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DYNAMICAL EFFECTS OF SELF-GENERATED MAGNETIC FIELDS IN COSMIC RAY MODIFIED SHOCKS

D. Caprioli[1], P. Blasi[2,3], E. Amato[2], M. Vietri[1]

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ABSTRACT

Recent observations of greatly amplified magnetic fields ($\delta B/B \sim 100$) around supernova shocks are consistent with the predictions of the non-linear theory of particle acceleration (NLT), if the field is generated upstream of the shock by cosmic ray induced streaming instability. The high acceleration efficiencies and large shock modifications predicted by NLT need however to be mitigated to confront observations, and this is usually assumed to be accomplished by some form of turbulent heating. We show here that magnetic fields with the strength inferred from observations have an important dynamical role on the shock, and imply a shock modification substantially reduced with respect to the naive unmagnetized case. The effect appears as soon as the pressure in the turbulent magnetic field becomes comparable with the pressure of the thermal gas. The relative importance of this unavoidable effect and of the poorly known turbulent heating is assessed. More specifically we conclude that even in the cases in which turbulent heating may be of some importance, the dynamical reaction of the field cannot be neglected, as instead is usually done in most current calculations.

Subject headings: acceleration of particles — shock waves — magnetic field

1. INTRODUCTION

The supernova remnant (SNR) paradigm for the origin of galactic cosmic rays is based on the assumption that at least $\sim 10-20\%$ of the kinetic energy of the expanding shell is converted into cosmic rays (CRs). Moreover, as recent observations have proved, the magnetic field in the shock vicinity is amplified by a large amount (e.g. Ballet (2006)) as would be expected if cosmic rays induce streaming instability (SI) upstream of the shock. We stress that such magnetic field amplification is required to accelerate protons up to $\sim 10^6$ GeV. The need for a satisfactory and self-consistent description of these points is sufficient to justify the development of a NLT of particle acceleration.

The developments of the theory are summarized in (Drury (1983), Jones & Ellison (1991), Malkov & Drury (2001)). The kinetic theory for arbitrary diffusion coefficients (Amato & Blasi (2005)), and even in the case of self-generated magnetic fields (Amato & Blasi (2006) and Vladimirov, Ellison & Bykov (2006)) has been recently developed.

All approaches to NL shock acceleration find that the large pressure of the accelerated particles decelerates the incoming gas, and leads to total compression factors that scale with the Mach number of the shock as $R_t \sim M_0^{3/4} \sim 20-50$. Such large shock modifications however are at odds with observations which are better fit by $R_t \sim 7-10$. The problem with larger values of R_t resides in both the estimated distance between the forward and reverse shocks (Warren et al (2005)) and in the fit to multifreqeuncy observations with concave spectra (VBK05 and references therein). The reduction in the compression factor is almost invariably attributed to turbulent heating (TH) in the precursor (Berezhko & Völk (1997) and later) as due to damping of waves on the background plasma (McKenzie & Völk (1982), hereafter MKV82). In fact this mechanism was originally proposed in order to keep the magnetic field amplification in the linear regime (e.g. $\delta B/B \ll 1$), but is now commonly applied to cases in which $\delta B/B \gg 1$. Unfortunately, the heating process is quite model dependent and even its applicability to situations of interest for SNRs can and should be seriously questioned. The effectiveness of the heating process can easily be reduced to negligible levels or artificially amplified to unphysical levels.

TABLE 1
Parameters for 5 well known SNRs.

SNR	$u_0(km/s)$	$B_2(\mu G)$	$P_{w2} \times 10^3$
Cas A	5200 (2500)	250–390	32 (36)
Kepler	5400 (4500)	210–340	23 (25)
Tycho	4600 (3100)	300–530	27 (31)
SN 1006	2900 (3200)	91–110	40 (42)
RCW 86	(800)	75–145	14-35 (16-42)

Note. — For the SNRs discussed by Parizot et al. (2006) we adopted $\rho_0 = 0.1\, m_p/cm^3$ in the case of SN 1006 and $\rho_0 = 0.5\, m_p/cm^3$ in the other cases, while VBK05 provide directly P_{w2}.

