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B_q^0-$\bar{B}_q^0$ Mixing and Matching with Fermilab Heavy Quarks

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We discuss the matching procedure for heavy-light 4-quark operators using the Fermilab method for heavy quarks and staggered fermions for light quarks. These ingredients enable us to construct the continuum-limit operator needed to determine the oscillation frequency of neutral B mesons. The matching is then carried out at the one-loop level. We also present an updated preliminary result for the SU(3)-breaking ratio ξ, based on calculations using the MILC Collaboration's ensembles of lattice gauge fields.

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1. Introduction

All neutral mesons—K^0, B^0, B_s, D^0—have been observed to oscillate from particle to antiparticle. The oscillation frequency ΔM tests the Standard Model's pattern of flavor violation. The phenomenology is especially simple for neutral B mesons (normal and strange), because the flavor-changing dynamics play out predominantly at distances much shorter than the scale of QCD. In the case of the B mesons, the width difference $\Delta\Gamma$ of the two propagating eigenstates also arises predominantly at short distances. It is especially intriguing (at least for now), because measurements of $\Delta\Gamma_s$ and the CP phase ϕ_s of the B_s are in imperfect agreement with the Standard Model [1, 2].

Neutral B mixing stems from $\Delta B = 2$ flavor-changing transitions. In the Standard Model these arise first at the one-loop level, so non-Standard contributions are conceivably of comparable size. The observables are then (approximately) $\Delta M = 2|M_{12}|$, $\Delta\Gamma = 2|\Gamma_{12}|\cos\phi$, and $\phi = \arg(-M_{12}/\Gamma_{12})$, where M_{12} and Γ_{12} are the off-diagonal elements of the mass and width matrices of the two-state systems:

$$M_{12} = \frac{G_F^2}{8\pi^2}\frac{M_W^2}{M_{B_q}^2}(V_{tq}^* V_{tb})^2 S_0(m_t^2/M_W^2)\eta_b(\mu)\langle B|\bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b|\bar{B}\rangle + \text{BSM}, \tag{1.1}$$

$$\Gamma_{12} = -\frac{G_F^2 m_b^2}{6\pi M_{B_q}}\left[G(V,\mu)\langle B|\bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b|\bar{B}\rangle + G_S(V,\mu)\langle B|\bar{q}_L b\bar{q}_L b|\bar{B}\rangle\right] + \text{BSM}, \tag{1.2}$$

where V is the CKM matrix, and S_0, η_b, G, and G_S are short-distance effects, computed in electroweak and QCD perturbation theory. Contributions beyond the Standard Model ("BSM") are not written out explicitly. Because of the $V-A$ structure of the electroweak interaction, only the left-handed (light) quark field $\bar{q}_L = \bar{q}\frac{1}{2}(1+\gamma_5)$ appears.

The remainder of this paper is organized as follows. Section 2 constructs lattice operators with staggered light quarks and Fermilab heavy quarks, corresponding to the 4-quark operators in Eqs. (1.1) and (1.2). (The construction suffices for any light quark with chiral symmetry and heavy quark with heavy-quark symmetry.) We give a status report of our numerical results in Sec. 3. Section 4 summarizes and presents some of our plans for the future.

2. Short-Distance Matching

To compute the hadronic matrix elements in Eqs. (1.1) and (1.2), one has to derive an expression in lattice gauge theory that approximates well $\bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b$ and $\bar{q}_L b\bar{q}_L b$. The lattice operators can then be computed, and the numerical and other uncertainties estimated, to determine M_{12} and Γ_{12}. Similar operators appear BSM, for which the following derivation serves as a template.

For the light valence quark we take naive asqtad propagators

$$\langle\Upsilon(x)\bar{\Upsilon}(y)\rangle_U = \Omega(x)\Omega^{-1}(y)\langle\chi(x)\bar{\chi}(y)\rangle_U, \tag{2.1}$$

where χ is the one-component staggered fermion field; Υ is a 4-component naive field, and $\langle\cdots\rangle_U$ denotes the fermion average in a fixed gauge field U. For the heavy quark we use

$$\Psi = [1 + d_1(m_0 a)\boldsymbol{\gamma}\cdot\boldsymbol{D}]\psi, \tag{2.2}$$

where ψ is the fermion field appearing in the Fermilab action [3] or an improved action with the same design features [4].

