Measurement of the top quark mass in two leptons

We present measurements of the top quark mass ($m_t$) in $t\bar{t}$ candidate events with two final state leptons using 1 fb$^{-1}$ of data collected by the D0 experiment. Our data sample is selected by requiring two fully identified leptons or by relaxing one lepton requirement to an isolated track if at least one
jet is tagged as a $b$ jet. The top quark mass is extracted after reconstructing the event kinematics under the $t\bar{t}$ hypothesis using two methods. In the first method, we integrate over expected neutrino rapidity distributions, and in the second we calculate a weight for the possible top quark masses based on the observed particle momenta and the known parton distribution functions. We analyze 83 candidate events in data and obtain $m_t = 176.2 \pm 4.8 \text{(stat)} \pm 2.1 \text{(sys)} \text{GeV}$ and $m_t = 173.2 \pm 4.9 \text{(stat)} \pm 2.0 \text{(sys)} \text{GeV}$ for the two methods, respectively. Accounting for correlations between the two methods, we combine the measurements to obtain $m_t = 174.7 \pm 4.4 \text{(stat)} \pm 2.0 \text{(sys)} \text{GeV}$. 

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I. INTRODUCTION

After the top quark was discovered in 1995 [1, 2], emphasis quickly turned to detailed studies of its properties including measuring its mass across all reconstructable final states. Within the standard model, a precise measurement of the top quark mass ($m_t$) and W boson mass ($M_W$) can be used to constrain the Higgs boson mass ($M_H$). In fact, these masses can be related by radiative corrections to $M_W$. One-loop corrections give $M_W^2 = \frac{3g^2m_t^2}{8\pi^2\alpha} + \Delta r$, where $\Delta r$ depends quadratically on $m_t$ and logarithmically on $M_H$ [3]. Beyond its relation to the Yukawa coupling, $Y_t$, for the top quark via $Y_t = m_t\sqrt{\alpha}/v$, where $v = 246 \text{ GeV}$ is the vacuum expectation value of the Higgs field [4]. Given that these couplings are not predicted by the theory, $Y_t = 0.995 \pm 0.007$ for the current $m_t$ [5] is curiously close to unity. One of several possible modifications to the mechanism underlying electroweak symmetry breaking suggests a more central role for the top quark. For instance, in top-color assisted technicolor [6, 7], the top quark plays a major role in electroweak symmetry breaking. These models entirely remove the need for an elementary scalar Higgs field in favor of new strong interactions that provide the observed mass spectrum. Perhaps there are extra Higgs doublets as in MSSM models [8]; measurement of the top quark mass may be sensitive to such models (e.g., Ref. [9]).

In the standard model, $BR(t \rightarrow Wb)$ is expected to be nearly 100%. So the relative rates of final states in events with top quark pairs, $t\bar{t}$, are dictated by the branching ratios of the $W$ boson to various fermion pairs. In approximately 10% of $t\bar{t}$ events, both $W$ bosons decay leptonically. Generally, only events that include the $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ modes yield final states with precisely reconstructed lepton momenta to be used for mass analysis. Thus, analyzable dilepton final states are $t\bar{t} \rightarrow e\ell^\prime \nu \bar{\nu} + b\bar{b}$, where $e, \ell' = e, \mu$. We measure $m_t$ in these dilepton events. The $W \rightarrow \tau\nu \rightarrow e(\mu)\nu\bar{\nu}$ decay modes cannot be separated from the direct $W \rightarrow e(\mu)\nu$ decays and are included in our analysis.

Dilepton channels provide a sample that is statistically independent of the more copious $t\bar{t} \rightarrow e\nu + q\bar{q} + b\bar{b}$ ($\ell+\text{jets}$) decays. The relative contributions of specific systematic effects are somewhat different between mass measurements from events with dilepton or $\ell+\text{jets}$ final states. The jet multiplicity and the dominant background processes are different. The measurement of $m_t$ in the dilepton channel also provides a consistency test of the $t\bar{t}$ event sample with the expected $t \rightarrow Wb$ decay. Non-standard decays of the top quark, such as $t \rightarrow H^\pm b$, can affect the final state particle kinematics differently in different $t\bar{t}$ channels. These kinematics affect the reconstructed mass significantly, for example in the $\ell+\text{jets}$ channel [10]. Therefore, it is important to precisely test the consistency of the $m_t$ measurements in different channels.

Previous efforts to measure $m_t$ in the dilepton channels have been pursued by the D0 and CDF collaborations. A frequently used technique reconstructs individual event kinematics using known constraints to obtain a relative probability of consistency with a range of top quark masses. The “matrix weighting” method (MWT) follows the ideas proposed by Dalitz and Goldstein [11] and Kondo [12]. It uses parton distribution functions and observed particle momenta to obtain a mass estimate for each dilepton event, and has previously been implemented by D0 [13, 14] and CDF [15]. The “neutrino weighting” method ($\nu$WT) was developed at D0 [13]. It integrates over expected neutrino rapidity distributions, and has been used by both the D0 [13, 14] and CDF [15] collaborations.

In this paper, we describe a measurement of the top quark mass in 1 fb$^{-1}$ of $p\bar{p}$ collider data collected using the D0 detector at the Fermilab Tevatron Collider. Events are selected in two categories. Those with one fully identified electron and one fully identified muon, two electrons, or two muons are referred to as “2$\ell$.” To improve acceptance, we include a second category consisting of events with only one fully reconstructed electron or muon and an isolated high transverse momentum ($p_T$) track as well as at least one identified $b$ jet, which we refer to as “$\ell+\text{track}$” events. We describe the detection, selection, and modeling of these events in Sections IV and V. Reconstruction of the kinematics of $t\bar{t}$ events proceeds by both the MWT and $\nu$WT approaches. These methods are described in Section VI. In Section VII we describe the maximum likelihood fits to extract $m_t$ from data. Finally, we discuss our results and systematic uncertainties in Section VIII.
II. DETECTOR AND DATA SAMPLE

A. Detector Components

The D0 Run II detector is a multipurpose collider detector consisting of an inner magnetic central tracking system, calorimeters, and outer muon tracking detectors. The spatial coordinates of the D0 detector are defined as follows: the positive $z$ axis is along the direction of the proton beam while positive $y$ is defined as upward from the detector's center, which serves as the origin. The polar angle $\theta$ is measured with respect to the positive $z$ direction and is usually expressed as the pseudorapidity, $\eta = -\ln[\tan(\theta/2)]$. The azimuthal angle $\phi$ is measured with respect to the positive $x$ direction, which points away from the center of the Tevatron ring.

The inner tracking detectors are responsible for measuring the trajectories and momenta of charged particles and for locating track vertices. They reside inside a superconducting solenoid that generates a magnetic field of 2 T. A silicon microstrip tracker (SMT) is innermost and provides precision position measurements, particularly in the azimuthal plane, which allow the reconstruction of displaced secondary vertices from the decay of long-lived particles. This permits identification of jets from heavy flavor quarks, particularly $b$ quarks. A central fiber tracker is composed of scintillating fibers mounted on eight concentric support cylinders. Each cylinder supports one axial and one stereo layer of fibers, alternating by $\pm 3^\circ$ relative to the cylinder axis. The outermost cylinder provides coverage for $|\eta| < 1.7$.

The calorimeter measures electron and jet energies, directions, and shower shapes relevant for particle identification. Neutrinos are also measured via the calorimeters’ hermeticity and the constraint of momentum conservation in the plane transverse to the beam direction. Three liquid-argon-filled cryostats containing primarily uranium absorbers constitute the central and endcap calorimeter systems. The former covers $|\eta| < 1.1$, and the latter extends coverage to $|\eta| = 4.2$. Each calorimeter consists of an electromagnetic (EM) section followed longitudinally by hadronic sections. Readout cells are arranged in a pseudo-projective geometry with respect to the nominal interaction region.

Drift tubes and scintillators are arranged in planes outside the calorimeter system to measure the trajectories of penetrating muons. One drift tube layer resides inside iron toroids with a magnetic field of 1.8 T, while two more layers are located outside. The coverage of the muon system is $|\eta| < 2$.

B. Data Sample

The D0 trigger and data acquisition systems are designed to accommodate instantaneous luminosities up to $3 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$. The Tevatron operates with 396 ns spacing between proton (antiproton) bunches and delivers a 2 MHz bunch crossing rate. For our data sample, each crossing yields on average 1.2 $p\bar{p}$ interactions.

Luminosity measurement at D0 is based on the rate of inelastic $p\bar{p}$ collisions observed by plastic scintillation counters mounted on the inner faces of the calorimeter endcap cryostats. Based on information from the tracking, calorimeter, and muon systems, the first level of the trigger limits the rate for accepted events to 2 kHz. This is a dedicated hardware trigger. Second and third level triggers employ algorithms running in processors to reduce the output rate to about 100 Hz, which is written to tape.

Several different triggers are used for the five decay channels considered in this measurement. We employ single electron triggers for the $ee$ and $e+\text{track}$ channels and single muon triggers for the $\mu\mu$ and $\mu+\text{track}$ channels. The $e\mu$ analysis employs all unprescaled triggers requiring one electron and/or one muon. We also use triggers requiring one lepton plus one jet for the $\ell+\text{track}$ channels. A slight difference between the $v\text{WT}$ and MWT analyses occurs because the latter excludes 2% of data collected while the single muon trigger was prescaled. While the effect on the kinematic distributions is negligible, this results in one less $\mu\mu$ candidate event in the final sample for the MWT analysis.

