SPACE CHARGE SUPPRESSION FOR UNEVEN EMITTANCES

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Abstract

The intensity of low and intermediate energy accelerators and storage rings is limited by space charge effects. An increase of the space charge tune shift may lead to lifetime degradation and coherent instability. A method to suppress the space charge effect is suggested for a beam with two uneven emittances.

1. INTRODUCTION

Conventionally, the space charge tune shift is calculated as [1]:

$$\Delta Q_y = -\frac{\lambda r_0}{2\pi\gamma_0^3\beta_0^2}\oint\frac{\beta_y ds}{a_y(a_x+a_y)} \quad , \qquad (1)$$

where λ is the beam linear density, r_0 is the classical radius for the beam particles, β_y is the beta-function, γ_0 and β_0 are the relativistic factors, s is a position around the ring, $a_{x,y}=\sqrt{\varepsilon_{x,y}\beta_{x,y}}$ are the beam rms radii.

For some purposes, a flat beam, $\varepsilon_y \ll \varepsilon_x$, is required. For example, flat ion beams are needed in ion-electron colliders in order to match the electron beams that are flat due do synchrotron radiation. One of the main obstacles to doing that is the space charge, since the tune shift $\Delta Q_y \propto \varepsilon_y^{-1/2}$ grows with reduction of the smaller emittance ε_y. With growing space charge tune shift, two different problems arise. First, coherent and incoherent resonances can be excited, leading to emittance growth and lifetime degradation. This typically prevents the increase of the space charge tune shift above 0.1-0.2. The second problem is a fast reduction of the Landau damping with growing space charge [2,3] that may lead to a beam transverse instability.

The idea of keeping a finite space charge tune shift for an arbitrary small lesser emittance was suggested by one of the authors [4]. Eq. (1) assumes that the beam optics is uncoupled or planar. Instead, the use of circular optics for a major part of the ring was proposed, where a mode with the large emittance (say, clockwise mode) determines the beam size, while a mode with the small emittance (counter-clockwise) determines a local difference in the transverse velocities [5, 6]. These types of beams were called canonical angular momentum dominated, or CAM-dominated beams [5]. The space charge tune shift is sensitive to the beam size, and is not sensitive to the local velocity spread. That is why, with circular optics, the space charge tune shift is independent of the small emittance, thus allowing it to be arbitrarily reduced.

In this paper, the space charge tune shift is calculated for a lattice with generally coupled optics in terms of coupled Twiss parameters of Ref. [7]. This result is then applied to the case of circular optics, yielding the same space charge tune shift for the both circular modes. This circular-mode tune shift is found to be equal to the planar-mode tune shift of the large-emittance mode, confirming an original idea of Ref. [4].

2. SPACE CHARGE TUNE SHIFT FOR COUPLED OPTICS

Following Ref. [7] and keeping all of its notations, the two single-particle eigenvectors can be generally presented in terms of 4D Twiss parameters:

$$\mathbf{V}_1 = \left(\sqrt{\beta_{1x}}\,,\ \frac{i(u-1)-\alpha_{1x}}{\sqrt{\beta_{1x}}}\,,\ \sqrt{\beta_{1y}}\,e^{i\nu_1}\,,\ \frac{-iu-\alpha_{1x}}{\sqrt{\beta_{1y}}}\,e^{i\nu_1}\right); \qquad (2)$$

$$\mathbf{V}_2 = \left(\sqrt{\beta_{2x}}\,e^{i\nu_2}\,,\ \frac{-iu-\alpha_{2x}}{\sqrt{\beta_{2x}}}\,e^{i\nu_2}\,,\ \sqrt{\beta_{2y}}\,,\ \frac{i(u-1)-\alpha_{1x}}{\sqrt{\beta_{2y}}}\right).$$

To determine the incoherent tune shift, the perturbation formalism of Ref. [8] can be used. The local action of space charge changes the 4D revolution matrix: $\mathbf{R}=\mathbf{R}^{(0)}+\mathbf{P}\cdot\mathbf{R}^{(0)}$, where $\mathbf{R}^{(0)}$ is its unperturbed value, and $\mathbf{P}$ is a 4D matrix of a local space charge perturbation. With that, the tune shift of the given mode $n=1,2$ is calculated as a diagonal matrix element of the perturbation in a basis of the 4D eigenvectors $\mathbf{V}$:

$$\Delta Q_n = -\frac{1}{4\pi}\mathbf{V}_n^{+}\cdot\mathbf{U}\cdot\mathbf{P}\cdot\mathbf{V}_n\,.$$

Assuming the beam to be Gaussian, with r. m. s. sizes a and b, tilted by an angle ψ, a local space charge kick is calculated as

