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ANALYTICAL DESCRIPTION OF TEVATRON INTEGRATED LUMINOSITY*

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Abstract

The recent record-setting performance of the Fermilab Tevatron is the culmination of a long series of efforts to optimize the many parameters that go into generating integrated luminosity for the colliding beams experiments. Here we take an analytical approach in an attempt to illustrate the most fundamental aspects of integrating and optimizing luminosity in the Tevatron. Essential features such as weekly integrated luminosity and store length optimization can be quickly analyzed in terms of antiproton stacking rate and observed beam emittance growth rates in the Tevatron. This optimization approach has been used to maximize the integrated luminosity for Run II.

INTRODUCTION

We will build up our understanding in two stages. First, we look at the condition where a collider uses two beams with unequal bunch intensities, and where particles are lost from the accelerator only due to collisions. Next, we introduce a beam lifetime which is independent of luminosity (for example, beam growth and subsequent loss due to noise sources) to arrive at a more realistic description.

UNEQUAL BUNCH INTENSITIES

In the Tevatron the proton beam is more intense than the antiproton beam by a factor of 3-4. Let the population of antiprotons per bunch be N_2 and that of the protons $N_1 > N_2$. The luminosity at a given collision point is

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{4\pi\sigma^{*2}} \cdot \mathcal{H}, \tag{1}$$

where σ^* is the transverse beam size (considered to be round) at the interaction point, f_0 is the revolution frequency, and $\mathcal{H}$ is a form factor taking into account crossing angles, "hour glass " effects, and so on. If particles were lost only due to collisions then a "perfect store," given enough time, would deliver an integrated luminosity, I, equal to the number of particles "consumed" divided by the interaction cross section; the luminosity delivered to each experiment would be this number divided by the number of experiments. The ultimate integrated luminosity for the store delivered to each experiment would be $I_0 = N_{total}/n\Sigma = BN_2^0/n\Sigma$, where B is the number of bunches per beam, n the number of interaction points, Σ the interaction cross section, and N_2^0 the initial bunch intensity of the less intense beam. I_0 is the maximum integrated luminosity one could hope for from a single store.

Assuming a one-to-one correspondence in the rate at which protons and antiprotons are consumed (*i.e.* particles are only lost due to collisions), we define $N_2(t) = N(t)$, $N_1(t) = N(t) + \Delta N$, where, $\Delta N = N_1^0 - N_2^0$. Here, N_1^0 is the initial bunch intensity of the more intense beam. The bunch intensity is governed by $B\ dN/dt = -\mathcal{L}\ \Sigma\ n$. Substituting Eq. 1 along with the definitions of $N_1(t)$ and $N_2(t)$ into this differential equation and integrating, we get

$$N(t) = \frac{N_2^0\ \Delta N}{N_1^0 e^{\Delta N k t} - N_2^0} \tag{2}$$

where $k \equiv n\mathcal{L}_0\Sigma/BN_1^0N_2^0 = \mathcal{L}_0/(I_0 N_1^0)$. Thus, the luminosity evolves with time according to

$$\begin{aligned} \mathcal{L}(t) &= \mathcal{L}_0 \frac{\Delta N^2 e^{\Delta N k t}}{(N_1^0 e^{\Delta N k t} - N_2^0)^2} \\ &\rightarrow \mathcal{L}_0 \left(1 - \frac{N_2^0}{N_1^0}\right)^2 e^{-(1-N_2^0/N_1^0)\mathcal{L}_0 t/I_0}, \end{aligned}$$

the last expression being for large values of t. The integrated luminosity, over a time period T, is then

$$I \equiv \int_0^T \mathcal{L}(t)dt = \frac{BN_2^0}{n\Sigma} \cdot \left(\frac{e^{\Delta N k T} - 1}{e^{\Delta N k T} - \frac{N_2^0}{N_1^0}}\right) \rightarrow I_0. \tag{3}$$

Once the less intense beam is depleted, there is no more luminosity(!), and the resulting asymptotic limit is I_0.

