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Interferometers as Holographic Clocks

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It is proposed that the spatial positions of bodies, relative to a classical metric and measured by interactions with classical radiation, are represented by directional quantum operators with Planck scale quantum conditions. With an antisymmetric commutator, the measured speed of light is the same at all frequencies, null fields are nondispersive, and phase is invariant on null sheets, but spacetime position measurements in different directions do not commute. This hypothesis leads to a new source of spatially coherent, Planckian "holographic noise" in relative measured phases of null fields propagating in different directions. Predicted phase correlations are estimated and compared with the sensitivities of current and planned interferometer experiments. Nearly co-located Michelson interferometers correlated at high frequency should be able to achieve the Planckian noise limit.

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INTRODUCTION

The Einsteinian notion of a pointlike spacetime event is a classical approximation. In a description of the world as a quantum-mechanical system, the position of an event should emerge from quantum mechanical operators that correspond in the classical limit to known behavior of matter and energy in spacetime. Quantum mechanics limits the precision with which classical observables, such as the interval between events described by the classical metric, can be defined [1–6]. In addition, gravitational theory argues for a fundamental minimum length or maximum frequency at the Planck scale that imposes a new kind of uncertainty, whose physical character is not known[7].

This paper posits particular properties for Planckian quantum limits on spacetime position measurements, and quantitatively evaluates some of their macroscopic consequences. The two main hypotheses here are that interactions of null fields with matter define spacetime position, and that position operators in different directions do not commute at the Planck scale. As a result, Planckian transverse uncertainty in spacetime position measurements accumulates over macroscopic times and distances, leading to a new kind of spacetime position indeterminacy with particular statistical properties, and thence to a new kind of noise in radiation fields of systems that are sensitive to transverse relative positions at large separations. It is then shown how this prediction can be precisely tested using correlated Michelson interferometers.

PHYSICAL INTERPRETATION OF NONCLASSICAL GEOMETRY

Consider an idealized world consisting of matter and radiation. We wish to establish an operational definition of position for matter. For definiteness, consider a reflecting surface. It forms a spacelike boundary condition ($\nabla\phi = 0$) for an electromagnetic field. Its position is defined by its effect on the field, which is how the position is measured: the field solution depends on the position and orientation of the surface. The system is classical: neither the surface nor the field is quantized. Since this measurement can include a large area that averages over many atoms, we can take the surface to be perfectly smooth. The field in vacuum obeys the standard classical relativistic wave equation, and propagates in a flat classical metric. The vacuum solutions of the field can be decomposed in the usual way into plane wave modes. These modes are not quantized, so we are not here considering quantization or photon noise in position measurement.

Position in each direction can be measured by the reflected phase of a field mode traveling in that direction. Let $x_i(t)$, where $i = 1, 2, 3$, denote the 3D position of some reference point on the surface in its rest frame, in rectilinear, orthonormal coordinates, in flat spacetime. Its position is defined by measurements based on configurations of reflected radiation. We wish to consider limits on the definition of relative position that may be imposed by fundamental physical limits on frequency at the Planck scale.

The new physics we seek to study is introduced by imposing quantum conditions on the geometry just defined. The correspondence between classical and quantum positions is posited to obey the following quantum conditions on

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position operators:

$$[\hat{x}_i, \hat{x}_j] = i(Ct_P)^2\theta_{ij}. \tag{1}$$

The scale is set by the Planck time, $t_P \equiv \sqrt{\hbar G_N/c^5} = 5.39 \times 10^{-44}$ seconds, with a coefficient C of the order of unity.

We have adopted a commutator θ_{ij} in three rather than four dimensions. Commutators with a time operator, with the physical interpretation used here, are are already constrained by experiments to be significantly less than the Planck length, as discussed below. The spatial off-diagonal elements however are harder to measure. They are not yet constrained by experiments at the Planck scale, but we will argue that Planckian sensitivity appears to be within reach of current technology.