As mentioned above, a breakthrough in the field has recently been provided by X-ray observations: the detection of X-ray bright filaments in the outskirts of some SNRs allows one to infer the strength of the magnetic field in these filaments, found to be $B \sim 100-500\mu G$. Such strong fields are generally attributed to the SI induced by CRs efficiently accelerated at the shock front, although alternative explanations have been proposed (Giacalone & Jokipii (2007)). In Tab. 1 we list some SNRs with estimated magnetic fields: u_0 is the shock velocity, B_2 is the magnetic field downstream of the shock as inferred from the X-ray brightness profile and $P_{w2} = B_2^2/(8\pi\rho_0 u_0^2)$ is the downstream magnetic pressure normalized to the bulk one. The values of the parameters are from Parizot et al. (2006) and from VBK05 (in parenthesis in Tab. 1).

[1] Scuola Normale Superiore, Pisa, Italy
[2] INAF/Osservatorio Astrofisico di Arcetri, Firenze, Italy
[3] Fermilab, Center for Particle Astrophysics, USA

We show below that for the field stregths inferred for SNRs, the magnetic pressure is comparable or even in excess of the thermal pressure of the background plasma and that whenever this happens the dynamical reaction of the field on the fluid is such that the compression factors are substantially reduced and fall in the range suggested by observations. It is crucial to keep in mind that, contrary to the TH, which can be either suppressed or amplified by simply changing parameters on which there is little or no control, the feedback of the self-generated turbulent magnetic field on the plasma is not model dependent and must be included.

2. DYNAMICS OF A MAGNETIZED CR MODIFIED SHOCK

The reaction of accelerated particles upstream of the shock leads to the formation of a precursor, in which the fluid speed gradually decreases while approaching the shock. One can describe this effect by introducing two compression factors $R_t = u_0/u_2$ and $R_s = u_1/u_2$, where u is the fluid speed and the indexes '0', '1' and '2' refer to quantities at upstream infinity, upstream and downstream of the subshock respectively.

The most general equations of conservation of mass, momentum and energy in the stationary case for a parallel shock are:

$$\frac{\partial}{\partial x}(\rho u) = 0, \tag{1}$$

$$\frac{\partial}{\partial x}\left(\rho u^2 + p + p_c + p_w\right) = 0, \tag{2}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma p u}{\gamma - 1} + F_w\right) = -u\frac{\partial p_c}{\partial x}. \tag{3}$$

As usual, ρ, u, p and γ stand for density, velocity, pressure and the ratio of specific heats of the gas, while p_w and F_w represent the pressure and energy flux in the form of Alfvén waves. p_c is the CR pressure. The continuity of the distribution function of the accelerated particles through the subshock implies that the CR pressure is also continuous ($p_{c1} = p_{c2}$), and that the term $\partial p_c/\partial x$ gives null contribution when Eq. 3 is integrated from $x = 0^-$ to $x = 0^+$. All previous equations at the subshock read as the usual Rankine-Hugoniot relations at a magnetized gaseous shock.

In order to treat the presence of Alfvén waves correctly, we use the approach of Vainio & Schlickeiser (1999) (hereafter VS99), considering two upstream wave trains with helicities $H_c = \pm 1$, and their respective downstream counterparts. If $\delta\vec{B}_i$ is a mode of the magnetic field perturbation, we write the velocity perturbation as $\delta\vec{u} = -H_c\frac{\delta\vec{B}}{\sqrt{4\pi\rho}}$. Neglecting the electric field contribution, which is of order $(u/c)^2$, the magnetic pressure and the energy flux, which for Alfvén waves is the sum of the normal component of Poynting vector $\vec{u}\times\delta\vec{B}\times\delta\vec{B}/4\pi$ and the transverse kinetic energy flux $\rho\delta\vec{u}^2/2$, are

$$p_w = \frac{1}{8\pi}\left(\sum_i \delta\vec{B}_i\right)^2 \;; \quad F_w = \sum_i \frac{\delta\vec{B}_i^2}{4\pi}\tilde{u}_i + p_w u\,, \tag{4}$$

having posed $\tilde{u}_i = u + H_{c,i}v_A$. The upstream magnetic turbulence typically shows two opposite helicities (Bell & Lucek (2001)), each of which splits into a reflected and a transmitted wave crossing the subshock. According to VS99, the transmission and reflection coefficients, in the limit $M_A^2 \gg R_s$ (large Alfvénic Mach number), do not depend on H_c and read

$$T \simeq (R_s + \sqrt{R_s})/2\,; \qquad R \simeq (R_s - \sqrt{R_s})/2\,. \tag{5}$$

For a typical supernova shock, the Alfvénic Mach number is $M_{A,1} = u_1/v_A \geq 100$, hence in the following we adopt these coefficients and neglect v_A with respect to the fluid velocity in Eq. 4. For each H_c we therefore have $\delta B_2/\delta B_1 = T + R = R_s$ and thus $p_{w2} = p_{w1}R_s^2$.