Events were collected with these triggers at D0 between April 2002 and February 2006 with $\sqrt{s} = 1.96 \text{ TeV}$. Data quality requirements remove events for which the tracker, calorimeter, or muon system are known to be functioning improperly. The integrated luminosity of the analyzed data sample is about 1 fb$^{-1}$.

C. Particle Identification

We reconstruct the recorded data to identify and measure final state particles, as described below. A more detailed description can be found in Ref. [17].

The primary event vertex (PV) is identified as the vertex with the lowest probability to come from a soft $p\bar{p}$ interaction based on the transverse momenta of associated tracks. We select events in which the PV is reconstructed from at least three tracks and with $|z_{\text{PV}}| < 60 \text{ cm}$. Secondary vertices from the decay of long-lived particles from the hard interaction are reconstructed from two or more tracks satisfying the requirements of $p_T > 1 \text{ GeV}$ and more than one hit in the SMT. We require each track to have a large impact parameter significance, $\text{DCA}/\sigma_{\text{DCA}} > 3.5$, with respect to the PV, where DCA is the distance of the track’s closest approach to the PV in the transverse plane.

High-$p_T$ muons are identified by matching tracks in the inner tracker with those in the muon system. The track requirements include a cut on DCA$< 0.02$ (0.2) cm for tracks with (without) SMT hits. Muons are isolated in the tracker when the sum of track momenta in a cone of radius $\Delta R(\text{muon,track}) = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.5$ around the muon’s matching track is small compared to
the track $p_T$. We also require isolated muons to have the sum of calorimeter cell energies in an annulus with radius in the range $0.1 < \Delta R < 0.4$ around the matched track to be low compared to the matching track $p_T$.

High-$p_T$ isolated tracks are identified solely in the inner tracker. We require them to satisfy track isolation requirements and to be separated from calorimeter jets by $\Delta R(\text{jet,track}) > 0.5$. These tracks must correspond to leptons from the PV, so we also require that $\text{DCA}/\sigma_{\text{DCA}} < 2.5$. We avoid double-counting leptons by requiring $\Delta R(\text{track, } \ell) > 0.5$.

Electrons are identified in the EM calorimeter. Cells are clustered according to a cone algorithm within $\Delta R < 0.2$ and then matched with an inner detector track. Electron candidates are required to deposit 90% of their energy in the EM section of the calorimeter. They must also satisfy an initial selection which includes a shower shape test ($\chi^2_{\text{max}}$) with respect to the expected electron shower shape, and a calorimeter isolation requirement summing calorimeter energy within $\Delta R < 0.4$ but excluding the cluster energy. To further remove backgrounds, a likelihood ($L_e$) selection is determined based on seven tracking and calorimeter parameters, including $\chi^2_{\text{max}}$, DCA, and track isolation calculated in an annulus of $0.05 < \Delta R < 0.4$ around the electron. The final electron energy calibration is determined by comparing the invariant mass of high $p_T$ electron pairs in $Z/\gamma \rightarrow e^+e^-$ events with the world average value of the $Z$ boson mass as measured by the LEP experiments.

In $t\bar{t}$ events, the leptons and tracks originate from the hard interaction. Therefore, we require their $z$ positions at the closest approach to the beam axis to match that of the PV within 1 cm.

We reconstruct jets using a fixed cone algorithm [18] with radius of 0.5. The four-momentum of a jet is measured as the sum of the four-momenta assigned to calorimeter cells inside of this cone. We select jets that have a longitudinal shower profile consistent with that of a collection of charged and neutral hadrons. We confirm jets via the electronically independent calorimeter trigger readout chain. Jets from $b$ quarks are tagged using a neural network $b$ jet tagging algorithm [19]. This combines the impact parameters for all tracks in a jet, as well as information about reconstructed secondary vertices in the jet. We obtain a typical efficiency of 54% for $b$ jets with $|\eta| < 2.4$ and $p_T > 30$ GeV for a selection which accepts only 1% of light flavor ($u, d, s$ quark or gluon) jets.

Because the $b$ jets carry away much of the rest energy of the top quarks, it is critical for the measurement of $m_t$ that the measurements of the energies of jets from top quark decay be well calibrated. Jet energies determined from the initial cell energies do not correspond to the energies of final state particles striking the calorimeter. As a result, a detailed calibration is applied [20, 21] in data and Monte Carlo separately. In general, the energy of all final state particles inside the jet cone, $E_j^\text{incl}$, can be related to the energy measured inside the jet cone, $E_j$, by $E_j^\text{incl} = (E_j - O)/(R \cdot S)$. Here, $O$ denotes an offset energy primarily from extra interactions in or out of time with an event. $R$ is the cumulative response of the calorimeter to all of the particles in a jet. $S$ is the net energy loss due to showering out of the jet cone. For a given cone radius, $O$ and $S$ are functions of the jet $\eta$ within the detector. $O$ is also a function of the number of reconstructed event vertices and the instantaneous luminosity. $R$ is the largest correction and reflects the lower response of the calorimeters to charged hadrons relative to electrons and photons. It also includes the effect of energy losses in front of the calorimeter. The primary response correction is derived in situ from $\gamma$+jet events and has substantial dependences on jet energy and $\eta$. For all jets that contain a non-isolated muon, we add the muon momenta to that of the jet. Under the assumption that these are $b$ quark semi-leptonic decays, we also add an estimated average neutrino momentum assumed to be collinear with the jet direction. The correction procedure discussed above does not correct all the way back to the original $b$ quark parton energy.

The event missing transverse energy, $E_T$, is equal in magnitude and opposite in direction to the vector sum of all significant transverse energies measured by the individual calorimeter cells. It is corrected for the transverse momenta of all isolated muons, as well as for the corrections to the electron and jet energies. The $E_T$ is also corrected if the track does not point to a jet, electron, or muon. In this case, we substitute the track $p_T$ for the calorimeter energy within a cone of radius $\Delta R = 0.4$ around the track.

### D. Signal and Background Simulation

An accurate description of the composition and kinematic properties of the selected data sample is essential to the mass measurement. Monte Carlo samples for the $t\bar{t}$ processes are generated for several test values of the top quark mass. The event generation, fragmentation, and decay are performed by PYTHIA 6.319 [22]. Background processes are called “physics” backgrounds when charged leptons originate from $W$ or $Z$ boson decay and when $E_T$ comes from high $p_T$ neutrinos. Physics backgrounds include $Z/\gamma \rightarrow \tau \tau$ with $\tau \rightarrow e, \mu$ and diboson ($WW$, $WZ$, and $ZZ$) production. The $Z/\gamma \rightarrow \tau \tau$ background processes are generated with ALPGEN 2.11 [23] as the event generator and PYTHIA for fragmentation and decay. We decay hadrons with $b$ quarks using EVTGEN [24]. To avoid double counting QCD radiation between ALPGEN and PYTHIA, the jet-parton matching scheme of Ref. 25 is employed in ALPGEN. The diboson backgrounds are simulated with PYTHIA. We use the CTEQ6L1 [26] parton distribution function (PDF). Monte Carlo events are then processed through a GEANT-based [27] simulation of the D0 detector. In order to accurately model the effects of multiple proton interactions and detector noise, data events from random $pp$ crossings are overlaid on the Monte Carlo events. Finally, Monte Carlo events are pro-
cessed through the same reconstruction software as used for data.

In order to ensure that reconstructed objects in these samples reflect the performance of the detector in data, several corrections are applied. Monte Carlo events are reweighted by the z coordinate of the PV to match the profile in data. The Monte Carlo events are further tuned such that the efficiencies to find leptons, isolated tracks, and jets in Monte Carlo events match those determined from data. These corrections depend on the $p_T$ and $\eta$ of these objects. The jet energy calibration derived for data is applied to jets in data, and the jet energy calibration derived for simulated events is applied to simulated events. We observe a residual discrepancy between jet energies in $Z/\gamma^*+jets$ events in data and Monte Carlo. We apply an additional correction to jet energies in the Monte Carlo to bring them into agreement with the data. This adjustment is then propagated into the $E_T$. We apply an additional correction to jet energies in Monte Carlo to reflect the performance of the detector in data, within $|\eta|<2.5$.

III. SELECTED EVENT SAMPLE

Events are selected for all channels by requiring either two leptons ($2\ell$) or a lepton and an isolated track ($\ell+\text{track}$), each with $p_T > 15$ GeV. Electrons must be within $|\eta| < 1.1$ or $1.5 < |\eta| < 2.5$; muons and tracks should have $|\eta| < 2.0$. An opposite charge requirement is applied to the two leptons or to the lepton and track. At least two jets are also required with pseudorapidity $|\eta| < 2.5$ and $p_T > 20$ GeV. We require the leading lepton to have $p_T > 40$ GeV. The dominant $\ell+\text{track}$ background arises from $Z \rightarrow e\mu$ production, so we design the event selection to reject these events.