Take $N_1^0 = 3 \times 10^{11}$ for the proton beam, and $N_2^0 = 8 \times 10^{10}$ for the antiproton beam. Other typical values are $\mathcal{H} = 0.6$, f_0 = 47.7 kHz, and σ^* = 25 μm. Then $\mathcal{L}_0 \approx 315 \times 10^{30}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ = 315 $\mu\mathrm{b}^{-1}$/sec = 1.1/pb/hr, and I_0 = 24/pb. Allow the store to yield 85% of its limit I_0, when the final luminosity would be roughly 10% of the initial luminosity. Then this store would need to last approximately 50 hr, and would integrate to $\sim$20/pb.

WORDS ON PARTICLE LIFETIME

In reality the Tevatron stores do not last this long, and do not integrate to the above I_0. Mechanisms other than collisions at the interaction points, such as scattering events with the residual gas in the vacuum chamber, various noise sources, *etc*, will cause particle loss due to diffusion. An equilibrium emittance, determined by the aperture, is approached and while the details of this process can be complex and nonlinear, in what follows we model this as a constant rate of single particle emittance growth.

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Let the normalized 95% emittance for a transverse coordinate, x, be $\epsilon \equiv 6\pi\gamma\langle x^2\rangle/\beta$, where γ is the Lorentz factor, and the angle brackets denote averaging over the particle distribution. If this degree-of-freedom is limited by an aperture at a distance a from the beam center, then the equilibrium emittance will be $\hat{\epsilon} \approx 0.92\pi\gamma a^2/\beta$. If in the absence of an aperture the beam emittance grows at a rate of $\dot{\epsilon} = (6\pi\gamma/\beta) \cdot d\langle x^2\rangle/dt$, then the asymptotic lifetime due to diffusion will be $\tau \approx 2\hat{\epsilon}/\dot{\epsilon}$.[1],[2] Typical emittance growth rates measured in the Tevatron are on the scale of $\dot{\epsilon} \approx 1\pi$ mm-mrad/hr and equilibrium emittances can be $\sim 12\pi$ mm-mrad, giving a particle lifetime due to diffusion of about 24 hours. We can now treat the evolution of beam intensity and luminosity over time in a straightforward analytical fashion.

LUMINOSITY WITH DIFFUSION

Following the above scenario, assume round beams at collision, with effective equilibrium emittances $\hat{\epsilon}$ and a common effective emittance growth rate, $\dot{\epsilon}$ due to diffusion, with $\tau = 2\hat{\epsilon}/\dot{\epsilon}$. Since $\mathcal{L} \propto N_1N_2$, the equations for the bunch population for the two beam species will be

$$\dot{N}_1 \quad = \quad = -kN_1N_2 - \frac{1}{\tau}N_1 \tag{4}$$

$$\dot{N}_2 \quad = \quad = -kN_1N_2 - \frac{1}{\tau}N_2 \tag{5}$$

with k as before. Subtracting and integrating, we find that

$$N_1(t) - N_2(t) = (N_1^0 - N_2^0)e^{-t/\tau} \tag{6}$$

so that solving for either $N_1(t)$ or $N_2(t)$ will automatically give the other. Let $N(t) \equiv N_2(t)$ and obtain

$$\dot{N} + \left(\frac{1}{\tau} + k\Delta N e^{-t/\tau}\right) N + kN^2 = 0, \tag{7}$$

the solution of which is

$$N(t) = N_2(t) = N_2^0 \; \frac{\Delta N \; e^{-t/\tau}}{N_1^0 e^{(1-e^{-t/\tau})\Delta N k\tau} - N_2^0}. \tag{8}$$

This rather formidable result (especially the exponential of the exponential of time in the denominator!) reduces to our result of the previous section in the limit where $\tau \to \infty$.

We immediately have our result for $N_1(t)$, which when combined with the above gives the luminosity,

$$\mathcal{L}(t) = \mathcal{L}_0 \; \frac{\Delta N^2 e^{-2t/\tau} e^{-(1-e^{-t/\tau})\Delta N k\tau}}{\left(N_1^0 - N_2^0 e^{-(1-e^{-t/\tau})\Delta N k\tau}\right)^2} \quad . \tag{9}$$

As before, this reduces to our previous result when $\tau \to \infty$. When τ is left finite but the bunch intensities become equal ($N_1^0 \to N_2^0 = N$), the result is

$$\mathcal{L}(t) \longrightarrow \frac{\mathcal{L}_0}{\left[Nk\tau - (1 + Nk\tau)e^{t/\tau}\right]^2},$$