As pointed out above, the subshock can be viewed as a simple shock in a magnetized gas, therefore the pressure jump is (VS99)

$$\frac{p_2}{p_1} = \frac{(\gamma+1)R_s - (\gamma-1) + (\gamma-1)(R_s-1)\Delta}{\gamma + 1 - (\gamma-1)R_s}\,, \tag{6}$$

with Δ defined as:

$$\Delta = \frac{R_s+1}{R_s-1}\frac{[p_w]}{p_1} - \frac{2R_s}{R_s-1}\frac{[F_w]}{p_1 u_1}\,. \tag{7}$$

Using the expressions for T and R (Eq. 5) we get

$$\Delta = (R_s-1)^2\frac{p_{w1}}{p_1} + R_s\,\frac{\vec{B}_-\cdot\vec{B}_+}{2\pi p_1}\,. \tag{8}$$

Following VS99, we assume that the two opposite–propagating waves carry magnetic fields $\vec{B}_\pm$ displaced in such a way that $\vec{B}_-\cdot\vec{B}_+ = 0$. This is not the most general configuration, but it is nevertheless rather common since it occurs when: 1) there is only one wave train, 2) when the two fields are orthogonal, and 3) on average, when the relative phase between the wave trains is arbitrary.

At this point we normalize all quantities to the ones at upstream infinity: $U(x) = u(x)/u_0$, $P_w(x) = p_w/\rho_0 u_0^2$ and $P(x) = p(x)/\rho_0 u_0^2 = \frac{U(x)^{-\gamma}}{\gamma M_0^2}$. In the latter, we used the assumption of adiabatic heating in the precursor and the conservation of mass.

Substituting Eq. 6, Eq. 8 and the above expression for $P(x)$ in the equation for momentum conservation, the compression factors R_s and R_t are related through the equation

$$R_t^{\gamma+1} = \frac{M_0^2 R_s^\gamma}{2}\left[\frac{\gamma+1-R_s(\gamma-1)}{1+\Lambda_B}\right], \tag{9}$$

which is the same as the standard relation apart from the factor $(1+\Lambda_B)$, with

$$\Lambda_B = W\left[1 + R_s\left(2/\gamma - 1\right)\right]\,, \tag{10}$$

and $W = P_{w1}/P_1$. It is clear that the net effect of the magnetic turbulence is to make the fluid less compressible, noticeably reducing R_t if $W = P_{w1}/P_1$ is of order 1. Moreover, the pressure and temperature jumps at the subshock are enhanced (Eq. 6).

We should notice that if one naively assumed that $F_w = 3up_w$ everywhere, neglecting the T and R coefficients needed to satisfy Maxwell equations at the subshock, one would obtain $\Delta' = [(R_s-1)^2 - 2R_s]W < \Delta$, leading to an incorrect pressure jump. This approach, adopted by Vladimirov, Ellison & Bykov (2006), also leads to a less marked decrease of R_t, since $\Lambda'_B = W\left[1 + R_s\left(3/\gamma - 2\right)\right]$.

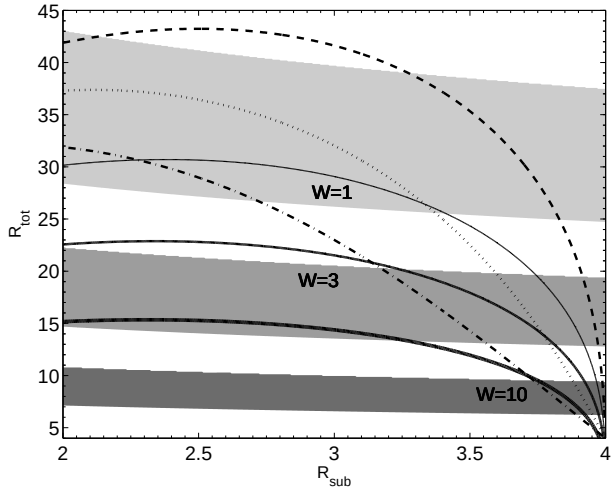

FIG. 1.— R_s-R_t for $W = 1, 3, 10$ (intersections of solid lines with corresponding shadowed regions) and for $\chi = 200, 500$ (dash-dotted and dotted lines). The dashed line represents the case $W = 0$ (see text for details).

