of phenomena that we have learned to describe in terms of perturbative QCD or the (near-)static nonperturbative domain of lattice QCD. The technology by which we apply QCD is incomplete, and still evolving. Many aspects of hadron phenomenology and spectroscopy are not yet calculable beginning from the QCD Lagrangian. Much analysis of experimental information relies on highly stylized, truncated pictures of the implications of the theory. While expanding the horizons, it is important to distinguish tests of QCD from tests of auxiliary assumptions.

The rest of the strong interactions, moreover, isn’t confined to common processes with large cross sections such as the “soft” particle production, elastic scattering, or diffraction. It may well be that interesting, unusual occurrences happen outside the framework of perturbative QCD—happen in some collective, or intrinsically nonperturbative, way. At the highest energies, well into the regime where the $pp$ total cross section grows as $\ln^2 s$, long-range correlations might show themselves in new ways. Quantum chromodynamics suggests new, modestly collective, effects such as multiple-parton interactions. The high density of partons carrying $p_z = 5–10$ GeV may give rise to hot spots in the spacetime evolution of the collision aftermath, and thus to thermalization or other phenomena not easy to anticipate from the QCD Lagrangian.

At lower energies, the basic features of the hadron spectrum have been reproduced in a convincing way. Some of the simplest hadronic transition amplitudes, needed to understand flavor physics, are in similarly good shape. The aspiration here is to compute many simple amplitudes with total errors that are 1% or smaller. Such precision will require nonperturbative matching and the charmed sea. Indeed, a next-generation assault on $B$ decays via $e^+e^- \rightarrow \Upsilon(4S)$ will hinge on such lattice QCD calculations (Ref. 1452). Calculations of similar difficulty
are related to moments of the parton distributions. Reliable lattice-QCD calculations would pin down predictions of signals and backgrounds at the LHC. The most crucial in this regard, and most challenging computationally, are moments of the gluon density inside the proton. See Ref. [375] and


Precision perturbative QCD and precision lattice QCD are important and challenging, yet programmatic. Other future avenues for research in QCD will explore its richness in ways that are harder to anticipate. QCD is frequently, and justifiably, hailed as a triumph of reductionist science, distilling the plethora of hadrons and their complicated properties into a simple Lagrangian field theory [Eq. (1)]. Now that QCD is accepted as a law of nature, however, it may be time to characterize QCD research by the phenomena that emerge from this tantalizing simple form. What are hadron masses and chiral symmetry breaking, if not emergent phenomena?

Many avenues offer themselves for quantitative and qualitative study. Although the spectrum of the lowest-lying conventional hadrons is well-computed, it remains a challenge to compute the masses of excited hadrons, and even the lowest-lying glueball, hybrid, and exotic states. While these masses tie into experimental programs, it would simply be intriguing to see towers of bound states emerge from the QCD Lagrangian. Another structure that emerges from QCD is a rich phase structure (see Sec. IV N). A fuller understanding will require experiments with heavy-ion collisions, including the higher-density probes of the Compressed Baryonic Matter experiment.


Complementary theoretical work will require both model studies and lattice QCD calculations, although a breakthrough in finite-density lattice QCD could relegate some model studies to secondary importance. The transition to (effectively) deconfined quarks at nonzero temperature and density may help explain why color cannot be isolated in the (zero-temperature) ground state of QCD. Finally, from the emergent phenomenon of hadrons emerges the whole field of nuclear physics. QCD is just beginning to answer questions about nuclear physics, and some nuclear physicists see the future of their field as QCD (see Sec. IV O).

To elucidate these features of QCD, it will help to study lightweight versions of Yang-Mills theories with quarks. For example, with one flavor there is no chiral symmetry to break—the anomaly represents an explicit breaking of the U(1) chiral symmetry (see Sec. III A 2)—presenting a laboratory to study confinement without spontaneous symmetry breaking.


What properties does this confining theory share with QCD? What does it lose along with the loss of chiral symmetry, spontaneously broken? An irony of nature’s version of QCD is that the up- and down-quark masses are much smaller than \( \Lambda_{\text{QCD}} \), so isospin is an excellent approximate symmetry. (More properly, isospin follows from \( m_u - m_d \ll \Lambda_{\text{QCD}} \).) Alternatively, one could imagine a theory with two quarks whose masses, and mass difference, are comparable to or larger than \( \Lambda_{\text{QCD}} \). Which dynamical features remain, and which are lost?

