FERMILAB-PUB-09-868-T

Non-universal gaugino masses from non-singlet F-terms in non-minimal unified models

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arXiv:0903.3568v2 [hep-ph] 22 Mar 2009

In phenomenological studies of low-energy supersymmetry, running gaugino masses are often taken to be equal near the scale of apparent gauge coupling unification. However, many known mechanisms can avoid this universality, even in models with unified gauge interactions. One example is an F-term vacuum expectation value that is a singlet under the Standard Model gauge group but transforms non-trivially in the symmetric product of two adjoint representations of a group that contains the Standard Model gauge group. Here, I compute the ratios of gaugino masses that follow from F-terms in non-singlet representations of $SO(10)$ and E_6 and their subgroups, extending well-known results for $SU(5)$. The $SO(10)$ results correct some long-standing errors in the literature.

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I. INTRODUCTION

The phenomenology of the Minimal Supersymmetric Standard Model (MSSM) [1] has most often been discussed within the confines of the minimal supergravity framework in which gaugino masses and scalar squared masses are assumed to be unified as running parameters at a very high renormalization scale. This scale is usually taken to be near the apparent unification of the gauge couplings at about 2×10^{16} GeV. However, it is well-known that qualitatively different types of models can arise if one abandons one or both of the gaugino mass or scalar mass unification assumptions. The dominant production and decay processes at colliders can be quite different and new mechanisms for obtaining the observed amount of cold dark matter become available in large regions of parameter space. Moreover, such models can be motivated by the fact that the supersymmetric "little hierarchy" problem can be ameliorated if the gluino mass parameter is significantly smaller than the wino mass parameter at the unification scale [2]-[4].

This paper is concerned with a mechanism for obtaining gaugino mass non-unification that can work even if the gauge couplings unify into a simple gauge group like $SU(5)$, $SO(10)$, or E_6. In general, gaugino masses in supergravity can arise from non-renormalizable dimension-5 operators involving F-terms that get a vacuum expectation value (VEV):

$$\mathcal{L} \;=\; -\frac{F^{ab}}{2M_{\rm Planck}}\lambda^a\lambda^b + \text{c.c.} \tag{1.1}$$

Here λ^a is the two-component field for the gaugino, and the indices a, b run over the adjoint representation of the gauge group. The resulting gaugino mass matrix is $\langle F^{ab}\rangle/M_{\rm Planck}$. The supersymmetry-breaking order parameter $\langle F^{ab}\rangle$ must transform as a singlet under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. In general, it can take independent values for the gluino, wino and bino, the gauginos corresponding to the three subgroups. However, one can make non-trivial predictions [5]-[8] in any model in which the Standard Model gauge group is embedded in a larger symmetry group, like a supersymmetric Grand Unified Theory (GUT) [9]. A non-zero contribution to the gaugino masses can arise from F^{ab} fields transforming in a representation in the symmetric part of the direct product of the adjoint representation of the unified group with itself.

The simplest example of this mechanism is the case of a GUT model based on $SU(5)$ [10]. There, one has[†]

$$(\mathbf{24} \times \mathbf{24})_S = \mathbf{1} + \mathbf{24} + \mathbf{75} + \mathbf{200}. \tag{1.2}$$

Each of the four irreducible representations on the right contains exactly one Standard Model singlet, so that each makes a unique prediction for the pattern of contributions to the bino, wino, and gluino mass parameters M_1, M_2, and M_3 at the breaking scale. The overall

[†] All group theory facts and notations followed in this paper can be found in the review article [11].

$SU(5)$	$M_1 : M_2 : M_3$
1	$1 : 1 : 1$
24	$-\frac{1}{2} : -\frac{3}{2} : 1$
75	$-5 : 3 : 1$
200	$10 : 2 : 1$

TABLE I: Ratios of gaugino masses for F-terms in representations of $SU(5)$, obtained in refs. [6],[8].

scale of each F-term VEV is an arbitrary input parameter, but the ratios of contributions to M_1, M_2, and M_3 are rational numbers fixed by the group theory. They were obtained in refs. [6],[8], and are listed in Table I. Only the $SU(5)$ singlet representation predicts universal gaugino masses. Depending on the model, one might suppose that one of the other three representations dominates the contribution to gaugino masses, or that two or more of the representations contribute in comparable amounts. The resulting phenomenology has been studied in many papers; for examples, see [12]-[31].