3. CONFRONTING OBSERVATIONS

Here we show that the magnetization levels estimated in SNRs as reported in Tab. 1 imply that $W \geq 1$, so that the dynamical feedback of the amplified magnetic field needs to be taken into account.

In Fig. 1 we plot R_t versus R_s for $M_0 = 100$: the three shadowed regions represent the relation between R_s and R_t for fixed $P_{w2} \in [0.02, 0.04]$ and $W = 1,\ 3,\ 10$:

$$P_{w2}/R_s^2 = P_{w1} = WP_{g1} = W\left(R_t/R_s\right)^{\gamma}/(\gamma M_0^2)\,. \quad (11)$$

The three solid lines represent the relation $R_t - R_s$ for the three given values of W as given by Eq. 9; the dashed line refers to $W = 0$, when p_w is not included.

The compression factor lies at the intersection between the curve and the shadowed region for a given value of W. If $W < 0.7$ there are no intersections. This implies that the values of the magnetic pressure inferred from observations require substantial magnetic field amplification upstream, and that the conservation equations are affected by the dynamical reaction of the field. Only values $W \geq 3$ are compatible with the whole range $0.02 \leq P_{w2} \leq 0.04$ inferred from observations. This means that in order to account for the inferred values of B_2, the magnetic pressure must be *at least* comparable to the gas pressure, and thus its dynamical role cannot be neglected. From Fig. 1 one also sees that the magnetic reaction leads to values of R_t lower by roughly a factor ~ 2 compared with the case $W = 0$. We will comment further on this point below.

Up until this point we never used the physically crucial point that the observed fields may be generated through a cosmic ray induced SI upstream of the shock. The instability may operate in the resonant (Bell (1978a)) and in the non-resonant (Bell (2004)) regime.

The growth rates of these different modes can be easily estimated only in the context of quasi-linear theory. Given the difficulty in deriving this information in the general non linear case, here we assume the following general relation between the pressures of CRs and wave pressure upstream of the subshock:

$$\xi_1 = \chi P_{w1}, \quad (12)$$

where $\xi(x) = p_{cr}(x)/\rho_0 u_0^2$ is the normalized CR pressure. For resonant SI, one has $\chi \simeq M_A = u_0/v_A$, while for non-resonant modes, $\chi \sim 4c/u_0$. In both cases, for typical values of the parameters, one obtains $200 \leq \chi \leq 500$.

From Eqs. 1 and 2 applied to the precursor, namely between upstream of the subshock ($x = 0^-$) and upstream infinity, we can write:

$$\frac{R_s}{R_t} + \frac{1}{\gamma M_0^2}\left[\left(\frac{R_t}{R_s}\right)^{\gamma} - 1\right] + P_{w1}(1+\chi) = 1. \quad (13)$$

The physical values of R_s and R_t for a given χ are obtained by determining the intersection of the corresponding curve with that obtained for a given value of W at the subshock. Whether the solution reproduces the estimated value of P_{w2} depends on whether the intersection falls within or outside the shadowed region for the same W in Fig. 1. The dash-dotted and the dotted line show the results for $\chi = 200$ and 500 respectively: it is evident that the chosen values of χ allow for a consistent explanation of the downstream magnetic pressures as inferred from observations, and, equally important, lead to compression factors which are much lower than those predicted by the standard NLT (Berezhko & Völk (1997) and papers that followed).

4. HEATING IN THE PRECURSOR

The strong shock modification predicted by NLT when the magnetic pressure is ignored is usually assumed to be somewhat mitigated by heating of the precursor as a result of damping of Alfven waves (Völk & McKenzie (1981),hereafter VMK81, and MKV82) on the background gas. Other phenomena (for instance acoustic instability) may also lead to heating of the precursor. In the original description, that remained basically unchanged to the present time, VMK81 assumed that the rate of damping (Γ) equals the rate of growth (σ) of Alfven waves. The main implication of this assumption is that the growth of the waves never reaches the non-linear regime, which is in fact the very reason why the mechanism was invoked in the 80's. The recent observations prove that waves can grow to $\delta B/B \gg 1$. It is therefore at least not self-consistent to apply the standard treatment for TH to situations in which magnetic field amplification to the non-linear regime takes place. In a minimal attempt to include faster growth one may assume that $\Gamma = \alpha\sigma$, with $\alpha < 1$. Following MKV82 and Berezhko & Ellison (1999) one can then obtain a generalized relation between R_t and R_s in the form

