

Overview of Neutrino Mixing Models and Their Mixing Angle Predictions¹

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Abstract. An overview of neutrino-mixing models is presented with emphasis on the types of horizontal flavor and vertical family symmetries that have been invoked. Distributions for the mixing angles of many models are displayed. Ways to differentiate among the models and to narrow the list of viable models are discussed.

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INTRODUCTION

Several hundred models of neutrino masses and mixings can be found in the literature which purport to explain the known oscillation data and predict the currently unknown quantities. We present an overview of the types of models proposed and discuss ways in which the list of viable models can be reduced when more precise data is obtained. This presentation is an update of one published in 2006 in collaboration with Mu-Chun Chen [1] and, due to space restrictions, is an abbreviated version of one appearing in [2] with complete model references there.

PRESENT OSCILLATION DATA AND UNKNOWNNS

The present data within 3σ accuracy as determined by Fogli et al. [3], for example, is given by

$$\begin{aligned}\Delta m_{32}^2 &= 2.39 \begin{matrix} +0.42 \\ -0.33 \end{matrix} \times 10^{-3} \text{ eV}^2, \\ \Delta m_{21}^2 &= 7.67 \begin{matrix} +0.52 \\ -0.53 \end{matrix} \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{23} &= 0.466 \begin{matrix} +0.178 \\ -0.135 \end{matrix}, \\ \sin^2 \theta_{12} &= 0.312 \begin{matrix} +0.063 \\ -0.049 \end{matrix}, \\ \sin^2 \theta_{13} &\leq 0.046, \quad (0.016 \pm 0.010), \quad (1)\end{aligned}$$

where the last figure in parenthesis indicates a departure of the reactor neutrino angle from zero with one σ accu-

racy determination. The data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins, and Scott [4],

$$U_{PMNS} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (2)$$

with $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.33$, and $\sin^2 \theta_{13} = 0$.

The reason for the plethora of models still in agreement with experiment of course can be traced to the inaccuracy of the present data and the imprecision of the model predictions in many cases. In addition, there are a number of unknowns that must still be determined: the hierarchy and absolute mass scales of the light neutrinos; the Dirac or Majorana nature of the neutrinos; the CP-violating phases of the mixing matrix; how close to zero the reactor neutrino angle, θ_{13} , lies; how near maximal the atmospheric neutrino mixing angle is; whether the approximate tri-bimaximal mixing is a softly-broken or an accidental symmetry; whether neutrino-less double beta decay will be observable, and how large charged lepton flavor violation will turn out to be. In this presentation we survey the models to determine what they predict for the mixing angles, and neutrino mass hierarchy.

THEORETICAL FRAMEWORK

The observation of neutrino oscillations implies that neutrinos have mass, with the mass squared differences given in Eq.(1). Information concerning the absolute neutrino mass scale has been determined by the combined WMAP, SDSS, and Lyman alpha data which place an upper limit on the sum of the masses [5],

$$\sum_i m_i \leq 0.17 - 1.2 \text{ eV}, \quad (3)$$

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depending upon the conservative nature of the bound extracted. An extension of the SM is then required, and possible approaches include one or more of the following:

- the introduction of dim-5 effective non-renormalizable operators;
- the addition of right-handed neutrinos with their Yukawa couplings to the left-handed neutrinos;
- the addition of direct mass terms with right-handed Majorana couplings;
- the addition of a Higgs triplet with left-handed Majorana couplings;
- the addition of a fermion triplet with Higgs doublet couplings.