The purpose of this paper is to extend these results to the case of unified groups $SO(10)$ and E_6, and all of their proper subgroups that embed $SU(3)_C \times SU(2)_L \times U(1)_Y$ in a way consistent with the Standard Model chiral fermion content. In each case, the F-terms can be classified by their transformation properties under both the full symmetry group and under subgroups that can be used to distinguish different Standard Model singlets. Then the object is to find the ratio of gaugino masses that can be produced by each distinct F-term representation. It is important to note that the subgroups are used only for distinguishing representations; they need not be the unbroken symmetry group for an effective theory at any scale. It is also possible that the full gauge symmetry is only a subgroup of the unified groups $SU(5)$, $SO(10)$ and E_6 in which the Standard Model gauge group can be embedded.

The method used to obtain the results below is as follows. For any given unified symmetry group, one starts with a field transforming as the symmetric product of the adjoint representation with itself, $\Phi^{ab} = \langle F^{ab} \rangle / M_{\rm Planck}$. Under a gauge transformation corresponding to a generator labeled by c, the VEV transforms by an amount proportional to:

$$\delta_c \Phi^{ab} = (t^c)^{aa'} \Phi^{a'b} + (t^c)^{bb'} \Phi^{ab'}, \tag{1.3}$$

where the adjoint representation generators are $(t^a)^{bc} = -i f^{abc}$, and f^{abc} are the structure constants of the Lie algebra. Since Φ^{ab} is required to be a Standard Model singlet, one can require that $\delta_c \Phi^{ab} = 0$ for each of the 12 generators $c = 1, 2, \ldots, 12$ of $SU(3)_C \times SU(2)_L \times U(1)_Y$. This reduces Φ^{ab} from 300 independent entries to only 4 for $SU(5)$, from 1035 entries to 9 for $SO(10)$, and from 3081 entries to 32 for E_6. This identifies the subspace of VEVs that are Standard Model singlets. Now, to decompose Φ^{ab} into irreducible representations of the full symmetry group, one can use the quadratic Casimir operator:

$$C^{(ab),(a'b')} \Phi^{a'b'} \equiv \left[(t^c t^c)^{aa'} \delta^{bb'} + \delta^{aa'} (t^c t^c)^{bb'} + 2 (t^c)^{aa'} (t^c)^{bb'} \right] \Phi^{a'b'}. \tag{1.4}$$

The matrix C, realized for $SU(5)$, $SO(10)$, and E_6 respectively as a 4×4, 9×9, and 32×32 dimensional matrix when restricted to the subspace of Standard Model singlets, has eigenvalues that are the quadratic Casimir invariants of the corresponding irreducible representations, and the corresponding eigenvectors are the Standard Model singlet members of the representations themselves. By restricting the implied sum over the index c in eq. (1.4) to subsets of the full list of generators, one can likewise identify the quadratic Casimir invariant eigenvalues and thus the irreducible representations for any desired subgroup. It is an important and fortunate fact that the quadratic Casimir invariant eigenvalues are in one-to-one correspondence with the irreducible representations that appear in all cases encountered below, so that ambiguities do not arise. One thus finds matrices Φ^{ab} that correspond to Standard Model singlets within any desired specific irreducible representations of the larger symmetry groups. These matrices Φ^{ab} are diagonal when the indices a, b are restricted to the subspace of $SU(3)_C \times SU(2)_L \times U(1)_Y$ generators, with eigenvalues proportional to the corresponding gaugino masses M_3, M_2, and M_1.

I use the following notation for left-handed chiral fermion fields as defined by their transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned}
&Q \sim (\mathbf{3}, \mathbf{2}, 1/6), \qquad \overline{u} \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3), \qquad \overline{d} \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3),\\
&L \sim (\mathbf{1}, \mathbf{2}, -1/2), \qquad \overline{e} \sim (\mathbf{1}, \mathbf{1}, 1), \qquad \overline{\nu} \sim (\mathbf{1}, \mathbf{1}, 0),\\
&H_u \sim (\mathbf{1}, \mathbf{2}, 1/2), \qquad H_d \sim (\mathbf{1}, \mathbf{2}, -1/2),\\
&h \sim (\mathbf{3}, \mathbf{1}, -1/3), \qquad \overline{h} \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3), \qquad N \sim (\mathbf{1}, \mathbf{1}, 0).
\end{aligned} \tag{1.5}$$

Here, $Q, \overline{u}, \overline{d}, L, \overline{e}$ are the notations for the Standard Model chiral fermions, and $\overline{\nu}$ for the additional Standard Model singlet found in the **16** of $SO(10)$. The remaining fields H_u, H_d, h, $\overline{h}$, N are the additional left-handed fermions found in the **27** of E_6.