Of particular interest is an antisymmetric θ_{ij}: the unit antisymmetric matrix, $\theta_{ij} = -\theta_{ji} = 1$ for $i \neq j$, with zero diagonal. This choice of quantum conditions is "holographic", in the sense that it imposes a Planckian limit on degrees of freedom in transverse spacelike directions defined by a null surface. Arguments originating in black hole thermodynamics suggest that the number of degrees of freedom of any system is given by the area of a bounding null surface in Planck units, a "holographic principle"[8–11]. An antisymmetric commutator in Eq. (1) imposes a Planckian limit on the degrees of freedom on light sheets. In the same way that conventional quantum conditions define a quantum of action in phase space, $\hbar$, the conditions given by Eq. (1) define a quantum of 2D Planck surface area. The numerical coefficient C in the commutator should naturally be set so that the information flux agrees with the entropy surface density of black hole event horizons[12–15].

There are other physical motivations for choosing antisymmetric θ_{ij}. One set of arguments for new physics at the Planck scale is the spontaneous formation of black holes; a particle at rest, confined to a Planck 3-volume, has enough mass-energy to create a black hole— it lies within its own Schwarzschild radius. However, a plane gravitational-wave metric perturbation can exceed the Schwarzschild energy density without forming a black hole. This suggests that particles delocalized in the transverse directions might be possible even above the Planck frequency, so that a longitudinal Planck cutoff is not even needed. The Planck cutoff may have a particular transverse character.

It is important that the new, nonclassical behavior is associated with directions in which positions are measured. A plane wave exactly aligned with a planar reflecting surface reflects in an exactly classical way; no new physics is detectable. Thus, a one-dimensional optical cavity that records phases of waves reflecting between parallel surfaces detects no new nonclassical effect, to first order. On the other hand, reflections of plane waves with orientations inclined to the surface depend on position components in those directions, and these do not commute. The state of the (otherwise classical) radiation field is affected by the (quantum) state of the boundary condition.

Indeed, nothing about photon propagation in vacuum is changed by adding the commutator, Eq.(1). The electromagnetic field still behaves as in a perfect classical spacetime with no new Planckian physics. The metric is not perturbed; the new effect is thus not the same as gravitational waves, or any quantization of a field mode. However, this classical spacetime on its own is not directly accessible to an actual position measurement. That requires interaction with matter at some position, and also a particular choice of frame and measurement direction. The position of the boundary condition with matter is where the new Planckian quantum behavior enters: it applies to the position of matter in the spacetime, as opposed to the unaltered metric. The boundary condition affects the radiation field in the usual way, so the configuration of the radiation field depends on the matter position state (and depends on the quantum position operator) even though its equation of motion in vacuum and the metric itself are not changed.

Even though this formulation is based on classical spacetime, radiation and matter, we have added a new quantum condition on the spacetime positions, which affects the radiation. The system can be placed by interactions into different states. We can thus speak of a measurement in a particular direction placing the system into an eigenstate of that direction. A measurement of a definite, measurable macroscopic configuration state of the field "collapses the wavefunction" in the usual way. In this situation, the relative transverse position is not fixed classically until the radiation is detected, which of course may be a macroscopic distance away. This holographic nonlocality does not violate causality, but it does correspond to a new kind of uncertainty in position that is shared coherently by otherwise unconnected bodies.

As noted previously, the usual one dimensional wave equation is obeyed in each direction. However, quantum operators that measure spacetime intervals, say by comparing ticks of a physical clock with the phase of a wave travelling between events, have an orientation in space. If the operators in different directions do not commute, a fundamental limit follows on the accuracy of position measurements compared in different spatial directions over macroscopic intervals. A new source of noise appears in devices that compare phases of null fields that propagate in different directions, at high frequencies (comparable to the inverse light travel time), across a macroscopic system extending in two spacelike dimensions. The noise resembles an accumulation of transverse Planck scale position errors over a light crossing time. The new behavior appears as a new kind of transverse jitter or displacement from a classical position.

Some properties of this "holographic noise" were previously derived[12–15] using wavefunctions with a fundamental Planck carrier frequency. The position-operator formulation presented here appears to describe equivalent physics. However, it allows more direct calculations of spatial correlations of the noise in general configurations, such as cross correlation in the case of displaced or misaligned interferometers, that are likely to be of value in experimental tests.