If we exclude the last possibility, the general 6×6 neutrino mass matrix in the $B(v_{\alpha L}, N_{\alpha L}^c)$ flavor basis of the six left-handed fields then has the following structure in terms of 3×3 submatrices:

$$\mathcal{M} = \begin{pmatrix} M_L & M_N^T \\ M_N & M_R \end{pmatrix}, \quad (4)$$

where M_N is the Dirac neutrino mass matrix, M_L the left-handed and M_R the right-handed Majorana neutrino mass matrices. With $M_L = 0$ and $M_N \ll M_R$ the type I seesaw formula,

$$m_\nu = -M_N^T M_R^{-1} M_N, \quad (5)$$

is obtained for the light left-handed Majorana neutrinos, while if $M_L \neq 0$ and $M_N \ll M_R$, one obtains the mixed type I + II seesaw formula,

$$m_\nu = M_L - M_N^T M_R^{-1} M_N. \quad (6)$$

There are two main approaches which we now describe that one can pursue to learn more about the theory behind the lepton mass generation.

Top - Down Approach

In the top-down approach one postulates the form of the mass matrix from first principles. The models will differ then due to the horizontal flavor symmetry chosen, the vertical family symmetry (if any) selected, and the fermion and Higgs representation assignments made.

The effective light left-handed Majorana mass matrix m_ν is constructed directly or with the seesaw formula once the Dirac neutrino matrix M_N and the Majorana neutrino matrices M_R (and M_L) are specified. Since m_ν is complex symmetric, it can be diagonalized by a unitary transformation, $U_{\nu L}$, to give

$$m_\nu^{diag} = U_{\nu L}^T m_\nu U_{\nu L} = \text{diag}(m_1, m_2, m_3), \quad (7)$$

with real, positive masses down the diagonal. On the other hand, the Dirac charged lepton mass matrix is diagonalized by a bi-unitary transformation according to

$$m_\ell^{diag} = U_{\ell R}^\dagger m_\ell U_{\ell L} = \text{diag}(m_e, m_\mu, m_\tau). \quad (8)$$

The neutrino mixing matrix [6], V_{PMNS} , is then given by

$$V_{PMNS} \equiv U_{\nu L}^\dagger U_{\nu L} = U_{PMNS} \Phi, \quad (9)$$

in the lepton flavor basis with $\Phi = \text{diag}(1, e^{i\alpha}, e^{i\beta})$. Note that the Majorana phase matrix Φ is required in order to compensate for any phase rotation on $U_{\nu L}$ needed to bring it into the Particle Data Book phase convention [7].

Bottom - Up Approach

On the other hand, with a bottom-up approach in the diagonal lepton flavor basis and with the general PMNS mixing matrix, one can determine the general texture of the light neutrino mass matrix to be

$$\begin{aligned} M_\nu &= U_{PMNS}^* \Phi^* M_\nu^{diag} \Phi^* U_{PMNS}^\dagger \\ &= U_{PMNS}^* \text{diag}(m_1, m_2 e^{-2i\alpha}, m_3 e^{-2i\beta}) U_{PMNS}^\dagger \\ &\equiv \begin{pmatrix} A & B & B' \\ \cdot & F' & E \\ \cdot & \cdot & F \end{pmatrix}, \end{aligned} \quad (10)$$

where the matrix elements are expressed in terms of the unknown neutrino masses, mixing angles and phases. By restricting the mixing matrix, one can learn that some of the matrix elements may not be independent.

MODELS AND MIXING ANGLE PREDICTIONS

After suggestions of atmospheric neutrino oscillations were found by the IMB and Kamiokande-II Collaborations [8] in the early 1990's, it became fashionable to assign texture zeros in different positions to m_ν with a top-down approach in hopes of identifying some flavor symmetry, but the procedure is basis dependent [9].

Another popular method invoked a $L_e - L_\mu - L_\tau$ lepton flavor symmetry [10]. The mass matrix then assumes the following form

$$m_\nu = \begin{pmatrix} 0 & * & * \\ \cdot & 0 & 0 \\ \cdot & 0 & 0 \end{pmatrix}, \quad (11)$$

which only leads to an inverted hierarchy.

By making use of a bottom-up approach instead, one is able to observe that a $\mu - \tau$ interchange symmetry

with $B' = B$, $F' = F$ in Eq. (10) leads to $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{13} = 0$ with $\sin^2 \theta_{12}$ arbitrary.