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$$\begin{pmatrix} dp_x \\ dp_y \end{pmatrix} = \frac{\lambda r_0 ds}{2\pi\gamma_0^3\beta_0^2 ab(a+b)} \cdot \mathbf{T} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\mathbf{T} \equiv \begin{pmatrix} a\sin^2\psi + b\cos^2\psi & (b-a)\cos\psi\sin\psi \\ (b-a)\cos\psi\sin\psi & a\cos^2\psi + b\sin^2\psi \end{pmatrix}$$

Elements of this 2D kick matrix **T** give the four nonzero elements of the 4D perturbation matrix **P**. This yields the space charge tune shift for an arbitrary 4D coupled lattice:

$$\Delta Q_1 = -\frac{\lambda r_0}{2\pi\gamma_0^3\beta_0^2}\oint F\,ds$$

$$F \equiv \frac{(b\beta_{1x} + a\beta_{1y})\cos^2\psi + (a\beta_{1x} + b\beta_{1y})\sin^2\psi + (b-a)\sqrt{\beta_{1x}\beta_{1y}}\cos\nu_1\sin 2\psi}{ab(a+b)}$$

In general, the beam sizes and tilt are expressed in terms of its 2 r. m. s. emittances $\varepsilon_{1,2}$, and the 4D Twiss parameters:

$$2a^2 = \langle x^2\rangle + \langle y^2\rangle + \sqrt{\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\langle xy\rangle^2}\,;$$

$$2b^2 = \langle x^2\rangle + \langle y^2\rangle - \sqrt{\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\langle xy\rangle^2}\,;$$

$$\tan 2\psi = \frac{2\langle xy\rangle}{\langle x^2\rangle - \langle y^2\rangle};$$

$$\langle x^2\rangle = \varepsilon_1\beta_{1x} + \varepsilon_2\beta_{2x}\,;$$

$$\langle y^2\rangle = \varepsilon_1\beta_{1y} + \varepsilon_2\beta_{2y}\,;$$

$$\langle xy\rangle = \varepsilon_1\sqrt{\beta_{1x}\beta_{1y}}\cos\nu_1 + \varepsilon_2\sqrt{\beta_{2x}\beta_{2y}}\cos\nu_2\,.$$

For a conventional uncoupled coasting beam with $\langle xy\rangle = 0,\ \beta_{2x} = \beta_{1y} = 0,\ \beta_{1x} = \beta_x,\ \beta_{2y} = \beta_y$, this yields the planar-mode result of Eq. (1).

For a round CAM-dominated beam [6] with circular optics $\langle xy\rangle = 0,\ \beta_{1x} = \beta_{1y} = \beta_{2x} = \beta_{2y} = \beta,\ \varepsilon_2 << \varepsilon_1 = \varepsilon$; the tune shifts for the two modes are identical and are determined by the highest emittance:

$$\Delta Q_1 = \Delta Q_2 = -\frac{r_0\lambda C}{2\pi\varepsilon_1\gamma_0^3\beta_0^2}\,; \qquad (3)$$

$$a = b = \sqrt{\beta\varepsilon_1}\,; \quad C \equiv \oint ds\,.$$

Thus, circular optics for the main part of the ring allows an arbitrarily large reduction of one of the two emittances without any limitation from the space charge tune shift, confirming the idea of Ref. [4]. Note that the circular-mode tune shift (3) is identical to the planar-mode tune shift (1) in the direction of the large emittance (x). So, the space charge effect can be suppressed for the flat beam by means of the circular optics for the main part of the ring. Optical transformation from the planar to circular optics and back can be provided by adapters based on skew-quadrupoles (see [6] and Refs. therein). It is worth noting that for circular optics, intra-beam scattering does not lead to a transfer of the large emittance into the small one, since both transverse velocities are determined by the same small emittance, while the large one is responsible only for the beam size [9].

3. FINAL REMARKS

It has been shown that for a beam with uneven emittances, use of the circular optics makes the space charge tune shift equal to its planar value in the large-emittance direction. This removes a limit on the smaller emittance from the space charge side. Thus, since flat beams can be extremely bright, they could find their use in various applications: ion-electron colliders, nuclear fusion, medicine, and others. One additional possibility for the use of these beams relates to the fact that in a matched solenoid the CAM-dominated beam is transformed into a parallel beam [5]. This transformation is essential for relativistic electron cooling of antiprotons at Fermilab [10]. The two transverse emittances can be made significantly different either by means of a special injection procedure (painting) [11], or by cooling [4], or by extraction of the beam from a magnetized gun [5]. In any case, use of the circular optics appears to open a special possibility for extremely bright beams.

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