

The Decay Constants f_{D_s} and f_{D^+} from Lattice QCD

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Received Day Month Year

Revised Day Month Year

Recent calculations of the decay constants in lattice QCD are reviewed and compared to experiment.

Keywords: lattice QCD; leptonic decay constant; charm.

PACS numbers: 11.25.Hf, 123.1K

1. Charm Systems and Lattice QCD

Charm experimental programs have produced abundant data ready for interpretation by theory. Lattice QCD is the only known *ab initio* method for computing the required masses and weak matrix elements. Control of systematic effects arising in simulation is key, and precise charm data are ideal for validating lattice QCD methods. Charm takes on added importance since many of the same techniques apply to bottom where the data are less precise. A major goal of lattice QCD is to compute weak matrix elements for the “gold plated” K , D and B leptonic and semileptonic decays plus mixings. Simulation in combination with experiment determine the CKM matrix elements. Charm and bottom decays are a major part of this program. Validation checks are critical since deviations from Standard Model expectations may signal new physics. Dobrescu and Kronfeld have discussed the possibility of non standard D_s leptonic decays.¹

Lattice QCD has already led to a successful prediction of the f_{D^+} decay constant.² Precision tests continue as both lattice and experiment improve. In this review, decay constants from lattice are directly compared to the experimental value computed from the branching ratio, lifetime and an assumed value for the CKM matrix element. Comparing instead the ratio $f^+(0)/f_{D_s}$, computed with the $D_s \rightarrow K\pi\ell\nu$ form factor, eliminates $|V_{cs}|$ providing a more stringent test of leptonic and semileptonic simulations.³

Table 1. Features of the lattice simulations.

Feature	FNAL/MILC	HPQCD	ETM
gauge ensembles	MILC	MILC	ETM
sea quark flavors n_f	2 + 1	2 + 1	2
sea quark action	asqtad	asqtad	twisted-mass
valence quark action	asqtad	HISQ	twisted-mass
sea / val quark error	$\alpha_s a^2$	$\alpha_s a^2 / 0.3\alpha_s a^2$	$a^2 m_q^2$
charm quark action	clover+FNAL	HISQ	twisted-mass
charm errors	$\alpha_s a^2 \Lambda^2, a^4 \Lambda^4$	$\alpha_s a^2 m_c^2$	$a^2 m_c^2$
lattice spacings [fm]	0.09, 0.12, 0.15	0.045, 0.06, 0.09, 0.12, 0.15	0.07, 0.09, 0.10
lightest pion [MeV]	~ 224	~ 224	~ 270

2. Decay Constants in Lattice QCD

We present the decay constants from recent studies by the HPQCD Collaboration^{4,5}, the Fermilab (FNAL) Lattice and MILC collaborations⁶ and from the European Twisted Mass (ETM) Collaboration.⁷ These studies include nearly complete analyses of statistical and systematic errors and hence are of the most interest to the experimental community. In Table 1 we list important features of the studies which we discuss below.

Sea quark effects are important to reproduce QCD. Quenched or zero-flavor QCD neglects vacuum polarization and leads to uncontrolled 10 to 15% errors while three-flavor simulations better reproduce experiment.⁸

Both HPQCD and FNAL/MILC groups use the 2 + 1 flavor MILC asqtad gauge ensembles. Each MILC ensemble has two degenerate light flavors of “asqtad” quarks, with a mass ranging between about $0.1m_s$ and m_s for the various ensembles, and a single strange quark flavor. Asqtad quarks are an improved form of staggered quarks. Asqtad improvement results in leading order $\alpha_s a^2$ discretization errors where a is the lattice spacing. Asqtad quarks are numerically less expensive than most other lattice quarks. This has allowed MILC to simulate three-flavor QCD with high statistics at five lattice spacings ranging from about 0.15 fm down to 0.045 fm and multiple sea quark masses. Multiple lattice spacings and reduced discretization errors are important for reliably taking the $a \rightarrow 0$, continuum limit.

The ETM gauge ensembles have twisted-mass sea quarks. When the the quark action is properly tuned, leading errors become $o(a^2 \mu_q^2)$ where μ_q is the mass. They have studied the decay constants at the three lattice spacings in Table 1. Note that the ETM ensembles have only two degenerate flavors and no strange quark which leads to a systematic error which is difficult to estimate *a priori*. All three studies have neglected charm sea quarks. The heavier charm mass, however, motivates a perturbative bound on such effects.⁴ HPQCD estimates this error to be $\ll 0.01\%$ for f_{D_s} . Both MILC⁹ and ETM are generating four flavor gauge ensembles while ETM already have preliminary decay constants.¹⁰

Valence quark types are listed in Table 1. HPQCD use Highly Improved Staggered Quarks (HISQ) quarks. HISQ errors are only 1/3 to 1/4 as large as asqtad