When the invariant mass of the lepton-track pair ($M_{b\ell}$) is in the range $70 < M_{b\ell} < 110$ GeV, the $E_T$ requirement is tightened to $E_T > 35$ (40) GeV for the $e+\text{track}$ ($\mu+\text{track}$) channel. Furthermore, we introduce the variable $E_T^{Z-H}$ that corrects the $E_T$ in $Z \rightarrow \ell\ell$ events for mismeasured lepton momenta. We rescale the lepton and track momenta according to their resolutions to bring $M_{b\ell}$ to the mass of the $Z$ boson (91.2 GeV) and then use these rescaled momenta to correct the $E_T$. Event selection based on this variable reduces the $Z$ background by half while providing 96% efficiency for $t\bar{t}$ events. The cuts on $E_T^{Z-H}$ are always identical to those on $E_T$.

At least one jet is required to be identified as a $b$ jet which provides strong background rejection for the $\ell+\text{track}$ channels. The $m_t$ precision is limited by signal statistics in the observed event sample when the background is reasonably low. The above selection is a result of an optimization which minimizes the statistical uncertainty on $m_t$. We do this in terms of $E_T$, $E_T^{Z-H}$, the transverse momenta of the leading two jets, and the b-tagging criteria by stepping through two or more different thresholds on these requirements. After considering all possible sets of selections, we choose the one which gives the best average expected statistical uncertainty on the $m_t$ measurement using many pseudoeperiments. The expected statistical uncertainty varies smoothly over a
15% range while the study is sensitive to 5% changes of the average statistical uncertainty.

We explicitly veto events satisfying the selection of any of the 2\ell channels, so the \ell+track channels are statistically independent of the 2\ell channels. We select eight events in the e+track channel and six events in the \mu+track channel.

C. Modeling Instrumental Backgrounds

Backgrounds can arise from instrumental effects in which the \ET is mismeasured. The main instrumental backgrounds for the ee, \mu\mu, e+\mu, and e+\mu+tracks, and \mu+track channels are the Z/\gamma\rightarrow ee and Z/\gamma\rightarrow \mu\mu processes. In these cases, apparent \ET results from tails in jet or lepton \PT resolutions. We use the NNLO cross section for Z/\gamma\rightarrow ee, \mu\mu processes, along with the Monte Carlo-derived efficiencies to estimate these backgrounds for the ee and \mu\mu channels. The Monte Carlo kinematic distributions, including the \ET, are verified to reproduce a data sample dominated by these processes. For the \ell+track channels, we normalize Drell-Yan Monte Carlo so that the total expected event yield in a \ell+track sample with low \ET equals the observed event yield in the data. We observe a slightly different \pt^2 distribution for simulated Z \rightarrow \ell\ell events in comparison with data. As a result, all Z boson simulated samples, including the Z \rightarrow \tau\tau physics background samples, are reweighted to the observed distribution of \pt^2 in data.

Another background arises when a lepton or a track within a jet is identified as an isolated lepton or track. We utilize different methods purely in data to estimate the level of these backgrounds for each channel. In all cases, however, we distinguish reconstructed muons and tracks as “loose” rather than “tight” by releasing the isolation criteria. We make an analogous distinction for electrons by omitting the requirement on the electron likelihood, \Lie, for “loose” electrons.

To determine the misidentified electron background yield in the ee and e\mu channels, we fit the observed distribution of \Lie in the data to a sum of the distributions from real isolated electrons and misidentified electrons. We determine the shape of \Lie for real electrons from a Z \rightarrow ee sample with \ET < 15 GeV. For the ee channel, we extract the shape for the misidentified electrons from a sample in which one “tag electron” is required to have both \chi^2_{\text{min}} and \Lie inconsistent with being an electron. We further require \Met < 60 GeV or \Met > 130 GeV and \ET < 15 GeV to reject Z and W boson events. The distribution of \Lie is obtained from a separate “probe electron” in the same events. In the e\mu channel, the \Lie distribution for misidentified electrons is obtained in a sample with a non-isolated muon and \ET < 15 GeV.

To estimate the background from non-isolated muons for the e\mu and \mu\mu channels, we use control samples to measure the fraction of muons, \f_{\mu}, with \PT > 15 GeV that appear to be isolated. To enhance the heavy flavor content which gives non-isolated muons, the control samples are selected to have two muons where a “tag” muon is required to be non-isolated. We use another “probe” muon to determine \f_{\mu}. The background yield for the e\mu channel is computed from the number of events having an isolated electron, a muon with no isolation requirement, and the same sign charge for the two leptons. We multiply the observed yield by \f_{\mu}.

We estimate the instrumental background for the \mu\mu and \ell+track channels by using systems of linear equations describing the composition of data samples with different “loose” or “tight” lepton and/or track selections. We relate event counts in these samples to the numbers of events with real or misidentified isolated leptons using the system of equations. These equations take as inputs the efficiencies for real or misidentified leptons and tracks to pass the tight identification requirements. For the \mu\mu and \ell+track channels, we determine the efficiencies for real leptons and tracks to pass the tight identification criteria using Z \rightarrow ee and Z \rightarrow \mu\mu events.

For the \ell+track channels, the probabilities for misidentified leptons and tracks to pass the tight selection criteria are determined from multijet data samples with at least one loose lepton plus a jet. We reject the event if two leptons of the same flavor satisfy tight criteria to suppress Drell-Yan events. We also reject events with one or more tight leptons with different flavor from the loose lepton. These tight lepton vetos allow some events with two loose leptons or a lepton and track in the sample. We further suppress resonant Z production by selecting events when \Mt > 100 GeV or \Mt > 70 GeV. We reject W+jets events and misreconstructed Z/\gamma\* events by requiring \ET < 15 GeV and \ET^{\text{JES}} < 25 GeV. Here, \ET^{\text{JES}} is the missing transverse energy with only jet energy corrections and no lepton corrections. We use the latter because loose leptons no longer adhere to standard resolutions. We calculate the probability for electrons or muons to be misidentified by dividing the number of tight leptons by the number of loose leptons. For the track probability, we combine the e+jet and \mu+jet samples and make the additional requirement that there be at least one loose track in the event. The tight track misidentification probability is again the number of tight tracks divided by the number of loose tracks.

To obtain samples dominated by misidentified isolated leptons for mass analysis, we select events with two loose leptons or tracks plus two jets. For the 2\ell channels, we additionally require same sign dilepton events.

D. Composition of Selected Samples

The expected numbers of background and signal events in all five channels (assuming a top quark production cross section of 7.0 pb) are listed in Table I along with the observed numbers of candidates. The \mu+track selection has half the efficiency of the e+track selection primarily due to the tight \mu\mu veto. The expected and observed event yields agree for all channels.
TABLE I: Expected event yields for $t\bar{t}$ (we assume $\sigma_{t\bar{t}} = 7.0$ pb) and backgrounds and numbers of observed events for the five channels. The $Z\ell\nu$ channel uncertainties include statistical as well as systematical uncertainties while the $e+\text{track}$ and $\mu+\text{track}$ uncertainties are statistical only.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$t\bar{t}$</th>
<th>Diboson</th>
<th>$Z\ell\nu$</th>
<th>Multijet/W+jets</th>
<th>Total</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e\mu$</td>
<td>$36.7 \pm 2.4$</td>
<td>$1.7 \pm 0.7$</td>
<td>$4.5 \pm 0.7$</td>
<td>$2.6 \pm 0.6$</td>
<td>$44.5 \pm 2.7$</td>
<td>$39$</td>
</tr>
<tr>
<td>$ee$</td>
<td>$11.5 \pm 1.4$</td>
<td>$0.5 \pm 0.2$</td>
<td>$2.3 \pm 0.4$</td>
<td>$0.6 \pm 0.2$</td>
<td>$14.8 \pm 1.5$</td>
<td>$17$</td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>$8.3 \pm 0.5$</td>
<td>$0.7 \pm 0.1$</td>
<td>$4.5 \pm 0.4$</td>
<td>$0.2 \pm 0.2$</td>
<td>$13.7 \pm 0.7$</td>
<td>$13$</td>
</tr>
<tr>
<td>$e+\text{track}$</td>
<td>$9.4 \pm 0.1$</td>
<td>$0.1 \pm 0.0$</td>
<td>$0.4 \pm 0.1$</td>
<td>$0.4 \pm 0.1$</td>
<td>$10.3 \pm 0.2$</td>
<td>$8$</td>
</tr>
<tr>
<td>$\mu+\text{track}$</td>
<td>$4.6 \pm 0.1$</td>
<td>$0.1 \pm 0.0$</td>
<td>$0.7 \pm 0.1$</td>
<td>$0.1 \pm 0.0$</td>
<td>$5.5 \pm 0.1$</td>
<td>$6$</td>
</tr>
</tbody>
</table>

FIG. 1: Comparison of the expected distributions from backgrounds and $t\bar{t}$ ($m_{t\bar{t}} = 170$ GeV) in the $\ell\+\text{track}$ channels. (a) $M_{t\bar{t}}$ for the $e+\text{track}$ channel without the requirement of the $b$-tag and with inverted $E_T$ cuts. (b) $E_T$ for the sum of both $\ell+\text{track}$ channels, again without the $b$-tag requirement. We assume $\sigma_{t\bar{t}} = 7.0$ pb.