On the other hand, with the assumption of exact tri-bimaximal mixing for which $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{13} = 0$, and $\sin^2 \theta_{12} = 0.333$, one finds in Eq. (10) that $B' = B$, $F' = F = \frac{1}{2}(A + B + D)$ and $E = \frac{1}{2}(A + B - D)$, so that just three unknowns are present.

With the realization in the past five years that neutrino mixing is well approximated by the tri-bimaximal mixing matrix, the name of the game has become one of finding what discrete horizontal flavor symmetry groups would lead naturally to this mixing pattern. Such flavor symmetries can then be used as starting points with soft breaking as the next approximation.

Discrete Horizontal Flavor Symmetry Groups

Of special interest are those groups containing doublet and triplet irreducible representations. We list several of the well-studied groups and pertinent features of each.

The permutation group of three objects, S_3 , contains 6 elements with 1, $1'$, and 2 dimensional irreducible representations (IR's). The same eigenstates occur as those for tri-bimaximal mixing, but there is a 2-fold neutrino mass degeneracy.

The group A_4 of even permutations of four objects has 12 elements with IR's labeled 1, $1'$, $1''$, and 3. A $U(1)_R$ symmetry [11] may also be included to fix the mass scale which is otherwise undetermined. Early attempts to extend this flavor group to the quark sector failed, as the CKM mixing matrix for the quarks remained diagonal.

The group T' is the covering group of A_4 , but interestingly A_4 is not one of its subgroups. It contains 24 elements with 1, $1'$, $1''$, 3, $2'$, $2''$ IR's, where the first four are identical to those in A_4 . While tri-bimaximal mixing is obtained for the leptons, due to the presence of the three doublet IR's, a satisfactory CKM mixing matrix can also be obtained for the quarks.

The permutation group of 4 objects, S_4 , has 24 elements with 1, $1'$, 2 , 3 , $3'$ IR's. Although this and higher dimensional discrete flavor groups can also yield tri-bimaximal mixing, it appears that models based on A_4 are the most economical ones for the lepton sector.

Examples Involving GUT Models

Studies of neutrino mixing models in the framework of grand unified theories with a vertical family symmetry were first pursued in the 1990's and more intensely following the discovery of atmospheric neutrino oscillations by the Super-Kamiokande Collaboration in 1998.

Examples exist of models based on $SU(5)$, $SO(10)$, and E_6 , where the $SO(10)$ models are generally of two types.

The so-called "minimal" $SO(10)$ models [12] involve Higgs fields appearing in the **10** and **126** IR's, but newer models of this type have been extended to include the **120**, **45**, and/or **54** IR's. They generally result in symmetric and/or antisymmetric contributions to the quark and lepton mass matrices.

On the other hand, $SO(10)$ models [13] with Higgs fields in the **10**, **16**, **$\overline{16}$** and **45** IR's result in "lopsided" down quark and charged lepton mass matrices due to the $SU(5)$ structure of the electroweak VEV's appearing in the **16** and **$\overline{16}$** representations.

For either type of GUT model, type I seesaws only lead to a stable normal hierarchy for the light neutrino masses [14], while type I + II seesaws can also result in an inverted hierarchy. Most of the $SO(10)$ models have a continuous and/or discrete flavor symmetry group produced with them, but no efforts were initially made to introduce a discrete flavor symmetry group from one of the types discussed earlier to achieve tri-bimaximal mixing. A few examples can now be found in the literature which combine an $SU(5)$, $SO(10)$ or E_6 GUT symmetry with a T' or A_4 flavor symmetry with some success [15].

SURVEY OF MIXING ANGLE AND HIERARCHY PREDICTIONS

The author has updated a previous survey [1] made in collaboration with Mu-Chun Chen in 2006 of models in the literature which satisfied the then current experimental bounds on the mixing angles and gave reasonably restrictive predictions for the reactor neutrino angle. The cutoff date for the present update is January 2009.

