

MUON COLLIDER LATTICE WITH LOCAL INTERACTION REGION CHROMATICITY CORRECTION*

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Abstract

Muon collider is a promising candidate for the next energy frontier machine [1]. In order to obtain peak luminosity 10^{34} - 10^{35} $\text{cm}^{-2}\text{s}^{-1}$ in the TeV energy range the collider lattice design must satisfy a number of stringent requirements. Taken together these requirements present a challenge that has never been met before. We offer a particular solution dubbed the “dipole first” scheme. Theoretical aspects and some options for this design are discussed.

INTRODUCTION

The Muon collider lattice design must meet a number of challenging and often contradictory requirements:

- low β^* ($\leq 1\text{cm}$),
- small circumference C (since luminosity $\sim 1/C$),
- momentum acceptance of up to 1% and sufficient dynamic aperture for normalized beam emittance of ~ 25 microns ,
- low momentum compaction ($\alpha_c < 10^{-4}$) to obtain small $\sigma_z \leq \beta^*$ with a reasonable RF voltage $U_{RF} < 1\text{GV}$,
- absence of long straights in order not to create "hot spots" of neutrino radiation,
- make provision for low beta quads and detectors protection from secondary particles (which limits field gradient of the quads and their proximity to IP)
- manageable sensitivity to errors which limits maximum values of β -functions.

The most difficulties are associated with chromatic correction sextupoles which may produce strong spherical aberrations. Different approaches to chromatic correction are reviewed in accompanying report [2]. Here we describe a design of $2 \times 0.75\text{TeV}$ muon collider which has the following distinctive features:

- chromatic compensation achieved with sextupoles and dispersion generating dipoles placed near the IR quadrupoles (not in a special section),
- low value of momentum compaction factor obtained by balancing positive contribution from the arcs with negative contribution from the suppressors of the generated in the IR dispersion.

LATTICE DESIGN

In the design we put rather conservative limits on magnet strength: $B=7.5\text{T}$ for dipoles at high-beta locations and 10T in the arcs, 200T/m for quadrupoles.

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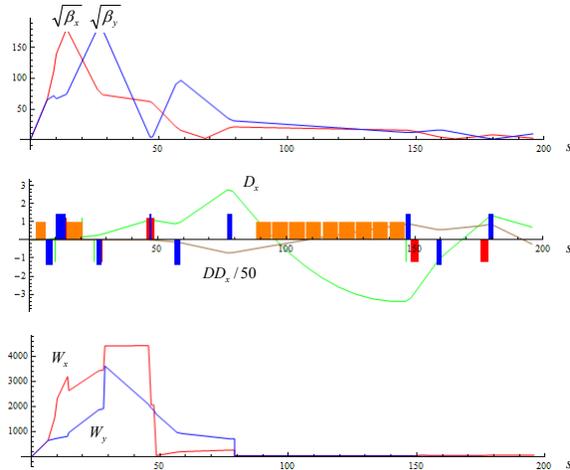


Figure 1. Optics and chromatic functions in IR and dispersion suppressor for $\beta^*=1\text{cm}$.

Interaction region

The distance from IP to the first quad was set at 6.5m as in earlier $2 \times 2\text{TeV}$ collider designs [3, 4]. In order to provide places with $\beta_x \gg \beta_y$ and $\beta_x \ll \beta_y$ (and sufficiently large dispersion D_x) for horizontal and vertical correction sextupoles the distance between the second and third final focus quadrupoles was increased to 12m (see Fig.1 middle plot where quads are depicted with blue rectangles).

In the initial design there was single dispersion generating dipole placed next to the 2nd quad (dipoles are shown in Fig.1 with orange rectangles). However, the dispersion appeared too small at the location of vertical correction sextupole near the 3rd quad resulting in high sextupole strength and unacceptable dynamic aperture even in the case of $\beta^* = 1\text{cm}$. There are three options:

- significant increase in the dipole field (may be possible with HTS),
- bringing the 1st quad closer to IP to limit β_{max} ,
- putting additional dipole in front of the 1st quad .

Decreasing the first quad distance from IP does not help much since its bore must accommodate shielding and can not be proportionally reduced.

The third option (our choice) is controversial: the 1st dipole reduces the amount of tungsten shielding, increases detector exposure to hard X-rays generated by decay electrons in its field, but at the same time deflects decay electrons and Bethe-Heitler muons (produced elsewhere) away from the most vulnerable central part of the detector. Only extensive simulations can show what is the overall effect.

As for the higher field promised by HTS, the “dipole first” scheme will profit from it as well: reduction in the required sextupole strength will allow for stronger perturbation, i.e. for smaller β^* .