Kinematic comparisons between data and the sum of the signal and background expectations provide checks of the content and properties of our data sample. Figure 1(a) shows the expected and observed distributions of $M_{t\bar{t}}$ in the $e+\text{track}$ channel without the $b$-tag requirement and for an inverted $E_T$ requirement. The $\mu+\text{track}$ distribution looks similar (not shown). The mass peak at $M_Z$ indicates the $e+\text{track}$ sample is primarily composed of $Z \to ee$ events before the final event selection. In Fig. 1(b), we show the $E_T$ distribution in the $\ell+\text{track}$ channels after all cuts except the $b$-tag requirement.

The expected numbers of background and signal events after all selections in all five channels are listed in Table I along with the observed numbers of candidates. We assume $\sigma_{t\bar{t}} = 7.0$ pb. We do not include systematic uncertainties for the $\ell+\text{track}$ channels. The small backgrounds mean their uncertainties have a negligible effect on the measured $m_{t\bar{t}}$ uncertainty. The $\mu+\text{track}$ selection has half the efficiency of the $e+\text{track}$ selection primarily due to the tight $\mu\mu$ veto. The expected and observed event yields agree for all channels. Figures 2(a) and (b) show the $E_T$ and leading lepton $p_T$ summed over all channels for the final candidate sample. We observe the data distributions to agree with our signal and background model.

IV. EVENT RECONSTRUCTION

Measurement of the dilepton event kinematics and constraints from the $t\bar{t}$ decay assumption allow a partial reconstruction of the final state and a determination of $m_t$. Given the decay of each top quark to a $W$ boson and a $b$ quark, with each $W$ boson decaying to a charged lepton and a neutrino, there are six final state particles: two charged leptons, two neutrinos, and two $b$ quarks. Each particle can be described by three momentum components. Of these eighteen independent parameters, we can directly measure only the momenta of the leptons. The leading two jets most often come from the $b$ quarks. Despite final state radiation and fragmentation, the jet momenta are highly correlated with those of the underlying $b$ quarks. We also measure the $x$ and $y$ components of the $\vec{E}_T$, $\vec{E}_z$ and $\vec{E}_y$, from the neutrinos. This leaves four quantities unknown. We can supply two constraints by relating the four-momenta of the leptons and neutrinos to the masses of the $W$ bosons:

$$
M_W^2 = (E_{\nu_1} + E_{\ell_1})^2 - (\vec{p}_{\nu_1} + \vec{p}_{\ell_1})^2
$$
$$
M_W^2 = (E_{\nu_2} + E_{\ell_2})^2 - (\vec{p}_{\nu_2} + \vec{p}_{\ell_2})^2,
$$

where the subscript indices indicate the $\ell\nu$ pair coming from one or another $W$ boson. Another constraint is supplied by requiring that the mass of the top quark and
the mass of the anti-top quark be equal:
\[
(E_{v_x} + E_{l_x} + E_{b_1})^2 - (\tilde{p}_{v_x} + \tilde{p}_{l_x} + \tilde{p}_{b_1})^2 =
\]
\[
(E_{v_y} + E_{l_y} + E_{b_2})^2 - (\tilde{p}_{v_y} + \tilde{p}_{l_y} + \tilde{p}_{b_2})^2.
\]

The last missing constraint can be supplied by a hypothesized value of the top quark mass. With that, we can solve the equations and calculate the unmeasured top quark and neutrino momenta that are consistent with the observed event. Usually, the dilepton events are kinematically consistent with a large range of \(m_t\). We quantify this consistency, or “weight,” for each \(m_t\) by testing measured quantities of the event (e.g., \(E_T\) or lepton and jet \(p_T\)) against expectations from the dynamics of \(t\bar{t}\) production and decay. This requires us to sample from relevant \(t\bar{t}\) distributions, yielding many solutions for a specific \(m_t\). We sum the weights for each solution for each \(m_t\). The distribution of weight vs. \(m_t\) is termed a “weight distribution” of a given event. Using parameters from these weight distributions, we can then determine the most likely value of \(m_t\).

Several previous efforts to measure \(m_t\) using dilepton events have used event reconstruction techniques. The differences between methods stem largely from which event parameters are used to calculate the event weight. We use the \(\nu\)WT and MWT techniques to determine the weighting as described below.

### A. Neutrino Weighting

The \(\nu\)WT method omits the measured \(E_T\) for kinematic reconstruction. Instead, we choose the pseudorapidities of the two neutrinos from \(t\bar{t}\) decay from their expected distributions. We obtain the distribution of neutrino \(\eta\) from several simulated \(t\bar{t}\) samples with a range of \(m_t\) values. These distributions can each be approximated by a single Gaussian function. The standard deviation specifying this function varies weakly with \(m_t\). Once the neutrino pseudorapidities are fixed and a value for \(m_t\) assumed, we can solve for the complete decay kinematics, including the unknown neutrino momenta. There may be up to four different combinations of solved neutrino momenta for each assumed pair of neutrino \(\eta\) values for each event. We assume the leading two jets are the \(b\) jets, so there are two possible associations of \(W\) bosons with \(b\) jets.

For each pairing of neutrino momentum solutions, we define a weight, \(w\), based on the agreement between the measured \(E_T\) and the sum of the neutrino momentum components in \(x\) and \(y\), \(p_{v_x}^\nu\) and \(p_{v_y}^\nu\). We assume independent Gaussian resolutions in measuring \(E_x\) and \(E_y\). The weight is calculated as

\[
w = \exp \left[ -\frac{(E_x^{\nu\text{calc}} - E_x^{\nu\text{obs}})^2}{2\sigma_x^{\nu\text{obs}}^2} \right] \exp \left[ -\frac{(E_y^{\nu\text{calc}} - E_y^{\nu\text{obs}})^2}{2\sigma_y^{\nu\text{obs}}^2} \right],
\]

reflecting the agreement between the measured and calculated \(E_x^{\nu}\) and \(E_y^{\nu}\). \(E_i^{\nu\text{obs}}\) \((i = x\text{ or } y)\) are the components of the measured event \(E_T\), and \(E_i^{\nu\text{calc}}\) are the components of the \(E_T\) calculated from the neutrino transverse momenta resulting from each solution. We calculate the quantities \(E_i^{\nu}\) to be the sums of the energies projected onto the \(i\) axes measured by all “unclustered” calorimeter cells – those cells not included in jets or electrons. The high \(p_T\) objects, leptons and jets, enter into the determination of both \(E_i^{\nu\text{calc}}\) and \(E_i^{\nu\text{obs}}\) whereas the unclustered energy \(E_i^{\nu}\) only enters into \(E_i^{\nu\text{obs}}\). Given the resolutions \(\sigma_i^{\nu}\) of the \(E_i^{\nu}\), we can therefore estimate the probability that the \(E_i^{\nu}\) are consistent with the \(E_i^{\nu\text{calc}}\) from the \(t\bar{t}\) hypothesis.

As parameters of the method, we determine \(\sigma_i^{\nu}\) using \(Z \to ee + 2\) jets data and Monte Carlo events. We calculate an unclustered scalar transverse energy, \(S_T^0\), as the scalar sum of the transverse energies of all unclustered calorimeter cells. Due to the azimuthal isotropy of the calorimeter, we observe that the independent \(x\) and \(y\) components of the \(\sigma_i^\nu\) depend on \(S_T^0\) in the same way within their uncertainties. Therefore, we combine results for both components to determine our resolution more precisely. We find agreement between data and simulation in the observed dependence of these parameters on \(S_T^0\). The distributions are shown for these combined resolutions in Fig. [3]. We fit the unclustered \(E_T\) resolutions.
obtained from simulation as

$$\sigma_t(S_T^0) = \sigma_{\ell}(S_T^0) = 4.38 \text{ GeV} + 0.52\sqrt{S_T^0} \text{ GeV},$$  \hspace{1cm} (5)$$

and use this parametrization for the unclustered missing energy resolution for both data and Monte Carlo in Eq. 4.

For each event, we consider ten different $\eta$ assumptions for each of the two neutrinos. We extract these values from the histograms appropriate to the $m_t$ being assumed. The ten $\eta$ values are the medians of each of ten ranges of $\eta$ which each represent 10% of the $t\bar{t}$ sample for a given $m_t$.

### B. Matrix Weighting

In the MWT approach, we use the measured momenta of the two charged leptons. We assign the measured momenta of the two jets with the highest transverse momenta to the $b$ and $\bar{b}$ quarks and the measured $E_T$ to the sum of the transverse momenta of the two neutrinos from the decay of the $t$ and $\bar{t}$ quarks. We then assume a top quark mass and determine the momenta of the $t$ and $\bar{t}$ quarks that are consistent with these measurements. We refer to each such pair of momenta as a solution for the event. For each event, there can be up to four solutions.