II. RESULTS FOR $SO(10)$ AND ITS SUBGROUPS

In this section, I consider the unified group $SO(10)$ [32], for which the adjoint representation is the **45**. The possible $SO(10)$ irreducible representations for the F-term are:

$$(\mathbf{45} \times \mathbf{45})_S = \mathbf{1} + \mathbf{54} + \mathbf{210} + \mathbf{770}. \tag{2.1}$$

The **1** and **54** each contain exactly one Standard Model singlet, but the **210** contains three and the **770** contains four. To uniquely distinguish the possible F-terms, one can additionally specify their transformation properties under a proper subgroup. It turns out that choosing any one of the maximal proper subgroups is sufficient to uniquely distinguish the possible F-terms in the $SO(10)$ case.

First, consider the "normal" embedding of $SU(5) \times U(1) \subset SO(10)$, with the fermions in a **16** of $SO(10)$ transforming under $SU(5)$ as $L, \overline{d} \sim \overline{\mathbf{5}}$ and $Q, \overline{u}, \overline{e} \sim \mathbf{10}$ and $\overline{\nu} \sim \mathbf{1}$, as in the original Georgi-Glashow $SU(5)$ GUT model [10]. The ratios of gaugino masses obtained

$SO(10)$	$SU(5)$	$M_1 : M_2 : M_3$
1	**1**	$1 : 1 : 1$
54	**24**	$-\frac{1}{2} : -\frac{3}{2} : 1$
210	**1**	$1 : 1 : 1$
	24	$-\frac{1}{2} : -\frac{3}{2} : 1$
	75	$-5 : 3 : 1$
770	**1**	$1 : 1 : 1$
	24	$-\frac{1}{2} : -\frac{3}{2} : 1$
	75	$-5 : 3 : 1$
	200	$10 : 2 : 1$

TABLE II: Ratios of gaugino masses for F-terms in representations of $SU(5) \subset SO(10)$, with the normal (non-flipped) embedding.

$SO(10)$	$[SU(5)' \times U(1)]_{\text{flipped}}$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, 0)$	$1 : 1 : 1$
54	$(\mathbf{24}, 0)$	$-\frac{1}{2} : -\frac{3}{2} : 1$
210	$(\mathbf{1}, 0)$	$-\frac{19}{5} : 1 : 1$
	$(\mathbf{24}, 0)$	$\frac{7}{10} : -\frac{3}{2} : 1$
	$(\mathbf{75}, 0)$	$-\frac{1}{5} : 3 : 1$
770	$(\mathbf{1}, 0)$	$\frac{77}{5} : 1 : 1$
	$(\mathbf{24}, 0)$	$-\frac{101}{10} : -\frac{3}{2} : 1$
	$(\mathbf{75}, 0)$	$-\frac{1}{5} : 3 : 1$
	$(\mathbf{200}, 0)$	$\frac{2}{5} : 2 : 1$

TABLE III: Ratios of gaugino masses for F-terms in representations of flipped $SU(5)' \times U(1) \subset SO(10)$.

for F terms in the various representations of $SU(5) \subset SO(10)$ are shown in Table II. Note that these results merely agree with those already listed for $SU(5)$ in Table I; this follows from the fact that the Standard Model generators are entirely embedded within the $SU(5)$ simple subgroup. In several similar situations below, where the gaugino mass ratios follow trivially from the results already obtained from a subgroup, I will just note this fact rather than record the results in tabular form.

The "flipped" embedding [33] of $SU(5)' \times U(1) \subset SO(10)$ yields different results, because the $U(1)_Y$ generator is a linear combination of the $U(1)$ generator inside $SU(5)'$ that commutes with $SU(3)_C \times SU(2)_L$ and the $U(1)$ generator outside of $SU(5)'$. In this case, the fermions in a **16** of $SO(10)$ transform under $SU(5)' \times U(1)$ as $L, \overline{u} \sim (\overline{\mathbf{5}}, 3)$ and $Q, \overline{d}, \overline{\nu} \sim (\mathbf{10}, -1)$ and $\overline{e} \sim (\mathbf{1}, -5)$. The results for gaugino mass ratios are shown in Table III. Note that the results for the ratio M_2/M_3 are the same as for the normal $SU(5)$ embedding, but the M_1/M_2 and M_1/M_3 ratios are different.