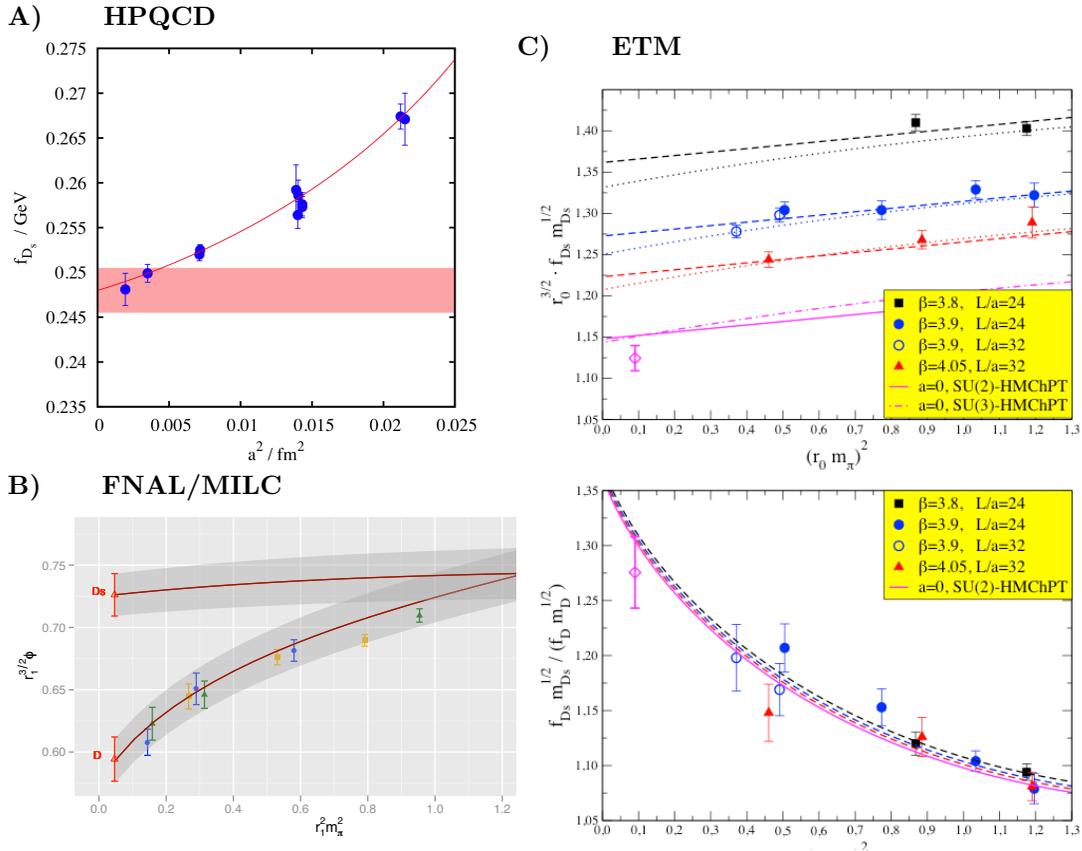


Fig. 1. Chiral and continuum extrapolations. Panel A (top left) shows the HPQCD extrapolation for f_{D_s} . Panel B (bottom left) shows the FNAL/MILC f_{D_s} (upper curve) and f_{D^+} (bottom curve) extrapolations. Panel C (two plots on the right) shows the ETM extrapolations for f_{D_s} (upper plot) and f_{D_s}/f_{D^+} (lower plot).

errors while formally both are of the same order. This added improvement comes at little extra cost. HPQCD also uses HISQ for charm where errors are $o(\alpha_s a^2 m_c^2)$ for a charm mass m_c . Twisted mass charm quarks are slightly less improved than HISQ with leading $a^2 m_c^2$ errors. FNAL/MILC use clover charm quarks with both leading $\alpha_s a^2 \Lambda^2$ and $a^4 \Lambda^4$ errors where $\Lambda \approx 0.7$ MeV.

Simulations are not (yet) performed with physical up and down quarks, hence a chiral extrapolation is necessary. Since the extrapolation combines results at finite lattice spacings, it must take into account artifacts and finite volume effects. Functional forms begin with the NLO chiral expansion for heavy-light decay constants, possibly adding (some) NNLO terms. Functions are modified to account for lattice artifacts. Control over the extrapolation is best when improved actions are used to minimize discretization errors, when results at multiple (small) lattice spacings are combined and with results close to the physical pion.