Noncommutative geometries[16, 17] and some of their observational consequences[18] have been extensively discussed in the literature. The two new features added here are the particular physical interpretation of position operators, and the particular choice of antisymmetric (holographic) θ_{ij}. The physical interpretatation proposed here— which leaves the classical geometry intact for the purpose of null field propagation, but attaches directional quantum conditions to the position of matter interactions— significantly affects macroscopic phenomenology. The macroscopic position noise derived from these features is qualitatively different from previously considered Planckian effects. It is independent of any parameters of effective field theory. Because of the transverse commutator, there are no dispersive effects, such as those potentially observable in cold-atom interferometers[19] or in cosmic photon propagation: null fields always propagate at exactly c, in agreement with current cosmic limits[20]. Holographic noise in interferometers also has spatial correlations that distinguish it from other Planckian noise sources predicted from quantum-gravitational fluctuations, quantization of very small scale spatial field modes, or spacetime foam[21–28]. Thus, *an experimental program can distinguish between different hypotheses about Planck scale physics.*

PLANCKIAN PHENOMENOLOGY OF INTERFEROMETERS

With a frequency-bounded system, the number of degrees of freedom is finite so the state of the system is specified by a countable set of numbers at the Shannon sampling density. There is thus no loss of generality in assuming that position operators are discrete[29–31]. Measurement of a position in any direction places a system into an eigenstate of that direction; measurement of position in another direction is then uncertain in the usual way for a conjugate variable. Continuous interaction of matter with null waves in two directions x_i, x_j resembles a series of such discrete measurements, with associated uncertainty, each of which takes about a Planck time. The accumulated uncertainty (the width of position wavepackets) after N measurements is

$$\Delta x_i \Delta x_j \approx \theta_{ij} N (C c t_P)^2 \tag{2}$$

where $\tau = N t_P$ can be a macroscopic time. This effect resembles the accumulation of quantum errors in atomic clocks, except that it refers to transverse spatial positions as measured by null waves. As in an atomic clock, the fractional error decreases with time, but the absolute error increases, like a random walk.

A holographic uncertainty relation for transverse position was derived previously using a wave description[12–15]. To agree with that theory, normalized by the black hole areal entropy density, we set $C = 1/2\pi$ in the numerical results below. The effect is based on null wavefront propagation so there is no physical distinction in using position or time. However, the construction here using directional operators shows more clearly that the effect is spatially coherent and inseparable from time measurement. A plane wave phase propagates nearly synchronously with other waves with the same orientation, even those separated on a macroscopic scale. The uncertainty is in definition of the spacetime frame rather than the positions of individual quantum particles, so there is a spatially coherent jitter in relative transverse phase displacement of amplitude $\approx C t_P \sqrt{N}$ on scale $N C t_P$. The range of the random jitter itself is microscopic (on the attometer scale for a laboratory-scale $N c t_P$), but is much larger than the Planck scale, and is potentially observable.

The new physics proposed here violates Lorentz invariance, but in a way that has not been previously tested to Planck precision. It can only be detected in an experiment that compares transverse positions over an extended spacetime volume to extremely high precision. The effect of the fluctuations is strongly suppressed in laboratory tests. Over time, average positions approach their classical values. The apparent fractional distortion in geometry is of order $\sqrt{t_P/\tau}$ for measurements averaged over time τ, about a factor of $\approx 10^7$ below the noise level of even the best atomic clocks. On the other hand, the required differential sensitivity in directional phase over an extended spacetime volume may be achieved by Michelson interferometry.

The optical elements and detectors of an interferometer create particular boundary conditions for the radiation field that make this effect detectable. In a simple Michelson interferometer, light propagates along two directions, say, x and y arms of length L. A single incoming wavefront is split into two noncommuting directions for a time $2L$. Light enters the apparatus prepared with a particular phase and orientation; the final signal depends on the position of the beamsplitter in two directions, at two different times separated by $2L$. When recombined the relative phases of the wavefronts have wandered apart from each other by $X \approx \sqrt{2CLct_P}$, just as if the beamsplitter had moved by this amount. The motion however is not a true motion; it is due to Planckian uncertainty in the position of matter.