We aim to construct lattice operators Q and Q_S such that

$$Q \doteq \bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b + \mathrm{O}(a^2), \tag{2.3}$$

$$Q_S \doteq \bar{q}_L b\bar{q}_L b + \mathrm{O}(a^2), \tag{2.4}$$

where $\doteq$ means "has the same matrix elements as." Here the $\mathrm{O}(a^2)$ term depends on $m_b a$. As long as one retains small corrections to heavy-quark symmetry, it remains bounded even as $m_b a \to \infty$; as long as certain Dirac off-diagonal improvements are consistently introduced [3, 4], they vanish as $a \to 0$. These two elements are the essence of the Fermilab method.

Our construction starts with the lattice operators $\bar{\Upsilon}_L\gamma_\mu\Psi\bar{\Upsilon}_L\gamma^\mu\Psi$ and $\bar{\Upsilon}_L\Psi\bar{\Upsilon}_L\Psi$. According to the HQET theory of cutoff effects [5, 6, 7], these lattice operators can be described by

$$\bar{\Upsilon}_L\gamma_\mu\Psi\bar{\Upsilon}_L\gamma^\mu\Psi \doteq 2C^{\mathrm{lat}}\bar{q}_L\gamma_\mu h^{(+)}\bar{q}_L\gamma^\mu h^{(-)} + 2\delta C^{\mathrm{lat}}\bar{q}_L h^{(+)}\bar{q}_L h^{(-)} + \sum_{i=1}^{5} B_i^{\mathrm{lat}}\mathscr{Q}_i + \cdots, \tag{2.5}$$

$$\bar{\Upsilon}_L\Psi\bar{\Upsilon}_L\Psi \doteq 2\delta C_S^{\mathrm{lat}}\bar{q}_L\gamma_\mu h^{(+)}\bar{q}_L\gamma^\mu h^{(-)} + 2C_S^{\mathrm{lat}}\bar{q}_L h^{(+)}\bar{q}_L h^{(-)} + \sum_{i=1}^{5} B_{Si}^{\mathrm{lat}}\mathscr{Q}_i + \cdots, \tag{2.6}$$

where $h^{(\pm)}$ are the heavy-quark fields of the heavy-quark effective theory (HQET), satisfying $h^{(\pm)} = \frac{1}{2}(1 \pm \gamma_4)h^{(\pm)}$. The sums are over five dimension-7, $\Delta B = 2$, four-quark operators, similar to those written out, but with an extra derivative. The series continues with operators of dimension 8 and higher. On the right-hand side of Eqs. (2.5) and (2.6) the operators are to be understood with some continuum regulator and renormalization scheme. Discretization effects are lumped into the short-distance coefficients $C_{(S)}^{\mathrm{lat}}$, $\delta C_{(S)}^{\mathrm{lat}}$, and $B_{(S)i}^{\mathrm{lat}}$, which depend on the couplings of the lattice action, as well as the lattice spacing a and the (renormalized) gauge coupling and quark masses.

The next step is to note that the target operators have a completely parallel description in HQET, namely

$$\bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b \doteq 2C\bar{q}_L\gamma_\mu h^{(+)}\bar{q}_L\gamma^\mu h^{(-)} + 2\delta C\bar{q}_L h^{(+)}\bar{q}_L h^{(-)} + \sum_{i=1}^{5} B_i\mathscr{Q}_i + \cdots, \tag{2.7}$$

$$\bar{q}_L b\bar{q}_L b \doteq 2\delta C_S\bar{q}_L\gamma_\mu h^{(+)}\bar{q}_L\gamma^\mu h^{(-)} + 2C_S\bar{q}_L h^{(+)}\bar{q}_L h^{(-)} + \sum_{i=1}^{5} B_{Si}\mathscr{Q}_i + \cdots, \tag{2.8}$$

where the (continuum HQET) operators on the right-hand sides of Eqs. (2.7) and (2.8) are precisely the same as those on the right-hand sides of Eqs. (2.5) and (2.6). The coefficients differ, however, because the lattice does not appear on the left-hand side of Eqs. (2.7) and (2.8).