$$R_t^{\gamma+1} = \frac{M_0^2 R_s^{\gamma}}{2}\left[\frac{\gamma + 1 - R_s(\gamma-1)}{(1+\Lambda_B)(1+\Lambda_{TH})}\right], \quad (14)$$

where

$$\Lambda_{TH} = \alpha(\gamma-1)\frac{M_0^2}{M_A}\left[1 - \left(\frac{R_s}{R_t}\right)^{\gamma}\right], \quad (15)$$

which becomes equivalent to the standard Eq. 50 of Berezhko & Ellison (1999) only for $\alpha = 1$. Now it is easy to check that for typical values of R_s and R_t $\Lambda_{TH} > \Lambda_B$ if $\alpha \gtrsim 3W\frac{M_A}{M_0^2}$. For instance for $M_A \sim 10^3$ and $M_0 \sim 100$ one requires α to be of order unity. In this case however

it is not easy to amplify the magnetic field to $\delta B \gg B_0$. If α is appreciably smaller than unity, the main process for the smoothening of the precursor is the dynamical reaction of the self-generated magnetic field. In both cases the role of TH can be seriously questioned.

A deeper look at the physical processes that may result in the heating of the precursor make the role of TH even more uncertain: in the original papers of VMK81 and MKV82 the Alfven heating was considered as a result of non-linear Landau damping in a gas in the hot coronal phase. The authors reached the conclusion that the damping is important if $u_0 \ll 4000 km/s (T_0/5 \times 10^5 K)^{1/2}$, where u_0 is the shock velocity and T_0 is the temperature of the unshocked gas. It is all but clear whether for the velocities and temperatures that apply to the SNRs in Tab. 1, non-linear Landau damping is such to lead to $\alpha \sim 1$. We stress that at the same time, α cannot be too close to unity, otherwise TH inhibits the growth of δB to the observed levels.

Other types of turbulent heating may be at work but a quantitative analysis of these phenomena is lacking at the present time. The expression for Λ_{TH} is however rather general, in that we did not specify the mathematical form of the growth and damping rates. Therefore we expect to draw similar conclusions in terms of the parameter α.

This section strongly suggests that, contrary to the common wisdom, the most likely reason for the smoothening of the precursor is the dynamical reaction of the generated magnetic fields rather than some form of non adiabatic heating in the precursor.

5. CONCLUSIONS

It is well known that the effect of a magnetic field is in general that of reducing the plasma compressibility. We show here that when applied to a cosmic ray modified shock, 1) this finding leads to conclude that SI generated by CRs is adequate to explain the levels of magnetization inferred from X-ray observations; 2) the downstream magnetic fields imply that $W \sim 1 - 10$, so that the field becomes dynamically important, since this happens whenever the magnetic pressure upstream becomes comparable with the gas pressure, namely when $W > 0.7 - 1$; 3) the dynamical reaction of the magnetic field reduces the compression in the precursor, leading to smaller (larger) values of R_t (R_s) in agreement with the values required to explain the distance between forward and reverse shock and the multifrequency observations of several SNRs; 4) this effect comes from first principles, though in our calculations we restricted our attention to the case of Alfven waves, and is not affected by the huge uncertainties typical of TH; 5) an efficient TH may lead to smoothening of the precursor if α is close to unity, but in this case it is likely to inhibit the growth of the field to $\delta B \gg B_0$.

Although the underlying physics is well known, the dynamical effect of the magnetic pressure has not been included in the calculations of the multifrequency emission from SNRs (Berezhko & Völk (1997) and successive papers), so that the strong modifications predicted by the NLT had to be compensated by assuming TH. The only exception that we are aware of is the recent work by Vladimirov, Ellison & Bykov (2006), in which the authors perform Monte Carlo simulations of the particle acceleration process including the pressure of self-generated magnetic fields. In such simulations, which represent the state of the art in the field, however, thermal and accelerated particles are treated in the same way, therefore the condition $W \sim 1$ could not be tested. We suspect that for this reason the smoothening of the precursor was attributed mostly to the backreaction of the accelerated particles on the field through injection. This effect is certainly present but as we showed here by using only a hydrodynamical approach, the smoothening is in fact mainly due to the reaction of the magnetic pressure on the background plasma.

The smoothening of the precursor also results in two important effects: 1) spectra of accelerated particles closer to power laws, though the concavity which is peculiar of NL DSA remains evident. 2) The maximum momentum of accelerated particles for given Mach number is predicted to be somewhat larger (see Blasi, Amato & Caprioli (2007) for a detailed discussion). Both these effects will be discussed in detail in a forthcoming paper.

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