Many models in the literature lack firm predictions for any of the mixing angles. For our analysis no requirement is made that the solar and atmospheric mixing angles or the mass differences be predicted, but if so, they must also satisfy the bounds given in Eq. (1). A complete listing of the 86 models which meet our criteria are referenced along with their predictions in [2].

Here we simply present the model predictions in the form of histograms plotted against $\sin^2 \theta_{13}$, where all models are assigned the same area, even if they extend across several basic intervals. The results are shown in Figs. 1 and 2 for the lepton flavor models and grand unified models, respectively. Two thirds of both types of models predict $0.001 \lesssim \sin^2 \theta_{13} \lesssim 0.05$, while the lepton flavor models have a much longer tail extending to very small reactor neutrino angles. The planned experiments involving Double Chooz and Daya Bay reactors [16] will reach down to $\sin^2 2\theta_{13} \lesssim 0.01$, so roughly two-thirds of the models will be eliminated if no $\bar{\nu}_e$ depletion is

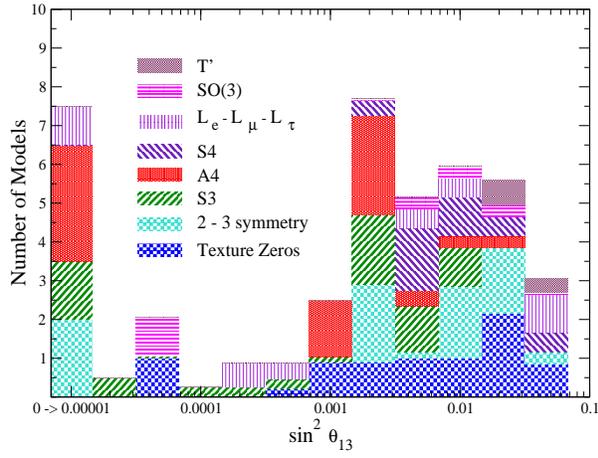


FIGURE 1. Lepton flavor model predictions for $\sin^2 \theta_{13}$.

observed. Both the T2K Collaboration at JPARC and the NOvA Collaboration at Fermilab are also expected to probe a similar reach with their ν_μ neutrino beams [17].

Even if $\bar{\nu}_e$ depletion is observed with some accuracy, it is apparent from the two histograms that the order of 10 - 20 models may survive which must still be differentiated. One suggestion is to make scatterplots of $\sin^2 \theta_{13}$ vs. $\sin^2 \theta_{12}$ and $\sin^2 \theta_{12}$ vs. $\sin^2 \theta_{23}$. We have attempted to do this in Figs. 3, 4, and 5 for both the lepton flavor models and grand unified models, where only the central value predictions are plotted. Most of the models considered favor central values of $\sin^2 \theta_{12}$ lying below 0.333, the value for exact tri-bimaximal mixing. This is in agreement with the present value extracted in Eq. (1), but central values for $\sin^2 \theta_{23} \geq 0.5$ are preferred, while the best extracted value is 0.466 from Eq. (1).

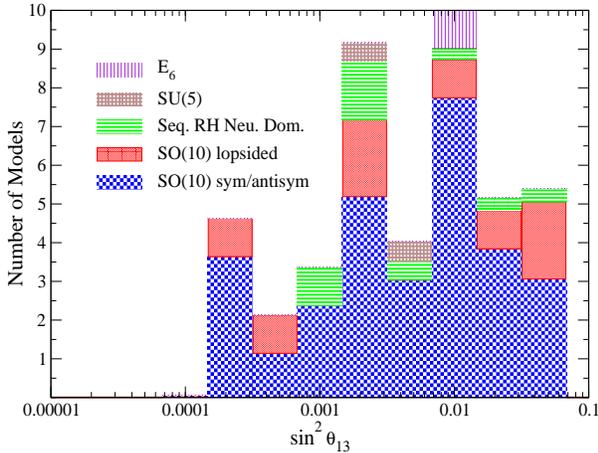


FIGURE 2. GUT model predictions for $\sin^2 \theta_{13}$.