Whatever results academic investigations bring, QCD will retain a strong and deep connection to particle physics, astrophysics and cosmology, and nuclear physics. Indeed, often QCD binds these fields to each other. As discussed above, QCD will always play a central role, within the standard model and beyond, for collider physics. A dream of particle theorists is to unify the strong, weak, and electromagnetic interactions. Further precision for \( \alpha_s \) and quark masses will inform and constrain this dream. Now that it is fairly well established that the up-quark mass cannot vanish, the strong CP problem demands other solutions. The most elegant proposal augments QCD with additional symmetry (Ref. [100]). The observable consequence is a pseudoscalar particle called the axion, which may comprise part of the “dark” matter of the universe (Ref. [102]).

A future challenge is to connect nuclei to QCD. Some aspects befuddle models, because some relevant properties are too hard to measure. For example, the three-nucleon interaction is an important missing piece to the puzzle of nuclear structure. Some questions are almost philosophical: How do \( \alpha_s \) and the quark masses lead to various happenstances of nuclear physics, some of which seem implausible, yet are necessary for carbon-based life to exist? Looking beyond Earth, details of the quark-gluon plasma influence the evolution of the early universe. Above the transition temperature, hadrons do not “dissolve” quite as fast as sometimes thought. Another interesting QCD calculation, which has not yet been carried out, is to determine the \( \Sigma^- \)-nucleon interaction. This is not a prosaic matter of hadronic physics, but a nuclear property that influences whether a supernova evolves to a neutron star or a black hole (Ref. [102]).

In summary, QCD is not our “most perfect theory” (Ref. [2]) merely because asymptotic freedom ensures its scope on towards the highest energies, temperatures, and densities. It is also a rich and varied physical theory, exhibiting qualitatively different behavior in different regimes, all stemming ultimately on the dynamics of quarks and gluons. For all the explanatory power of QCD, it still provides problems for physicists to work on.
Acknowledgments

We thank Thomas Becher, Johan Bijnens, Lance Dixon, Claudia Glasman, Maxime Gouzevitch, Kenichi Hatekeyama, Frithjof Karsch, Stefan Kluth, Christina Mesropian, and Heath O’Connell for assistance in the preparation of this Resource Letter.

Fermilab is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy.

Appendix A: Links to Basic Resources

1. Journals

New research papers on QCD are published in journals of elementary particle physics and of nuclear physics. The principal particle physics journals are


The principal nuclear physics journals are


Journals with review articles:

- Annual Reviews of Nuclear and Particle Science, available on-line at http://arjournals.annualreviews.org/loi/nucl

These websites provide electronic versions (e.g., pdf files) of most—in some cases all—papers published in the corresponding journal. Often a personal or institutional subscription, or the payment of a fee, is necessary.

2. Electronic archives

Most research papers and conference proceedings appear first in the physics e-print archives:

- http://arxiv.org/archive/hep-ex/ contains e-prints on experimental high-energy (elementary particle) physics, many of which concern QCD;
- http://arxiv.org/archive/hep-lat/ contains e-prints on lattice gauge theory, most of which address nonperturbative QCD;

The arXiv provides free downloads. The arXiv version of this Resource Letter provides hyperlinks to arXiv.org where possible, and otherwise provides a hyperlink to the digital object identifier (doi) of other electronically published sources. One should bear in mind, however, that the versions in journals are usually definitive; arXiv.org provides doi links.
3. Pedagogical web sites

For a very approachable introduction to the ideas of contemporary particle physics, see the


and the accompanying

\[521\] The Charm of Strange Quarks: Mysteries and Revolutions of Particle Physics, R. M. Barnett, H. Muehry, and H. R. Quinn (Springer, Heidelberg, 2000). (E)

The ideas of nuclear science are presented in a wall chart and teacher’s guide, available at


The Particle Data Group (Ref. 8) maintains a web site as comprehensive as its review, with updates online midway between the biennial editions. Students and the general public should enjoy their Particle Adventure, http://www.particleadventure.org/

Visualizations of some of the main elements of nonperturbative QCD, with helpful explanations, may be found at the URL in Ref. 26.

The laboratories and major experiments in particle and nuclear physics maintain web sites that feature educational materials.

* E-mail address: ask@fnal.gov
† E-mail address: quigg@fnal.gov