Dispersion suppressor

Strong dipoles at high-beta locations generate a large dispersion “invariant”

$$J_x = \frac{D_x^2 + (\beta_x D'_x + \alpha_x D_x)^2}{\beta_x} \approx \beta_x \phi^2 \quad (1)$$

ϕ being the dipole bend angle. The value this invariant obtains in IR exceeds by many times the value it has in the arcs, therefore a reverse dispersion suppression is needed. It can be achieved with centripetal bends if the sign of the dispersion function is inverted.

In the result the dispersion suppressor helps to reduce the ring circumference and – which is no less important – gives negative contribution to the momentum compaction factor

$$\alpha_c = \frac{1}{C} \int \frac{D_x}{\rho} ds \quad (2)$$

where ρ is instantaneous radius of curvature. This allows us to obtain small total α_c values using simple FODO cells in the arcs. Typical DS contribution is $\alpha_c^{(DS)} \sim -10^{-3}$.

At the present stage of the design we did not try to provide dispersion-free regions for RF cavities but placed them symmetrically around the points where dispersion function crosses zero.

Arc cells

The major factors which determine the choice of arc cells are: the required contribution $\alpha_c^{(arc)} \approx -\alpha_c^{(DS)}$, maximum dipole packing factor $\kappa_d = l_{dipole}/l_{total}$ and minimum adverse effect on particle stability.

The number of classical FODO cells ($\mu_x = \mu_y = \mu$) which is required to obtain the target value of $\alpha_c^{(arc)}$ depends on the phase advance per cell μ as

$$N_{cell} = \frac{\pi}{\sin \frac{\mu}{2}} \left(\frac{2\pi\rho}{\alpha_c^{arc} C \kappa_d} \right)^{1/2} \quad (3)$$

The quadrupole and sextupole integrated strengths rather weakly depend on the phase advance μ :

$$k_1 l_{quad} = \pm 2 \left(\frac{\phi_{arc} \kappa_d}{\alpha_c^{arc} C \rho} \right)^{1/2}, \quad (4)$$

$$k_2 l_{sext} = \pm \frac{2}{1 \pm \frac{1}{2} \sin \frac{\mu}{2}} \left(\frac{\phi_{arc}}{\alpha_c^{arc} C} \right)^{3/2} \left(\frac{\kappa_d}{\rho} \right)^{1/2}$$

where ϕ_{arc} is the total bending angle in the arcs. Since the gradients are determined by technical considerations, their lengths l_{quad} , l_{sext} also weakly depend on μ so that the total length occupied by the arc quadrupoles and sextupoles goes with μ as N_{cell} . For the dipole packing factor we have

$$\kappa_d^{-1} = 1 + \frac{l_{quad} + l_{sext}}{\sin \frac{\mu}{2}} \left(\frac{\phi_{arc}}{\alpha_c^{arc} C \rho} \right)^{1/2} \quad (5)$$

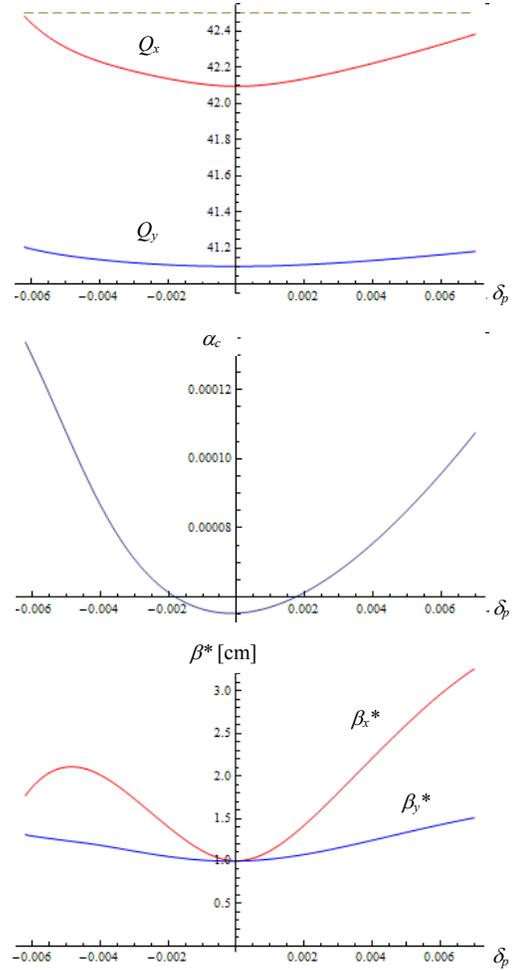


Figure 2. Tunes (top), momentum compaction factor (middle), and IP β -functions (bottom) vs. momentum deviation

which shows that in order to minimize the arc length the phase advance μ should be chosen as large as possible.