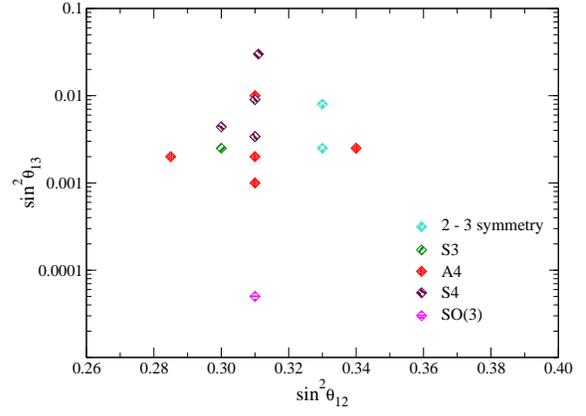


FIGURE 3. Central value distributions of $\sin^2 \theta_{23}$ vs. $\sin^2 \theta_{12}$ for the discrete flavor symmetry models.

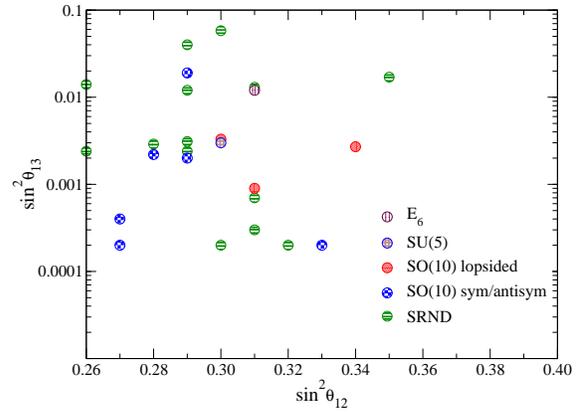


FIGURE 4. Central value distributions of $\sin^2 \theta_{23}$ vs. $\sin^2 \theta_{12}$ for the grand unified models.

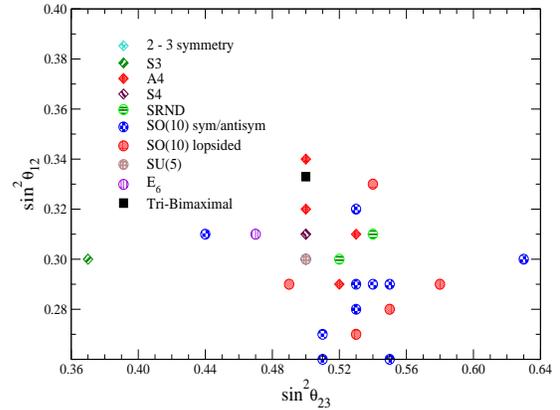


FIGURE 5. Central value distributions of $\sin^2 \theta_{12}$ vs. $\sin^2 \theta_{23}$ for both types of models.

Neutrino-less double beta decay can serve as a valuable probe of the neutrino mass hierarchy observed in Nature. In fact, the effective mass plot for perturbed tri-bimaximal mixing in Fig. 6 shows a clear separation of the normal and inverted ordering distributions, so accurate neutrino-less double beta decay experiments should be decisive in determining the hierarchy.

SUMMARY

We have made a survey of neutrino mixing models based on some horizontal lepton flavor symmetry and those based on GUT models having a vertical family symmetry and a flavor symmetry. We have tried to differentiate the models on the basis of their neutrino mass hierarchy and mixing angles. Most of the models allow either mass hierarchy with the exceptions being just normal for the type I seesaw models and only inverted for the conserved $L_e - L_\mu - L_\tau$ models.