In some models, it is more useful to distinguish the possible F-terms by their transfor-

$SO(10)$	$SU(4)\times SU(2)_R$	$M_1 : M_2 : M_3$
1	$(\mathbf{1},\mathbf{1})$	$1:1:1$
54	$(\mathbf{1},\mathbf{1})$	$-\frac{1}{2}:-\frac{3}{2}:1$
210	$(\mathbf{1},\mathbf{1})$	$-\frac{3}{5}:1:0$
	$(\mathbf{15},\mathbf{1})$	$-\frac{4}{5}:0:1$
	$(\mathbf{15},\mathbf{3})$	$1:0:0$
770	$(\mathbf{1},\mathbf{1})$	$\frac{19}{10}:\frac{5}{2}:1$
	$(\mathbf{1},\mathbf{5})$	$1:0:0$
	$(\mathbf{15},\mathbf{3})$	$1:0:0$
	$(\mathbf{84},\mathbf{1})$	$\frac{32}{5}:0:1$

TABLE IV: Ratios of gaugino masses for F-terms in representations of $SU(4)\times SU(2)_L\times SU(2)_R\subset SO(10)$.

mations under the maximal proper subgroup $SU(4)\times SU(2)_L\times SU(2)_R$, where the $SU(4)$ is the Pati-Salam unification [34] of color and $B-L$. Here the fermions in a **16** of $SO(10)$ transform under $SU(4)\times SU(2)_L\times SU(2)_R$ as $Q,L\sim(\mathbf{4},\mathbf{2},\mathbf{1})$ and $\overline{u},\overline{d},\overline{\nu},\overline{e}\sim(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$. The results for gaugino mass ratios for F-terms classified by representations within this subgroup are shown in Table IV.

The above is a complete description of the situation for $SO(10)$, since the three maximal proper subgroup embeddings listed above are the only ones compatible with the Standard Model fermion assignments, and the resulting gaugino mass ratios are completely determined in each case. Classification of F-terms in representations of smaller subgroups of $SO(10)$ are obtained simply as special cases of the above results.

Results equivalent to those of lines 2, 3, and 6 of Table IV in the present paper have already been given (presented in the $SU(4)\times SU(2)_L\times SU(2)_R$ basis for gaugino masses rather than the MSSM basis $SU(3)_C\times SU(2)_L\times U(1)_Y$ basis M_3,M_2,M_1 used here) in ref. [35], see Table 3 and eq. (31).

The results given in this section do not agree with those reported in ref. [36], which have been used as the basis for several studies in the literature. In ref. [36], it was claimed that an F-term in the **54** representation with a breaking chain including $SU(4)\times SU(2)_L\times SU(2)_R$ would give $M_1:M_2:M_3=-1:-3/2:1$. However, there is only one singlet within the **54** of $SU(10)$ that can give masses to the MSSM gauginos, and that singlet also resides within the **24** of the Georgi-Glashow $SU(5)$. Therefore, as shown in Tables I-IV above, it must give gaugino mass ratios that agree with the $SU(5)$ **24** case, regardless of the VEV classification scheme or the breaking pattern. Ref. [36] also gives a result for breaking through $SU(2)\times SO(7)$, but this is impossible to reconcile with the chiral fermion assignments of the Standard Model, since $SO(7)$ spinor representations are real. Finally, it is claimed in ref. [36] that for the **210** of $SO(10)$ with the breaking chain through flipped $SU(5)'\times U(1)$, the gaugino mass ratios are $M_1:M_2:M_3=-96/25:1:1$, but this should actually be $-19/5:1:1$, as seen in the third line of Table III above.

III. RESULTS FOR E_6 AND ITS SUBGROUPS

In this section, I consider the unified group E_6 [37], for which the adjoint representation is the **78**. The possible E_6 irreducible representations for the F-term are:

$$(\mathbf{78} \times \mathbf{78})_S = \mathbf{1} + \mathbf{650} + \mathbf{2430}. \tag{3.1}$$

The **1**, the **650**, and the **2430** contain respectively 1, 11, and 20 Standard Model singlets. To uniquely distinguish the possible F-terms, one can specify their transformation properties under various subgroups, as in the $SO(10)$ case. Many of these possible F-term representations are not capable of giving any masses to MSSM gauginos; they can be recognized as those that are charged under any $U(1)$ subgroup of E_6, or transform non-trivially under the subgroup $SU(2)_X$ under which (L, H_d) and $(\overline{d}, \overline{h})$ and $(\overline{\nu}, N)$ are doublets. Representations that do not contribute to MSSM gaugino masses are omitted from the tables below.