We tried two values of μ , 108° ($3\pi/5$) and 135° ($3\pi/4$). With $\mu=108^\circ$ the total of 120 arc cells was required. Increasing μ to 135° allows to reduce this number to 80 and increase the dipole packing factor κ_d from 63% to 71%. Corresponding reduction in the machine circumference and gain in luminosity is $\sim 10\%$. However, the nonlinear effects became more pronounced so we have chosen $\mu=108^\circ$. Another possibility to reduce N_{cell} without increasing μ is modulation of the dispersion function as discussed in [2].

NONLINEAR CORRECTION

The arc sextupoles are used for correction of linear chromaticity of the tunes. With $\mu=108^\circ$ they give little contribution to other effects due to small values of β -functions in the arcs.

Sextupoles in IR and dispersion suppressor (some are visible in Fig.1 as red rectangles) provide compensation of chromatic functions (Fig.1 bottom plot) and 2nd order

dispersion (Fig.1 middle plot). The latter is important for control of $d\alpha_c/d\delta_p$. The IR sextupoles are interleaved and produce large second order detuning with amplitude, especially the cross term $\partial Q_x/\partial E_y \sim 2 \cdot 10^8 \text{ m}^{-1}$, where E_y is the Courant-Snyder invariant.

To compensate this strong detuning as well as the 2nd order chromaticity a number of octupoles is used.

Optimization of nonlinear correctors is a multistage process, which has not been automated yet. Chromatic dependence of betatron tunes, momentum compaction factor and IP β -functions with the best set of corrector parameters found so far is shown in Fig. 2.

$\partial Q_x/\partial E_x$	$0.252 \cdot 10^8 \text{ m}^{-1}$
$\partial Q_x/\partial E_y$	$0.196 \cdot 10^8 \text{ m}^{-1}$
$\partial Q_y/\partial E_y$	$0.185 \cdot 10^8 \text{ m}^{-1}$

Table 1. Detuning coefficients after correction

According to MAD8 STATIC command the cross-detuning coefficient was reduced with the help of octupoles by an order of magnitude (see Table 1) resulting in significant improvement on dynamic aperture (DA).

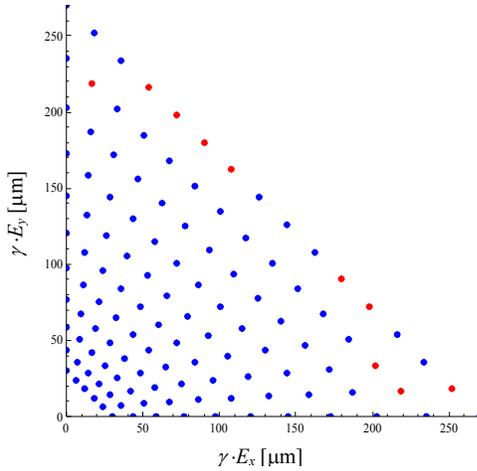


Figure 3. Survival plot for 1024 turns. Initial conditions for lost particles are shown in red.

The results of tracking for 1024 turns in ideal lattice are presented in Fig.3. The dynamic aperture is 3σ for normalized emittance $\varepsilon_{\perp N} = 25 \mu\text{m}$. In the presence of errors it will certainly be smaller.

SUMMARY & OUTLOOK

The basic parameters of “dipole first” design are given in Table 2. The design meets stability requirements for muon beam parameters envisaged in the baseline scheme of the muon collider [1]: energy spread 0.1% and normalized transverse emittance $\varepsilon_{\perp N} = 25 \mu\text{m}$.

β -functions have moderate maximum values ($< 33 \text{ km}$) making the lattice less sensitive to magnet errors. Still, the tolerance on quadrupole errors - 10^{-4} - is challenging.

Both the momentum acceptance and dynamic aperture can be increased further with stronger octupoles: there was no evidence of the resonance excitation by them thus far. However, it seems more prospective to modify the chromatic correction scheme as outlined in [2] so that the IR sextupoles formed non-interleaved pairs with pseudo -/ transformation between sextupoles belonging to a pair.

Since the vertical correction sextupole pair straddles the IP there is a problem with phase advance perturbation by the beam-beam interaction. A radical solution to this problem – complete suppression of the beam-beam interaction by over-dense plasma – was considered in [5].

Beam energy, TeV	0.75
Maximum B , T	10
Number of IPs	2
Circumference, km	3.11
β^* , cm	1
β_{max} , km	32.7
Betatron tunes	42.095/41.1
Momentum compaction α_c	$5.53 \cdot 10^{-5}$
Momentum acceptance	$\pm 0.63\%$
DA for $\varepsilon_{\perp N} = 25 \mu\text{m}$	3σ

Table 2. Collider ring parameters

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