We assign a weight to each solution, analogous to the $\nu$WT weight of Eq. 3 given by

$$w = f(x)f(\bar{x})p(E_\ell^t|m_t)p(E_\bar{\ell}^\bar{t}|m_t),$$  \hspace{1cm} (6)$$

where $f(x)$ is the PDF for the proton for the momentum fraction $x$ carried by the initial quark, and $f(\bar{x})$ is the corresponding value for the initial antiquark. The quantity $E_\ell^t$ is the observed lepton energy in the top quark rest frame. We use the central fit of the CTEQ6L1 PDFs and evaluate them at $Q^2 = m_t^2$. The quantity $p(E_\ell^t|m_t)$ in Eq. 6 is the probability that for the hypothesized top quark mass $m_t$, the lepton $\ell$ has the measured $E_\ell^t$:

$$p(E_\ell^t|m_t) = \frac{4m_tE_\ell^t(m_t^2 - m_\ell^2 - 2m_tE_\ell^t)}{(m_t^2 - m_\ell^2)^2 + M_W^4(m_t^2 - m_\ell^2) - 2M_W^4},$$  \hspace{1cm} (7)$$

### C. Total Weight vs. $m_t$

Equations 4 and 5 indicate how the event weight is calculated for a given top quark mass in the $\nu$WT and MWT methods. In each method, we consider all solutions and jet assignments to get a total weight, $w_{\text{tot}}$, for a given $m_t$. In general, there are two ways to assign the two jets to the $b$ and $\bar{b}$ quarks. There are up to four solutions for each hypothesized value of the top quark mass. The likelihood for each assumed top quark mass $m_t$ is then given by the sum of the weights over all the possible solutions:

$$w_{\text{tot}} = \sum_i \sum_j w_{ij},$$  \hspace{1cm} (8)$$

where $j$ sums over the solutions for each jet assignment $i$. We repeat this calculation for both the $\nu$WT and MWT methods for a range of assumed top quark masses from 80 GeV through 330 GeV.

For each method, we also account for the finite resolution of jet and lepton momentum measurements. We repeat the weight calculation with input values for the measured momenta (or inverse momenta for muons) drawn from normal distributions centered on the measured values with widths equal to the known detector resolutions. We then average the weight distributions obtained from $N$ such variations:

$$w_{\text{tot}}(m_t) = N^{-1} \sum_{n=1}^{N} w_{\text{tot},n}(m_t),$$  \hspace{1cm} (9)$$

where $N$ is the number of samples. One important benefit of this procedure is that the efficiency of signal events to provide solutions increases. For instance, the $\nu$WT efficiency to find a solution for $t\bar{t} \rightarrow e\mu$ events is 95.9% without resolution sampling, while 99.5% provide solutions when $N = 150$. For the MWT analysis, events with $m_t = 175$ GeV yield an efficiency of 90% without resolution sampling. This rises to over 99% when $N = 500$. We use $N = 150$ and 500 for $\nu$WT and MWT, respectively.

Examples of a single event weight distribution for $\nu$WT and MWT are shown in Fig. 3 for two different simulated events. The sensitivities of the two methods are similar on average, but the different widths of the weight distributions vary significantly in both approaches on an event-by-event basis. This can be caused by an overall insensitivity of an event’s kinematic quantities to $m_t$, or to a different sensitivity when using those kinematic quantities with specific event reconstruction techniques.

Properties of the weight distribution are strongly correlated with $m_t$ if the top quark decay is as expected in
the standard model. For instance, Fig. 5(a) illustrates the correlation of the mean of the $\nu$WT weight distribution, $\mu_w$, with the generated top quark mass from the Monte Carlo. The relationship between the root-mean-square of the weight distribution, $\sigma_w$, and $\mu_w$ also varies with $m_t$, as shown in Fig. 5(b). There is the potential for non-standard decays of the top quark. For $m_t = 170$ GeV and assuming $\text{BR}(H^\pm \to \tau \nu) \sim 100\%$, we observe $\mu_w$ ($\nu$WT) to shift systematically upward when a $H^\pm$ boson of mass 80 GeV is present in the decay chain instead of a $W$ boson. When $\text{BR}(t \to H^\pm b) = 100\%$, this shift is 10%. Thus, the measurements of this paper are strictly valid only for standard model top quark decays.

V. EXTRACTING THE TOP QUARK MASS

We cannot determine the top quark mass directly from $\mu_w$ or from the most probable mass from the event weight distributions, $\text{max}_w$. Effects such as initial and final state radiation systematically shift these quantities from the actual top quark mass. In addition, the presence of background must be taken into account when evaluating events in the candidate sample. We therefore perform a maximum likelihood fit to distributions ("templates") of characteristic variables from the weight distributions. This fit accounts for the shapes of signal templates and for the presence of background.

A. Measurement Using Templates

We define a set of input variables characterizing the weight distribution for event $i$, denoted by $\{x_i\}_N$, where $N$ is the number of variables. Examples of $\{x_i\}_N$ might be the integrated weight in bins of a coarsely binned template, or they might be the moments of the weight distribution. A probability density histogram for simulated signal events, $h_s$, is defined as an $(N+1)$-dimensional histogram of input top quark mass vs. $N$ variables. For background, $h_b$ is defined as an $N$-dimensional histogram of the $\{x_i\}_N$. Both $h_s$ and $h_b$ are normalized to unity:

$$\int h_s(\{x_i\}_N | m_t) \, d\{x_i\}_N = 1, \quad (10)$$

$$\int h_b(\{x_i\}_N) \, d\{x_i\}_N = 1. \quad (11)$$

An example of a template for the MWT method is shown on Fig. 6. We measure $m_t$ from $h_s(\{x_i\}_N, m_t)$ and $h_b(\{x_i\}_N)$ using a maximum likelihood method. For each event in a given data sample, all $\{x_i\}_N$ are found and used for the likelihood calculation. We define a likelihood $\mathcal{L}$ as

$$\mathcal{L}(\{x_i\}_N, m_t, N_{\text{obs}} | m_t) = \prod_{i=1}^{N_{\text{obs}}} \frac{n_s h_s(\{x_i\}_N, m_t) + n_b h_b(\{x_i\}_N)}{n_s + n_b}, \quad (12)$$

where $N_{\text{obs}}$ is the number of events in the sample, $n_b$ is the number of background events, and $n_s$ is the signal event yield. We obtain a histogram of $-\ln \mathcal{L}$ vs. $m_t$ for the sample. We fit a parabola that is symmetric around the point with the highest likelihood (lowest $-\ln \mathcal{L}$). The fitted mass range is several times larger than the expected statistical uncertainty. It is chosen a priori to give the best sensitivity to the top quark mass using Monte Carlo pseudoexperiments, and is typically around $\pm 20$ GeV.

We obtain measurements of $m_t$ for several channels by multiplying the likelihoods of these channels:

$$-\ln \mathcal{L} = \sum_{\text{ch}} (-\ln \mathcal{L}_{\text{ch}}), \quad (13)$$

where "ch" denotes the set of channels. In this paper, we calculate overall likelihoods for the 2$\ell$ subset, $\text{ch} \in \{e\mu, ee, \mu\mu\}$; the $\ell$+track subset, $\text{ch} \in \{e+\text{track}, \mu+\text{track}\}$; and the five channel dilepton set, $\text{ch} \in \{e\mu, ee, \mu\mu, e+\text{track}, \mu+\text{track}\}$.

B. Choice of Template Variables

The choice of variables characterizing the weight distributions has been given some consideration in the past. For example, the D0 MWT analysis and CDF $\nu$WT analyses have used $\text{max}_w^{14,15}$. Earlier D0 $\nu$WT analyses
employed a multiparameter probability density technique using the coarsely binned weight distribution to extract a measure of $m_t$ \cite{13,14}.

For the MWT analysis described here, we use the single parameter approach. In particular, to extract the mass, we use Eq. (12) where $x_i = \{\text{max}_w\}$. We determine the values of $n_s$ and $n_b$ by scaling the sum of the expected numbers of signal ($\bar{n}_s$) and background ($\bar{n}_b$) events in Table I to the number observed in each channel. We fit the histogram of $-\ln L$ to a parabola using a 40 GeV wide mass range centered at the top quark mass with the minimal value of $-\ln L$ for all channels.

For the $\nu$WT analysis, we define the optimal set of input variables for the top quark mass extraction to be the set that simultaneously minimizes the expected statistical uncertainty and the number of variables. The coarsely binned weight distribution approach exhibits up to 20% better statistical performance than single parameter methods for a given kinematic reconstruction approach by using more information. However, over five bins are typically needed, and the large number of variables and their correlations significantly complicate the analysis. We study the performance of the method with many different choices of variables from the weight distributions. These include single parameter choices such as $\text{max}_w$ or $\mu_w$, which provide similar performance in the range $140 \text{ GeV} < m_t < 200 \text{ GeV}$.

Vectors of multiple parameters included various coarsely binned templates, or subsets of their bins. For the $\nu$WT analysis, we observe that individual event weight distributions have fluctuations which are reduced by considering bulk properties such as their moments. The most efficient parameters are the first two moments ($\mu_w$ and $\sigma_w$) of the weight distribution. This gives 16% smaller expected statistical uncertainty than using $\text{max}_w$ or $\mu_w$ alone. The improvement of the performance comes from the fact that $\sigma_w$ is correlated with $\mu_w$ for a given input top quark mass. This is shown in Fig. 5(b) for three different input top quark masses. The value of $\sigma_w$ helps to better identify the range of input $m_t$ that is most consistent with the given event having a specific $\mu_w$. This ability to de-weight incorrect $m_t$ assignments results in a narrower likelihood distribution and causes a corresponding reduction in the statistical uncertainty. No other choice of variables gives significantly better performance. The use of the weight distribution moments, $x_i = \{\mu_w, \sigma_w\}$, in $h_s$ and $h_b$ with Eq. (12) is termed $\nu$WT$_h$.