First, consider the "normal" embeddings of $SO(10) \subset E_6$, with the chiral fields transforming as $L, \overline{d}, Q, \overline{u}, \overline{e}, \overline{\nu} \sim \mathbf{16}$ and $H_d, \overline{h}, H_u, h \sim \mathbf{10}$ and $N \sim \mathbf{1}$ under $SO(10)$. Then there are three possible distinct embeddings of the chiral fields into maximal proper subgroups of $SO(10)$, corresponding to the cases discussed in the previous section: normal $SU(5)$, flipped $SU(5)' \times U(1)$, and $SU(4) \times SU(2)_L \times SU(2)_R$. To find the ratios of gaugino masses, it is only necessary to know how the relevant representations within E_6 break down into $SO(10)$ representations:

$$\mathbf{1} \rightarrow \mathbf{1}, \tag{3.2}$$

$$\mathbf{650} \rightarrow \mathbf{1} + \mathbf{54} + \mathbf{210} + \left(\mathbf{10} + \mathbf{10} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{45} + \mathbf{144} + \overline{\mathbf{144}}\right), \tag{3.3}$$

$$\mathbf{2430} \rightarrow \mathbf{1} + \mathbf{210} + \mathbf{770} + \left(\mathbf{16} + \overline{\mathbf{16}} + \mathbf{45} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{560} + \overline{\mathbf{560}}\right). \tag{3.4}$$

The representations enclosed in parentheses cannot contribute to MSSM gaugino masses in this embedding, even though all except the **10**'s contain at least one Standard Model singlet. For the other representations of $SO(10)$, namely the **1**, **54**, **210**, and **770**, the corresponding gaugino mass ratio contributions simply follow immediately from the results of Tables II, III, and IV, in a way just analogous to how Table II follows from Table I.

Next, consider the "flipped $SO(10)$" embedding [38] within E_6, for which the chiral fields transform under $SO(10)' \times U(1)$ as $Q, \overline{h}, N, H_u, \overline{d}, \overline{\nu} \sim (\mathbf{16}, 1)$ and $h, L, \overline{u}, H_d \sim (\mathbf{10}, -2)$ and $\bar{e} \sim (\mathbf{1}, 4)$. This does not reduce to the results of the preceding section, because the Standard Model $U(1)_Y$ generator is not contained within the $SO(10)'$ subgroup. There are two distinct subcases, in which the F-terms are classified by their representations under a maximal proper subgroup of $SO(10)'$, either $SU(5)'' \times U(1)'$ or $SU(4)' \times SU(2)_L \times SU(2)_X$.

In the first subcase of flipped $SO(10)$, the chiral fields transform under $SU(5)'' \times U(1)'$ as $Q, \overline{h}, N \sim (\mathbf{10}, -1)$ and $H_u, \overline{d} \sim (\overline{\mathbf{5}}, 3)$ and $h, L \sim (\mathbf{5}, 2)$ and $\overline{u}, H_d \sim (\overline{\mathbf{5}}, -2)$ and $\overline{e} \sim (\mathbf{1}, -5)$. The results for the gaugino mass ratios for F-term representations classified by this subgroup are shown in Table V.

In the second subcase of flipped $SO(10)$, the chiral fields transform under $SU(4)' \times$

E_6	$[SO(10)' \times U(1)]_{\rm flipped}$	$SU(5)''$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, 0)$	**1**	$1 : 1 : 1$
650	$(\mathbf{1}, 0)$	**1**	$-\frac{22}{5} : 1 : 1$
	$(\mathbf{45}, 0)$	**1**	$1 : 0 : 0$
		24	$1 : 0 : 0$
	$(\mathbf{54}, 0)$	**24**	$\frac{1}{10} : -\frac{3}{2} : 1$
	$(\mathbf{210}, 0)$	**1**	$-\frac{1}{5} : 1 : 1$
		24	$-\frac{1}{5} : -\frac{3}{2} : 1$
		75	$-\frac{1}{5} : 3 : 1$
2430	$(\mathbf{1}, 0)$	**1**	$\frac{122}{5} : 1 : 1$
	$(\mathbf{45}, 0)$	**1**	$1 : 0 : 0$
		24	$1 : 0 : 0$
	$(\mathbf{210}, 0)$	**1**	$-\frac{1}{5} : 1 : 1$
		24	$-\frac{1}{5} : -\frac{3}{2} : 1$
		75	$-\frac{1}{5} : 3 : 1$
	$(\mathbf{770}, 0)$	**1**	$1 : 1 : 1$
		24	$\frac{5}{2} : -\frac{3}{2} : 1$
		75	$-\frac{1}{5} : 3 : 1$
		200	$\frac{2}{5} : 2 : 1$

TABLE V: Ratios of gaugino masses for F-terms in representations of $SU(5)'' \times U(1)' \times U(1) \subset [SO(10)' \times U(1)]_{\rm flipped} \subset E_6$.