For short time intervals, the x axis light can be regarded as a reference clock, equivalent to defining a frame. Relative to this phase, the y axis light experiences phase fluctuations that appear as noise in the output. For time differences τ up to $2L/c$, Eq.(2) suggests that there is noise in the phase comparison of the light from the two arms, equivalent to a variance in beamsplitter position $\sigma_X^2(\tau) = \tau t_P/2\pi$ at time lag τ:

$$\Delta y(x)\Delta y(x + c\tau) = c^2\tau t_P/2\pi. \tag{3}$$

The same result can be obtained using the y axis light as a phase reference. For larger time differences $\tau > 2L/c$, the phase does not continue to drift apart, since the wavefronts from the two directions are not actually independent, but constrained by the finite apparatus size. Phase differences at intervals $\tau > 2L/c$ represent independent samplings of a distribution about the classical position. The distribution has a variance $\sigma^2 = 2Lt_P/2\pi c$, with a mean that approaches the classical expectation value of arm length difference.

The effect is nonlocal and depends on measurement with macroscopic spacelike extent in two directions. For experiments, this nonlocality provides a powerful diagnostic technique using cross correlation. Two nearly co-located and co-aligned interferometers that share an overlapping volume of spacetime, but otherwise have no physical connection, experience common mode holographic fluctuations, since the wavefunctions of the spacetime volumes they measure must collapse into the same state. If they are offset or misaligned from each other, the cross correlation is reduced.

It seems quite strange that the positions of bodies in a given rest frame and a given direction share the same displacement, even if there is no physical connection between them. In the classical situation, with zero commutator, this coherence is of course taken for granted; everything has zero holographic displacement. The coherence is perhaps most easily understood in terms of departure from the classical behavior: the new transverse jitter in any direction only becomes apparent between paths with a significant transverse separation. If two parallel paths are much longer than the transverse separation between them, they will measure almost the same total transverse displacement when compared with a single, similarly long transverse path. The mean square displacement difference grows linearly with transverse separation. This is a consequence of Planckian random walks occuring transversely relative to light sheets, rather than a fixed laboratory rest frame. It is coherent because the amplitude of the holographic jitter grows with scale; once again, the effect is different from microscopic quantum fluctuations, which average out in a macroscopic system. Indeed, this averaging is the key to reducing quantum noise enough to allow macroscopic phase measurements in an interferometer with such precision. The coherence is needed for holographic jitter to be detectable at all; entire macroscopic optical elements of the interferometers "move" almost coherently. It is also the reason that holographic noise has escaped detection up to now, since it is harder to detect on small scales.

Let us estimate the correlation properties. Let X_A, X_B denote the apparent arm length difference in each of two interferometers A and B. The cross correlation is defined as the limiting average,

$$\Xi(\tau)_\times = \lim_{T\to\infty}(2T)^{-1}\int_{-T}^{T} dt X_A(t)X_B(t+\tau). \tag{4}$$

Eqs.(3,4) can be used to determine the cross-correlation of two interferometers, including the cases when they are displaced from each other or misaligned. Based on the above interpretation of the uncertainty, we adopt the following rule for estimating correlations. Transverse holographic displacements are the same everywhere on a null plane wavefront; thus, the differential phase perturbations in the two machines are the same when both pairs of laser wavefronts are traveling in the same direction at the same time in the lab frame. If they are displaced or misaligned the correlation is reduced by appropriate directional and overlap projection factors.

For aligned interferometers displaced by ΔL along one axis, the cross correlation of measured phase displacement (in length units) then becomes

$$\Xi_\times(\tau) = (ct_P/2\pi)(2L - 2\Delta L - c\tau), \qquad 0 < c\tau < 2L - 2\Delta L \tag{5}$$

$$= 0, \qquad c\tau > 2L - 2\Delta L. \tag{6}$$

For two interferometers with co-located beamsplitters, but misaligned by angle θ, there are two projection factors, one for the amount of time with parallel propagation in both, the other for the angular projection of the in-common component of displacement noise:

$$\Xi_\times(\tau) = (ct_P/2\pi)[2(L\cos\theta) - c\tau]\cos\theta, \qquad 0 < c\tau < 2(L\cos\theta) \tag{7}$$

$$= 0, \qquad c\tau > 2(L\cos\theta). \tag{8}$$

These formulae provide concrete predictions for experimental tests of the hypothesis (1). Assuming the theory is normalized by black hole thermodynamics, there are no parameters in the predictions.