With Eqs. (2.5)–(2.8) the desired construction of Q and Q_S is immediate:

$$Q = Z\bar{\Upsilon}_L\gamma_\mu\Psi\bar{\Upsilon}_L\gamma^\mu\Psi + \delta Z\bar{\Upsilon}_L\Psi\bar{\Upsilon}_L\Psi + \sum_i b_i Q_i, \tag{2.9}$$

$$Q_S = Z_S\bar{\Upsilon}_L\Psi\bar{\Upsilon}_L\Psi + \delta Z_S\bar{\Upsilon}_L\gamma_\mu\Psi\bar{\Upsilon}_L\gamma^\mu\Psi + \sum_i b_{Si} Q_i, \tag{2.10}$$

where the Q_i are lattice discretizations of the $\mathscr{Q}_i$, such that $Q_i \doteq C_{ij}^{\mathrm{lat}}\mathscr{Q}_j + \text{dimension 8}$. Simple algebra then shows that if

$$Z = \left[CC_S^{\mathrm{lat}} - \delta C\delta C_S^{\mathrm{lat}}\right] / \left[C^{\mathrm{lat}}C_S^{\mathrm{lat}} - \delta C^{\mathrm{lat}}\delta C_S^{\mathrm{lat}}\right], \tag{2.11}$$

$$\delta Z = \left[\delta C - Z\delta C^{\mathrm{lat}}\right] / C_S^{\mathrm{lat}}, \tag{2.12}$$

$$b_i = \left[B_j - ZB_j^{\mathrm{lat}} - \delta ZB_{Sj}^{\mathrm{lat}}\right] C^{\mathrm{lat}\,-1}_{\;\;ji}, \tag{2.13}$$

then Eq. (2.3) is satisfied. Similar expressions exist for Z_S, δZ_S, and b_{Si}, such that Eq. (2.4) is satisfied. From the structure of Eqs. (2.11)–(2.13) it is clear that the regulator and renormalization scheme dependence of the HQET drops out of $Z_{(S)}$, $\delta Z_{(S)}$, and $b_{(S)i}$.

Let us close this section with a few remarks. The enumeration of the operators $\mathscr{Q}_i$, and further operators of dimension 8, is an easy extension of Ref. [6]. In perturbation theory $C_{(S)}$ ($\delta C_{(S)}$ and the B_i) start at tree (one-loop) level, but they could also be determined nonperturbatively, adapting schemes such as that of Ref. [8]. Because of the way Fermilab lattice actions are constructed [3, 4], starting with Wilson fermions, one has $\lim_{a\to 0} C^{\rm lat} = C$, *etc.*, without fine tuning. (In lattice NRQCD this is possible only with fine tuning.) Although our derivation hinges on the HQET description of cutoff effects, one could also (for $m_b a \ll 1$) use the Symanzik theory; the results for $Z_{(S)}$, $\delta Z_{(S)}$, and $b_{(S)i}$ would be the same.

We have embarked on a one-loop calculation of $Z_{(S)}$ and $\delta Z_{(S)}$. At present they are being checked. As with currents [6, 7], it may prove prudent to write

$$Z_{(S)} = Z_{V_{bb}} Z_{V_{qq}} \rho_{(S)}, \tag{2.14}$$

where $Z_{V_{bb}}$ and $Z_{V_{qq}}$ are nonperturbatively determined matching factors for the vector current. The remaining factor $\rho_{(S)}$ could have a tamer perturbative expansion, because of cancellation among diagrams. We do not expect the cancellation to be as good as in the case of currents, because 4-quark operators have new diagrams in which a gluon is exchanged from one bilinear to the other.

With the rotation of Eq. (2.2), the $b_{(S)i}$ in Eqs. (2.9) and (2.10) are of order α_s and are not available. The calculations of the 4-quark operator matrix elements described below thus have discretization errors of the form

$$\frac{B_{(S)i}\langle \mathscr{Q}_i\rangle}{\langle \bar{q}_L\gamma_\mu b \bar{q}_L \gamma^\mu b\rangle} \sim a\Lambda \frac{\alpha_s}{2(1+m_0 a)}, \tag{2.15}$$

$$\text{dim 8 ops} \sim a^2\Lambda^2 f(m_0 a), \tag{2.16}$$

where the mass dependence of the $B_{(S)i}$ is an Ansatz with the correct asymptotic behavior as $m_0 \to \infty$ and as $m_0 a \to 0$ for the Fermilab action. The functions $f(m_0 a)$ multiplying the $\mathrm{O}(a^2)$ discretization effects are known for the Fermilab action [6, 9].