Table 2. Decay constants (in MeV) from three-flavor (HPQCD^{4,5} and FNAL/MILC⁶) and two-flavor (ETM⁷) simulations. FNAL/MILC results are preliminary.

three flavor results				two flavor results			
collaboration	f_{D_s}	f_{D^+}	f_{D_s}/f_{D^+}	collaboration	f_{D_s}	f_{D^+}	f_{D_s}/f_{D^+}
HPQCD	248.0(25)	213(4)	1.164(18)	ETM	244(8)	197(9)	1.24(3)
FNAL/MILC	261(9)	220(9)	1.19(2)				

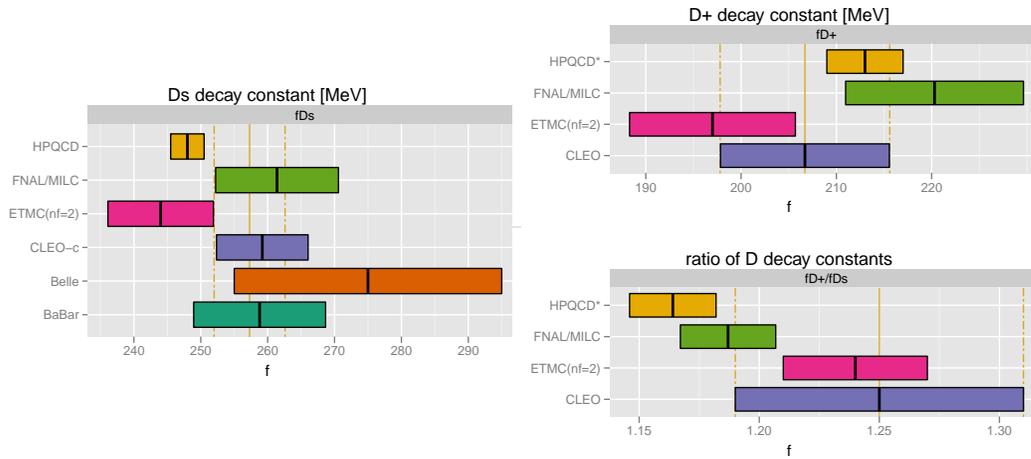


Fig. 2. Comparison of lattice results and recent experiment for the D system. Experimental values are from Ref. 11 and Ref. 12. Lattices results are from Ref. 4, Ref. 6 and Ref. 7.

Figure 1 shows the chiral extrapolations. The HPQCD f_{D_s} extrapolation (figure panel A) has results at five lattice spacings and it clearly shows the a^2 dependence. Multiple points at fixed a^2 indicate a (mild) sea quark dependence. Panel B shows the FNAL/MILC extrapolations for f_{D_s} and f_{D^+} arising from different limits of a single fit to all their points. Only a small subset of their points are visible in this figure. Panel C shows the ETM extrapolations. The upper figure compares f_{D_s} extrapolations in both $SU(2)$ and $SU(3)$ flavor theories. Extrapolations in the lower figure lead to f_{D_s}/f_{D^+} . The bulk of many systematic effects are expected to cancel in the ratio.

3. Summary and Comparison with Experiment

The decay constants are tabulated in Table 2 and plotted in Figure 2. The most precise f_{D_s} value is from HPQCD. It is about 2σ higher than their previous result. The change is due to a more precise determination of the lattice spacing and better tuning of the quark masses. They have updated f_{D^+} using the new f_{D_s} and their older f_{D_s}/f_{D^+} ratio which is expected to be less sensitive to mistuning of the lattice spacing and masses. The preliminary FNAL/MILC f_{D_s} value is about 1.4σ higher than the HPQCD result but with a larger error. The f_{D^+} values, however, are in

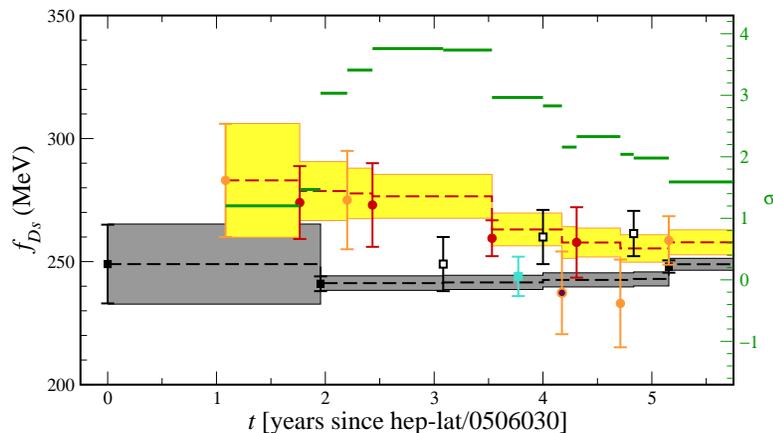


Fig. 3. A brief history of f_{D_s} from the lattice and from experiment.¹³

better agreement. FNAL/MILC expect to finalize their results once the charm quark mass tuning is complete. The two flavor ETM f_{D^+} value is about 1.6σ lower than the HPQCD value while f_{D_s} is in better agreement. It is not clear how much of the difference is from neglecting the strange sea quark, given the errors. Lattice and experiment differ most significantly for f_{D_s} . Figure 3 shows Kronfeld’s (updated) history of f_{D_s} .¹³ The yellow bands depict the evolution of the experimental average while the three-flavor lattice average is shown in grey. The right-hand scale and green lines show the differences in sigmas. The 3.8σ discrepancy around $t \approx 2$ provoked the “ f_{D_s} puzzle”. That discrepancy has now shrunk to 1.6σ . Future lattice and experiment will be decisive.

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