Because the templates are two-dimensional for background and three-dimensional for signal, a small number of bins are unpopulated. We employ a constant extrap-
olation for unpopulated edge bins using the value of the populated bin closest in \( m_t \) but having the same \( \mu_w \) and \( \sigma_w \). For empty bins flanked by populated bins in the \( m_t \) direction but with the same \( \mu_w \) and \( \sigma_w \), we employ a linear interpolation. We fit the histogram of \( -\ln L \) vs. \( m_t \) with a parabola. When performing the fit, the \( \nu WT_b \) approach determines \( n_s \) to be \( n_s = N_{\text{obs}} - \bar{n}_b \). Fit ranges of 50, 40, and 30 GeV are used for \( \ell + \tau \), \( 2\ell \), and all dilepton channels, respectively.

C. Probability Density Functions

In both methods described above, there are finite statistics in the simulated samples used to model \( h_s \) and \( h_b \), leading to bin-by-bin fluctuations. We address this in the \( \nu WT \) analysis by performing fits to \( h_s \) and \( h_b \) templates. We term this version of the \( \nu WT \) method \( \nu WT_f \). For the signal, we generate a probability density function \( f_s \) by fitting \( h_s \) with the functional form

\[
f_s(\mu_w, \sigma_w | m_t) = (\sigma_w + p_{13})^{p_s} \exp[-p_T (\sigma_w + p_{13})^{p_s}] \times \left[ (1-p_9) \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(\mu_w - m)^2}{2\sigma^2}\right] + p_9 \frac{p_{11}}{(1+p_{12})} (\mu_w - m) \right]^{p_{12}} \exp[-p_{11}(\mu_w - m)] \times \Theta(\mu_w - m) \right] \times \left\{ \int_0^\infty (x + p_{13})^{p_s} \exp[-p_T (x + p_{13})^{p_s}] dx \right\}^{-1}.
\]

The parameters \( m \) and \( \sigma \) are linear functions of \( \sigma_w \) and \( m_t \):

\[
m = p_0 + p_1 (\sigma_w - 36 \, \text{GeV}) + p_2 (m_t - 170 \, \text{GeV}),
\]

\[
\sigma = p_3 + p_4 (\sigma_w - 36 \, \text{GeV}) + p_5 (m_t - 170 \, \text{GeV}).
\]

Equations (13) and (15) are ad hoc functions determined empirically. A typical \( \chi^2 \) with respect to \( h_s \) is found in the \( e\mu \) channel which yields 4.0 per degree of freedom. The linear relationship between \( \sigma_w \) and \( \mu_w \) is shown in Fig. 7(a), which is an example of the probability density vs. \( \sigma_w \) and \( \mu_w \) for fixed input top quark mass of 170 GeV. The dependence of \( f_s \) on \( \sigma_w \) is expressed in the first line of Eq. (14). The second and third lines contain a Gaussian plus an asymmetrical function to describe the dependence on \( \mu_w \). The factors \( 1/(\sigma \sqrt{2\pi}) \) and \( p_{11} \right) / \Gamma(1 + p_{12}) \) and the integral in the fourth line normalize the probability density function to unity. Examples of two-dimensional slices of three-dimensional signal histograms for fixed input \( m_t = 170 \, \text{GeV} \) and \( \sigma_w = 30 \, \text{GeV} \) are shown in Fig. 7(a) and (c), respectively. The corresponding slices of the fit functions are shown in Fig. 7(b) and (d), respectively.

The background probability density function \( f_b(\mu_w, \sigma_w) \) is obtained as the normalized two-dimensional function of \( \mu_w \) and \( \sigma_w \) of simulated background events:

\[
f_b(\mu_w, \sigma_w) = \frac{\exp\left[-(p_1 \mu_w + p_2 \sigma_w - p_9)^2 - (p_4 \mu_w + p_5 \sigma_w - p_{13})^2 \right]}{\int_0^\infty \int_0^\infty \exp[-(p_1 x + p_2 y - p_9)^2 - (p_4 x + p_5 y - p_{13})^2] dx dy}
\]

This is also an ad hoc function determined empirically. The fit is performed to \( h_b \) containing the sums of all backgrounds for each channel and according to their expected yields. A typical \( \chi^2 \) with respect to \( h_b \) is found in the \( e\mu \) case which yields 5.2 per degree of freedom. Examples of \( h_b \) and \( f_b \) are shown in Fig. 8(a) and (b), respectively.

To measure \( m_t \), we begin by adding two extra terms to the likelihood \( L \) of Eq. (12). The first term is a constraint that requires that the fitted sum of the number of signal events \( n_s \) and the number of background events \( n_b \) within Poisson fluctuations with the number of observed events \( N_{\text{obs}} \):

\[
L_{\text{poisson}}(n_s + n_b, N_{\text{obs}}) \equiv \frac{(n_s + n_b)^{N_{\text{obs}}} \exp[-(n_s + n_b)]}{N_{\text{obs}}!}.
\]

The second term is a Gaussian constraint that requires agreement between the fitted number of background events \( n_b \) and the number of expected background events \( \bar{n}_b \) within Gaussian fluctuations, where the width of the Gaussian is given by the estimated uncertainty \( \delta_b \) on \( \bar{n}_b \):

\[
L_{\text{gauss}}(n_b, \bar{n}_b, \delta_b) \equiv \frac{1}{\sqrt{2\pi} \delta_b} \exp[-(n_b - \bar{n}_b)^2/2\delta_b^2].
\]

The total likelihood for an individual channel is given by

\[
L(\mu_{t1}, \sigma_{t1}, \bar{n}_b, N_{\text{obs}} | m_t, n_s, n_b) = \frac{N_{\text{obs}}}{\prod_{i=1}^{N_{\text{obs}}} n_s f_s(\mu_{t1}, \sigma_{t1} | m_t) + n_b f_b(\mu_{t1}, \sigma_{t1})}.
\]

The product extends over all events in the data sample. The maximum of the likelihood corresponds to the measured top quark mass. We simultaneously minimize \( -\ln L \) with respect to \( m_t, n_s, \) and \( n_b \) using MINUIT [30, 31]. The fitted sample composition is consistent with the expected one. This yields an analytic function of \( -\ln L \) vs. \( m_t \). Its statistical uncertainty is found by fixing \( n_s \) and \( n_b \) to their optimal values and taking half of distance between the points at which the \( -\ln L \) value is 0.5 units greater than its minimum value. We obtain measurements of \( m_t \) for several channels by minimizing the combined \( -\ln L \) simultaneously with respect to \( m_t \) and the numbers of signal and background events for the channels considered.

D. Pseudoexperiments and Calibration

The maximum likelihood fits attempt to account for the presence of background and for the signal and background shapes of the templates. For a precise measurement of \( m_t \), we must test for any residual effects that can
FIG. 7: Slices of probability density histograms \( h_s \) and fit functions \( f_s \) for the \( \nu WT \) method in the \( e\mu \) channel. Probability densities vs. \( \sigma_w \) and \( \mu_w \) for \( m_t = 170 \text{ GeV} \) are shown for (a) \( h_s \) and (b) \( f_s \). Probability densities vs. \( m_t \) and \( \mu_w \) for \( \sigma_w = 30 \text{ GeV} \) are shown for (c) \( h_s \) and (d) \( f_s \).

cause a shift in the relationship between the fitted and actual top quark masses. We test our fits and extract correction factors for any observed shifts by performing pseudoexperiments.

A pseudoexperiment for each channel is a set of simulated events of the same size and composition as the selected dataset given in Table I. We compose it by randomly drawing simulated events out of the large Monte Carlo event pool. Within a given pool, each Monte Carlo event has a weight based on production information and detector performance parameters. An example of the latter is the \( b \)-tagging efficiency which depends on jet \( p_T \) and \( \eta \), and an example of former is the weight with which each event is generated by aLPGEN. We choose a random event, and then accept or reject it by comparing the event weight to a random number. In this way, our pseudoexperiments are constructed with the mix of events that gives the correct kinematic distributions.