For both types of models our study indicates that the upcoming Double Chooz and Daya Bay reactor experiments will be able to eliminate roughly two-thirds of the models surveyed, if their planned sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.01$ and no depletion of the $\bar{\nu}_e$ flux is observed. However, no smoking gun apparently exists to rule out many types of models based on accurate data for $\sin^2 \theta_{13}$ alone, should evidence for a depletion be found. Of the order of 10 - 20 models have similar values for this mixing angle in the 0.001 - 0.05 interval. Only the lepton flavor models appear to lead to extremely small values of $\sin^2 \theta_{13} \lesssim 10^{-4}$.

Most models prefer $\sin^2 \theta_{12} \lesssim 0.31$ rather than 0.333 for tri-bimaximal mixing in agreement with the present best value of 0.312. On the other hand, most models prefer $\sin^2 \theta_{23} \geq 0.50$ compared with a best fit value of 0.466. One notable exception is the model of Stech and Tavartkiladze [18] based on the E_6 family group with $SU(3) \times Z_2$ flavor symmetry.

It is clear that very accurate determinations of the neutrino mass hierarchy, the three mixing angles and eventually the three CP-violating phases will be required to pin down the most viable models.

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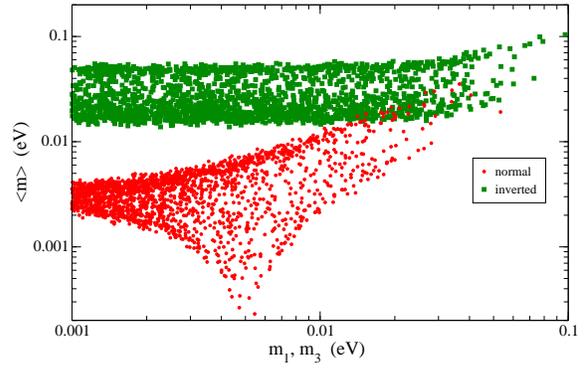


FIGURE 6. Effective mass plot for neutrino-less double beta decay in the case of perturbed tri-bimaximal mixing.

REFERENCES

1. C. H. Albright and M.-C. Chen, Phys. Rev. D **74**, 113006 (2006).
2. C. H. Albright, in Proceedings of the XIII International Workshop on Neutrino Telescopes (Venice, 2009), edited by Milla Baldo Ceolin, arXiv:0905.0146 [hep-ph].
3. G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. Lett. **101**, 141801 (2008).
4. P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **535**, 163 (2002).
5. O. Host, O. Lahav, F. B. Abdalla and K. Eitel, Phys. Rev. D **76**, 113005 (2007).
6. B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) [*Sov. Phys. JETP* **6**, 429 (1957)]; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
7. C. Amsler *et al.*, [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
8. D. Casper *et al.*, [IMB-3 Collab.], Phys. Rev. Lett. **66**, 2561 (1991); K. S. Hirata *et al.*, [Kamiokande-II Collab.], Phys. Lett. B **280**, 146 (1992).
9. P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. **B406**, 19 (1993).
10. R. N. Mohapatra, J. High Energy Phys. **10**, 027 (2004).
11. C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
12. K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).
13. K. S. Babu and S. M. Barr, Phys. Lett. B **381**, 202 (1996); C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. **81**, 1167 (1998).
14. C. H. Albright, Phys. Lett. B **599**, 285 (2004).
15. M.-C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007); G. Altarelli, F. Feruglio and C. Hagedorn, JHEP **0803**, 052 (2008); F. Bazzocchi, M. Frigerio and S. Morisi, Phys. Rev. D **78**, 116018 (2008).
16. C. E. Lane, [Double Chooz Collab.], in Proceedings of the 34th International Conference on High Energy Physics (Philadelphia, 2008); M.-C. Chu [Daya Bay Collab.], *ibid.*
17. I. Kato, [T2K Collab.], J. Phys.: Conf. S. **136**, 022018 (2008); R. Ray, [NOVA Collab.], J. Phys.: Conf. S. **136**, 022019 (2008).
18. B. Stech and Z. Tavartkiladze, Phys. Rev. D **77**, 076009 (2008).