COMPARISON WITH EXPERIMENTS

It is interesting to compare this Planckian directional position error with the best atomic clocks. Over a time τ the holographic uncertainty limit corresponds to a standard deviation of phase in orthogonal directions,

$$\frac{\Delta\nu(\tau)}{\nu} = \Delta t(\tau)/\tau = \sqrt{\frac{5.39\times 10^{-44}\mathrm{sec}}{2\pi\tau}} = 9.26\times 10^{-23}/\sqrt{\tau/\mathrm{sec}}. \tag{9}$$

For comparison, atomic clock frequency inaccuracy is currently [32] $\Delta\nu(\tau)/\nu = 2.8\times 10^{-15}/\sqrt{\tau/\mathrm{sec}}$. Thus the holographic limit is far beyond the currently practicable level of time measurements using atomic clocks. It is not possible for example to measure Planckian phase variations relative to a local time standard.

However, over short (but still macroscopic) time intervals, Planckian holographic noise in relative phase anisotropy in different directions may be detectable using interferometers. For times $\approx 2L/c$, interferometers are, in this limited differential sense, by far the most stable clocks. The sensitivities attainable by current and planned experiments are shown in Figure (1), along with the holographic noise prediction, Eq. (9).

At very low frequencies, interferometers are of limited use even as differential clocks, since they are susceptible to environmental influences. Spaceborne interferometers such as LISA, in isolated parts of the solar system, could achieve near-holographic precision at frequencies as low as a millihertz, but even then, their sensitivity is likely to be limited by ubiquitous gravitational waves.

Existing gravitational wave interferometers, such as LIGO, VIRGO, and GEO-600, have approximately the required phase sensitivity to reach the level in Eq.(9). The lower plotted experimental points are derived by taking published noise curves[33, 34] at the most sensitive frequency, and evaluating the corresponding rms arm-difference fluctuation in a single wave cycle at that frequency. In the case of LIGO, this leads to a value (labeled "gravitational waves" in Figure 1) that is actually below the holographic noise curve. The fact that as LIGO does not see excess noise at this level is an approximate bound on time-space commutation: new terms of that kind must apparently be significantly smaller than the Planck length. While this estimate is only approximate, it appears that LIGO can already impose a profound constraint on the interpretation of noncommutative geometry, even well beyond the Planck scale.

However, there is another factor that must be included to compare these experiments with the holographic prediction. Because they are designed to find gravitational waves, GEO600 and LIGO are both optimized to measure displacements at rather low frequencies, about a kilohertz and below, two orders of magnitude below the light-crossing frequency characteristic of their holographic jitter. To compare with the holographic noise prediction, we must estimate what level of jitter at frequency $c/2L$ would match the maximum instrument sensitivity at the measured sub-kilohertz frequencies. For intervals longer than L/c, the long-time average displacement approaches to its classical value; each light-crossing time represents an independent sampling of a classical position, so that the distribution of the time average position get narrower with time, instead of growing (as it would in measurements by a larger apparatus). In effect, the whole apparatus "moves together" so the measured phase is not affected by longer wavelength transverse modes. Therefore we should multiply the rms values just quoted by another factor of $\sqrt{c/2Lf_*}$, where f_* is the frequency at the minimum of the noise curve. This averaging factor makes the LIGO and GEO600 sensitivities to holographic jitter worse by factors of about 15 and 20 respectively, as shown by the upper points in Figure 1. (This factor was not included in earlier estimates of these instruments' responses, e.g. [12, 13]). When it is included, holographic noise is not expected to be a detectable contribution in the current noise budget of either experiment. The factor does not apply to LISA, which is designed to measure displacements at frequencies comparable to c/L.

It appears that current interferometer technology is adequate to detect the effect, but that a new experiment must be built to achieve a convincing detection or limit. The design should optimized to extract a holographic noise signature that would allow it to be distinguished from other noise sources at high frequencies comparable to c/L, particularly the dominant photon shot noise.

One way to isolate the holographic noise would be to cross-correlate two nearly-collocated interferometers at high frequencies. Because of their overlapping spacetime volumes, their holographic displacements are correlated (as in Eq. 5), whereas their photon shot noise is independent. With a long integration, a time-averaged holographic correlation emerges above uncorrelated photon shot noise. This is similar to the correlation technique used with LIGO at lower frequencies for isolating gravitational-wave stochastic backgrounds. (The LIGO correlations however do not themselves constrain holographic noise, because the interferometers being correlated are not co-located— indeed, they are kept separate to avoid other sources of cross correlation at low frequency.)