For MWT, we compose pseudoexperiments by drawing Monte Carlo events from signal and background samples with probabilities proportional to the numbers of events expected, \( \bar{n}_s \) and \( \bar{n}_b \). Thus, we draw events for each source based on a binomial probability. In the \( \nu WT \) pseudoexperiments, the number of background events of each source is Poisson fluctuated around the expected yields of Table I. The remaining events in the pseudoexperiment are signal events. If the sum of backgrounds totals more than \( N_{\text{obs}} \), then the extra events are dropped and \( n_s = 0 \). In this way, we do not use the \( tt \) production cross section, which is a function of \( m_t \).
To establish the relationship between the fitted top quark mass, $m_t^{\text{fit}}$, and the actual generated top quark mass $m_t$, we assemble a set of many pseudoexperiments for each input mass. For the νWT method, we choose 300 pseudoexperiments because that is the point at which the Monte Carlo statistics begin to be oversampled, and all pseudoexperiments can still be considered statistically independent. We average $m_t^{\text{fit}}$ for each input $m_t$ for each channel. We combine channels according to Eq. 13, and we fit the dependence of this average mass on $m_t$ with

$$\langle m_t^{\text{fit}} \rangle = \alpha (m_t - 170 \text{ GeV}) + \beta + 170 \text{ GeV}. \quad (20)$$

The calibration points and fit functions are shown in Fig. 9. The results of the fits are summarized in Table III. Ideally, $\alpha$ and $\beta$ should be unity and zero, respectively. The $m_t^{\text{fit}}$ of each pseudoexperiment and data measurement is corrected for the slopes and offsets given in Table III by

$$m_t^{\text{meas}} = \alpha^{-1}(m_t^{\text{fit}} - \beta - 170 \text{ GeV}) + 170 \text{ GeV}. \quad (21)$$

The pull is defined as

$$\text{pull} = m_t^{\text{meas}} - m_t^{\text{fit}} / \sigma(m_t^{\text{meas}}), \quad (22)$$

where $\sigma(m_t^{\text{meas}}) = \alpha^{-1}\sigma(m_t^{\text{fit}})$ is the measured statistical uncertainty after the calibration of Eq. 21. The ideal pull distribution has a Gaussian shape with a mean of zero and a width of one. The pull widths from pseudoexperiments are given in Table III. A pull width larger (less) than one indicates an underestimated (overestimated) statistical uncertainty. The uncertainty of the data measurement is corrected for deviations of the pull width from one as well as for the slope of the calibration curve. The mean of the distribution of calibrated and pull-corrected statistical uncertainties yields the expected statistical uncertainty (see Table III). Figure 10 shows the pull-width-corrected distribution of statistical uncertainties for the $m_t$ measurements from the ensemble testing. The expected uncertainty on the combined measurement for all channels is 5.1, 5.3, and 5.8 GeV for νWT$_h$, νWT$_f$, and MWT, respectively.

VI. RESULTS

The calibrated mass and statistical uncertainties for the 2ℓ, $\ell$+track, and their combination are shown in Table III for each of the three methods. The $-\ln \mathcal{L}$ fits from the νWT$_h$, νWT$_f$, and MWT methods, including data points, are shown in Fig. 11. There are no data points for the νWT$_f$ fit since the corresponding curve is a one-dimensional slice of an analytic three-dimensional fit function, $f_s$. The calibrated statistical uncertainties determined in data from these likelihood curves are shown by arrows in Fig. 11. The statistical uncertainties agree with the expectations from ensemble testing.

A. Systematic Uncertainties

The top quark mass measurement relies substantially on the Monte Carlo simulation of $t\bar{t}$ signal and backgrounds. While we have made adjustments to this model to account for the performance of the detector, residual uncertainties remain. The limitations of modeling of physics processes may also affect the measured mass. There are several categories of systematic uncertainties: modeling of physics processes, modeling of the detector response, and the method. We have estimated each of...
FIG. 9: The combined calibration curves corresponding to the (a) $\nu WT_h$, (b) $\nu WT_f$, and (c) MWT methods. Overlaid is the result of the linear fit as defined in Eq. 20. The uncertainties are small and corresponding bars are hidden by the markers.

TABLE II: Slope ($\alpha$) and offset ($\beta$) from the linear fit in Eq. 20 to the pseudoexperiment results of Fig. 9 for the 2$\ell$, $\ell$+track, and combined dilepton channel sets using the MWT and $\nu WT$ methods.

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>$\nu WT_h$</td>
<td>2$\ell$</td>
<td>0.98 ± 0.01</td>
<td>-0.04 ± 0.11</td>
<td>1.02 ± 0.02</td>
<td>5.8</td>
</tr>
<tr>
<td>$\nu WT_h$</td>
<td>$\ell$+track</td>
<td>0.92 ± 0.02</td>
<td>2.28 ± 0.27</td>
<td>1.04 ± 0.02</td>
<td>13.0</td>
</tr>
<tr>
<td>$\nu WT_h$ combined</td>
<td>0.99 ± 0.01</td>
<td>-0.04 ± 0.11</td>
<td>1.03 ± 0.02</td>
<td></td>
<td>5.1</td>
</tr>
<tr>
<td>$\nu WT_f$</td>
<td>2$\ell$</td>
<td>1.03 ± 0.01</td>
<td>-0.32 ± 0.15</td>
<td>1.06 ± 0.02</td>
<td>5.8</td>
</tr>
<tr>
<td>$\nu WT_f$</td>
<td>$\ell$+track</td>
<td>1.07 ± 0.03</td>
<td>-0.04 ± 0.37</td>
<td>1.07 ± 0.02</td>
<td>12.9</td>
</tr>
<tr>
<td>$\nu WT_f$ combined</td>
<td>1.04 ± 0.01</td>
<td>-0.45 ± 0.13</td>
<td>1.06 ± 0.02</td>
<td></td>
<td>5.3</td>
</tr>
<tr>
<td>MWT</td>
<td>2$\ell$</td>
<td>1.00 ± 0.01</td>
<td>0.95 ± 0.05</td>
<td>0.98 ± 0.01</td>
<td>6.3</td>
</tr>
<tr>
<td>MWT</td>
<td>$\ell$+track</td>
<td>0.99 ± 0.01</td>
<td>0.64 ± 0.12</td>
<td>1.06 ± 0.01</td>
<td>13.8</td>
</tr>
<tr>
<td>MWT combined</td>
<td>0.99 ± 0.01</td>
<td>0.97 ± 0.05</td>
<td>0.99 ± 0.01</td>
<td></td>
<td>5.8</td>
</tr>
</tbody>
</table>

these as follows.

1. Physics Modeling

   a. *b fragmentation* A systematic uncertainty arises from the different models of $b$ quark fragmentation, namely the distribution of the fraction of energy taken by the heavy hadron. The standard D0 simulation used for this analysis utilized the default PYTHIA tune in the Bowler scheme [32]. We reweighted our $t\bar{t}$ simulated samples to reach consistency with the fragmentation model measured in $e^+e^- \rightarrow Z \rightarrow bb$ decays [33]. A systematic uncertainty is assessed by comparing the measured $m_t$ in these two scenarios.

   b. Underlying event model An additional systematic uncertainty can arise from the underlying event model. We compare measured top quark masses for the PYTHIA tune DW [34] with the nominal model (tune A).

c. Extra jet modeling Extra jets in top quark events from gluon radiation can affect the $t\bar{t}$ $p_T$ spectrum, and therefore the measured $m_t$. While our models describe the data within uncertainties for all channels, the ratio of the number of events with only two jets to those with three or more jets is typically four in the Monte Carlo and three in the data. To assess the affect of this difference, we reweight the simulated events with a top quark mass of 170 GeV so that this ratio is the same. Pseudoexperiments with reweighted events are compared to the nominal pseudoexperiments to determine the uncertainty.

d. Event generator There is an uncertainty in event kinematics due to the choice of an event generator. This can lead to an uncertainty in the measured top quark mass. To account for variations in the accuracy of $t\bar{t}$ generators, we compare pseudoexperiment results using $t\bar{t}$ events generated with ALPGEN to those generated with PYTHIA for $m_t = 170$ GeV. The difference between the two estimated masses is corrected by subtracting the expected statistical uncertainty divided by the square
FIG. 10: Distribution of statistical uncertainties for top quark mass measurements of pseudoexperiments for the combination of all channels for simulated events with $m_t = 170$ GeV for the (a) $\nu W T_h$, (b) $\nu W T_f$, and (c) MWT methods. The uncertainties are corrected by the calibration curve and for the pull width. The arrows indicate the statistical uncertainties for the measured top quark mass.

TABLE III: Calibrated fitted $m_t$ for the $\nu W T_h$, $\nu W T_f$, and MWT methods. All uncertainties are statistical.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\nu W T_h$ [GeV]</th>
<th>$\nu W T_f$ [GeV]</th>
<th>MWT [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\ell$</td>
<td>177.5 ± 5.8</td>
<td>176.1 ± 5.8</td>
<td>176.6 ± 5.8</td>
</tr>
<tr>
<td>$\ell +\text{track}$</td>
<td>170.7 ± 12.3</td>
<td>174.6 ± 13.8</td>
<td>165.0 ± 8.5</td>
</tr>
<tr>
<td>combined</td>
<td>176.3 ± 4.9</td>
<td>176.0 ± 5.3</td>
<td>173.2 ± 4.9</td>
</tr>
</tbody>
</table>

root of the number of pseudoexperiments.

e. PDF variations The top quark mass measurement relies on Monte Carlo events generated with a particular PDF set (CTEQ6L1). Moreover, this PDF set is used directly by the MWT method. We estimate the resulting uncertainty on $m_t$ by reweighting the Monte Carlo according to the different eigenvectors of the CTEQ6L1 PDFs. For each choice, we measure a new mass and the difference between the mass obtained with reweighting and a nominal mass is computed. The resulting uncertainty is the sum in quadrature of all above uncertainties.

f. Background template shape The uncertainties on the background kinematics can affect the template shapes and consequently the measured top quark mass. The uncertainty from the background template shape is found by substituting simulated WW events for all $Z$ backgrounds (including $Z \rightarrow \tau\tau$) in all pseudoexperiments. The uncertainty is taken as the difference between the average measured top quark mass with this assumption and the nominal value.