$SU(2)_L \times SU(2)_X \subset SO(10)$ as $Q, H_u \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $\overline{d}, \overline{\nu}, \overline{h}, N \sim (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $\overline{u}, h \sim (\mathbf{6}, \mathbf{1}, \mathbf{1})$ and $L, H_d \sim (\mathbf{1}, \mathbf{2}, \mathbf{2})$ and $\overline{e} \sim (\mathbf{1}, \mathbf{1}, \mathbf{1})$. The results for the gaugino mass ratios for F-term representations classified by this subgroup are shown in Table VI.

Another useful way to classify the possible F-terms is through representations of the trinification [39] subgroup of E_6, $SU(3)_C \times SU(3)_L \times SU(3)_R$. The chiral fields transform under this group as $Q, h \sim (\mathbf{3}, \mathbf{3}, \mathbf{1})$ and $\overline{d}, \overline{u}, \overline{h} \sim (\overline{\mathbf{3}}, \mathbf{1}, \overline{\mathbf{3}})$ and $L, H_u, H_d, \overline{e}, \overline{\nu}, N \sim (\mathbf{1}, \overline{\mathbf{3}}, \mathbf{3})$. The results for the ratios of gaugino masses are given in Table VII. In this classification scheme, there are two distinct singlets of $SU(3)_C \times SU(3)_L \times SU(3)_R$ within the **650** of E_6, which cannot be distinguished by their transformations under this subgroup. The results given in the table represent an arbitrary choice of basis for these two representations, denoted by subscripts 1 and 2. The results of lines 2, 3, and 7 of Table VII are equivalent to those obtained previously in [35] (see Table 4 and eq. (32); this reference presents the gaugino mass contributions in the $SU(3)_C \times SU(3)_L \times SU(3)_R$ basis rather than the MSSM $SU(3)_C \times SU(2)_L \times U(1)_Y$ basis M_3, M_2, M_1 used here).

Finally, one can classify F-term representations by their transformations under the maximal proper subgroup $SU(6) \times SU(2) \subset E_6$. There are three distinct possible embeddings of the chiral fields within this group, which can be treated as subcases.

In the first subcase, one classifies possible F-term representations by $SU(6) \times SU(2)_X$,

E_6	$[SO(10)' \times U(1)]_{\text{flipped}}$	$SU(4)'$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, 0)$	**1**	$1 : 1 : 1$
650	$(\mathbf{1}, 0)$	**1**	$-\frac{22}{5} : 1 : 1$
	$(\mathbf{45}, 0)$	**15**	$1 : 0 : 0$
	$(\mathbf{54}, 0)$	**1**	$\frac{1}{10} : -\frac{3}{2} : 1$
	$(\mathbf{210}, 0)$	**1**	$0 : 1 : 0$
		15	$-\frac{1}{5} : 0 : 1$
2430	$(\mathbf{1}, 0)$	**1**	$\frac{122}{5} : 1 : 1$
	$(\mathbf{45}, 0)$	**15**	$1 : 0 : 0$
	$(\mathbf{210}, 0)$	**1**	$0 : 1 : 0$
		15	$-\frac{1}{5} : 0 : 1$
	$(\mathbf{770}, 0)$	**1**	$\frac{1}{10} : \frac{5}{2} : 1$
		84	$\frac{8}{5} : 0 : 1$

TABLE VI: Ratios of gaugino masses for F-terms in representations of $SU(4)' \times SU(2)_L \times SU(2)_X \times U(1) \subset [SO(10)' \times U(1)]_{\text{flipped}} \subset E_6$. These F terms are all singlets under $SU(2)_X$.