Assuming a photon shot noise limit, this design (labeled Holometer in Fig.1) should achieve better than Planckian sensitivity for holographic noise. For this purpose, nearly co-located interferometers must be able to record correlated signals at high frequencies, that is, $\approx c/2L \approx 3.74$ MHz$(40\mathrm{m}/L)$, and distinguish other external sources of cross

correlation at high frequencies. Such an experiment should be able to achieve better than Planckian sensitivity to transverse components of θ_{ij}.

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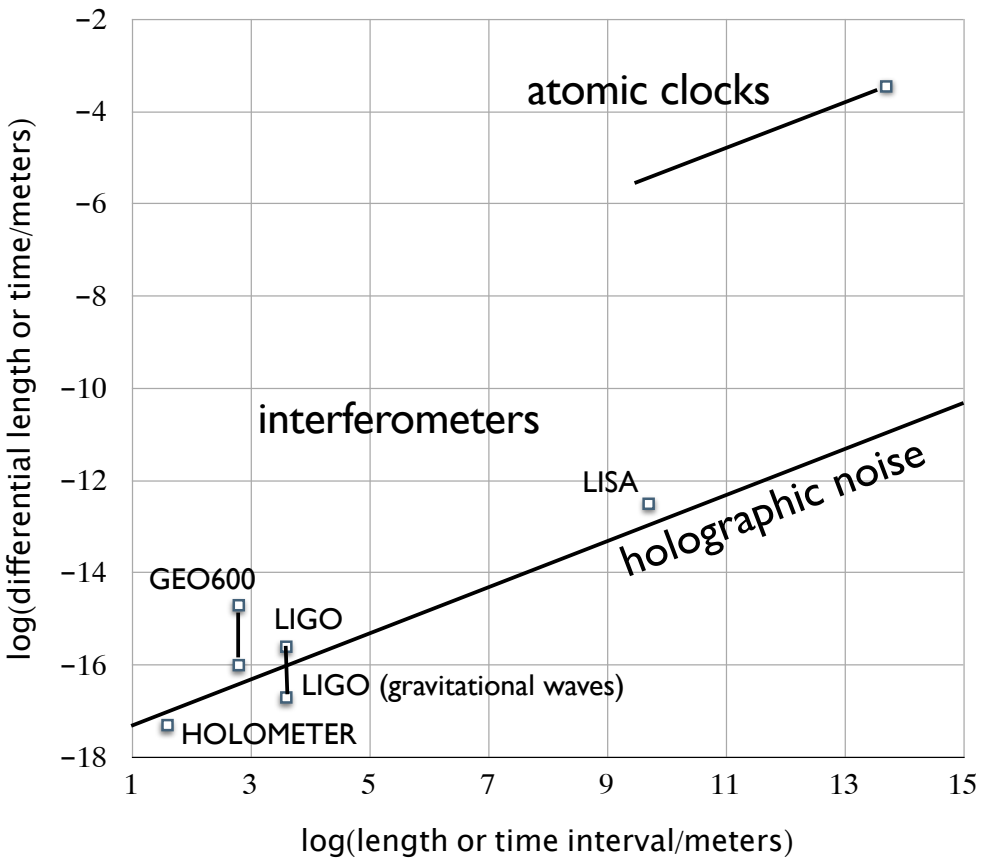

FIG. 1: Sensitivities of spacetime fluctuation experiments. Differential length or time is plotted as a function of system scale or duration, both with decimal log scales in meters. The holographic noise line shows the transverse displacement amplitude estimated in Eq.(9), as a function of time or length. Atomic clocks are shown with the currently best-measured accuracies over a range of frequencies[32]. Current (LIGO, GEO600) and planned (LISA) interferometer sensitivities show the rms sensitivity to displacement in a single period at the frequency of the minimum of the noise curve, as a function of the instrument size. In the case of LIGO and GEO600, the higher points are the ones appropriate for holographic noise comparisons; these take into account differences in apparatus response between holographic noise and gravitational waves. The point labeled Holometer shows the estimated photon-shot-noise limit for two 40-meter, correlated, co-located interferometers at 2000 watt cavity power and 1 hour integration time. Interferometers with similar parameters may detect or rule out transverse, Planckian holographic noise.