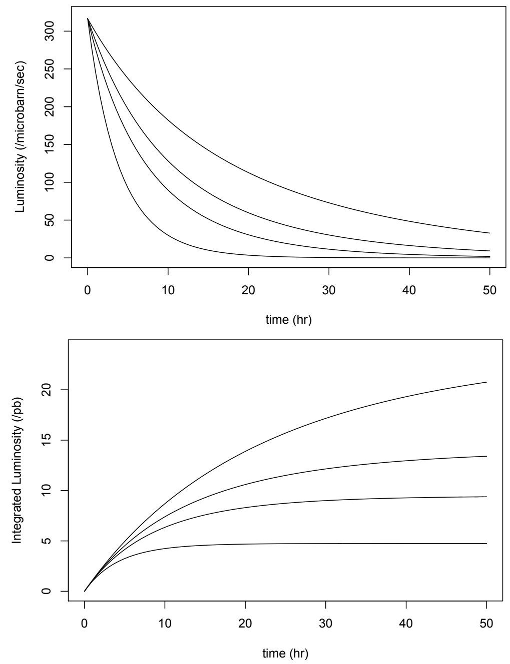

Figure 1: Instantaneous (top) and integrated (bottom) luminosity *vs.* time for values of τ = 10 hr, 25 hr, 50 hr and $\tau = \infty$, using typical Tevatron parameters as in the text.

and when $\tau \to \infty$ we get the usual result, namely $\mathcal{L}(t) \to \mathcal{L}_0/[1 - (\mathcal{L}_0 t/I_0)]^2$.

Finally, we can obtain an expression for the integrated luminosity during a store by integrating Eq. 9 to get

$$I(t) = I_0 \; \left[1 - \frac{(N_1^0 - N_2^0)e^{-t/\tau}}{N_1^0 e^{(1-e^{-t/\tau})\Delta N k\tau} - N_2^0} - \frac{I_0}{\mathcal{L}_0\tau} \; \frac{N_1^0}{N_2^0} \; \ln\left(\frac{N_1^0 - N_2^0 e^{-(1-e^{-t/\tau})\Delta N k\tau}}{N_1^0 - N_2^0}\right)\right] \tag{10}$$

which reproduces our previous result as $\tau \to \infty$.

Plots of luminosity and integrated luminosity during stores for various values of τ are shown in Figure 1, using parameters as in our earlier numerical examples.

APPLICATION TO TEVATRON

There have been a few weeks of Tevatron operation during which all stores were ended intentionally (rather than due to failures, lightening storms, *etc.*). For example, the week of 30 Dec 2006 through 7 Jan 2007 delivered about 45/pb to each experiment using six stores. The average store length was about 25 hours and the average initial luminosity, $\langle\mathcal{L}\rangle$, was 234/$\mu$b/sec. Additionally, $\langle I\rangle \approx 7.5$/pb, $\langle N_2^0\rangle \approx 72.5 \times 10^9$, $\langle N_1^0\rangle = 231 \times 10^9$, and β^* = 30 cm, $\hat{\epsilon} = 12\pi$ mm-mrad and $\mathcal{H} = 0.59$ at the beginning of a store. The typical "shot set-up time" between stores was about two hours. To produce the required 324×10^{10} antiprotons during the time between the start of neighboring stores, the *average* antiproton production rate was about 12×10^{10}/hr (max. rate at low intensity of 17×10^{10}/hr). Along with the above information, our analytical model has one last free parameter, namely the emittance growth rate, $\dot{\epsilon}$. Adjusting to make our model adhere to 7.5/pb per store, the best value is roughly $\dot{\epsilon}$ = 0.8π mm-mrad/hr — again, totally consistent with experience. The analytical result with this set of parameters is plotted in Figure 2 along with the actual integrated luminosity data logged by the accelerator controls system for this week of January.

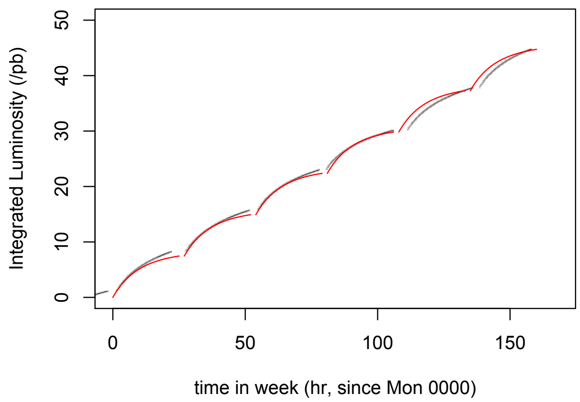

Figure 2: Integrated luminosity for the week of 30 Dec 2006. Black dots are data, red (color) curve is the model.