3. Long-Distance Matrix Elements

To compute the matrix elements we use a data-object called the open-meson propagator [10]. Valence quark propagators are started at an origin $(\boldsymbol{x}_0, t_0)$, where the 4-quark operator sits, out to all $(\boldsymbol{x}, t)$. Since, for this problem, we are interested only in zero-momentum pseudoscalars, at each t the Dirac indices are contracted with γ_5, and this contraction is summed over all $\boldsymbol{x}$. On the other hand, M_{12} and Γ_{12} require two (several) Dirac structures in (beyond) the Standard Model. Therefore we leave the Dirac and color indices free at $(\boldsymbol{x}_0, t_0)$, writing out one $12 \times 12 \times N_4$ data-object per configuration, where N_4 is the total number of time slices. Three-point functions are formed by contracting open-meson propagators at times t_i and t_f with the Dirac structure of each 4-quark operator. Two-point functions from t_0 to t are used to normalize the matrix elements and to provide a cross-check with our separate calculations of B-meson decay constants [11].

Our calculations are carried out on several ensembles of lattice gauge fields with a realistic sea of 2+1 flavors, made available by the MILC Collaboration [12, 13]. The ensembles used here are listed in Table 1 together with the valence quark masses. The sea quarks are simulated with the asqtad action for staggered quarks, and with the fourth-root procedure to reduce the number of species from 4 to 1.

To discuss the analysis, it is helpful to introduce some notation. The four-quark matrix elements are written

$$\langle B_q^0|\bar{\Upsilon}_L\gamma_\mu\Psi\bar{\Upsilon}_L\gamma^\mu\Psi|\bar{B}_q^0\rangle = \tfrac{2}{3}M_{B_q}\beta_q^2, \tag{3.1}$$

where the quantity β_q is well-behaved in the heavy-quark limit. We extract β_s and β_d from 2- and 3-point functions. With staggered valence quarks these correlators have contributions from wrong-parity states with time dependence $(-1)^{t/a}$. We are careful to disentangle these states. To isolate the ground state we use Bayesian fits, varying the number of states.

We then carry out a partially-quenched (i.e., m_q and m_l varying independently) chiral extrapolation of β_q/β_s to obtain β_d/β_s, using rooted staggered chiral perturbation theory for β_q [14, 15]. With more valence masses than sea masses, the effects of partial quenching constrain the parameters of χPT more stringently than would unitary ($m_q = m_l$) data alone. Fitting the ratio β_d/β_s yields smaller statistical errors than fitting $r_1^{3/2}\beta_q$ directly. We also carry out a chiral extrapolation of $r_1^{3/2}\beta_s$, which is mild, because it depends only on the sea masses (am_l, am_h).

In the phenomenology of B-$\bar{B}$ mixing it is conventional to write the matrix element as

$$\langle B_q^0|\bar{q}_L\gamma_\mu b\bar{q}_L\gamma^\mu b|\bar{B}_q^0\rangle = \tfrac{2}{3}f_{B_q}^2 M_{B_q}^2 B_{B_q}. \tag{3.2}$$

Neglecting $Z-1$ and δZ in Eq. (2.9) one sees that $\beta_q = f_{B_q}\sqrt{M_{B_q}B_{B_q}}$. Of special importance is

$$\xi = f_{B_s}B_{B_s}^{1/2}/f_{B_d}B_{B_d}^{1/2} = (M_{B_d}/M_{B_s})^{1/2}(\beta_s/\beta_d), \tag{3.3}$$

where, again, the right-most expression neglects $Z-1$ and δZ. We use the experimentally measured meson masses and our chirally extrapolated β_s and β_d/β_s to obtain $f_{B_s}B_{B_s}^{1/2}$ and ξ. The light-quark-mass dependence is shown in Fig. 1. Further plots can be found in Ref. [16].

A preliminary, but comprehensive, error budget is given in Table 2. The B^*-B-π coupling $g_{B^*B\pi}$ enters the expressions for the chiral extrapolation. The data are not precise enough to determine

a (fm)	Lattice	$N_{\rm confs}$	Sea (am_l, am_h)	Valence am_q
0.12	$24^3\times 64$	529	$(0.005, 0.05)$	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415
"coarse"	$20^3\times 64$	833	$(0.007, 0.05)$	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415
	$20^3\times 64$	592	$(0.01, 0.05)$	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415
	$20^3\times 64$	460	$(0.02, 0.05)$	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415
0.09	$28^3\times 96$	557	$(0.0062, 0.031)$	0.0031, 0.0044, 0.062, 0.0124, 0.0272, 0.031
"fine"	$28^3\times 96$	534	$(0.0124, 0.031)$	0.0031, 0.0042, 0.062, 0.0124, 0.0272, 0.031

Table 1: Input parameters for the numerical calculations. The lattice spacings listed are approximate mnemonics. The heavier sea mass m_h is close to the strange mass, which then is subject to retuning *a posteriori*, yielding the last value of am_q for the coarse ensembles.