2. Detector Modeling

a. Jet energy scale Because the $b$ jets carry the largest share of the energy in top quark events, their calibration has the largest affect on the uncertainty on $m_t$. Ideally, the procedure to calibrate jet energies in data and Monte Carlo achieves the same energy scale in both. However, each procedure yields a systematic uncertainty. We estimate the resulting uncertainty by repeating the pseudoexperiments with simulated events in which the jet energies are shifted up and down by the known $p_T$ and $\eta$ dependent uncertainty, obtained by
FIG. 11: Negative log-likelihood $-\ln \mathcal{L}$ vs. $m_t$ for the combination of all channels before calibration for the (a) $\nu WT_h$, (b) $\nu WT_f$, and (c) MWT methods.
summing in quadrature uncertainties on the data and Monte Carlo jet energy scales. The probability density histograms \((h_s, h_b)\) and functions \((f_s, f_b)\) are left with the nominal calibration.

b. \(b/\)light quark response ratio This uncertainty arises from the fact that the jets in signal events are primarily \(b\) jets. These have a different detector response than the light quark and gluon jets which dominate the \(\gamma+\text{jet}\) sample used to derive the overall jet energy calibration. By applying this calibration to the \(b\) jet sample, a 1.8\% shift in jet \(p_T\) is observed \[5\]. We adjust the jets in the Monte Carlo for this and propagate the correction into the \(E_T\). This causes a shift in the measured \(m_t\) which is taken as an uncertainty.

c. Sample dependent jet energy scale After the initial calibration, a residual shift in jet \(p_T\) distributions is observed in \(Z+\)jets events when comparing data and Monte Carlo. We adopt a further calibration that improves agreement in these distributions and apply it to all of our background samples. Because this correction may not apply to \(tt\) events, we take the shift in the measured \(m_t\) to be a systematic uncertainty.

d. Object resolution The jet resolution from the simulation is better than that observed in data. To improve the agreement, we apply an additional smearing to Monte Carlo events. A residual difference between data and Monte Carlo jet resolutions can lead to a mass bias. To estimate the effect on the \(m_t\) measurement, we repeat the pseudoexperiments by adjusting this smearing up and down within its uncertainty while keeping \(h_s\) and \(h_b\) with the nominal resolutions. We estimate the systematic uncertainties arising from the muon, isolated track, and electron \(p_T\) resolutions in a similar way.

e. Jet identification Jet reconstruction and identification efficiency in Monte Carlo is corrected to match the data. We propagate the uncertainty on the correction factor to the top quark mass measurement.

f. Monte Carlo corrections Residual uncertainties on the Monte Carlo corrections exist for triggering, luminosity profiles, lepton identification, and \(b\)-tagging. These uncertainties affect the top quark mass uncertainties. In each case, a respective systematic uncertainty on \(m_t\) is found by reweighting events according to the uncertainties of Monte Carlo correction factors introduced to reproduce the data.

3. Method

a. Background yield Due to the limited statistics of the simulation, there is some uncertainty in the event yields for the background processes. This uncertainty affects the likelihood and the measured top quark mass via Eq. \[12\]. To estimate the effect of the uncertainty on background event yield, we vary the total background yield by its known uncertainty up and down keeping the relative ratios of individual background processes constant.

b. Template statistics The templates used in the MWT and \(\nu\text{WT}_b\) methods have finite statistics. Local fluctuations in these templates can cause local fluctuations in the individual likelihood fits and the top quark mass. We obtain an uncertainty in \(m_t\) by varying the \(-\ln\mathcal{L}\) points from the data ensemble within their errors. The width of the \(m_t\) distribution provides the systematic uncertainty.

For the \(\nu\text{WT}_f\) method, the \(f_s\) function depends on fifteen parameters, each of which has a corresponding uncertainty. Consequently, there is some uncertainty on the shape of this function. There is a corresponding uncertainty on the parameters of the \(f_b\). The uncertainty on the shape causes the fit uncertainty on the measured top quark mass. We find the impact of this uncertainty on the data sample by varying the parameters of \(f_s\) and \(f_b\) within their uncertainties. For each such variation, we remeasure \(m_t\) for the data sample. The fit uncertainty is the width of this distribution.

c. Monte Carlo calibration There is an uncertainty on fitting the parameters (slope and offset) of the calibration curve. This uncertainty causes an uncertainty in the calibrated top quark mass. The calibration uncertainty is obtained as the uncertainty of the offset.

A summary of estimated systematic errors for the combined dilepton channels is provided in Table \[IV\]. We assumed the systematic uncertainties for all three methods to be completely correlated for each source of systematic uncertainty and uncorrelated among different sources. The correlations of statistical uncertainties are given in the next Section. All uncertainties are corrected for the slope of the mass scale calibration. The total uncertainty is found by assuming all the contributions are independent and adding them in quadrature.

B. Combined Results

Because the statistical use of the \(\nu\text{WT}\) moments template is different between the \(\nu\text{WT}_b\) and \(\nu\text{WT}_f\) methods, ensemble tests show that these two measurements are only 85\% correlated. We combine them using the best linear unbiased estimator (BLUE) method \[36\] to form a final \(\nu\text{WT}\) measurement. Applying the correlation to the measurements from data, we obtain a final \(\nu\text{WT}\) measurement of \(m_t = 176.2 \pm 4.8\text{(stat)} \pm 2.1\text{(sys)}\) GeV for the combination of all five channels. We treat all systematic uncertainties as 100\% correlated across methods except for the Monte Carlo calibration and template
TABLE IV: Summary of systematic uncertainties for the combined analysis of all dilepton channels. The $\nu WT_h$, $\nu WT_f$, and MWT method results are shown.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>$\nu WT_h$ [GeV]</th>
<th>$\nu WT_f$ [GeV]</th>
<th>MWT [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ fragmentation</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Underlying event modeling</td>
<td>0.3</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Extra jets modeling</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Event generator</td>
<td>0.6</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>PDF variation</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Background template shape</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Jet energy scale (JES)</td>
<td>1.5</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>$b$/light response ratio</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Sample dependent JES</td>
<td>0.4</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Muon/track $p_T$ resolution</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Electron energy resolution</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Jet identification</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>MC corrections</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Background yield</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Template statistics</td>
<td>0.8</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>MC calibration</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>2.1</td>
<td>2.3</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Statistics uncertainties. These are treated as uncorrelated and 85% correlated, respectively. The individual systematic uncertainties on the $\nu WT$ combination are the same as those for the $\nu WT_h$ method to the precision given in Table IV.

The final MWT measurement is $m_t = 173.2 \pm 4.9$(stat) $\pm 2.0$(sys) GeV for the combination of all dilepton channels. The total systematic uncertainties are 2.0 GeV and 2.4 GeV for the 2$\ell$ and $\ell$+track MWT results, respectively. The $\nu WT$ and MWT approaches use partially different information from each $t\bar{t}$ event; the two results are measured to be 61% correlated. Therefore, we use the BLUE method to determine an overall measurement of $m_t = 174.7 \pm 4.4$(stat) $\pm 2.0$(sys) GeV for the combination of all dilepton channels. We treat all systematic uncertainties as 100% correlated across methods except for two uncertainties. The Monte Carlo calibration and template statistics systematic uncertainties are treated as uncorrelated and 61% correlated, respectively. The channel-specific results for both measurement combinations are given in Table V.

VII. CONCLUSIONS

In 1 fb$^{-1}$ of $p\bar{p}$ collision data from the Fermilab Tevatron collider, we employed two mass extraction methods to measure $m_t$ in $t\bar{t}$ events with two high $p_T$ final state leptons. We analyzed three channels with two fully identified leptons ($ee$, $\mu\mu$, and $e\mu$) and two channels with relaxed lepton selection and a $b$-tagged jet ($e+\text{track}$ and $\mu+\text{track}$). Using the $\nu WT$ event reconstruction, we perform a maximum likelihood fit to the first two moments of the resulting distribution of relative weight vs. $m_t$ to measure

$$m_t = 176.2 \pm 4.8$(stat) $\pm 2.1$(sys) GeV. \hspace{1cm} (23)$$

We also employ the MWT method using a fit to the mass giving the maximum weight. We measure

$$m_t = 173.2 \pm 4.9$(stat) $\pm 2.0$(sys) GeV. \hspace{1cm} (24)$$

Accounting for correlations in these two measurements, we obtain a final combined result of

$$m_t = 174.7 \pm 4.4$(stat) $\pm 2.0$(sys) GeV. \hspace{1cm} (25)$$

Our result is consistent with the current world average value of $m_t$ \[37\].

We thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF and KOSEF (Korea); CONICET and UBACyT (Argentina); FOM (The Netherlands); STFC (United Kingdom); MSMT and GACR (Czech Republic); CRC Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); The Swedish Research Council (Sweden); CAS and CNSF (China); and the Alexander von Humboldt Foundation (Germany).
TABLE V: Final results for the $\nu$WT method and $\nu$WT+MWT combination.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\nu$WT [GeV]</th>
<th>$\nu$WT+MWT [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$\ell$</td>
<td>177.1 ± 5.4(stat) ± 2.0(sys)</td>
<td>176.9 ± 4.8(stat) ± 1.9(sys)</td>
</tr>
<tr>
<td>$\ell$+track</td>
<td>171.2 ± 12.3(stat) ± 2.7(sys)</td>
<td>165.7 ± 8.4(stat) ± 2.4(sys)</td>
</tr>
<tr>
<td>combined</td>
<td>176.2 ± 4.8(stat) ± 2.1(sys)</td>
<td>174.7 ± 4.4(stat) ± 2.0(sys)</td>
</tr>
</tbody>
</table>