E_6	$SU(3)_L \times SU(3)_R$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, \mathbf{1})$	$1 : 1 : 1$
650	$(\mathbf{1}, \mathbf{1})_1$	$-\frac{3}{5} : 1 : 0$
	$(\mathbf{1}, \mathbf{1})_2$	$-\frac{4}{5} : 0 : 1$
	$(\mathbf{1}, \mathbf{8})$	$1 : 0 : 0$
	$(\mathbf{8}, \mathbf{1})$	$-\frac{1}{5} : 1 : 0$
	$(\mathbf{8}, \mathbf{8})$	$1 : 0 : 0$
2430	$(\mathbf{1}, \mathbf{1})$	$1 : 1 : 1$
	$(\mathbf{1}, \mathbf{8})$	$1 : 0 : 0$
	$(\mathbf{8}, \mathbf{1})$	$-\frac{1}{5} : 1 : 0$
	$(\mathbf{8}, \mathbf{8})$	$1 : 0 : 0$
	$(\mathbf{1}, \mathbf{27})$	$1 : 0 : 0$
	$(\mathbf{27}, \mathbf{1})$	$\frac{9}{5} : 1 : 0$

TABLE VII: Ratios of gaugino masses for F-terms in representations of the trinification subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R \subset E_6$.

where the chiral fields are assigned to representations $\overline{d}, \overline{h}, \overline{\nu}, N, L, H_d \sim (\overline{\mathbf{6}}, \mathbf{2})$ and $Q, H_u, h, \overline{u}, \overline{e} \sim (\mathbf{15}, \mathbf{1})$. To uniquely distinguish the possible F-terms, it is sufficient to also give the representation under the $SU(3)_L$ subgroup of $SU(6)$ that commutes with $SU(3)_C$. This $SU(3)_L$ contains the weak isospin group $SU(2)_L$. The results for the ratios of gaugino masses are given in Table VIII. As noted earlier, the only representations that can give non-zero gaugino masses are singlets under $SU(2)_X$.

In the second subcase, one classifies F-terms by their representations under $SU(6)' \times$

E_6	$SU(6) \times SU(2)_X$	$SU(3)_L$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : 1 : 1$
650	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : 1 : 1$
	$(\mathbf{35}, \mathbf{1})$	**1**	$-\frac{1}{5} : -1 : 1$
		8	$\frac{3}{5} : 1 : 0$
	$(\mathbf{189}, \mathbf{1})$	**1**	$-3 : 1 : 1$
		8	$-1 : 1 : 0$
2430	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : 1 : 1$
	$(\mathbf{189}, \mathbf{1})$	**1**	$-3 : 1 : 1$
		8	$-1 : 1 : 0$
	$(\mathbf{405}, \mathbf{1})$	**1**	$\frac{33}{5} : 1 : 1$
		8	$\frac{19}{5} : 1 : 0$
		27	$\frac{9}{5} : 1 : 0$

TABLE VIII: Ratios of gaugino masses for F-terms in representations of $SU(3)_C \times SU(3)_L \times U(1) \times SU(2)_X \subset SU(6) \times SU(2)_X \subset E_6$.

$SU(2)_R$, where the chiral fields are assigned to representations $\overline{u}, \overline{d}, \overline{e}, \overline{\nu}, H_u, H_d \sim (\overline{\mathbf{6}}, \mathbf{2})$ and $Q, L, h, \overline{h}, N \sim (\mathbf{15}, \mathbf{1})$. To uniquely distinguish the different possible F-terms, it is again sufficient to also give the representation under the $SU(3)_L$ subgroup of $SU(6)'$ that commutes with $SU(3)_C$. The results for the ratios of gaugino masses are given in Table IX.

The last subcase identifies the $SU(2)$ with the weak isospin of the Standard Model, so that the chiral fields transform under $SU(6)'' \times SU(2)_L$ as $Q, L, H_u, H_d \sim (\mathbf{6}, \mathbf{2})$ and $\overline{u}, \overline{d}, \overline{\nu}, \overline{e}, h, \overline{h}, N \sim (\overline{\mathbf{15}}, \mathbf{1})$. To uniquely distinguish the different possible F-terms in this case, it is sufficient to also give the representation under the $SU(3)_R$ subgroup of $SU(6)''$ that commutes with $SU(3)_C$. The results for the ratios of gaugino masses are given in Table X.

This concludes the discussion of E_6, since these are the only maximal proper subgroup embeddings that are consistent with the Standard Model fermion content. Here, we do not count as distinct any embeddings that differ from the above ones by exchanging the identities of $(H_d,\ L)$ or $(\overline{\nu}, N)$ or $(\overline{d}, \overline{h})$, since these pairs of fields have the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers. Such embeddings will give the same results for gaugino mass ratios as the ones listed above.