$g_{B^*B\pi}$, so it must be set with a prior distribution in the chiral fits. A range that encompasses phenomenological and quenched lattice estimates is $g_{B^*B\pi} = 0.35 \pm 0.14$. The error in Table 2 corresponds to this range, while the prior width in the fits is ± 0.28.

Until the perturbation theory has been checked, we prefer not to report a value for $f_{B_s} B_{B_s}^{1/2}$. The matching corrections nearly cancel in the ratio β_q/β_s; the results with and without $Z-1$ and δZ are nearly the same, as shown in Fig. 1b. With the error budget discussed above we find

$$\xi = 1.205 \pm 0.037_{\text{stat}} \pm 0.034_{\text{syst}}, \tag{3.4}$$

unchanged since *Lattice 2008* [15].

4. Future Prospects

When the perturbative matching has been completely checked, we will be in a position to present final results. We can also compare different strategies, in particular, whether the perturbative expansion seems to work better for $\rho_{(S)}$ or $Z_{(S)}$ (cf. Eq. (2.14)).

In the longer term, we plan to obtain results for 4-quark operators that enter beyond the Standard Model. Furthermore, the MILC ensembles now not only have much higher statistics than the

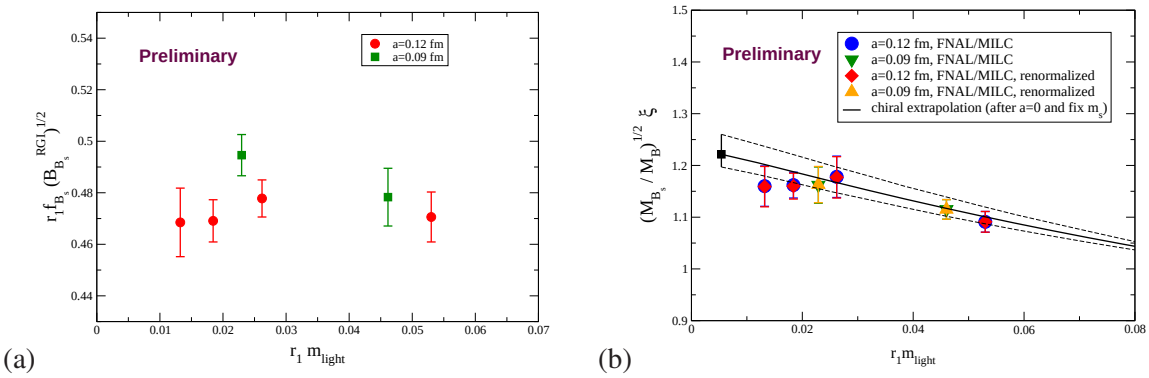

Figure 1: Light-quark-mass dependence of $f_{B_s} B_{B_s}^{1/2}$ and ξ. The curve in the right plot is a fit to all partially-quenched data, not just the shown unitary data.

Source	β_s	β_d	ξ
Statistics	2.7	4.0	3.1
Scale (r_1)	3.0	3.1	0.2
Sea and valence quark masses	0.3	0.5	0.7
b-quark hopping parameter	≤ 0.5	≤ 0.1	≤ 0.1
χPT + light-quark discretization	0.4	2.5	2.8
$g_{B^*B\pi}$	0.3	0.6	0.3
Heavy-quark discretization	2	2	0.2
Matching (perturbation theory)	~ 4	~ 4	≤ 0.5
Finite volume	≤ 0.5	≤ 0.5	≤ 0.1
Total	6.1	7.3	4.3

Table 2: Preliminary error budget. Entries in percent.

current project at $a = 0.12$ and 0.09 fm, but also extend to smaller lattice spacings, $a = 0.06$ and 0.045 fm. New runs with higher statistics and five lattice spacings (also 0.15 fm) are underway.

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