STORE LENGTH OPTIMIZATION

With a consistent set of parameters for our analytical description, optimization of the Tevatron operation can be examined, balancing the rate of antiproton production against the lifetime of the instantaneous luminosity along with the length of time that it takes to end one store and produce the next one. Suppose antiprotons are produced at the rate $\mathcal{R}$ leading to N_2^0 antiprotons per bunch at the beginning of each new store. If $\mathcal{R}$ is fast enough, then enough antiprotons will always be ready for the next store. If not, then the store must continue until N_2^0 is reached, and the integration of luminosity may be low during this time period. Thus, there will be an optimal store length for a particular production rate, other parameters being constant.

Figure 3 shows the results for this line of reasoning, using our analytical expressions. The antiproton accumulation rate is varied and the average weekly integrated luminosity is plotted as a function of initial number of accumulated antiprotons per store. A delivery efficiency from accumulation to store conditions of 83% is assumed, as well as a finite lifetime for stored antiprotons of 35 hr.

As can be seen, optimal initial antiproton intensities – and thus the corresponding store lengths – are evident. We note that the maximum at high production rates can be rather broad, and thus the accelerator complex would tend to operate at the left-hand side of the optimum, to mitigate premature loss of accumulated antiprotons. The current Tevatron running conditions are indicated. The model can also be used to examine the role of the emittance growth rate. It tells us that for today's antiproton accumulation rate, if the emittance growth rate could be diminished by 33%, for example, then one would contemplate going to higher initial antiproton intensities – 450×10^{10}, say – and the integrated luminosity could increase by $\sim$20%.

CONCLUDING REMARKS

The essential features of a Tevatron store can be described in terms of the initial beam conditions and by the introduction of a non-luminous particle lifetime. Several details such as the possible time dependence of the form factor, the influence of the beams on each other due to beam-beam interactions, *etc.*, have been left out. However, the overall features of Tevatron stores and the integrated luminosity per week can be nicely demonstrated analytically. Using this as a guide, along with more detailed models of the operation yielding similar results [4], this optimization has been used to maximize the integrated luminosity of Tevatron Run II for record-setting performance.

REFERENCES

[1] Edwards, D. A., and Syphers, M. J., *An Introduction to the Physics of Particle Accelerators*, John Wiley, and Sons, New York (1993).

[2] Syphers, M.J., Fermilab Report, Beams-doc-1478 (2004) http://beamdocs.fnal.gov .

[3] Syphers, M.J., Fermilab Report Beams-doc-2798 (2007).

[4] Gattuso, et al., these proceedings.

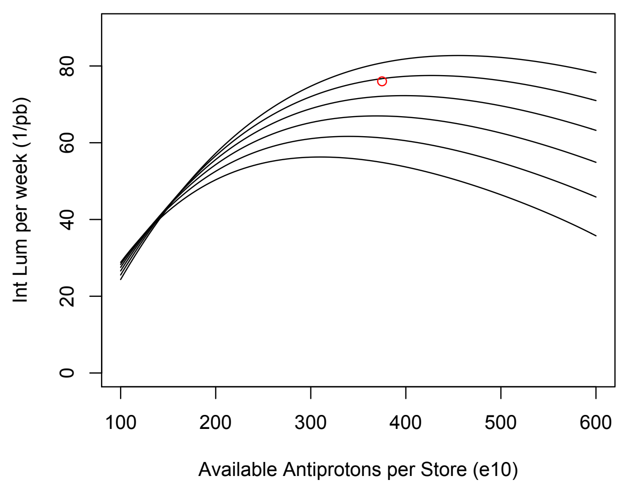

Figure 3: Curves of weekly integrated luminosity *vs.* number of antiprotons accumulated during a store for given values of the low intensity accumulation rate, $\mathcal{R} = 20, 22, \ldots, 30 \times 10^{10}$/hr. For this plot, τ = 30 hr. The red (color) circle indicates the present level of operation.