E_6	$SU(6)' \times SU(2)_R$	$SU(3)_L$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : 1 : 1$
650	$(\mathbf{1}, \mathbf{1})$	**1**	$-\frac{13}{5} : 1 : 1$
	$(\mathbf{35}, \mathbf{1})$	**1**	$-\frac{1}{5} : -1 : 1$
		8	$-\frac{3}{5} : 1 : 0$
	$(\mathbf{35}, \mathbf{3})$	**1**	$1 : 0 : 0$
		8	$1 : 0 : 0$
	$(\mathbf{189}, \mathbf{1})$	**1**	$-\frac{3}{5} : 1 : 1$
		8	$\frac{1}{5} : 1 : 0$
2430	$(\mathbf{1}, \mathbf{1})$	**1**	$\frac{41}{15} : 1 : 1$
	$(\mathbf{1}, \mathbf{5})$	**1**	$1 : 0 : 0$
	$(\mathbf{35}, \mathbf{3})$	**1**	$1 : 0 : 0$
		8	$1 : 0 : 0$
	$(\mathbf{189}, \mathbf{1})$	**1**	$-\frac{3}{5} : 1 : 1$
		8	$\frac{1}{5} : 1 : 0$
	$(\mathbf{405}, \mathbf{1})$	**1**	$\frac{9}{5} : 1 : 1$
		8	$-\frac{11}{5} : 1 : 0$
		27	$\frac{9}{5} : 1 : 0$

TABLE IX: Ratios of gaugino masses for F-terms in representations of $SU(3)_C \times SU(3)_L \times U(1) \times SU(2)_R \subset SU(6)' \times SU(2)_R \subset E_6$.

E_6	$SU(6)'' \times SU(2)_L$	$SU(3)_R$	$M_1 : M_2 : M_3$
1	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : 1 : 1$
650	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : -5 : 1$
	$(\mathbf{35}, \mathbf{1})$	**1**	$-\frac{4}{5} : 0 : 1$
	$(\mathbf{189}, \mathbf{1})$	**1**	$0 : 0 : 1$
		8	$1 : 0 : 0$
2430	$(\mathbf{1}, \mathbf{1})$	**1**	$1 : \frac{35}{9} : 1$
	$(\mathbf{189}, \mathbf{1})$	**1**	$0 : 0 : 1$
		8	$1 : 0 : 0$
	$(\mathbf{405}, \mathbf{1})$	**1**	$\frac{12}{5} : 0 : 1$
		8	$1 : 0 : 0$
		27	$1 : 0 : 0$

TABLE X: Ratios of gaugino masses for F-terms in representations of $SU(3)_C \times SU(3)_R \times U(1) \times SU(2)_L \subset SU(6)'' \times SU(2)_L \subset E_6$.

IV. CONCLUDING REMARKS

In this paper, I have derived the ratios of gaugino masses that follow from non-singlet F-term VEVs in various representations of the unified groups $SO(10)$ and E_6, and their subgroups, in which the Standard Model gauge group can be embedded. One interesting facet of these results is that in models with $SO(10)$ symmetry, if one views the most likely large deviation from universality as coming from the smallest non-singlet representation, namely the **54**, then the contribution to the gaugino mass ratios is necessarily the same as that found from the **24** of $SU(5)$, which is distinguished by the same criterion. However, clearly any desired deviation from universality can be achieved by taking an F-term VEV in an appropriate linear combination of representations, in a variety of ways.

Some of the results found here (lines 2, 3, and 6 of Table IV and lines 2, 3, and 7 of Table VII) agree with those obtained earlier and presented in a different form in ref. [35]. However, my results disagree with those found in ref. [36].

In this paper, I have not relied on any particular symmetry breaking pattern, instead using the representations of unified and partially unified symmetry groups only as a classification scheme for the possible F terms. How these results might be realized in particular models is a separate and detailed dynamical question. It should be noted that in most viable supersymmetric GUT models, the gauge coupling above the unification scale probably is quickly driven so large by renormalization group running as to render perturbative analyses problematic. Threshold corrections can also be quite significant [40], and in general they will have a different structure than the tree-level gaugino mass ratios given here. There are many other ways that gaugino mass non-universality can be realized in model building. However, if the deviation from universality observed in gaugino masses is sufficiently dramatic, and can be correlated with other observed features of the MSSM, then the ratios of gaugino masses following from non-singlet F-terms may yield an important insight into the structure of supersymmetry breaking.

Acknowledgments: This work was supported in part by National Science Foundation grant PHY-0757325.

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